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Fuzzy Control of Inverted Robot Arm with Perturbed Time-Delay Affine Takagi-Sugeno Fuzzy Model

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***Abstract*—A stability analysis and controller synthesis methodology for an inverted robot arm system is proposed in this paper. This uncertain system is modeled by a state space Takagi-Sugeno (T-S) fuzzy model with linear nominal part and structure bounded parameter uncertainties in the state equations of each fuzzy rule. First, a sufficient condition on robust stability of the Continuous Perturbed Time-Delay Affine T-S (CPTDATS) fuzzy models of inverted robot arm is proposed. Then, H_∞-disturbance attenuation performance of the fuzzy models is analyzed. At last, a numerical example shows the use of the proposed approach on the stabilization and H_∞-disturbance attenuation for the inverted robot arm systems.**

I. INTRODUCTION

The inverted robot arm is an ubiquitous example of nonlinear control systems analysis and design. The standard problem related to inverted robot arm is stabilization of either downward or upright equilibriums at a prescribed position. The purpose of this paper aims to make the inverted robot arm balance; afterwards, the inverted robot arm could get the dynamic balance. The track of the inverted robot arm can be horizontal or gradient. In this paper, we propose the state feedback fuzzy control based on the T-S model for the inverted robot arm.

In recent years, the T-S fuzzy control [1-8] has become one of the useful control approaches for complex nonlinear control systems. It is well known that time delay often occurs in many dynamical systems. Therefore, time delay could be considered as an important issue in T-S fuzzy control systems. In the time-delay T-S fuzzy model [8], local dynamics in different state space regions are represented by linear models. The overall T-S fuzzy model of the system is achieved by fuzzy "blending" of these time-delay linear subsystems.

The control design is carried out based on the so-called Parallel Distributed Compensation (PDC) [1-8] scheme. The idea is that for each local linear model, a linear feedback control is designed. The resulting overall controller which is in general nonlinear is also a fuzzy blending of each individual linear controller. The control approach is to design linear feedback gains for each local linear model and to let the overall control input can be blended by these linear feedback gains.

One of the most important requirements for a control system is the so-called robustness. Those can solve the disturbance and perturbations problems from inherent uncertainties in the real system. In this paper, the H_∞ control scheme [9] is used to deal with the robust performance design problems in CPTDATS fuzzy models. It can provide the guaranteed H_∞ performance for the attenuation γ, which can cope with the worst disturbances in systems. We also propose the issue of robust stability in the presence of norm-bounded uncertainty. The uncertain models are described by a state-space model and time-varying norm-bounded parameter uncertainty in the system matrices.

In general, based on Linear Matrix Inequalities (LMI) [1] methods, one can find suitable linear feedback gains for each fuzzy rule for closed-loop homogeneous T-S fuzzy systems. However, the synthesis of the CPTDATS fuzzy models is a difficult problem for the designers because the closed-loop stability conditions are not LMI formulations but Bilinear Matrix Inequalities (BMI) ones. The BMI conditions cannot be easily solved via a convex optimization algorithm. For this reason, an Iterative LMI (ILMI) [6-8] algorithm is applied to solve the proposed BMI problem in this paper.

II. DYNAMIC MODEL OF INVERTED ROBOT ARM

In this section, we first introduce the mathematical model of inverted robot arm system. Referring to Fig. 1, we proposed a simplified dynamic model to describe open-loop inverted robot arm system as follows [10].

$$\ddot{\theta}(t) = -\frac{g}{l}\sin(\theta(t)) - \frac{k}{m}\dot{\theta}(t) + e\,v(t) \tag{1}$$

where l is the length of the rod, m is the mass of the bob, g is the acceleration due to gravity, k is the friction, $\theta(t)$ is the angle by the rod and the vertical axis, $v(t)$ denotes the disturbances.

The torque is the control input and it is assumed that the control object is to maintain a constant angle $\theta(t)=\beta$. In order to maintain $\theta(t)=\beta$, the torque must have a steady-state component T_{ss} that satisfies

$$-a\sin(\beta) + cT_{ss} = 0 \text{ or } T_{ss} = a\sin(\beta)/c \tag{2}$$

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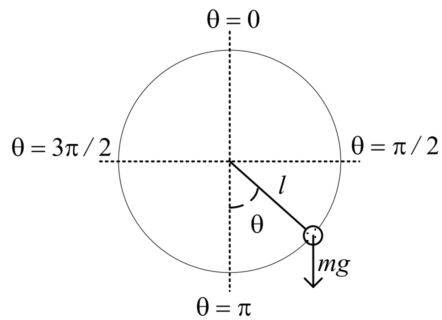

