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Generation and Local Stabilization of Fixed Point Based on a Stability Mechanism of Passive Walking

Yoshito Ikemata, Akihito Sano, and Hideo Fujimoto

Abstract— **A passive walker with knees can walk down gentle slope in a natural gait and can exhibit a stable limit cycle. Though the passive walker is simple, it is a sort of hybrid system which combines the continuous dynamics of leg-swing motion and the discrete event of leg-exchange. We focus on the mechanisms of generation and stabilization of a fixed point of passive walking. We propose a generation method of fixed point based on its physical structure. We derive the local stabilization control method from a stability mechanism of a fixed point of passive walking.**

I. Introduction

Passive walking [1] can be regarded as a physical phenomenon generated by the hybrid system, which consists of continuous dynamics of leg-swing motion and discrete event of leg exchange. Gait generation and its stability must be analyzed from the hybrid system. Passive walking can exhibit a stable limit cycle. When the state keeps on the stable limit cycle, walking system is stable.

McGeer [1][2] first studied the passive walking from viewpoint of discrete-time system. He demonstrated the stability of fixed point from Jacobian matrix obtained by linearizing the discrete-time state equation (called "step to step equation"). Goswami et al. [3], Coleman et al. [4], and Garcia et al. [5] studied the stability of fixed point of various passive walking in detail. However, these studies have not demonstrated why the fixed point of passive walking becomes stable.

Many dynamical systems reach an equilibrium state which condition is minimum or local minimum point of energy function. On the other hand, the fixed point of passive walking is known that it keeps a balance between the energy supplied by potential energy and the energy lost by heel-strike [3][5][6]. However, it is nothing but the result obtained by observing the phenomenon. Many researchers have not considered the physical structure of fixed point. They merely searched the fixed point by numerical method.

In recent years, several researchers [7]–[12] have studied walking robots based on passive walking. The robots can walk on level ground with efficient. These studies assume that a stable fixed point of passive walking exists. In some cases, fixed point of passive walking is not always generated, and is not always stable. The studies except for [11] have not considered the stabilization of passive walking.

Passive walking has not only a stable fixed point but also a unstable fixed point. When 1-periodic gait turns 2-periodic gait, a stable fixed point of 1-periodic gait becomes unstable. Several researchers [13]–[15] have proposed the stabilization control method based on the existing control method. These stabilization control methods are not particularly effective stabilization method of passive walking because these don't consider the stability mechanism of a fixed point of passive walking.

Y. Ikemata, A. Sano and H. Fujimoto are with Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya, Aichi, 466-8555, JAPAN ikemata@vier.mech.nitech.ac.jp

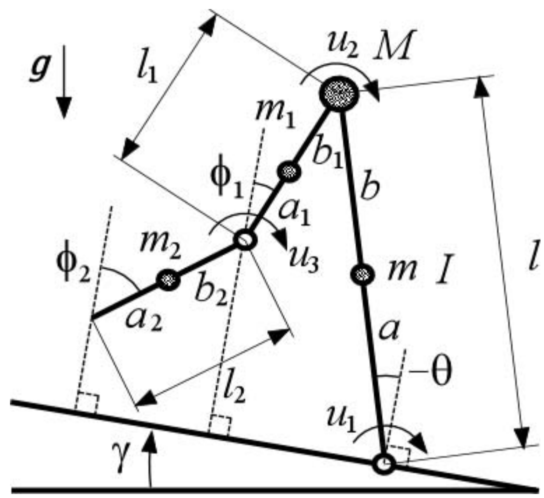


Fig. 1. Model of passive walker with knees

In this paper, we focus on the mechanisms of generation and stabilization of a fixed point in passive walking. At first, we demonstrate the physical structure of a fixed point, and propose a generation method of a fixed point based on its physical structure. Secondly, we derive the local stabilization control method from a stability mechanism of a fixed point. Though our stabilization control method is very simple, the highest local stability of a fixed point can be achieved. Finally, the validity of our proposed methods of generation and stabilization is confirmed by the simulation.

