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Minimization of Constraint Forces in Industrial Manipulators*

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Abstract— **Constraint force minimization is essential to improve the dynamic performance of an industrial manipulator. An optimization method is proposed to minimize the constraint forces using the concept of dynamically equivalent system of point-masses. It is shown that for the six-DOF PUMA robot the constraint forces are substantially reduced.**

I. INTRODUCTION

The constraint forces are indispensable in modeling of all constrained system, e.g., industrial manipulators. In robotic literature, a lot emphasis has been given to eliminate the constraint forces from the equations of motion [1-4] in order to perform inverse and forward dynamics suitable for the robot control and simulation, respectively. This is based on the fact that constraint forces do not contribute to the motion of the system under study. However, they significantly influence the design of the mechanical components of a robot. Moreover, if the phenomena like friction, etc. play a significant role in the dynamics then constraint forces, etc. need to be considered for the calculation of the driving forces. Hence, the minimization of the constraint forces is also important from this point of view. Note that the friction forces are dependent on the normal reactions, i.e., constraint force, which are not constant but depend on the robot configurations. In [3] and others, however, these force were taken constant such simplification is not preferred in a realistic dynamic model. In all practical industrial robotic systems constraint forces are very high and their effect in the equations of motion can not be eliminated. Their reduction will not only help reducing the power losses due to friction but also weaken coupling of applied and constraint forces. In addition, this will reduce the noise and wear, improve the dynamic performances, and extend the fatigue life of the manipulators.

When the dimensions of a manipulator and its joint trajectories are given, the inertia-induced constraint forces depend on only upon the mass distribution of the moving links, i.e., the link masses, their mass center locations, and the inertias of the moving links [5]. To minimize the constraint forces it is required to optimally distribute the masses of the links. This problem can be treated by the dynamically equivalent system of point masses, which is a convenient way to represent the inertia properties. The dynamically equivalent system is also called *equimomental system* [6, 7]. The concept of equimomental system of point-masses is used in [8, 9] to minimize the shaking force and shaking moment of multiloop planar mechanisms. The concept is extended here to minimize the constraint forces in industrial manipulators.

First, the links of a manipulator under study are represented by the equimomental system of point masses. Then the equations of motion in the point-mass parameters are derived that state the equivalence between the given system and the set of point-masses. An optimization problem formulation is proposed to minimize the constraint forces due to inertia forces at the joints of the serial manipulator. Essentially what is done is that the masses and the inertias of links are represented by a collection of point masses, whose magnitudes are optimally distributed to reduce the inertia-induced forces and moments. This will minimize the constraint forces in the joints. If one needs to apply in an existing robot one should use counter-weight balancing in which the counterweights are represented as equimomental point-masses. The effectiveness of the methodology is shown by applying it to a six-DOF PUMA robot.

II. SEVEN POINT-MASS MEDEL

Let us consider the *i*th rigid link moving in the three-dimensional Cartesian space as shown in Fig. 1. It has mass, m_i, the mass center at $(\bar{x}_i, \bar{y}_i, \bar{z}_i)$, the moment of inertia ($I_{i,xx}, I_{i,yy}, I_{i,zz}$), and the product of inertia ($I_{i,xy}$, $I_{i,yz}, I_{i,zx}$), where the mass center and inertias are referred to the link fixed frame, $O_i\ X_i\ Y_i\ Z_i$. In order to optimally distribute link masses, each link is treated as a dynamically equivalent system of point masses. A set of seven point-masses as discussed below is defined to dynamically represent the links. Any other set of point-masses can be taken provided it satisfies the equimomental conditions [7]. It is assumed that point-masses m_{ij} are located at the vertices of a rectangular parallelepiped. The center of the parallelepiped is at the origin point, O_i, and has sides $2h_{ix}$, $2h_{iy}$, and $2h_{iz}$, as shown in Fig. 1. The point masses are rigidly fixed to the frame, $O_i\ X_i\ Y_i\ Z_i$.

