2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

Fully-isotropic Three-degree-of-freedom Parallel Wrists

G. Gogu

Abstract—**This paper presents the structural synthesis of fully-isotropic parallel wrists (PWs) with three degrees of freedom. The mobile platform has 3 rotations (3R) driven by three actuators mounted on the fixed base. A method is proposed for structural synthesis of 3R-PWs with uncoupled motions and fully-isotropic based on the theory of linear transformations. A one-to-one correspondence exists between the actuated joint velocity space and the operational velocity space of the moving platform. The Jacobian matrix mapping the three vector spaces of 3R-PWs with uncoupled motions is a 3×3 diagonal matrix. We use the condition number and the manipulability ellipsoids for their performance analysis. The Jacobian matrix of the fully-isotropic 3R-PWs presented in this paper is the 3×3 identity matrix throughout their entire workspace. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission capabilities. As far as we are aware, this paper presents for the first time solutions of fully-isotropic three-degree-of-freedom parallel wrists.**

I. INTRODUCTION

Parallel manipulators (PMs) consists of an output link (mobile platform) connected to the base (fixed platform) by at least two kinematic chains called legs. With respect to serial manipulators, such mechanisms can offer advantages in terms of stiffness, accuracy, load-to-weight ratio, dynamic performances. Their disadvantages include smaller workspace, complex command and a lower dexterity due to a high motion coupling and multiplicity of singularities inside their workspace. Uncoupled and fully-isotropic parallel manipulators can overcome these disadvantages. They have a very simple command and could achieve high energy-saving due to the fact that for a unidirectional motion only one motor works and the others are locked.

Isotropicity of a robotic manipulator is related to the condition number of its Jacobian matrix, which can be calculated as the ratio of the largest and the smallest singular values. A robotic manipulator is fully-isotropic if its Jacobian matrix is isotropic throughout the entire workspace, i.e., the condition number of the Jacobian matrix is equal to one. We know that the Jacobian matrix of a robotic manipulator is the matrix mapping (i) the actuated joint velocity space and the end-effector velocity space, and (ii) the static load on the end-effector and the actuated joint forces or torques. Thus, the condition number of the Jacobian matrix is a useful performance indicator characterizing the distortion of a unit ball under this linear mapping. The condition number of the Jacobian matrix was first used to design mechanical fingers [1] and developed later as a kinetostatic performance index of robotic mechanical systems [2]. The isotropic design aims at ideal kinematic and dynamic performance of the robotic manipulator [3].

Three-degree-of-freedom (3-DoF) parallel wrists (PWs) are used in many applications that require orienting a body in space and enabling three independent rotations (3R) of the mobile platform about a fixed point [4].

Three general architectures of 3-DoF PWs are known today: spherical, non-spherical and wrists with a passive leg. The solutions based on spherical mechanisms use only revolute joints with intersecting axes in a common point that is the centre of the sperical motion [5-10]. Agile Eye [9] used as camera-orienting device is an overconstrained PW that achieves the spherical motion of the platform by using the common constraints of spherical mechanisms. The main drawback of this overconstrained architecture is that the mechanism jams or high internal stresses arise in the links when geometric errors occur. Argos [11] is a 3-DoF spherical remote-centre-of–motion PW for force reflexion in a haptic device with a non-spherical architecture. It is based on pantograph mechanisms integrated in the three parallel drive chains that connect the end-effector to the base. Various solutions of 3-DoF PWs with non-spherical architecture have been proposed in [12-17]. Solutions of PWs with a passive leg reduced to a single spherical joint and three actuated legs have been presented in [4], [18]. We note that all solutions of 3-DoF PWs presented in the literature have coupled motions. Just some solutions of 2-DoF PWs with uncoupled motions and fully-isotropic have been recently proposed [19].

As far as we are aware, no solutions of 3-DoF fully-isotropic PWs have been presented in the literature. The main aims of this paper is to present two new families of 3-degree-of-freedom PWs with uncoupled motions and fully-isotropic and to emphasize on the structural synthesis approach that allowed us to obtain them.

The general methods used for structural synthesis of parallel mechanisms can be divided into three approaches: the method based on displacement group theory [20]-[22], the methods based on screw algebra [15-16], [23]-[28] and

G. Gogu is with the Mechanical Engineering Research Group, French Institute of Advanced Mechanics and Blaise Pacal University, Clermont-Ferrand (phone: + 33 4 73 28 80 22; fax: 33 4 73 28 81 00; e-mail: Grigore.Gogu@ ifma.fr).

This work is supported by the CNRS (The French National Council of Scientific Research) in the framework of the projects ROBEA-MAX (2002-2004) and ROBEA-MP2 (2005-2006).