II. Model of passive walker with knees

A. Leg–swing motion

Figure 1 shows the model of passive walker with knees. The model consists of stance and swing legs. Knee of the stance leg is locked straight. The motion is assumed to be constrained to saggital plane. For the purpose of simplicity and clarity of analysis, assumptions are given as follows:

$$M \gg m, \quad M \gg m_1, \quad M \gg m_2 \tag{1}$$

1) Motion equation of 3 links (with Knees): Stance leg is assumed to be fixed on the ground without any slippage or take off. The equation of leg-swing motion of 3 links can be written as

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K)\ddot{\boldsymbol{\theta}}_K + \boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) + \boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \boldsymbol{E}_K\boldsymbol{\tau}_K \tag{2}$$

where

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K) = \begin{bmatrix} l^2 & 0 & 0 \\ -(b_1 l + pll_1)\cos(\theta-\phi_1) & b_1^2 + pl_1^2 & pb_2l_1\cos(\phi_1-\phi_2) \\ -b_2 l\cos(\theta-\phi_2) & b_2 l_1\cos(\phi_1-\phi_2) & b_2^2 \end{bmatrix}$$

$$\boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) = \begin{bmatrix} 0 \\ (b_1 l + pll_1)\sin(\theta-\phi_1)\dot{\theta}^2 + pb_2l_1\sin(\phi_1-\phi_2)\dot{\phi}_2^2 \\ b_2 l\sin(\theta-\phi_2)\dot{\theta}^2 - b_2l_1\sin(\phi_1-\phi_2)\dot{\phi}_1^2 \end{bmatrix}$$

$$\boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \begin{bmatrix} -l\sin(\theta+\gamma) \\ (b_1 + pl_1)\sin(\phi_1+\gamma) \\ b_2\sin(\phi_2+\gamma) \end{bmatrix} g$$

$$\boldsymbol{E}_K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{\tau}_K = \begin{bmatrix} u_1/M \\ u_2/m_1 \\ u_3/m_2 \end{bmatrix}$$

$\boldsymbol{\theta}_K (= [\theta, \phi_1, \phi_2]^T)$ is the vector of joint angles. g is the acceleration of gravity. p is $p = m_2/m_1$. Setting $a = \{m_1(l_2 + a_1) + m_2a_2\}/(m_1+m_2)$ and $I = m_2(l_2-b_2-a)^2 + m_1(l_2+a_1-a)^2$, stance leg is equal to swing leg.

2) Equations of knee-lock: Knee-lock occurs when the swing leg becomes straight ($\phi_1 = \phi_2 = \phi$). Assuming that the swing knee locks instantaneously, angular momentum is conserved through the knee-lock for the whole walker about the contact point of stance foot, and the swing leg about the hip. Angular velocities of stance and swing legs just after knee-lock can be obtained from these conservations of angular momentum as

$$\dot{\theta}^+ = \dot{\theta}^- \tag{3}$$

$$\dot{\phi}^+ = \frac{(b_1^2 + pl_1^2 + pl_1b_2)\dot{\phi}_1^- + (pb_2^2 + pb_2l_1)\dot{\phi}_2^-}{b_1^2 + p(l_1+b_2)^2} \tag{4}$$

The "+" superscript means "just after knee-lock," and the "−" superscript means "just before knee-lock".

3) Motion equation of 2 links (Compass-type): After knee-lock, the model can be regarded as compass-like biped model. The equation of leg-swing motion of 2 links can be written as

$$\boldsymbol{M}_C(\boldsymbol{\theta}_C)\ddot{\boldsymbol{\theta}}_C + \boldsymbol{H}_C(\boldsymbol{\theta}_C, \dot{\boldsymbol{\theta}}_C) + \boldsymbol{G}_C(\boldsymbol{\theta}_C, \gamma) = \boldsymbol{E}_C\boldsymbol{\tau}_C \tag{5}$$

where

$$\boldsymbol{M}_C(\boldsymbol{\theta}_C) = \begin{bmatrix} l^2 & 0 \\ -(1+p)bl\cos(\theta-\phi) & \bar{I} + (1+p)b^2 \end{bmatrix}$$

$$\boldsymbol{H}_C(\boldsymbol{\theta}_C, \dot{\boldsymbol{\theta}}_C) = \begin{bmatrix} 0 \\ (1+p)bl\sin(\theta-\phi)\dot{\theta}^2 \end{bmatrix}$$

$$\boldsymbol{G}_C(\boldsymbol{\theta}_C, \gamma) = \begin{bmatrix} -l\sin(\theta+\gamma) \\ (1+p)b\sin(\phi+\gamma) \end{bmatrix} g$$

$$\boldsymbol{E}_C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \boldsymbol{\tau}_C = \begin{bmatrix} u_1/M \\ u_2/m_1 \end{bmatrix}$$

$\boldsymbol{\theta}_C$ ($=[\theta, \phi]^T$) is the vector of joint angles after knee-lock. $\bar{I}$ is I/m_1.