Now the two systems, i.e., the rigid link and the system of seven point-masses, are dynamically equivalent if the following conditions are satisfied [7]:

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$$\sum_{j=1}^{7} m_{ij} = m_i \tag{1}$$

$$(m_{i1} + m_{i2} - m_{i3} - m_{i4} - m_{i5} + m_{i6} + m_{i7})h_{ix} = m_i \bar{x}_i \tag{2}$$

$$(m_{i1} + m_{i2} + m_{i3} + m_{i4} - m_{i5} - m_{i6} - m_{i7})h_{iy} = m_i \bar{y}_i \tag{3}$$

$$(m_{i1} - m_{i2} - m_{i3} + m_{i4} + m_{i5} + m_{i6} - m_{i7})h_{iz} = m_i \bar{z}_i \tag{4}$$

$$(m_{i1} + m_{i2} - m_{i3} - m_{i4} + m_{i5} - m_{i6} - m_{i7})h_{ix}h_{iy} = I_{i,xy} \tag{5}$$

$$(m_{i1} - m_{i2} - m_{i3} + m_{i4} - m_{i5} - m_{i6} + m_{i7})h_{iy}h_{iz} = I_{i,yz} \tag{6}$$

$$(m_{i1} - m_{i2} + m_{i3} - m_{i4} - m_{i5} + m_{i6} - m_{i7})h_{iz}h_{ix} = I_{i,zx} \tag{7}$$

$$\sum_{j=1}^{7} m_{ij}(h_{iy}^2 + h_{iz}^2) = I_{i,xx} \tag{8}$$

$$\sum_{j=1}^{7} m_{ij}(h_{iz}^2 + h_{ix}^2) = I_{i,yy} \tag{9}$$

$$\sum_{j=1}^{7} m_{ij}(h_{ix}^2 + h_{iy}^2) = I_{i,zz} \tag{10}$$

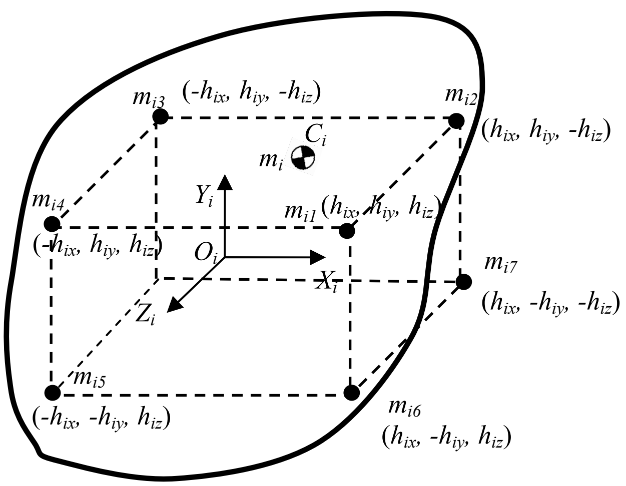


Fig. 1 Equimomental system of seven point-masses

Equation (1) ensures that the total mass of the equimomental system of point-masses is equal to the mass of the rigid link. Equations (2)-(4) satisfy the conditions of the mass center location, whereas (5)-(10) ensure the same inertia tensor for the equivalent point mass system and the link about the point, O_i. Rearranging (8)-(10) yields

$$h_{ix}^2 = (-I_{i,xx} + I_{i,yy} + I_{i,zz})/(2m_i) \tag{11}$$

$$h_{iy}^2 = (I_{i,xx} - I_{i,yy} + I_{i,zz})/(2m_i) \tag{12}$$

$$h_{iz}^2 = (I_{i,xx} + I_{i,yy} - I_{i,zz})/(2m_i) \tag{13}$$

where (1) is used. The moments of inertias, $I_{i,xx}$, $I_{i,yy}$, and $I_{i,zz}$, are such that the sum of any two of them is always greater than the third one [6], which implies that

$(-I_{i,xx} + I_{i,yy} + I_{i,zz}) > 0$; $(I_{i,xx} - I_{i,yy} + I_{i,zz}) > 0$; and

$$(I_{i,xx} + I_{i,yy} - I_{i,zz}) > 0 \tag{14}$$

Hence, h_{ix}, h_{iy}, and h_{iz} will never have imaginary values. Knowing these parameters, the remaining unknowns, m_{ij}, can be determined easily using (1)-(7), which are linear in the point-masses. Such a system of point masses is dynamically identical to the rigid link.