Fig. 1 Inverted robot arm system

Choose the state variables as $x_1(t) = \theta(t) - \beta$, $x_2(t) = \dot{\theta}(t)$, and the control variable as $u(t) = T(t) - T_{ss}$. Then, the new equilibrium point is $x_1(t) = 0$, $x_2(t) = 0$ and $u(t) = 0$. The inverted robot arm equation (1) can be thus represented as

$$\dot{x}_1(t) = x_2(t) \tag{3a}$$

$$\dot{x}_2(t) = -a\{sin(x_1(t) + \beta) - sin(\beta)\} + c\,u(t) + e\,v(t) \tag{3b}$$

Then, we will consider this class including premise nominal parameter uncertainty:

$$\dot{x}_1(t) = \varphi(t) \tag{4a}$$

$$\begin{aligned}\dot{x}_2(t) = -a\{&sin(x_1(t) + \beta) + 0.01cos(t)x_1(t) - sin(\beta)\}\\ &c\,u(t) + e\,v(t)\end{aligned} \tag{4b}$$

$$\begin{aligned}\varphi(t) = &(\rho + 0.02\,sin(t))x_2(t)\\ &+((1-\rho) + 0.01\,sin(t))x_2(t - \tau(t))\end{aligned} \tag{4c}$$

where $\varphi(t) \in \Re^n$ is a time-delay weighting function and $\rho \in [0,\ 1]$ is the weighting coefficient.

III. THE AFFINE T-S FUZZY MODEL OF INVERTED ROBOT ARM

Generally, the main feature of the T-S fuzzy system can be expressed by joining dynamics of each fuzzy rule of linear subsystems. Given a pair of $(x(t),\ u(t))$, the perturbed time-delay affine T-S fuzzy model of inverted robot arm system (1) can be inferred as follows [8]:

$$\begin{aligned}\dot{x}(t) = &\\ &\sum_{i=1}^{r} h_i(z(t))\{(\mathbf{A}_i + \Delta\mathbf{A}_i)x(t) + (\mathbf{A}_{id} + \Delta\mathbf{A}_{id})x(t - \tau(t))\}\\ &+\sum_{i=1}^{r} h_i(z(t))\{(\mathbf{B}_i + \Delta\mathbf{B}_i)u(t) + (\mathbf{a}_i + \Delta\mathbf{a}_i)\} + \mathbf{E}\,v(t)\end{aligned} \tag{5}$$

where

$$z(t) = [z_1(t),\ z_2(t),\ \dots,\ z_p(t)],\ \omega_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t))$$

$$h_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))},\ h_i(z(t)) \ge 0 \text{ and } \sum_{i=1}^{r} h_i(z(t)) = 1 \tag{6}$$

The quantities $\mathbf{A}_i$, $\mathbf{A}_{id}$, $\mathbf{B}_i$, $\mathbf{a}_i$ and $\mathbf{E}$ are constant matrices. Besides, $\Delta\mathbf{A}_i$, $\Delta\mathbf{A}_{id}$, $\Delta\mathbf{B}_i$ and $\Delta\mathbf{a}_i$ are time-varying matrices with appropriate dimensions and they are structured in the following norm-bounded form:

$$[\Delta\mathbf{A}_i\ \ \Delta\mathbf{A}_{id}\ \ \Delta\mathbf{B}_i\ \ \Delta\mathbf{a}_i] = \mathbf{D}_i\mathbf{\Delta}_i(t)[\mathbf{Q}_{1i}\ \ \mathbf{Q}_{2i}\ \ \mathbf{Q}_{3i}\ \ \mathbf{Q}_{4i}] \tag{7}$$

where $\mathbf{D}_i$, $\mathbf{Q}_{1i}$, $\mathbf{Q}_{2i}$, $\mathbf{Q}_{3i}$ and $\mathbf{Q}_{4i}$ are known real constant matrices of appropriate dimensions, and $\mathbf{\Delta}_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $\mathbf{\Delta}_i^T(t)\mathbf{\Delta}_i(t) \le \mathbf{I}$.

For a nonlinear T-S fuzzy system represented by (5), a fuzzy controller is designed to share the same fuzzy sets with the plant. It is based on the PDC concept [1]. The output of the PDC-based fuzzy controller is determined by the summation such as

$$u(t) = -\sum_{i=1}^{r} h_i(z(t))\{\mathbf{F}_i\,x(t) + \boldsymbol{\mu}_i\} \tag{8}$$