B. Leg-exchange

It supplies a leg-exchange rule when the swing foot hits the ground. Collision occurs when the geometric condition

$$2\theta - \phi = 0 \tag{6}$$

is met. For an inelastic no-sliding collision with the ground, angular momentum is conserved through the collision for the whole walker about the contact point of swing foot, and the former stance leg about the hip [16]. Relational expression can be obtained from these conservations of angular momentum as

$$\boldsymbol{Q}^+(\alpha)\dot{\boldsymbol{\theta}}_C^+ = \boldsymbol{Q}^-(\alpha)\dot{\boldsymbol{\theta}}_C^- \tag{7}$$

where

$$\boldsymbol{Q}^+(\alpha) = \begin{bmatrix} l^2 & 0 \\ -bl\cos\alpha & b^2 + \dfrac{\bar{I}}{1+p} \end{bmatrix}$$

$$\boldsymbol{Q}^-(\alpha) = \begin{bmatrix} l^2\cos\alpha & 0 \\ -ab + \dfrac{\bar{I}}{1+p} & 0 \end{bmatrix}$$

The "+" superscript means "just after heel-strike," and the "−" superscript means "just before heel-strike". α is inter-leg angle at heel-strike. α_k and α_{k+1} are assumed that $0 < \alpha_k < \pi/2$ and $0 < \alpha_{k+1} < \pi/2$.

From Eq. (7), the vector of angular velocity just after heel-strike can be given as

$$\dot{\boldsymbol{\theta}}_C^+ = (\boldsymbol{Q}^+(\alpha))^{-1}\boldsymbol{Q}^-(\alpha)\dot{\boldsymbol{\theta}}_C^- \tag{8}$$

III. Fixed point of passive walking class

A. Physical structure of fixed point

Walking system generates a cyclic trajectory. When the one cyclic trajectory is closed, the state just after heel-strike is fixed as one point. This point is called "fixed point". In this section, we demonstrate the physical structure of a fixed point.

One cycle is defined as the period from the state just after heel-strike to the next state. We focus on the fixed point of 1–periodic gait. Torques of ankle, hip, and knee of k steps are assumed to be constant as follows:

$$\frac{u_1}{M} = \tau_{1k}, \quad \frac{u_2}{m_1} = \tau_{2k}, \quad \frac{u_3}{m_2} = \tau_{3k} \tag{9}$$

By the assumption of $M \gg m$, term that includes m is much smaller than term that includes M. Term that includes u_2, u_3 is much smaller than term that includes u_1.

The state just after heel-strike consists of inter-leg angle α_k, angular velocities of stance, and swing legs $\dot{\theta}_k^+$, $\dot{\phi}_k^+$. From energy conservation law and Eq. (7), discrete-time state equation of $\dot{\theta}_k^{+2}$ can be derived as

$$\dot{\theta}_{k+1}^{+2} = e_{k+1}^2\left(\dot{\theta}_k^{+2} + \frac{2g}{l}\left\{\cos\left(\frac{\alpha_k}{2} - \gamma\right) - \cos\left(\frac{\alpha_{k+1}}{2} + \gamma\right)\right\} + \frac{\tau_{1k}}{l^2}(\alpha_k + \alpha_{k+1})\right) \quad (10)$$

where $e_{k+1} = \cos\alpha_{k+1}$. We call e_{k+1} $(0 < e_{k+1} < 1)$ loss coefficient. e_{k+1}^2 means residual ratio of energy at heel-strike.