III. Equations of Motion

Referring to the ith rigid link of a serial manipulator (Fig. 2a), points O_i and O_{i+1} are the joints where the link is connected to its previous, (i-1)st, and the subsequent, (i+1)st, links, respectively. The Newton-Euler (NE) equations of motion of the ith link in a fixed inertial frame are given by [10]

$$\mathbf{M}_i \dot{\mathbf{t}}_i + \mathbf{W}_i \mathbf{M}_i \mathbf{E}_i \mathbf{t}_i = \mathbf{w}_i \tag{15}$$

where the 6×6 matrices $\mathbf{M}_i$, $\mathbf{W}_i$ and $\mathbf{E}_i$ are the mass, angular velocity, and coupling matrices, respectively, and defined as

$$\mathbf{M}_i \equiv \begin{bmatrix} \mathbf{I}_i & m_i \tilde{\mathbf{d}}_i \\ -m_i \tilde{\mathbf{d}}_i & m_i \mathbf{1} \end{bmatrix}; \mathbf{W}_i \equiv \begin{bmatrix} \tilde{\boldsymbol{\omega}}_i & \mathbf{O} \\ \mathbf{O} & \tilde{\boldsymbol{\omega}}_i \end{bmatrix}; \quad \mathbf{E}_i \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} \tag{16}$$

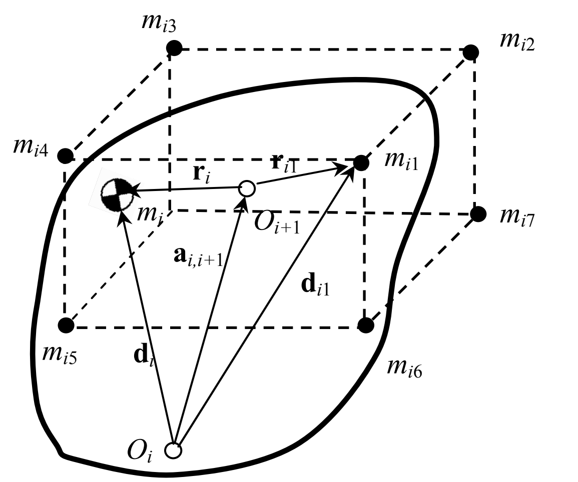


Fig. 2 Definitions of vectors

in which $\mathbf{I}_i$ is the inertia tensor of the link with respect to O_i, whereas the 3×3 skew symmetric matrices, $\tilde{\mathbf{d}}_i$ and $\tilde{\boldsymbol{\omega}}_i$, are associated with the 3-vectors, $\mathbf{d}_i$ and $\boldsymbol{\omega}_i$, respectively, i.e., $\tilde{\mathbf{d}}_i \mathbf{x} = \mathbf{d}_i \times \mathbf{x}$, and $\tilde{\boldsymbol{\omega}}_i \mathbf{x} = \boldsymbol{\omega}_i \times \mathbf{x}$, for the 3-vector, $\mathbf{x}$. The matrix, $\mathbf{O}$, is the zero matrix of appropriate size based on where it appears. Furthermore, the 6-vectors of twist, $\mathbf{t}_i$, twist rate, $\dot{\mathbf{t}}_i$, and wrench, $\mathbf{w}_i$, are defined as

$$\mathbf{t}_i \equiv \begin{bmatrix} \boldsymbol{\omega}_i \\ \mathbf{v}_i \end{bmatrix}; \quad \dot{\mathbf{t}}_i \equiv \begin{bmatrix} \dot{\boldsymbol{\omega}}_i \\ \dot{\mathbf{v}}_i \end{bmatrix}; \ \mathbf{w}_i \equiv \begin{bmatrix} \mathbf{n}_i \\ \mathbf{f}_i \end{bmatrix} \tag{17}$$

where $\boldsymbol{\omega}_i$, $\mathbf{v}_i$, $\mathbf{n}_i$ and $\mathbf{f}_i$, are the angular velocity, linear velocity, the resultant moment, and the resultant force acting on the ith link, respectively, at O_i of the link.