1-4244-0602-1/07/$20.00 ©2007 IEEE.

the methods based on the theory of linear transformations [19], [29]-[34]. The approach proposed in this paper represents an extension of the methods founded on the theory of linear transformations to structural synthesis of 3-DoF fully-isotropic PWs. This approach integrates the new formulae of mobility, connectivity, redundancy and overconstraints of parallel manipulators proposed in [35], [36] and demonstrated via the theory of linear transformations.

II. Structural Synthesis

The basic kinematic structure of a 3R-type PW discussed in this paper is obtained by using three open kinematic chains A_i $(1\equiv 0\text{-}\ldots\text{-}n_{Ai}\equiv n)$, $i=1,2,3$. The first link of each leg is the fixed platform ($1_{Ai}\equiv 0$) and the final link is the moving platform ($n_{Ai}\equiv n$). The first joint of each kinematic chain A_i ($i=1,2,3$) is actuated. We denote by q_i and $\dot{q}_i$ ($i=1,2,3$) the finite displacements and the velocities in the actuated joints and we consider α,β,δ and $\omega_1=\dot{\alpha}$, $\omega_2=\dot{\beta}$, $\omega_3=\dot{\delta}$ the finite displacements and the angular velocities of the mobile platform.

The linear mapping between the actuated joint space and the operational space of the moving platform of a 3R-type PWs is given by

$$[\boldsymbol{\omega}]=[J][\dot{\boldsymbol{q}}] \tag{1}$$

where : $[\boldsymbol{\omega}]=[\dot{\alpha}\ \dot{\beta}\ \dot{\delta}]^T$ is the angular velocity of the moving platform, $[\dot{\boldsymbol{q}}]=[\dot{q}_1\ \dot{q}_2\ \dot{q}_3]^T$ are the velocities of the actuated joints and $[J]$ is the Jacobian matrix.

We distinguish four types of PWs: (i) fully-isotropic PWs, when the Jacobian J is an diagonal matrix with identical diagonal elements throughout the entire workspace, (ii) PWs with uncoupled motions if J is a diagonal matrix with different diagonal elements, (iii) PWs with decoupled motions, if J is a triangular matrix and (iv) PWs with coupled motions if J is neither a triangular nor a diagonal matrix.

The mechanism associated to a PW with uncoupled motions is denoted by Q. The existence of this mechanism involves the following conditions for the connectivity (spatiality) $S^Q_{n/1}$ between the moving and the fixed platforms (n and $1\equiv 0$) and for the base $(R^Q_{n/1})$ of the vector space of relative velocities of the moving platform:

a) general conditions for any position of the mechanism when $\dot{q}_1\neq 0$, $\dot{q}_2\neq 0$ and $\dot{q}_3\neq 0$

$$S^Q_{n/1}=3, \tag{2}$$

$$(R^Q_{n/1})=(\vec{\omega}_1,\vec{\omega}_2,\vec{\omega}_3), \tag{3}$$

b) particular conditions when $\dot{q}_1=0$

$$S^Q_{n/1}=2, \tag{4}$$

$$(R^Q_{n/1})=(\vec{\omega}_2,\vec{\omega}_3), \tag{5}$$

b) particular conditions when $\dot{q}_2=0$

$$S^Q_{n/1}=2, \tag{6}$$

$$(R^Q_{n/1})=(\vec{\omega}_1,\vec{\omega}_3), \tag{7}$$

c) particular conditions when $\dot{q}_3=0$

$$S^Q_{n/1}=2, \tag{8}$$

$$(R^Q_{n/1})=(\vec{\omega}_1,\vec{\omega}_2). \tag{9}$$

The base $(R^Q_{n/1})$ mentioned above must be unique base of the vector space of the relative velocities of the mobile platform. This base is given by:

$$(R^Q_{n/1})=(R^{A1}_{nA1/1}\cap R^{A2}_{nA2/1}\cap R^{A3}_{nA3/1}). \tag{10}$$

We recall that the connectivity (spatiality) $S^Q_{n/1}$ between the moving and the fixed platforms in the mechanism Q represents the number of relative independent infinitesimal displacements or velocities allowed by the mechanism between the two platforms. It is given by the dimension of the vector space $R^Q_{n/1}$ of the relative velocities between the two platforms [35]:

$$S^Q_{n/1}=\dim(R^Q_{n/1}). \tag{11}$$

If the Jacobian J is a diagonal matrix, the singular values are equal to the diagonal elements. The Jacobian J of a fully isotropic mechanism has non zero identical singular values and unit condition number. Consequently, a PW with uncoupled motions is fully isotropic if all diagonal elements of its Jacobian matrix are identical. In this case Eq. (1) becomes

$$[\omega]=\lambda[I][\dot{q}] \tag{12}$$

where λ is the value of the diagonal elements, $[I]$ is the 3×3 identity matrix. The mechanism Q respecting Eq. (12) is fully-isotropic and implicitly it has uncoupled motions. The mechanism Q achieves a homothetic transformation of coefficient λ between the velocity of the actuated joints and the velocity of the moving platform. When $\lambda=1$ the Jacobian matrix ($J=\lambda[I]$) becomes the 3×3 identity matrix. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission. We focus on this particular case in section 4.