In the fixed point, $\alpha_{k+1} = \alpha_k$, $\dot{\theta}_{k+1}^+ = \dot{\theta}_k^+$, and $\dot{\phi}_{k+1}^+ = \dot{\phi}_k^+$ hold. Angular velocity of stance leg $\dot{\theta}_k^+$ is derived from discrete-time state equation (10) as follows:

$$\dot{\theta}_k^+ = \sqrt{\frac{2e_k^2}{l(1-e_k^2)}\left(2g\sin\frac{\alpha_k}{2}\sin\gamma + \frac{\tau_{1k}\alpha_k}{l}\right)} \quad (11)$$

Equation (11) can be rewritten as

$$\frac{1}{2}Ml^2\dot{\theta}_k^{+2}\left(\frac{1}{e_k^2} - 1\right) = 2Mgl\sin\frac{\alpha_k}{2}\sin\gamma + M\tau_{1k}\alpha_k \quad (12)$$

Left part of Eq. (12) is denoted for the energy lost by heel-strike [1]. Right part of Eq. (12) is denoted for the energies supplied by gravitational potential and ankle torque of stance leg. Eq. (12) represents energy balance in one cycle.

From leg-exchange equation (7), equation can be obtained as follows:

$$\dot{\phi}_k^+ = q(\alpha_k)\dot{\theta}_k^+ \quad (13)$$

where

$$q(\alpha_k) = \frac{-ab + \dfrac{\bar{I}}{1+p} + bl\cos^2\alpha_k}{\left(b^2 + \dfrac{\bar{I}}{1+p}\right)\cos\alpha_k}$$

The states just after heel-strike have a physical structure constrained by Eq. (13). From Eqs. (11) and (13), the angular velocity of swing leg $\dot{\phi}_k^+$ can be derived as

$$\dot{\phi}_k^+ = q(\alpha_k)\sqrt{\frac{2e_k^2}{l(1-e_k^2)}\left(2g\sin\frac{\alpha_k}{2}\sin\gamma + \frac{\tau_{1k}\alpha_k}{l}\right)} \quad (14)$$

From Eqs. (11) and (14), the fixed point is represented by α_k, $\dot{\theta}_k^+(\alpha_k)$, and $\dot{\phi}_k^+(\alpha_k)$. The fixed point is generated if α_{k+1}=α_k. α_{k+1} is dependent on leg–swing motion.

As mentioned above, the fixed point is formed by the physical structures of energy balance, leg-exchange, and leg-swing motion.

[1] Energy lost by knee-lock is negligible by assumptions of Eq. (1).

B. Generation method of fixed point

Linearized equations of energy balance (11), leg-exchange (13), and leg–swing motion (2)–(5) are given as follows:

$$\dot{\theta}_k^+ = \sqrt{\frac{2e_k^2\alpha_k}{l(1-e_k^2)}\left(g\gamma + \frac{\tau_{1k}}{l}\right)} \quad (15)$$

$$\dot{\phi}_k^+ = \frac{-ab + \dfrac{\bar{I}}{1+p} + bl}{b^2 + \dfrac{\bar{I}}{1+p}}\dot{\theta}_k^+ \quad (16)$$

and

$$\boldsymbol{M}_{KL}(\boldsymbol{\theta}_K)\ddot{\boldsymbol{\theta}}_K + \boldsymbol{H}_{KL}(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) + \boldsymbol{G}_{KL}(\boldsymbol{\theta}_K, \gamma) = \boldsymbol{E}_K\boldsymbol{\tau}_K \quad (17)$$

where

$$\boldsymbol{G}_{KL}(\boldsymbol{\theta}_K, \gamma) = \begin{bmatrix} -l(\theta+\gamma) \\ (b_1 + pl_1)(\phi_1 + \gamma) \\ b_2(\phi_2 + \gamma) \end{bmatrix} g$$

and

$$\boldsymbol{M}_{CL}(\boldsymbol{\theta}_C)\ddot{\boldsymbol{\theta}}_C + \boldsymbol{H}_{CL}(\boldsymbol{\theta}_C, \dot{\boldsymbol{\theta}}_C) + \boldsymbol{G}_{CL}(\boldsymbol{\theta}_C, \gamma) = \boldsymbol{E}_C\boldsymbol{\tau}_C \quad (18)$$

where

$$\boldsymbol{G}_{CL}(\boldsymbol{\theta}_C, \gamma) = \begin{bmatrix} -l(\theta+\gamma) \\ (1+p)b(\phi+\gamma) \end{bmatrix} g$$

Due to limitations of space, the terms except for the gravity term in Eqs. (17) and (18) are not written.