Substituting (8) into (5), one can obtain the corresponding closed-loop system

$$\begin{aligned}\dot{x}(t) = &\frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\hat{x}(t))h_j(\hat{x}(t))\\ &\{(\bar{\mathbf{A}}_{ij} + \mathbf{D}_i\Delta\bar{\mathbf{A}}_{ij}) + (\bar{\mathbf{A}}_{ji} + \mathbf{D}_j\Delta\bar{\mathbf{A}}_{ji})\}\tilde{x}(t) + \mathbf{E}\,v(t)\end{aligned} \tag{9}$$

where

$\tilde{x}(t) = [x^T(t)\ \ x^T(t - \tau(t))\ \ 1]^T$, $\bar{\mathbf{A}}_{ij} = [\mathbf{G}_{ij} \mid \mathbf{A}_{id} \mid \mathbf{g}_{ij}]$, $\mathbf{G}_{ij} = \mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j$, $\mathbf{g}_{ij} = \mathbf{a}_i - \mathbf{B}_i\boldsymbol{\mu}_j$ and $\Delta\bar{\mathbf{A}}_{ij} = \mathbf{\Delta}_i(t)[\mathbf{Q}_{1i} - \mathbf{Q}_{3i}\mathbf{F}_j \mid \mathbf{Q}_{2i} \mid \mathbf{Q}_{4i} - \mathbf{Q}_{3i}\boldsymbol{\mu}_j]$.

Base on the PDC type fuzzy controller (8), a sufficient condition for ensuring delay-independent stability of controlled time-delay affine T-S fuzzy model (9) is introduced in this paper. Moreover, a H_∞ control performance with $\gamma > 0$ is also considered in this paper. This constraint is of the following form.

$$\int_0^{t_f} x^T(t)\mathbf{S}x(t)dt < \gamma^2 \int_0^{t_f} v^T(t)v(t)dt \tag{10}$$

with zero initial condition for all $v(t) \in L_2[0, t_f]$, where γ is a prescribed value which denotes the worst case effect of $v(t)$ on $x(t)$. Besides, $\mathbf{S} = \mathbf{S}^T > 0$ is a positive-definite weighting matrix and $\mathbf{S} \in \Re^{n\times n}$. The purpose of this paper is to find satiable fuzzy controllers (8) such that the closed-loop system (9) is robustly stable with satisfying the H_∞ constraint (10).

IV. SUFFICIENT CONDITIONS OF ROBUST FUZZY CONTROLLER DESIGN

A fuzzy controller is designed to share the same fuzzy sets with the affine T-S fuzzy model (5) based on the PDC scheme. In this section, the delay-independent stability conditions for the CPTDATS fuzzy model (9) are described in the following theorem.

Theorem 1

Given a H_∞ attenuation parameter $\gamma > 0$. The CPTDATS fuzzy system described by (9) is quadratically stable in the large and the H_∞ control performance (10) is guaranteed for an attenuation γ if there exist common positive definite matrices $\mathbf{P} > 0$, $\mathbf{S} > 0$, $\mathbf{N}_1 > 0$, control gains $\mathbf{F}_i$, $\boldsymbol{\mu}_i$ and scalars $\xi_{ijq} \ge 0$ such that

$$\begin{cases} \Upsilon_{ij}^{crt} \triangleq \mathbf{\Gamma}_{ij}^{c} + \mathbf{\Theta}_{ij}^{R} < 0 \\ \mathbf{P} \ge \mathbf{Q}_{2i}^T\mathbf{Q}_{2i}/2 + \mathbf{Q}_{2j}^T\mathbf{Q}_{2j}/2 + \mathbf{N}_1 \end{cases} \quad \text{for} \quad i \in \hat{I}_0 \tag{11a}$$

$$\begin{cases} \bar{\Upsilon}_{ij}^{crt} \triangleq \bar{\mathbf{\Gamma}}_{ij}^{c} + \bar{\mathbf{\Theta}}_{ij}^{R} < 0 \\ \mathbf{P} \ge \mathbf{Q}_{2i}^T\mathbf{Q}_{2i}/2 + \mathbf{Q}_{2j}^T\mathbf{Q}_{2j}/2 + \mathbf{N}_1 \end{cases} \quad \text{for} \quad i \in \hat{I}_1 \tag{11b}$$

where

$$\mathbf{\Gamma}_{ij}^{c} = \left(\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right)^T \mathbf{P} + \mathbf{P}\left(\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right) \tag{12}$$

$$\bar{\mathbf{\Gamma}}_{ij}^{c} = \left[\begin{array}{c|c} \mathbf{\Gamma}_{ij}^{c} - \sum_{q=1}^{p} \xi_{ijq}\mathbf{T}_{ijq} & * \\ \hline \left(\frac{\mathbf{g}_{ij} + \mathbf{g}_{ji}}{2}\right)^T \mathbf{P} - \sum_{q=1}^{p} \xi_{ijq}\mathbf{n}_{ijq}^T & -\sum_{q=1}^{p} \xi_{ijq} v_{ijq} \end{array}\right] \tag{13}$$