Referring to Fig. 2b, the 3-vectors, $\mathbf{d}_{ij}$ and $\mathbf{r}_{ij}$, are the the positions the point-mass, m_{ij}, from the origins O_i and O_{i+1}, respectively. Subscripts i and j denote the ith link and its jth point-mass, respectively. Using (2)-(4), vector $\mathbf{d}_i$ locating the total mass center in terms of $\mathbf{d}_{ij}$'s is obtained as

$$\mathbf{d}_i = \frac{1}{m_i}\sum_{j=1}^{7} m_{ij}\mathbf{d}_{ij} \tag{18}$$

Denoting $\mathbf{d}_{ij} \equiv [d_{ijx}, d_{ijy}, d_{ijz}]^T$, the 3×3 skew-symmetric matrix, $\tilde{\mathbf{d}}_i$, associated with the vector, $\mathbf{d}_i$, is given by

$$\tilde{\mathbf{d}}_i = \frac{1}{m_i}\begin{bmatrix} 0 & -\sum_{j=1}^{7} m_{ij}d_{ijz} & \sum_{j=1}^{7} m_{ij}d_{ijy} \\ \sum_{j=1}^{7} m_{ij}d_{ijz} & 0 & -\sum_{j=1}^{7} m_{ij}d_{ijx} \\ -\sum_{j=1}^{7} m_{ij}d_{ijy} & \sum_{j=1}^{7} m_{ij}d_{ijx} & 0 \end{bmatrix} \tag{19}$$

Using the conditions of equality for each component of the inertia tensor, i.e., (5)-(10), the inertia tensor, $\mathbf{I}_i$, about O_i, as in (16), in terms of the point mass parameters has the following representation:

$$\mathbf{I}_i = \begin{bmatrix} \sum_{j=1}^{7} m_{ij}(d_{ijy}^2 + d_{ijz}^2) & -\sum_{j=1}^{7} m_{ij}d_{ijx}d_{ijy} & -\sum_{j=1}^{7} m_{ij}d_{ijx}d_{ijz} \\ & \sum_{j=1}^{7} m_{ij}(d_{ijz}^2 + d_{ijx}^2) & -\sum_{j=1}^{7} m_{ij}d_{ijy}d_{ijz} \\ Sym & & \sum_{j=1}^{7} m_{ij}(d_{ijx}^2 + d_{ijy}^2) \end{bmatrix} \tag{20}$$

Equations (18)-(20) define the mass matrix, $\mathbf{M}_i$, of the ith link in terms of the parameters of the equimomental seven point masses. Next, the joint reactions are determined using a recursive algorithm, e.g., the one proposed in [10]. To minimize these reactions an optimization method is proposed in Section IV.

IV. Optimization Problem

A Optimality criteria

There are many possible criteria by which the joint reactions, i.e., the constraint forces at each joint can be minimized. For example, one criterion could be the combination of the root mean squares (RMS) of the constraint forces. Besides, RMS values there are other ways to specify the dynamic quantities, namely, by maximum values, or by the amplitude of the specified harmonics, or by the amplitudes at certain point in the cycle. The choice of course depends on the requirements. Here, the RMS value is preferred over others as it gives equal emphasis on the results of every time instances of the cycle, and every harmonic component. In order to reduce constraint forces at the joints, the following objective function is proposed based on the RMS values:

$$z = \sum_{i=1}^{n} w_{i1}\tilde{f}_i^c + w_{i2}\tilde{n}_i^c \tag{21}$$

where w_{i1} and w_{i2} are the weighting factors whose values may vary depending on an application, whereas $\tilde{f}_i^c$ and $\tilde{n}_i^c$ are the RMS values of the constraint force, $f_i^c = \left|\mathbf{f}_i^c\right|$, and moment, $n_i^c = \left|\mathbf{n}_i^c\right|$, at the ith joint, respectively. For example, if $w_{1i} = 0$ the function, z of (21), will minimize the constraint moments only, whereas $w_{2i} = 0$ will minimize the constraint forces only. Moreover, $0 < w_{1i} > 1$, $0 < w_{2i} > 1$, and $w_{1i} + w_{2i} = 1$.

B Design variables and constraints

Based on the equations of motion presented in Section III, the constraint forces and moments are expressed as the function of the parameters of the point-masses. The point masses, m_{i1}, …, m_{i7}, per link are taken as the design variables. Note that the locations of the point-masses for each link are fixed in the link-fixed frame. For a manipulator having n moving links, the $7n$-vector of the design variables, $\mathbf{x}$, is then defined as

$$\mathbf{x} \equiv [\mathbf{m}_1^T, ..., \mathbf{m}_n^T]^T \tag{22}$$

where the 7-vector, $\mathbf{m}_i$, is as follows:

$$\mathbf{m}_i \equiv \begin{bmatrix} m_{i1} & m_{i2} & m_{i3} & m_{i4} & m_{i5} & m_{i6} & m_{i7} \end{bmatrix}^T$$

in which m_{ij} is defined in Section II. Considering the minimum, $m_{i,\min}$, and the maximum mass, $m_{i,\max}$, of the ith link, the problem of minimization of constraint forces is finally stated as

$$\text{Minimize } z(\mathbf{x}) = \sum_{i=1}^{n} w_{1i}\tilde{f}_i^c + w_{2i}\tilde{n}_i^c \tag{23a}$$

$$\text{Subject to } \quad m_{i,\min} \le \sum_{j=1}^{7} m_{ij} \le m_{i,\max} \tag{23b}$$

for i=1, …,n. Note here that the 3-vector, $\mathbf{d}_{ij}$, is constant in a link-fixed frame. If the link-fixed frame is located at O_{i+1} and its axes are parallel to the sides of the parallelepiped, as shown in Fig. 1, then the vector, $\mathbf{d}_{ij}$, is given by

$$\mathbf{d}_{ij} = \mathbf{a}_{i,i+1} + \mathbf{r}_{ij} \tag{24}$$

where the components of $\mathbf{r}_{ij}$ are as follows:

	$\mathbf{r}_{i1}$	$\mathbf{r}_{i2}$	$\mathbf{r}_{i3}$	$\mathbf{r}_{i4}$	$\mathbf{r}_{i5}$	$\mathbf{r}_{i6}$	$\mathbf{r}_{i7}$
r_{ijx}	h_{ix}	h_{ix}	$-h_{ix}$	$-h_{ix}$	$-h_{ix}$	h_{ix}	h_{ix}
r_{ijy}	h_{iy}	h_{iy}	h_{iy}	h_{iy}	$-h_{iy}$	$-h_{iy}$	$-h_{iy}$
r_{ijz}	h_{iz}	$-h_{iz}$	$-h_{iz}$	h_{iz}	h_{iz}	h_{iz}	$-h_{iz}$

Furthermore, the minimum moments of inertia of the ith link in the link- fixed frame are as follows:

$$I_{ixx,\min} = \sum_{j=1}^{7} m_{ij}(d_{ijy}^2 + d_{ijz}^2) = m_{i,\min}[(a_{i,i+1y} + r_{ijy})^2 + (a_{i,i+1z} + r_{ijz})^2] \quad (25)$$

$$I_{iyy,\min} = \sum_{j=1}^{7} m_{ij}(d_{ijz}^2 + d_{ijx}^2) = m_{i,\min}[(a_{i,i+1z} + r_{ijz})^2 + (a_{i,i+1x} + r_{ijx})^2] \quad (26)$$

$$I_{izz,\min} = \sum_{j=1}^{7} m_{ij}(d_{ijx}^2 + d_{ijy}^2) = m_{i,\min}[(a_{i,i+1x} + r_{ijx})^2 + (a_{i,i+1y} + r_{ijy})^2] \quad (27)$$

Similarly, one can find expression for the product of inertias also. It is now evident from (25)-(27) that the minimum moment of inertia is always positive and depends on the limits of the link masses. Hence, the optimization problem formulated finds a value for each point mass while the total mass of each link is subjected to a lower and an upper limit. The optimization method is task based and optimizes the dynamic performance of the robotic manipulator. The dynamic performance is measured by the weighted combination of the root-mean-squared values of the constraint forces and moments generated during the execution of the specified trajectories. In the method, it is necessary for the designer to provide the appropriate constraints for the design parameters. These constraints directly affect the results.

The optimization problem is solved here using the optimization toolbox of MATLAB. From the optimized values of point-masses (m_{ij}^*), optimized total mass (m_i^*), the location of the mass center ($\bar{x}_i^*, \bar{y}_i^*, \bar{z}_i^*$), and the inertias ($I_{i,xx}^*, I_{i,yy}^*, I_{i,zz}^*, I_{i,xy}^*, I_{i,yz}^*, I_{i,zx}^*$) of each link are determined using the equimomental conditions, (1)-(10).