Torque vectors $\boldsymbol{\tau}_K$, $\boldsymbol{\tau}_C$ are set to

$$\boldsymbol{\tau}_K = \begin{bmatrix} l(\gamma' - \gamma) \\ -(b_1 + pl_1 + pb_2)(\gamma' - \gamma) \\ -b_2(\gamma' - \gamma) \end{bmatrix} g \quad (19)$$

$$\boldsymbol{\tau}_C = \begin{bmatrix} l(\gamma' - \gamma) \\ -(1+p)b(\gamma' - \gamma) \end{bmatrix} g \quad (20)$$

where γ' is constant number. The equations of the fixed point are equivalent to the ones of passive walking in slope angle γ. The equations are applicable to downhill ($\gamma > 0$), level ground ($\gamma = 0$), and uphill ($\gamma < 0$). By inputting the constant torques as shown in Eqs. (19) and (20), the same fixed point as passive walking can be generated. We call the fixed point "a fixed point of passive walking class". It is generated by same energy as passive walking.

As an example, we generate the fixed point of passive walking class on level ground ($\gamma = 0$). The model parameters are set to l=0.7[m], l_1=l_2=0.35[m], a=b=0.35[m], a_1=b_1=a_2=b_2=0.175[m], and p=0.4. γ' is set to 0.073[rad]. $\boldsymbol{\tau}_K$ and $\boldsymbol{\tau}_C$ are given by Eqs. (19) and (20) as $\boldsymbol{\tau}_K$ = [0.50078, –0.275429, –0.125195]^T and $\boldsymbol{\tau}_C$ = [0.50078, –0.275429]^T[Nm/kg]. Finally, the fixed point of long period gait can be obtained as α_f=0.73750[rad] and $\dot{\theta}_f^+$=1.35140[rad/s]. The fixed point of short period gait can be obtained as α_f=0.68592[rad] and $\dot{\theta}_f^+$=1.44662[rad/s].

A fixed point of passive walking class is not particular class; hence it is equivalent to the virtual passive walking proposed by Asano [8]. Main aim of gait generation method [8][12] is to reproduce the mechanical energy, and a fixed point is accordingly generated. Our proposed method is to generate a fixed point directly by inputting the constant torques. Also, it has feature that data of the fixed point is integrated into our stabilization control method as mentioned in Section IV.

IV. Dynamic-based stabilization method

A. Stabilization mechanism of fixed point

In this section, we demonstrate the structure of stabilization mechanism of a fixed point. The state quantities of the state just after heel-strike are expressed as $\boldsymbol{x}_k^+ = [\alpha_k,\ \dot{\theta}_k^+]^T$. Successive state is related as

$$\boldsymbol{x}_{k+1}^+ = f(\boldsymbol{x}_k^+) \tag{21}$$

The fixed point is expressed as $\boldsymbol{x}_f^+$. The fixed point is related as $\boldsymbol{x}_f^+ = f(\boldsymbol{x}_f^+)$. For a small perturbation $\Delta\boldsymbol{x}_k^+$ around the fixed point, f is expressed in term of Taylor series expansion as

$$\boldsymbol{x}_{k+1}^+ = f(\boldsymbol{x}_f^+) + \left.\frac{\partial f}{\partial \boldsymbol{x}^+}\right|_{\boldsymbol{x}^+=\boldsymbol{x}_f^+} \Delta\boldsymbol{x}_k^+ \tag{22}$$

From Eqs.(21) and (22), linear discrete-time state equation is derived as

$$\Delta\boldsymbol{x}_{k+1}^+ = \left.\frac{\partial f}{\partial \boldsymbol{x}^+}\right|_{\boldsymbol{x}^+=\boldsymbol{x}_f^+} \Delta\boldsymbol{x}_k^+ \equiv \boldsymbol{J}_f \Delta\boldsymbol{x}_k^+ \tag{23}$$

$(\partial\dot{\theta}_{k+1}^+/\partial\alpha_k)|_f$ and $(\partial\dot{\theta}_{k+1}^+/\partial\dot{\theta}_k^+)|_f$ can be derived from Eq. (10) as

$$\left.\frac{\partial\dot{\theta}_{k+1}^+}{\partial\alpha_k}\right|_f = a_f \left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f + b_f \tag{24}$$

$$\left.\frac{\partial\dot{\theta}_{k+1}^+}{\partial\dot{\theta}_k^+}\right|_f = a_f \left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f + c_f \tag{25}$$

See the appendix for the detail of a_f, b_f, and c_f. From Eqs. (24) and (25), Jacobian matrix $\boldsymbol{J}_f$ in Eq. (23) can be obtained as

$$\boldsymbol{J}_f = \begin{bmatrix} \left.\dfrac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f & \left.\dfrac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f \\ a_f\left.\dfrac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f + b_f & a_f\left.\dfrac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f + c_f \end{bmatrix} \tag{26}$$

If all absolute values of eigenvalues of Jacobian matrix are less than one, the fixed point is local-asymptotically stable.