$$\mathbf{\Theta}_{ij}^{R} = \bar{\mathbf{U}}_{ij} + \mathbf{P} + \mathbf{P}\mathbf{E}\left(\mathbf{I}/\gamma^2\right)\mathbf{E}^T\mathbf{P} + \mathbf{S} \tag{14}$$

$$\bar{\mathbf{\Theta}}_{ij}^{R} = \left[\begin{array}{c|c} \mathbf{\Theta}_{ij}^{R} & * \\ \hline 0 & \mathbf{U}_{ij} \end{array}\right] \tag{15}$$

$$\begin{aligned} \bar{\mathbf{U}}_{ij} &= \left(\mathbf{P}\mathbf{A}_{id}\mathbf{N}_1^{-1}\mathbf{A}_{id}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{jd}\mathbf{N}_1^{-1}\mathbf{A}_{jd}^T\mathbf{P}\right)/2 \\ &\quad + \mathbf{P}\begin{bmatrix}\mathbf{D}_i & \mathbf{D}_j\end{bmatrix}\{3\mathbf{I}/2\}\begin{bmatrix}\mathbf{D}_i & \mathbf{D}_j\end{bmatrix}^T\mathbf{P} \\ &\quad + \left(\mathbf{Q}_{1i} - \mathbf{Q}_{3i}\mathbf{F}_j\right)^T\left(\mathbf{Q}_{1i} - \mathbf{Q}_{3i}\mathbf{F}_j\right)/2 \\ &\quad + \left(\mathbf{Q}_{1j} - \mathbf{Q}_{3j}\mathbf{F}_i\right)^T\left(\mathbf{Q}_{1j} - \mathbf{Q}_{3j}\mathbf{F}_i\right)/2 \end{aligned} \tag{16}$$

$$\begin{aligned} \mathbf{U}_{ij} &= \left(\mathbf{Q}_{4i} - \mathbf{Q}_{3i}\boldsymbol{\mu}_j\right)^T\left(\mathbf{Q}_{4i} - \mathbf{Q}_{3i}\boldsymbol{\mu}_j\right)/2 \\ &\quad + \left(\mathbf{Q}_{4j} - \mathbf{Q}_{3j}\boldsymbol{\mu}_i\right)^T\left(\mathbf{Q}_{4j} - \mathbf{Q}_{3j}\boldsymbol{\mu}_i\right)/2 \end{aligned} \tag{17}$$

From Theorem 1, it can be noted that the matrix inequalities in $\mathbf{P}$, $\mathbf{F}_i$ and $\boldsymbol{\mu}_j$ belong to the class of BMIs and the controller synthesis cannot be solved with ease by a convex optimization algorithm. In order to solve the present robust fuzzy controller design problem, it is necessary to rewrite the conditions of Theorem l. In next section, a new theorem is provided to introduce new stability conditions which can be solved by an ILMI algorithm.

V. ROBUST FUZZY CONTROLLER DESIGN VIA ILMI ALGORITHM

In this section, an ILMI algorithm is provided to get a suitable solution for the stability conditions of Theorem 1. The decay rate α is considered in the stability conditions in order to relax the LMI search procedure and make it feasible.

Theorem 2

The stability conditions (11) described in Theorem 1 are held and the CPTDATS is quadratically stable in the large if there exists a decay rate $\alpha < 0$, positive definite matrices $\mathbf{P} > 0$, $\mathbf{S} > 0$, $\mathbf{N}_1 > 0$, control gains $\mathbf{F}_i$, $\boldsymbol{\mu}_i$ and scalars $\xi_{ijq} \ge 0$ such that

$$\begin{cases} \mathbf{\Gamma}_{ij}^{ctd} < 0 \\ \mathbf{P} \ge \mathbf{Q}_{2i}^T\mathbf{Q}_{2i}/2 + \mathbf{Q}_{2j}^T\mathbf{Q}_{2j}/2 + \mathbf{N}_1 \end{cases} \quad \text{for} \quad i \in \hat{I}_0 \tag{18a}$$

$$\begin{cases} \bar{\mathbf{\Gamma}}_{ij}^{ctd} < 0 \\ \mathbf{P} \ge \mathbf{Q}_{2i}^T\mathbf{Q}_{2i}/2 + \mathbf{Q}_{2j}^T\mathbf{Q}_{2j}/2 + \mathbf{N}_1 \end{cases} \quad \text{for} \quad i \in \hat{I}_1 \tag{18b}$$