III. Kinematic Structures with Uncoupled Motions

We consider that the first joint of each leg A_i ($i=1,2,3$) is actuated (the underlined joint) and we denote by M_{Ai} the mobility of A_i-leg. The simplest architecture for leg A_1 is a serial kinematic chain with three revolute joints of type R-R-R having orthogonal and concurrent axes. This architecture has $(R^{O}_{n/1})=(\vec{\omega}_1, \vec{\omega}_2, \vec{\omega}_3)$ and $M_{A1}=S^{A1}_{nA1/1}=3$. It respects the general conditions (2-3) and the particular conditions (4-5). The simplest architectures of open kinematic chains A_2 have $(R^{A2}_{nA2/1})=(\vec{v}_x, \vec{v}_y, \vec{v}_z, \vec{\omega}_1, \vec{\omega}_2, \vec{\omega}_3)$ and $M_{A2}=S^{A2}_{nA2/1}=6$, where $\vec{v}_x, \vec{v}_y, \vec{v}_z$ represent the linear velocities of point O_0 situated on the output link $n_{A2}\equiv 5$. These architectures respect the general conditions (2-3) and the particular conditions (6-7) and they are of type P'-R'-R'-S and P'-R'-P-S. Just revolute (R), prismatic (P) and spherical joint (S) are used in these solutions in which two consecutive revolute and prismatic joints have parallel or perpendicular axes. The revolute and prismatic joints marked by apostrophe have parallel axes/directions. The simplest architecture of open kinematic chain A_3 is of type R-HJ with $(R^{A3}_{nA3/1})=(\vec{\omega}_1, \vec{\omega}_2, \vec{\omega}_3)$ and $M_{A3}=S^{A3}_{nA3/1}=6$. It respects the general conditions (2-3) and the particular conditions (8-9) and. This leg integrates a homokinetic joint (HJ) and centre O_0 of the spherical motion is not reachable (Figs. 1 and 2). It is easier to get to this point if we use a leg of type R-HJ-P-HJ. This leg integrates two homokinetic joints connected by a telescopic shaft (Figs. 3 and 4) and has $M_{A3}=S^{A3}_{nA3/1}=6$ with $(R^{A2}_{nA2/1})=(\vec{v}_x, \vec{v}_y, \vec{v}_z, \vec{\omega}_1, \vec{\omega}_2, \vec{\omega}_3)$. Just the input and the output shafts are indicated in figures for each homokinetic joint. The intermediary members of the homokinetic joints are not indicated. Various types of homokinetic joints could be used: Tracta, Weiss, Bendix, Dunlop, Rzeppa, Birfield, Glaenzer, Thompson, Triplan, Tripode, UF (undercut-free) ball joint, AC (angular contact) ball joint, VL plunge ball joint, DO (double offset) plunge ball joint, AAR (angular