Eigenvalues of Jacobian matrix $\boldsymbol{J}_f$ are derived as follows:

$$R_f = A_f \pm \sqrt{D_f} \tag{27}$$

where

$$A_f = \frac{1}{2}\left(\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f + a_f\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f + c_f\right) \tag{28}$$

$$D_f = \frac{1}{4}\left(\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f + a_f\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f + c_f\right)^2 + b_f\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f - c_f\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f \tag{29}$$

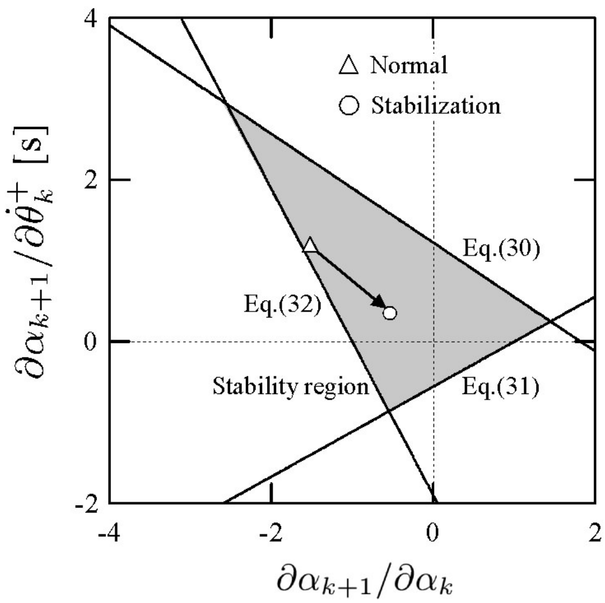


Fig. 2. Relationship between stability region and fixed point

Equations of stability condition of the fixed point are derived from Eq. (27) as follows:

$$-b_f\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f + c_f\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f < 1 \tag{30}$$

$$(a_f + b_f)\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f - (c_f - 1)\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f + c_f < 1 \tag{31}$$

$$-(a_f - b_f)\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f - (c_f + 1)\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f - c_f < 1 \tag{32}$$

$(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ denote each rate of change of inter-leg angle at heel-strike α_{k+1} for small perturbations of inter-leg angle α_k and angular velocity of stance leg $\dot{\theta}_k^+$. When $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ meet Eqs. (30), (31), and (32), the fixed point is local-asymptotically stable.

Figure 2 shows the stability region derived from Eqs. (30), (31), and (32) as the shaded area. The fixed point (long period gait) is mentioned in section III-B. Horizontal and vertical axes denote each $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$.

$(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$, which are obtained by numerical analysis, are overlaid as the small triangle in Fig. 2. If the small triangle is included in the stability region, the fixed point is stable. As shown in Fig. 2, the fixed point is stable because the small triangle is included in the stability region.

B. Local stabilization control method

In this section, we derive the stabilization control method based on the stability mechanism of a fixed point of passive walking.

Local stabilization of a fixed point is that $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ are placed in proper stability region. In this paper, we assume that a fixed point of passive walking class and stability region are fixed.

$(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ can be written as

$$\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f \approx \frac{\alpha_{k+1}-\alpha_f}{\alpha_k-\alpha_f},\quad \left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f \approx \frac{\alpha_{k+1}-\alpha_f}{\dot{\theta}_k^+-\dot{\theta}_f^+} \tag{33}$$

By changing α_{k+1} for $\alpha_k-\alpha_f$ and $\dot{\theta}_k^+-\dot{\theta}_f^+$, the placement of $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ can be changed. Inter-leg angle at heel-strike α_{k+1} can be controlled by inputting the hip torque τ_{2k}. We propose the stabilization control method as follows:

$$\tau_{2k} = K_\alpha(\alpha_k-\alpha_f) + K_{\dot{\theta}}(\dot{\theta}_k^+-\dot{\theta}_f^+) + \tau_{2f} \tag{34}$$

where K_α and $K_{\dot{\theta}}$ are control coefficients. τ_{2f} is the torque to generate a fixed point of passive walking class. $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ can separately be controlled by setting each K_α and $K_{\dot{\theta}}$.