where

$$\mathbf{\Gamma}_{ij}^{ctd} \triangleq \begin{bmatrix} \mho_R - \alpha\mathbf{P} & * & * & * \\ \mathbf{L}_{4ij}^T & -\mathbf{I} & * & * \\ \mathbf{L}_{5ij}^T\mathbf{P} & 0 & -\mathbf{I}/2 & * \\ \mathbf{L}_{6ij}^T\mathbf{P} & 0 & 0 & -\mathbf{N}_1 \\ \mathbf{L}_{7ij}^T\mathbf{P} & 0 & 0 & 0 \\ \mathbf{L}_{8ij} & 0 & 0 & 0 \\ \mathbf{L}_{9ij} & 0 & 0 & 0 \\ \mathbf{E}^T\mathbf{P} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ -\mathbf{I}/3 & * & * & * \\ 0 & -\mathbf{I} & * & * \\ 0 & 0 & -\mathbf{I} & * \\ 0 & 0 & 0 & -\gamma^2\mathbf{I}/2 \end{bmatrix} < 0, \tag{19}$$

$$\bar{\boldsymbol{\Gamma}}_{ij}^{ctd} \triangleq \begin{bmatrix} \mho_{R11}-\alpha\mathbf{P} & * & * & * & * & * & * & * & * & * & * & * \\ \mho_{R21} & \mho_{R22} & * & * & * & * & * & * & * & * & * & * \\ 0 & \mathbf{L}_{1ij}^T & -\mathbf{I} & * & * & * & * & * & * & * & * & * \\ 0 & \mathbf{L}_{2ij} & 0 & -\mathbf{I} & * & * & * & * & * & * & * & * \\ 0 & \mathbf{L}_{3ij} & 0 & 0 & -\mathbf{I} & * & * & * & * & * & * & * \\ \mathbf{L}_{4ij}^T & 0 & 0 & 0 & 0 & -\mathbf{I} & * & * & * & * & * & * \\ \mathbf{L}_{5ij}^T\mathbf{P} & 0 & 0 & 0 & 0 & 0 & -\mathbf{I}/2 & * & * & * & * & * \\ \mathbf{L}_{6ij}^T\mathbf{P} & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{N}_1 & * & * & * & * \\ \mathbf{L}_{7ij}^T\mathbf{P} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{I}/3 & * & * & * \\ \mathbf{L}_{8ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{I} & * & * \\ \mathbf{L}_{9ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{I} & * \\ \mathbf{E}^T\mathbf{P} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2\mathbf{I}/2 \end{bmatrix} < 0, \tag{20}$$

and $\mathbf{L}_{1ij} = \begin{bmatrix} \boldsymbol{\mu}_i^T & \boldsymbol{\mu}_j^T \end{bmatrix}$, $\mathbf{L}_{2ij} = \mathbf{Q}_{4i} - \mathbf{Q}_{3i}\boldsymbol{\mu}_j$, $\mathbf{L}_{3ij} = \mathbf{Q}_{4j} - \mathbf{Q}_{3j}\boldsymbol{\mu}_i$, $\mathbf{L}_{4ij} = \begin{bmatrix} \mathbf{F}_i^T & \mathbf{F}_j^T \end{bmatrix}$, $\mathbf{L}_{5ij} = \begin{bmatrix} \mathbf{B}_i & \mathbf{B}_j \end{bmatrix}$, $\mathbf{L}_{6ij} = \begin{bmatrix} \mathbf{A}_{id} & \mathbf{A}_{jd} \end{bmatrix}$, $\mathbf{L}_{7ij} = \begin{bmatrix} \mathbf{D}_i & \mathbf{D}_j \end{bmatrix}$, $\mathbf{L}_{8ij} = \mathbf{Q}_{1i} - \mathbf{Q}_{3i}\mathbf{F}_j$, $\mathbf{L}_{9ij} = \mathbf{Q}_{1j} - \mathbf{Q}_{3j}\mathbf{F}_i$

$$\mho_{R11} = \mho_R - \sum_{q=1}^{p} 2\xi_{ijq}\mathbf{T}_{ijq} \tag{21}$$

$$\begin{aligned} \mho_{R21} &= \left(\mathbf{a}_i - \mathbf{B}_i\mathbf{y}_j\right)^T \mathbf{P} + \left(\mathbf{a}_j - \mathbf{B}_j\mathbf{y}_i\right)^T \mathbf{P} \\ &+ \left(\mathbf{y}_j - \boldsymbol{\mu}_j\right)^T \mathbf{z}_i + \left(\mathbf{y}_i - \boldsymbol{\mu}_i\right)^T \mathbf{z}_j - \sum_{q=1}^{p} 2\xi_{ijq}\mathbf{n}_{ijq}^T \end{aligned} \tag{22}$$

$$\begin{aligned} \mho_{R22} &= \mathbf{y}_i^T\mathbf{y}_i + \mathbf{y}_j^T\mathbf{y}_j - \mathbf{y}_i^T\boldsymbol{\mu}_i - \boldsymbol{\mu}_i^T\mathbf{y}_i \\ &- \mathbf{y}_j^T\boldsymbol{\mu}_j - \boldsymbol{\mu}_j^T\mathbf{y}_j - \sum_{q=1}^{p} 2\xi_{ijq} \mathrm{v}_{ijq} \end{aligned} \tag{23}$$