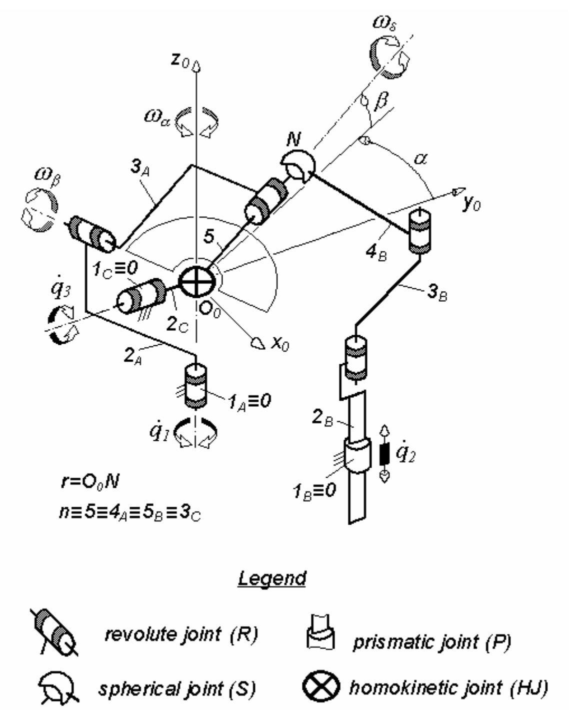

Fig. 1. Kinematic structure RRR-PRRS-RHJ-type with uncoupled motions

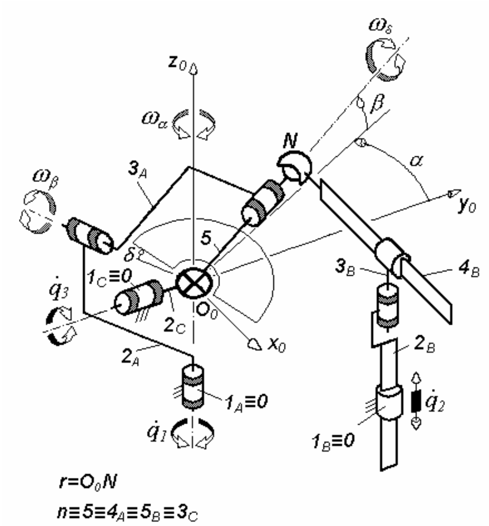

Fig. 2. Kinematic structure RRR-PRPS-RHJ-type with uncoupled motions

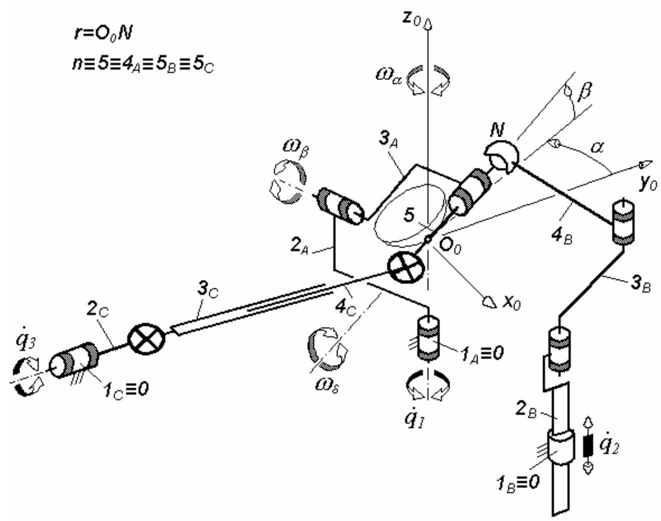

Fig. 3. Kinematic structure RRR-PRRS-RHJPHJ-type with uncoupled motions

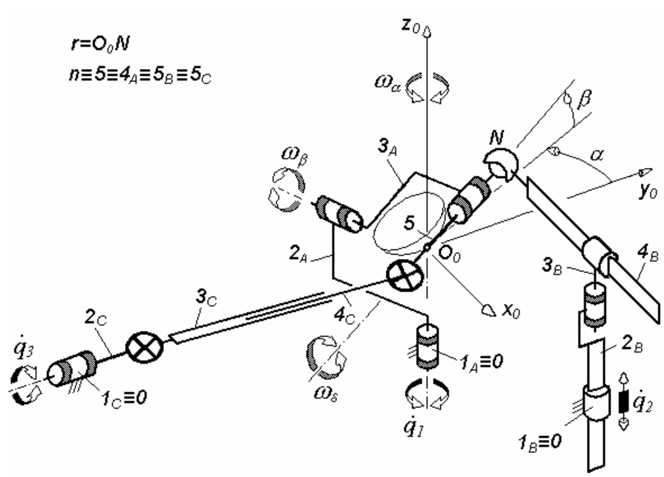

Fig. 4. Kinematic structure RRR-PRPS-RHJPHJ-type with uncoupled motions

adjusted roller), helical flexure U-joints, etc. [37]. We can see that legs A_1 and A_3 are actuated by the revolute motors and the leg A_2 by a linear motor.

By connecting the legs A_i ($i=1,2,3$) to the output link $n \equiv n_{A1} \equiv n_{A2} \equiv n_{A3}$ we obtain the architectures of PWs with uncoupled motions presented in Figs. 1-4 for solutions with non accessible (Figs. 1 and 2) and accessible (Figs. 3 and 4)centre of the spherical motion. To simplify the notations of links e_{Ai} ($i=1,2,3$ and $e=1,2,\ldots,n$) by avoiding the double index in Fig. 1 and the following figures we have denoted by e_A the elements belonging to leg A_1 ($e_A \equiv e_{A1}$), by e_B and e_C the elements of legs A_2 ($e_B \equiv e_{A2}$) and A_3 ($e_C \equiv e_{A3}$). The solutions from Figs. 1 and 3 have no unactuated prismatic joints and the solutions from Figs. 2 and 4 have one unactuated prismatic joint.

In all cases, the three rotational motions are uncoupled, that is each rotation of the moving platform is achieved by only one actuator.

The linear mapping (1) becomes:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}, \quad a = \frac{1}{r\cos\beta}, \tag{13}$$

where r is the length of the output link ($r=O_0N$). Point N represents the centre of the spherical joint of A_2-leg (see Figs. 1 and 2). We can see that $\omega_1 = \dot{q}_1$, $\omega_2 = a\dot{q}_2$ and $\omega_3 = \dot{q}_3$.