In this study, $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ are placed in position that eigenvalues R_f are zero. As seen in Eq. (27), $|R_f|$=0 holds if $A_f = 0$ and $D_f = 0$. $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ are derived from Eqs. (28) and (29) as follows:

$$\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_f = -\frac{b_f c_f}{b_f + a_f c_f} \tag{35}$$

$$\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_f = -\frac{c_f^2}{b_f + a_f c_f} \tag{36}$$

As mentioned above, the design procedure of generation and stabilization of a fixed point is given as follows:

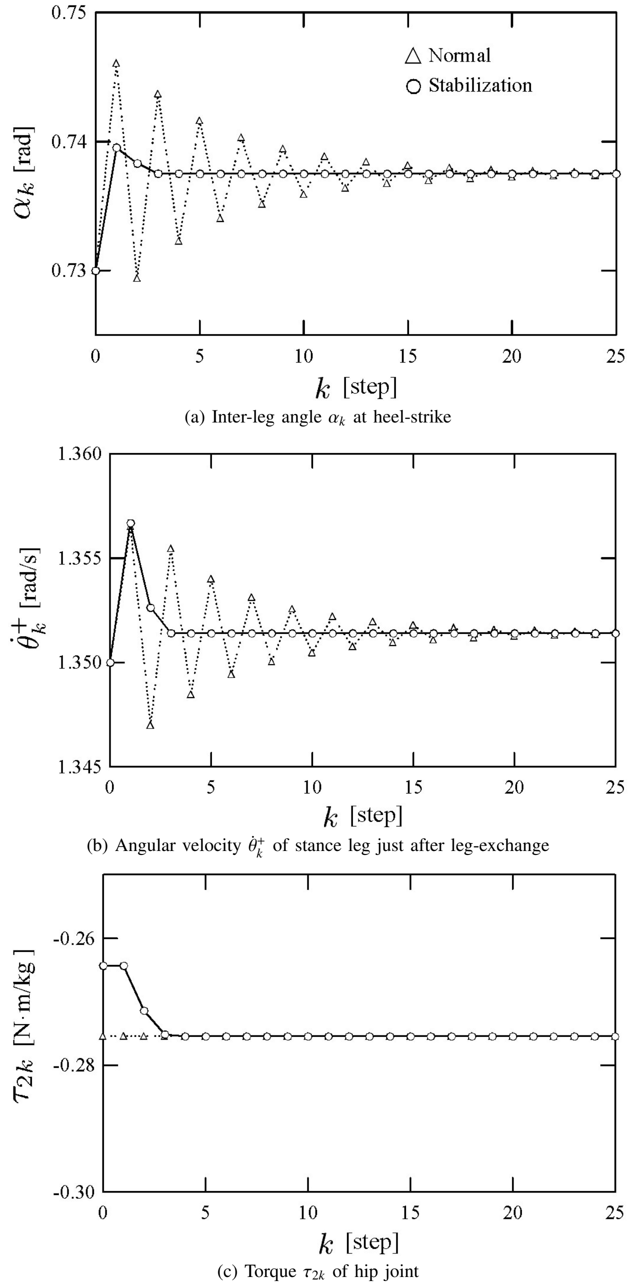
Step 1 γ' correspond to slope angle is set.
Step 2 Torque vectors $\boldsymbol{\tau}_K$, $\boldsymbol{\tau}_C$ are calculated with Eqs. (19) and (20).
Step 3 Fixed points α_f, $\dot{\theta}_f^+$ are calculated with Eq. (12) of energy balance, Eq. (13) of leg-exchange, Eqs. (2)–(5) of leg-swing motion.
Step 4 a_f, b_f, and c_f are calculated with Eqs. (37), (38), and (39), respectively.
Step 5 $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ are calculated with Eqs. (35) and (36).
Step 6 K_α, $K_{\dot{\theta}}$ of the stabilization control method (34) are set to coincide with the numerical value obtained by **Step 5**.

Our proposed control method (34) has the similar structure of the control method based on OGY method [13]. However, our proposed stabilization control method is inevitably led by the stabilization mechanism of a fixed point of passive walking, and it can achieve the highest local stability of discrete-time system.