$$\begin{aligned} \mho_R &= \mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i + \mathbf{A}_j^T\mathbf{P} + \mathbf{P}\mathbf{A}_j + \mathbf{Y}_{ij}^T\mathbf{Y}_{ij} + \mathbf{Y}_{ji}^T\mathbf{Y}_{ji} \\ &- \mathbf{Y}_{ij}^T\left(\mathbf{B}_i^T\mathbf{P} + \mathbf{F}_j\right) - \left(\mathbf{P}\mathbf{B}_i + \mathbf{F}_j^T\right)\mathbf{Y}_{ij} - \mathbf{Y}_{ji}^T\left(\mathbf{B}_j^T\mathbf{P} + \mathbf{F}_i\right) \\ &- \left(\mathbf{P}\mathbf{B}_j + \mathbf{F}_i^T\right)\mathbf{Y}_{ji} + \mathbf{z}_i^T\mathbf{z}_i + \mathbf{z}_j^T\mathbf{z}_j - \mathbf{z}_i^T\mathbf{B}_i^T\mathbf{P} - \mathbf{P}\mathbf{B}_i\mathbf{z}_i - \mathbf{z}_j^T\mathbf{B}_j^T\mathbf{P} \\ &- \mathbf{P}\mathbf{B}_j\mathbf{z}_j + 2\mathbf{P} + 2\mathbf{S} \end{aligned} \tag{24}$$

in which

$$\mathbf{Y}_{ij} = \mathbf{B}_i^T\mathbf{P} + \mathbf{F}_j,\ \mathbf{z}_i = \mathbf{B}_i^T\mathbf{P},\ \mathbf{y}_i = \boldsymbol{\mu}_i \tag{25}$$

According to the conditions of Theorem 2, the solutions of fuzzy control problem of CPTDATS fuzzy systems can be obtained by applying ILMI algorithm, which is developed on the LMI technique. The flowchart of ILMI algorithm, which can be used to solve the conditions of Theorem 2, is introduced as follows.

<ILMI Algorithm>

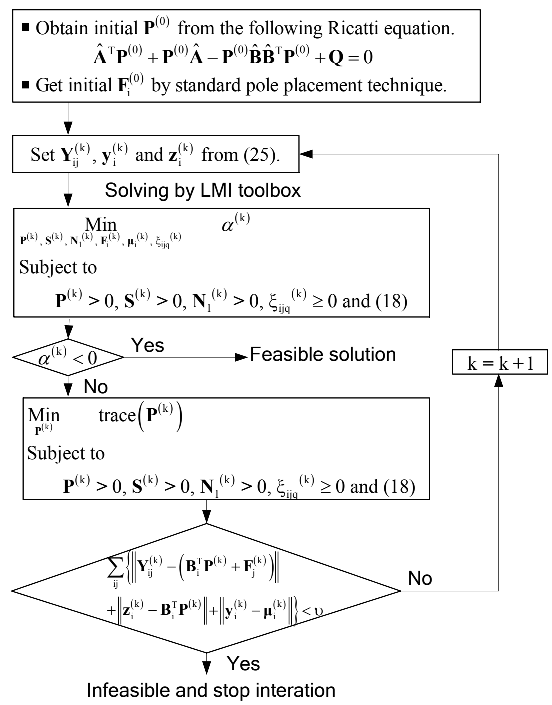

Fig. 2 ILMI Algorithm

In which $\hat{\mathbf{A}} = \frac{1}{r}\sum_{i=1}^{r}\mathbf{A}_i$, $\hat{\mathbf{B}} = \frac{1}{r}\sum_{i=1}^{r}\mathbf{B}_i$ and $\mathbf{Q} > 0$, υ is a predetermined small. Applying the above ILMI algorithm, one can obtain a fuzzy controller (8) to stabilize the CPTDATS fuzzy systems (5) with satisfying the H_∞ performance constraint (10). In next section, a numerical example is provided to demonstrate the usefulness and effectiveness of the proposed design approach.