V. NUMERICAL EXAMPLE

The six-DOF PUMA robot is considered here to minimize the constraint forces and moments using the methodology proposed in this paper. The dimensions and configuration of the robot is defined using the Denavit-Hartenberg (DH) parameters provided in Appendix. The DH parameters, and mass and inertias of the links of the robot, which are taken from [11], are given in Table 1. Note here that the off-diagonal elements of the inertia tensors, namely, I_{ixy}, I_{iyz}, I_{izx}, are taken zeros, whereas, the inertias of the links are given in the local frame. In Table 1, r_{ix}, r_{iy} and r_{iz} denote the vector $\mathbf{r}_i$ for link i, Fig. 2, in its local frame. The joint trajectories are taken as given in the local frames. In Table 1, r_{ix}, r_{iy} and r_{iz} denote

$$\theta_i = \theta_i(0) + \frac{\theta_i(T) - \theta_i(0)}{T}\left[t - \frac{T}{2\pi}\sin\left(\frac{2\pi}{T}t\right)\right] \quad (28)$$

where T=10 sec, $\theta_i(0) = 0$, $\theta_i(T) = 180^o$. Choosing equal weight to the constrain force and the constraint moment, some suitable limits on the link masses, as given by (23), then the optimization problem for the PUMA robot for the specified trajectories, (28), is formally stated as

$$\text{Minimize } z(\mathbf{x}) = \sum_{i=1}^{6} 0.5\tilde{f}_i + 0.5\tilde{n}_i \quad (29a)$$

$$\text{Subject to } \quad m_i^o \le \sum_{j=1}^{7} m_{ij} \le 5m_i^o \quad (29b)$$

where m_i^o is the original mass of the ith link. The comparison of the RMS values of the constraint forces and moments for the optimized and the original manipulator is shown in Table 2, whereas their reactions are shown in Figs. 3, 4. Table 3 shows the optimized values for the masses and inertias of the links. Note that The RMS values of constraint moments for first four joints is reduced drastically compared to for the last two joints for which the values are insignificant. The constraint forces fluctuation due to inertia forces reduce to their static constraint forces due to gravity which cannot be decreased for a given system of masses. However, constraint forces due to the inertia forces have reduced, as clearly evident for the first four joints shown in Fig. 3(a-d).

TABLE 1 DH parameters, and mass and inertia properties

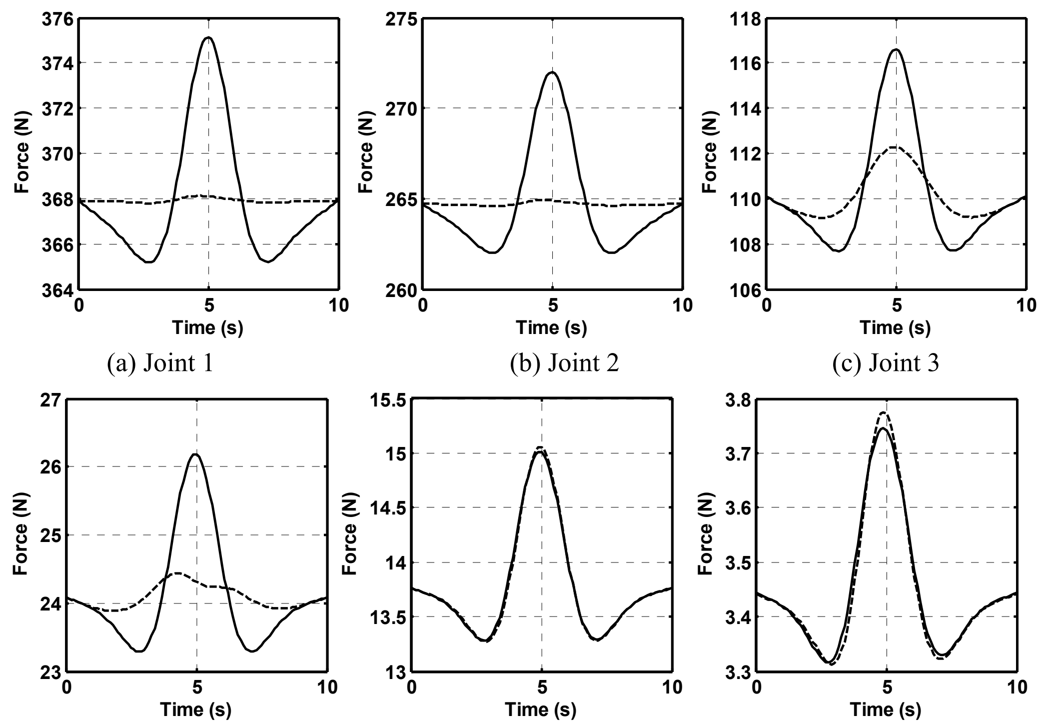
i	a_i	b_i	α_i	θ_i	m_i	r_{ix}	r_{iy}	r_{iz}	I_{ixx}	I_{iyy}	I_{izz}
	(m)	(m)	(deg)	(deg)	(Kg)		(m)			(Kg-m^2)	
1	0	0	-90	θ_1	10.521	0	0	-0.054	1.612	1.612	0.5091
2	0.432	0.149	0	θ_2	15.761	-0.292	0	0	0.4898	8.0783	8.2672
3	0.02	0	90	θ_3	8.767	-0.02	0	0.197	3.3768	3.3768	0.3009
4	0	0.432	-90	θ_4	1.052	0	-0.057	0	0.181	0.1273	0.181
5	0	0	90	θ_5	1.052	0	0	0.007	0.0735	0.1273	0.0735
6	0	0.056	0	θ_6	0.351	0	0	-0.019	0.0071	0.0071	0.0141