V. Simulation

In this section, the validity of our proposed method is demonstrated by the simulation.

In case of the fixed point of passive walking class of long period gait mentioned in section III-B, $\max|R_f|$ is obtained as 0.83. The condition, which $|R_f| = 0$ holds, can be derived from Eqs. (35) and (36) as $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$=–0.54861 and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$= 0.36845 [s] (small circle in Fig .2). In this case, K_α and $K_{\dot{\theta}}$ are derived as K_α=–1.7582 and $K_{\dot{\theta}}$=1.4508 by numerical calculation.



(a) Inter-leg angle α_k at heel-strike

(b) Angular velocity $\dot{\theta}_k^+$ of stance leg just after leg-exchange

(c) Torque τ_{2k} of hip joint

Fig. 3. Simulation results of finite time settling (level ground)

Figure 3 shows the simulation results. Initial conditions are set to α_0=0.73[rad] and $\dot{\theta}_0^+$=1.35[rad/s]. Figure 3 (a) shows the variation of inter-leg angle at heel-strike α_k. Figure 3 (b) shows the variation of angular velocity of stance leg just after heel-strike $\dot{\theta}_k^+$. Figure 3 (c) shows the variation of hip torque τ_{2k}. In these figures, the small triangle denotes the normal walking. The small circle denotes the walking stabilized by stabilization control method (34).

As shown in Figs. 3 (a) and (b), in case of normal walking, convergence steps are many steps. While, in case of stabilized walking, convergence steps are only 3 steps. As shown in Fig. 3 (c), hip torque τ_{2k} is inputted by the stabilization control. Finally, τ_{2k} becomes hip torque of the fixed point τ_f.

As mentioned above, the validity of our proposed methods of generation and stabilization of a fixed point was demonstrated. In this paper, a fixed point of long period gait could be stabilized by our stabilization method. In addition, an unstable fixed point of short period gait, an unstable fixed point after bifurcation can be stabilized by our stabilization method. Our stabilization method functions effectively around the fixed point. We must derive a global stabilization method of a fixed point [17].

VI. Conclusion

In this paper, we derived a generation method of a fixed point from its physical structure, which is formed by energy balance, leg-exchange, and leg-swing motion. On level ground and uphill, a fixed point of passive walking class can be generated only by inputting the constant torques.

In local stabilization control of a fixed point, it is desired to maximize the stability. The stability of a fixed point is dependent on the leg-swing motion (inter-leg angle at heel-strike). Local stabilization of a fixed point is that $(\partial\alpha_{k+1}/\partial\alpha_k)|_f$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_f$ are placed in proper stability region. We derived a simple stabilization method, which can easily control the placement of these partial differentiations.

To achieve the highest local stability of discrete-time system, the pole must be placed in the original point. Corresponding placement of partial differentiations can be derived. Our stabilization control method can realize the highest local stability of discrete-time system. The validity of our proposed methods was demonstrated by the walking simulation.

We assumed that next state just after heel–strike exists. However, this assumption may not always hold in experiment. For example, passive walker falls down by stubbing its toe. We must solve this problem.

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Appendix

a_f, b_f, and c_f are given as follows:

$$a_f = \frac{1}{d_f}\frac{\partial e_f}{\partial \alpha_f}\left(\dot{\theta}_f^{+2} + \frac{4g}{l}\sin\frac{\alpha_f}{2}\sin\gamma + \frac{2\tau_{1f}\alpha_f}{l^2}\right) + e_f\frac{g}{2ld_f}\left(\sin\frac{\alpha_f}{2}\cos\gamma + \cos\frac{\alpha_f}{2}\sin\gamma\right) + e_f\frac{\tau_{1f}}{2l^2 d_f} \tag{37}$$

$$b_f = e_f\frac{g}{2ld_f}\left(-\sin\frac{\alpha_f}{2}\cos\gamma + \cos\frac{\alpha_f}{2}\sin\gamma\right) + e_f\frac{\tau_{1f}}{2l^2 d_f} \tag{38}$$

$$c_f = e_f\frac{\dot{\theta}_f^+}{d_f} \tag{39}$$

where

$$d_f = \sqrt{\dot{\theta}_f^{+2} + \frac{4g}{l}\sin\frac{\alpha_f}{2}\sin\gamma + \frac{2\tau_{1f}\alpha_f}{l^2}}$$