VI. NUMERICAL SIMULATIONS

To consider the time delay effect in the actuality situation, it is assumed that the sensor for exploring the $x_2(t) = \dot{\theta}(t)$ is perturbed by time delay given as:

$$\Delta\mathbf{A}_i = \mathbf{D}_i \boldsymbol{\Delta}_i(t)\mathbf{Q}_{1i} \tag{26a}$$

$$\Delta\mathbf{A}_{id} = \mathbf{D}_i \boldsymbol{\Delta}_i(t)\mathbf{Q}_{2i} \tag{26b}$$

where

$$\mathbf{D}_i = \begin{bmatrix} 0 & 0.01 \\ 0.01 & 0 \end{bmatrix},\ \boldsymbol{\Delta}_i(t) = \begin{bmatrix} cos(t) & 0 \\ 0 & sin(t) \end{bmatrix},$$

$$\mathbf{Q}_{1i} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},\ \mathbf{Q}_{2i} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

To obtain the CPTDATS fuzzy model of the inverted robot arm system, it is necessary to apply the linearization technique [12]. Let us choose three operating points as follows:

$$\left(x^+,\ x_d^+,\ u^+\right)_{\text{oper1}} = \left(58^\circ \quad 0 \;\vdots\; 0^\circ \quad 0 \;\vdots\; 0\right),$$

$$\left(x,\ x_d,\ u\right)_{\text{oper2}} = \left(0^\circ \quad 0 \;\vdots\; 0^\circ \quad 0 \;\vdots\; 0\right),$$

and $$\left(x^-,\ x_d^-,\ u^-\right)_{\text{oper3}} = \left(-118^\circ \quad 0 \;\vdots\; 0^\circ \quad 0 \;\vdots\; 0\right) \tag{27}$$

Then, three linear subsystems can be constructed by these three operating points. In which, $\left(x,\ x_d,\ u\right)_{\text{oper2}}$ is the maintain equilibrium point and the others are the off-equilibrium points. Through constructing the above three linear subsystems and defining membership functions as Fig. 3, one can obtain the time-delay affine T-S fuzzy model, which is composed by three rules as follows:

Rule 1: IF $x_1(t)$ is about M_{11}

$$\text{THEN } \dot{x}(t) = \left(\mathbf{A}_1 + \Delta\mathbf{A}_1\right)x(t) + \left(\mathbf{A}_{1d} + \Delta\mathbf{A}_{1d}\right)x\left(t-\tau(t)\right) + \mathbf{B}_1 u(t) + \mathbf{a}_1 + \mathbf{E}v(t) \tag{28a}$$

Rule 2: IF $x_1(t)$ is about M_{21}

$$\text{THEN } \dot{x}(t) = \left(\mathbf{A}_2 + \Delta\mathbf{A}_2\right)x(t) + \left(\mathbf{A}_{2d} + \Delta\mathbf{A}_{2d}\right)x\left(t-\tau(t)\right) + \mathbf{B}_2 u(t) + \mathbf{a}_2 + \mathbf{E}v(t) \tag{28b}$$

Rule 3: IF $x_1(t)$ is about M_{31}

$$\text{THEN } \dot{x}(t) = \left(\mathbf{A}_3 + \Delta\mathbf{A}_3\right)x(t) + \left(\mathbf{A}_{3d} + \Delta\mathbf{A}_{3d}\right)x\left(t-\tau(t)\right) + \mathbf{B}_3 u(t) + \mathbf{a}_3 + \mathbf{E}v(t) \tag{28c}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0.85 \\ 2.2495 & 0 \end{bmatrix},\ \mathbf{A}_2 = \begin{bmatrix} 0 & 0.85 \\ -7.0711 & 0 \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} 0 & 0.85 \\ 3.91 & 0 \end{bmatrix},\ \mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_3 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\Delta\mathbf{A}_1 = \Delta\mathbf{A}_2 = \Delta\mathbf{A}_3 = \begin{bmatrix} 0 & 0.02\,sin(t) \\ 0.01\,cos(t) & 0 \end{bmatrix},$$

$$\mathbf{A}_{1d} = \mathbf{A}_{2d} = \mathbf{A}_{3d} = \begin{bmatrix} 0 & 0.15 \\ 0 & 0 \end{bmatrix},$$

$$\Delta\mathbf{A}_{1d} = \Delta\mathbf{A}_{2d} = \Delta\mathbf{A}_{3d} = \begin{bmatrix} 0 & 0.01\,sin(t) \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{a}_1 = \begin{bmatrix} 0 \\ -4.9498 \end{bmatrix},\ \mathbf{a}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \mathbf{a}_3 = \begin{bmatrix} 0 \\ 27.05 \end{bmatrix},\ \mathbf{E} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$$