To compare the singular values of the Jacobian matrix of linear mapping (13), the elements of this matrix should have the same units. From (13), the elements of the first and the third column of the Jacobian matrix J are non-dimensional. The second column has the unit of length^{-1}. The characteristic length of the manipulator, i.e., l, is used to homogenize the elements of the Jacobian matrix so that the condition number is non-dimensional. The characteristic length renders the Jacobian dimensionally homogeneous and optimally conditioned, i.e., with a minimum condition number [2]. For joint rates belonging to a unit sphere the operational velocities of the moving platform belong to an ellipsoid. The eigenvectors of the matrix $(JJ^T)^{-1}$ define the direction of the principal axes of this ellipsoid. The square

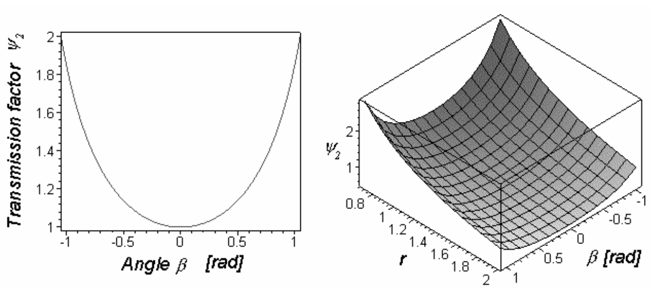

Fig. 5. Variation of the transmission factor ψ_2 with the rotation angle: for the characteristic length $r=l=1$ (a) and for various values of the platform length (b).

roots ξ_i ($i=1,2,3$) of the eigenvalues of $(JJ^T)^{-1}$ are the lengths of the aforementioned principal axes. The velocity transmission factors in the directions of the principal axes are defined by $\psi_i = 1/\xi_i$. These transmission factors can be used to define the joint limits [38]. The PWs presented in Figs. 1-4 have $\psi_1 = 1$, $\psi_2 = 1/(r\cos\beta)$ and $\psi_3 = 1$. The variation of the transmission factor ψ_2 with the rotation angle of the moving platform is presented in Fig. 5. In Fig. 5 (a) we considered that the platform length is equal to the characteristic length ($r=l=1$). In Fig. 5 (b) various values of platform length are considered. For $1 \le r \le 2$ and $\beta \in [-60°, 60°]$ the transmission factor is $0.4 \le \psi_2 \le 2$.

The isotropic configuration of the PWs presented in Figs. 1 and 4 is obtained when $\beta = 0$ and $l=r=1$. In this configuration, the Jacobian becomes the 3×3 identity matrix and (13) maps the joint rates belonging to a unit sphere into operational velocities belonging to unit sphere too.

IV. Fully-isotropic Kinematic Structures

Fully-isotropic PWs can be obtained by using legs A_1 and A_3 presented in the previous section and a leg A_2 actuated by a rotary motor by respecting the general conditions(2-3) and the particular conditions (6-7)–see Figs. 6-9. The simplest architectures of leg A_2 have $M_{A2} = S^{A2}_{nA2/1} = 6$ and $(R^{A2}_{nA2/1}) = (\vec{v}_x, \vec{v}_y, \vec{v}_z, \vec{\omega}_1, \vec{\omega}_2, \vec{\omega}_3)$. These architectures are of type Pa-R'-R'-S and Pa-R-P-S. A planar parallelogram loop (Pa) is integrated in each leg. The axes of the four revolute pairs of this parallelogram mechanism are parallel to xy- plane. Leg A_2 is actuated by a rotary motor fixed on the base and integrated in the parallelogram loop. In this way, a complex kinematic chain integrating the parallelogram loop Pa is associated with each leg A_2. By connecting legs A_i to output link $n \equiv n_{A1} \equiv n_{A2} \equiv n_{A3}$ we obtain