Table 2 The RMS values of the constraint moments and forces

	RMS values: $(\tilde{f}_1, \tilde{f}_2, \tilde{f}_3, \tilde{f}_4, \tilde{f}_5, \tilde{f}_6, \tilde{n}_1, \tilde{n}_2, \tilde{n}_3, \tilde{n}_4, \tilde{n}_5, \tilde{n}_6)$
Original	(367.975 264.788 110.219 24.138 13.797 3.452 73.119 75.613 14.474 5.433 0.111 0.0755)
Optimized	(367.912 264.702 110.123 24.089 13.797 3.450 1.947 1.867 1.227 1.048 0.238 0.0673)

TABLE 3 Optimized mass and inertia properties

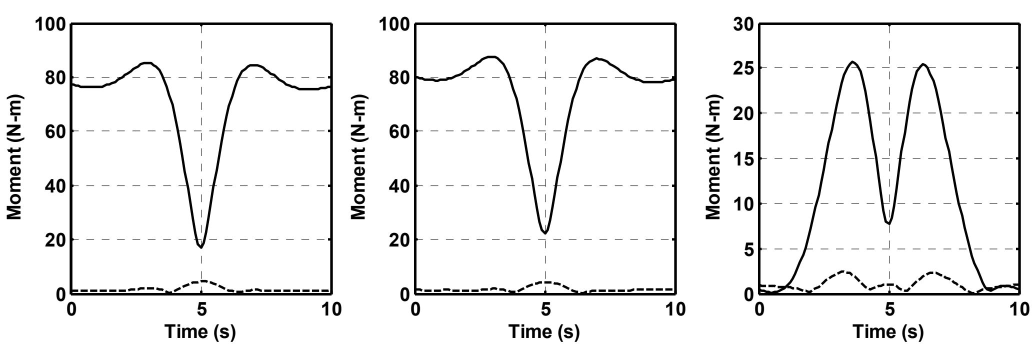
i	m_i^*	r_{ix}^*	r_{iy}^*	r_{iz}^*	I_{ixx}^*	I_{iyy}^*	I_{izz}^*	I_{ixy}^*	I_{iyz}^*	I_{izx}^*
	(kg)		(m)					Kg/m^2		
1	10.521	-0.001	-0.005	-0.002	1.6423	1.6423	0.5088	0.0142	-0.0761	0.0399
2	15.767	-0.737	0.000	-0.253	0.4896	9.4224	9.6112	-1.2058	-0.1971	-0.9016
3	8.767	-0.025	0.01	0.027	3.7168	3.7203	0.3044	-0.1440	1.0677	-0.3595
4	1.052	0.051	1.087	-0.017	0.1844	0.1273	0.1844	-0.7766	0.3608	-0.0124
5	1.052	-0.007	0.014	0.007	0.0736	0.1274	0.0735	-0.0438	0.0699	-0.1936
6	0.351	-0.014	0.008	-0.055	0.0072	0.0072	0.0141	0.0290	0.0020	-0.0048