According to the membership functions defined in Fig. 3, the *S*-procedure is presented as follows. For ***Rule***$_s$ 11, i.e., $3^\circ \le x_1(t) \le 60^\circ$, the matrices of *S*-procedure are given as follows:

$$\mathbf{T}_{111} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \mathbf{n}_{111} = \begin{bmatrix} -\frac{1}{2}\left(3\pi/180 + 60\pi/180\right) \\ 0 \end{bmatrix} \text{ and}$$

$$\text{v}_{111} = \left(3\pi/180\right)\times\left(60\pi/180\right) \tag{29}$$

For ***Rule***$_s$ 33, i.e., $-120^\circ \le x_1(t) \le -3^\circ$, the matrices of *S*-procedure are given as follows:

$$\mathbf{T}_{331} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \mathbf{n}_{331} = \begin{bmatrix} -\frac{1}{2}\left(-3\pi/180 - 120\pi/180\right) \\ 0 \end{bmatrix} \text{ and}$$

$$\text{v}_{331} = \left(-3\pi/180\right)\times\left(-120\pi/180\right) \tag{30}$$

For the above CPTDATS fuzzy model (28), the fuzzy controller can be designed by applying Theorem 2 and the ILMI algorithm. In this example, it is assumed that the H_∞ control performance is guaranteed for an attenuation $\gamma^2 = 0.01$. Besides, the disturbance is $v(t) = 0.5\,sin(t)$. Then, we can get a feasible solution after four iterations of the fuzzy controller design procedure. The final decay rate α is -0.5782 and the feasible solutions are obtained as follows:

$$\mathbf{P} = \begin{bmatrix} 282.4378 & 11.4783 \\ 11.4783 & 12.1012 \end{bmatrix},\ \mathbf{S} = \begin{bmatrix} 444.0919 & 473.4944 \\ 473.4944 & 505.6346 \end{bmatrix},$$

$$\mathbf{N}_1 = \begin{bmatrix} 9.5678e9 & 0 \\ 0 & 9.5678e9 \end{bmatrix},$$

$$\xi_{111} = 10.0226,\ \xi_{331} = 5.7973 \tag{31}$$

And, the fuzzy controller has the following form:

Rule 1: IF $x_1(t)$ is about M_{11}

THEN $u(t) = -[200.3008 \quad 221.9395]x(t) - 0.4599$ (32a)

Rule 2: IF $x_1(t)$ is about M_{21}

THEN $u(t) = -[200.2968 \quad 221.9349]x(t)$ (32b)

Rule 3: IF $x_1(t)$ is about M_{31}

THEN $u(t) = -[200.3648 \quad 221.9972]x(t) - 1.5482$ (32c)

The simulation results are shown in Fig. 4 and Fig. 5. From the simulated results, one can find that the controlled nonlinear time-delay inverted robot arm system (4) is globally stable.

VII. Conclusions

In this paper, we have shown that the perturbed inverted robot arm system with disturbance can be controlled by the T-S fuzzy controllers. The proposed fuzzy controller design approach was developed via PDC method and ILMI algorithm. Finally, the simulation results showed that the fuzzy controller designed in this paper can stabilized the nonlinear inverted robot arm subject to satisfying H_∞ performance constraint.

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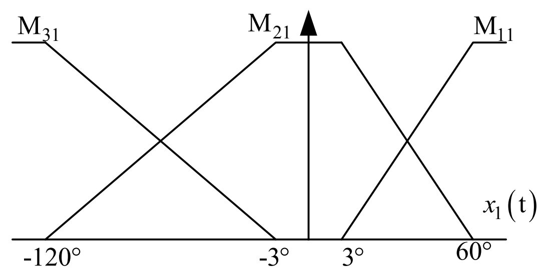

Fig. 3 Membership functions of $x_1(t)$

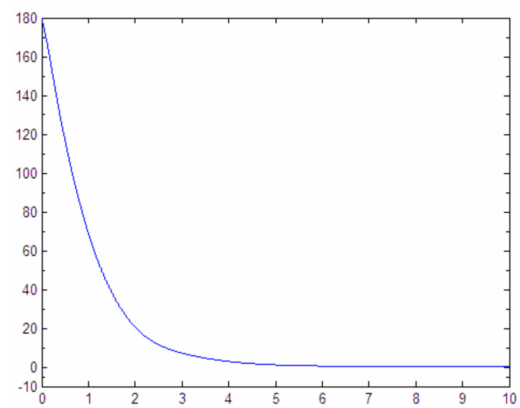

Fig. 4 Responses of $x_1(t)$

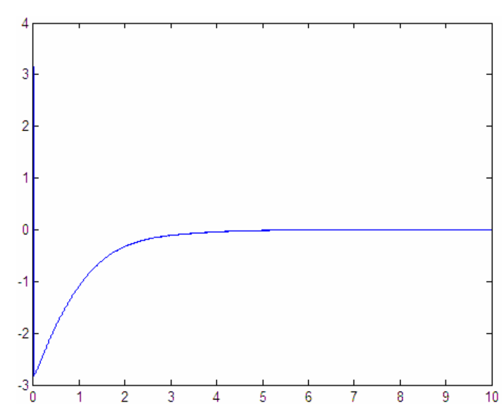

Fig. 5 Responses of $x_2(t)$