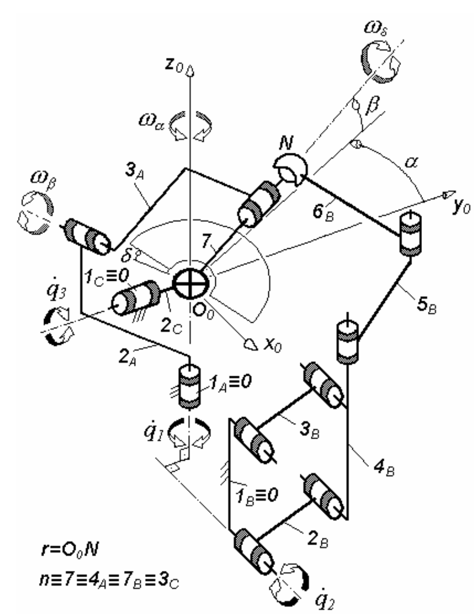

Fig. 6. Kinematic structure of fully-isotropic PW *RRR-PaRRS-RHJ*-type

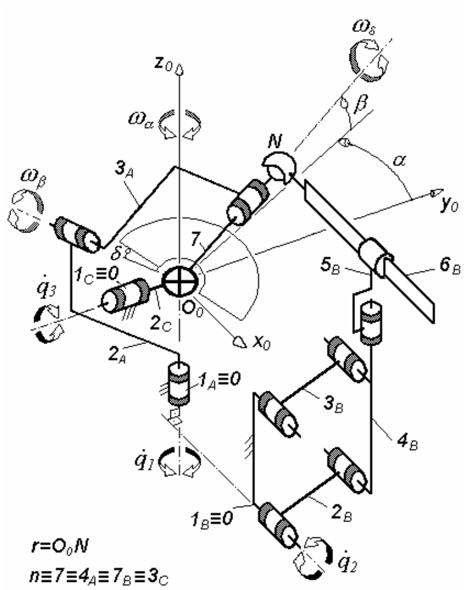

Fig. 7. Kinematic structure of fully-isotropic PW *RRR-PaRPS-RHJ*-type

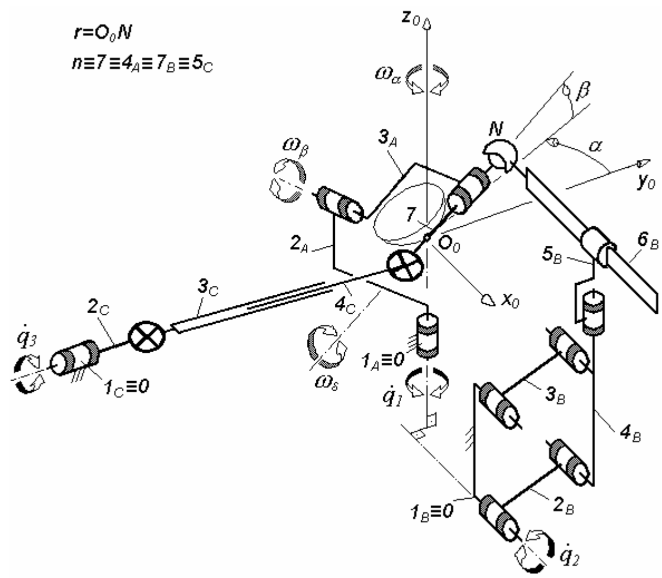

Fig. 9. Kinematic structure of fully-isotropic PW *RRR-PaRPS-RHJPHJ*-type

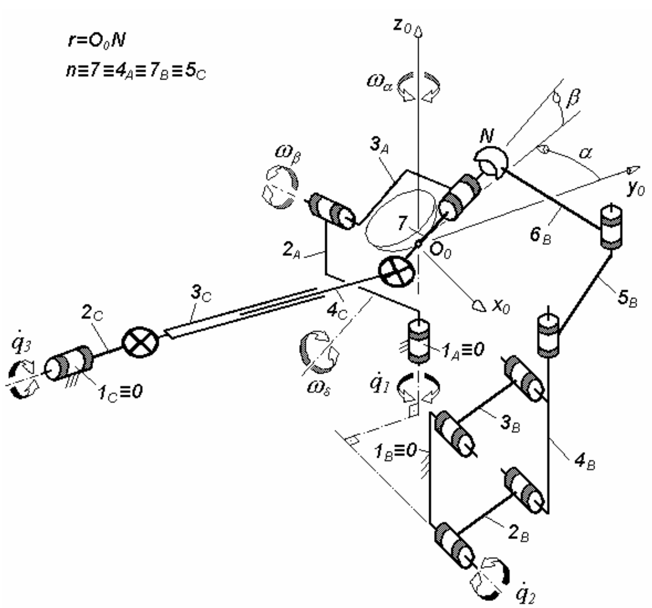

Fig. 8. Kinematic structure of fully-isotropic PW *RRR-PaRRS-RHJPHJ*-type

the architectures of fully-isotropic PWs presented in Figs. 6-9. We can see that these solutions of fully-isotropic PWs are obtained from the solutions of PWs with uncoupled motions presented in the previous section by replacing the first prismatic pair of leg A_2 by a parallelogram loop. In this way, the three legs are actuated by rotary motors. The Jacobian matrix of these solutions is the 3×3 identity matrix throughout their entire workspace.

Advantages of these fully-isotropic solutions include: (i) high stiffness enabling orientation of large loads with high angular velocities and accelerations, (ii) simplification of inverse kinematic computation, (iii) the three actuators are situated on the fixed base. Moreover, solutions in Figs. 8 and 9 have the ability to position the working device at the geometric centre of rotation thereby reducing inertia. The workspace of these solutions must be correlated with the angular capability of the homokinetic joints and translational capability of the telescopic shafts.