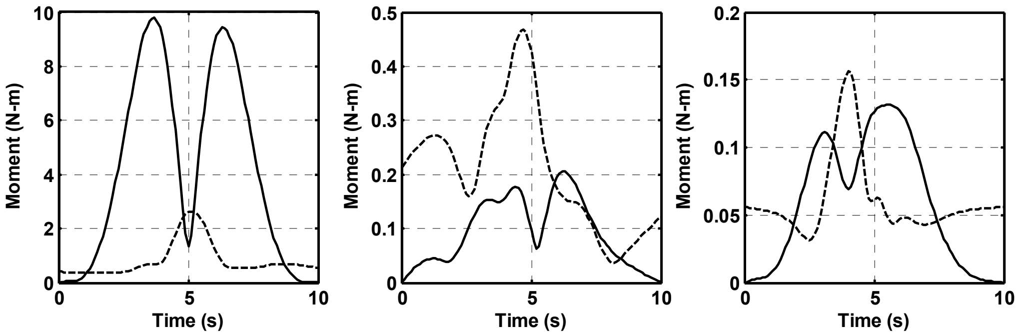
(a) Joint 1 (b) Joint 2 (c) Joint 3

(d) Joint 4 (d) Joint 5 (f) Joint 6

Fig. 3 Constraint forces ——— Original; ------ Optimized



(a) Joint 1 (b) Joint 2 (c) Joint 3



(d) Joint 4 (e) Joint 5 (f) Joint 6

Fig. 4 Constraint moments, —— Original; ------ Optimized

VI. CONCLUSION

Realizing the importance of the constraint forces in an industrial manipulator, their minimization problem is posed as an optimization problem. For this, the dynamic modeling of the manipulators is presented in terms of the equimomental system of point-masses. The solution of the optimization problem provides the redistribution of the link masses such that the constraint forces of joints is reduced to minimum. Such results have not been reported in the robotics literature and one of the major contribution of this paper, along with dynamic and optimization formulations. The effectiveness of the proposed methodology is illustrated using the six-DOF PUMA robot. A significant reduction in constraint forces due to inertia-induced forces is obtained. The method is generic and can be used for any spatial robotic system.

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Appendix: DH Parameters

With each link, namely, the (i-1)st one, a Cartesian coordinate system, $O_iX_iY_iZ_i$, denoted by F_i , is attached at O_i. Point O_i is the origin of the ith coordinate system and located at the intersection of X_i and Z_i. The axis, Z_i, is along the ith joint axis, whereas X_i is the common perpendicular between consecutive axes Z_{i-1} to Z_i directed from the former to the latter. The axis Y_i is then defined to complete a right-handed system $O_iX_iY_iZ_i$. The distance between Z_i and Z_{i+1} is defined as a_i, which is positive. The Z_i coordinate of the intersection point on Z_i with X_{i+1} is denoted by b_i. The angles between, Z_i and Z_{i+1}, and X_i and X_{i+1}, are α_i, and θ_i, measured about the positive direction of X_{i+1} and Z_i, respectively. For a revolute joint, only the parameter θ_i varies and is called the joint variable. The parameters a_i, b_i, α_i, and θ_i, for each link are DH parameters, as shown in Fig. 6.

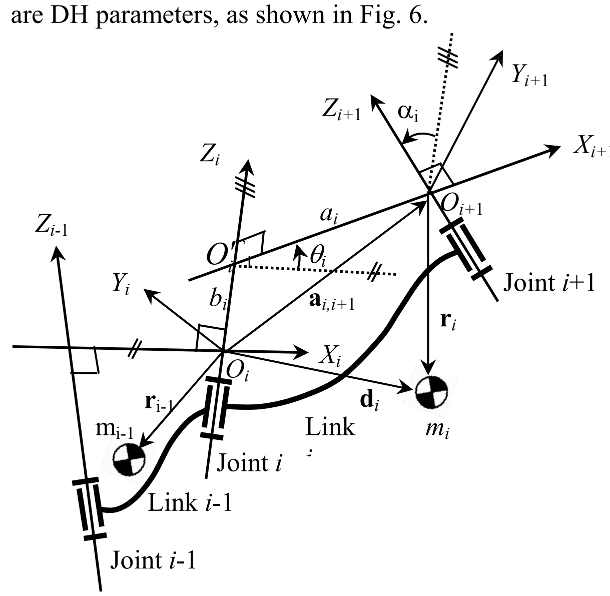

Fig. 5 Coordinate frames and associated parameters