V. Conclusions

An approach has been proposed for structural synthesis of three degree-of-freedom parallel wrists that are fully-isotropic throughout their entire workspace. The Jacobian matrix mapping the joint and the operational vector spaces of the fully-isotropic PWs presented in this paper is the 3×3 identity matrix throughout the entire workspace. Fully-isotropic PWs presented in this paper give a one-to-one mapping between the actuated joint velocity space and the operational velocity space. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission. Moreover, the actuators are mounted directly on the base, effectively contributing to the reduction of the weight of the moving parts. The solutions presented in this paper overcome many disadvantages usually affecting parallel manipulators. These PWs can support a payload at the center of rotation providing a large workspace, simplified kinematic computations, reduced inertia and reduction of interference between the payload and the mechanism within the working space. Special legs have been conceived to achieve fully-isotropic condition. Examples of parallel wrists with uncoupled motions and fully-isotropic parallel wrists are presented in this paper to illustrate the proposed approach. As far as we are aware, this paper presents for the first time fully-isotropic parallel wrists with three degrees of freedom and an innovative method for their structural synthesis.

REFERENCES

[1] J. K. Salsbury and J. J. Craig, "Articulated hands: force and kinematic issues," *The International Journal of Robotics Research,* vol. 1, no. 1, pp1-17, 1982.

[2] J. Angeles, *Fundamentals of Robotic Mechanical Systems: Theory, Methods, and Algorithms*. New York, Springer, 1987.

[3] A. Fattah and A. M. Hasan Ghasemi, "Isotropic design of spatial parallel manipulators," *The International Journal of Robotics Research,* vol. 21, no. 9, pp. 811-824, 2002.

[4] J. P. Merlet, *Les robots parallèles*. Paris, Hermès, 2nd ed. 1997.

[5] H. Asada and J. A. Cor Granito, "Kinematic and static characterization of wrist joints and their optimal design," in *Proc. IEEE International Conference on Robotics and Automation,* St. Louis, 1985, vol. 2, pp. 244–250.

[6] C. M. Gosselin and J. Angeles, "The optimum kinematic design of a spherical three-degree-of-freedom parallel manipulator," *Journal of Mechanisms, Transmissions, and Automation in Design,* vol. 111, pp. 202–207, 1989.

[7] D. Cox and D. Tesar, "The dynamic model of a three degree of freedom parallel robotic shoulder model," in *Proc. 4th ICAR*, Columbus, 1989, pp. 475-487.

[8] C. Innocenti and V. Parenti-Castelli, "Echelon form solution of direct kinematics for the general fully-parallel spherical wrist," *Mechanism and Machine Theory,* vol. 28, no. 4, pp. 553–561, 1993.

[9] C. M. Gosselin and J.-F. Hamel, "The Agile Eye: A high performance three-degree-of-freedom camera- orienting device," in *Proc. IEEE International Conference on Robotics and Automation*, San Diego, 1994, pp. 781–786.

[10] J. M. Wiitala and M. M. Stanišić, "Design of an overconstrained dextrous spherical wrist," *Transactions of the ASME, Journal of Mechanical Design,* vol. 122, pp. 347-353, 2000.

[11] P. Vischer, R. Clavel, "Argos: a novel 3-DoF parallel wrist mechanism," *The International Journal of Robotics Research,* vol. 19, no. 1, pp. 5-11, 2000.

[12] M. Karouia and J. M. Hervé, "A three-dof tripod for generating spherical rotation," in *Advances in Robot Kinematics*, J. Lenarčič and M. M. Stanišić, (Eds.), Kluwer Academic Publishers, Dordrecht, pp. 395-402, 2000.

[13] M. Karouia, and J.M. Hervé, " Enumération de mécanismes parallèles sphériques isostatiques," in *Proc. 16ème Congrès Français de Mécanique*, Nice, 2003.

[14] M. Karouia and J. M. Hervé, "New parallel wrists: special limbs with motion dependency," in *On Advances in Robot Kinematics*, J. Lenarčič and C. Galletti, (Eds.), Kluwer Academic Publishers, Dordrecht, pp. 371-380, 2004.

[15] X. Kong and C. M. Gosselin, "Type synthesis of 3-DOF spherical parallel manipulators based on screw theory," *Transactions of the ASME, Journal of Mechanical Design,* vol. 126, pp. 101-108, 2004.

[16] X. Kong and C. M. Gosselin, "Type synthesis of three-degree-of-freedom spherical parallel manipulators," *The International Journal of Robotics Research,* vol. 23, no. 3, pp. 237-245, 2004.

[17] M. Karouia and J. M. Hervé, "Asymmetrical 3-dof spherical parallel mechanisms," *European Journal of Mechanics A/Solids,* vol. 24, no. 1, pp. 47-57, 2005.

[18] J. Sefrioui and C. M. Gosselin, "Etude et représentation des lieux de singularités des manipulateurs parallèles sphériques à trois degrés de liberté avec des actionneurs prismatiques," *Mechanism and Machine Thoery,* vol. 29, no. 4, pp. 559-579, 1994.

[19] G. Gogu, "Fully-isotropic over-constrained parallel wrists with two degrees of freedom," in *Proc. of 2005 IEEE International Conference on Robotics and Automation*, Barcelona, pp. 4025-4030.

[20] J. M. Hervé, "Design of parallel manipulators via the displacement group," in *Proc. 9th World Congress on the Theory of Machines and Mechanisms*, Milan, 1995, pp. 2079-2082.

[21] Q. Li, Z. Huang, J.M. Hervé, "Type synthesis of 3R2T 5-DOF parallel mechanisms using the Lie group of displacements," *IEEE Trans. Robotics and Automation*, vol. 20, no. 2, pp. 173-180, 2004.

[22] J. Angeles, "The qualitative synthesis of parallel manipulators," *ASME Trans*., *Journal of Mechanical Design,* vol. 126, pp. 617-624, 2004.

[23] Y. Fang and L.-W. Tsai, "Structural synthesis of a class of 4-dof and 5-dof parallel manipulators with identical limb structures," *The International Journal of Robotics Research,* vol. 21, no. 9, pp. 799-810, 2002.

[24] Z. Huang and Q. C. Li, "General methodology for type synthesis of symmetrical lower-mobility parallel manipulators and several novel manipulators," *The International Journal of Robotics Research,* vol. 21, no. 2, pp. 131-145, 2002.

[25] Z. Huang and Q. C. Li, "Type synthesis of symmetrical lower-mobility parallel mechanisms using the constraint-synthesis method," *The International Journal of Robotics Research,* vol. 22, no. 1, pp. 59-79, 2003.

[26] A. Frisoli, D. Checcacci, F. Salsedo and M. Bergamasco, "Synthesis by screw algebra of translating in-parallel actuated mechanisms," in *Advances in Robot Kinematics*, J. Lenarčič and M. M. Stanišić, Eds., Dordrecht: Kluwer Academic Publishers, pp. 433-440, 2000.

[27] X. Kong and C. M. Gosselin, "Type synthesis of 3-dof translational parallel manipulators based on screw theory," *ASME Trans., Journal of Mechanical Design*, vol. 126, pp. 83-92, 2004.

[28] M. Carricato, "Fully isotropic four-degree-of-freedom parallel mechanisms for Schönflies motions," *The International Journal of Robotics Research,* vol. 24, no. 5, pp. 397-414, 2005.

[29] G. Gogu, "Structural synthesis of fully-isotropic translational parallel robots via theory of linear transformations," *European Journal of Mechanics– A/Solids*, vol. 23, no. 6, pp. 1021-1039, 2004.

[30] G. Gogu, "Fully-isotropic T3R1-type parallel manipulators," in *On Advances in Robot Kinematics*, Lenarčič, J., Galletti, C. (Eds.), Dordrecht, Kluwer Academic Publishers, pp. 265-274, 2004.

[31] G. Gogu, "Fully-isotropic over-constrained planar parallel manipulators," in *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Sendai, 2004, pp. 3519-3520.

[32] G. Gogu, "Singularity-free fully-isotropic parallel manipulators with Schönflies motions," in *Proc. 12th International Conference on Advanced Robotics*, Seattle, 2005, pp. 194-201.

[33] G. Gogu, "Fully-isotropic parallel robots with four degrees of freedom T2R2-type," in *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems* Edmonton, 2005, pp. 1190-1195.

[34] G. Gogu, "Fully-isotropic T1R2-type parallel manipulators with three degrees of freedom," *Proc. International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, Long Beach, 2005, Paper DETC2005-84313.

[35] G. Gogu, "Mobility and spatiality of parallel robots revisited via theory of linear transformations," *European Journal of Mechanics / A –Solids*, vol. 24, no. 5, pp. 690-711, 2005.

[36] G. Gogu, "Mobility criterion and overconstraints of parallel manipulators," *Proc. International Workshop on Computational Kinematics* Cassino, Paper 22-CK 2005.

[37] F. Dudiţă, D. Diaconescu, M. Lateş and M. Neagoe, *Cuplaje mobile podomorfe*. Braşov, Ed. Trisedes Press., 2001.

[38] D. Chablat and P. Wenger, "Architecture optimization of a 3-DOF translational parallel mechanism for machining applications, the Orthoglide," *IEEE Transactions on Robotics and Automation*, vol. 19, no. 3, pp. 403-410, 2003.