2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

Uncalibrated Dynamic Visual Tracking of Manipulators*

Hesheng Wang and Yun-Hui Liu

Dept. of Mechanical and Automation Engineering
The Chinese University of Hong Kong
Shatin, NT, Hong Kong
{hswang, yhliu}@mae.cuhk.edu.hk

Abstract - **This paper presents a new controller for controlling a number of feature points on a robot manipulator to trace desired trajectories specified on the image plane of a fixed camera. The controller is designed to cope with the case when the intrinsic and extrinsic parameters of the camera as well as the robot parameters are not calibrated. The controller employs the depth-independent image Jacobian to map the errors on the image plane onto the joint space. By using the depth-independent image Jacobian, it is possible to linearly parameterize the unknown camera parameters in the closed loop dynamics of the system. A new algorithm is developed to estimate unknown parameters on-line. We have proved asymptotic convergence of the image errors by Lyapunov method with a full consideration of dynamic responses of the robot manipulator and demonstrated the performance by experiments.**

Index Terms **-** *Visual Servoing , Tracking Control, Adaptive Control, Uncalibrated Parameters*

I. INTRODUCTION

IMAGE-BASED visual servoing is a problem of realizing motion control of robots by controlling a set of feature points using the visual feedback from a camera, which can be either mounted on the end-effector [1][2] or fixed at a position near the robot [3]. There are two kinds of image-based visual servoing problems, namely regulation and tracking control of the feature points. In the regulation problem, the controller moves the feature points to desired positions on the image plane, while in the tracking problem the feature points are to trace time-varying trajectories.

Tremendous efforts have been made to the regulation problem of visual servoing since late 1980's [4][5]. However, works on the visual tracking problem are limited. Papanikolopoulos et al. [6] developed a visual tracking algorithm based on on-line estimation of the depth of the target. Hsu et al. [3] proposed an adaptive visual tracking controller for a planar robot manipulator. Zergeroglu et al. [7] designed a tracking controller for planar robots with uncertain parameters. Xiao et al. [8] developed a hybrid controller for tracing trajectory on a plane in an uncalibrated workspace with a fixed camera. We developed a dynamic tracking controller in [9][10][11] by assuming camera parameters are unknown. In this paper, we will extend our work to cope with the case when the robot parameters are not calibrated.

In this paper, we address the problem of controlling a number of feature points on a robot manipulator to trace desired trajectories specified on the image plane of a fixed camera when the intrinsic and extrinsic parameters of the camera as well as the robot physical parameters are unknown. A new adaptive controller, taking into account the nonlinear robot dynamics, will be presented. The difference between the real and estimated projection is included in the adaptive law to guarantee that the estimated camera parameters will converge to the real one. To cope with nonlinear dependence of the image Jacobian on the unknown camera parameters, this controller employs a matrix called depth-independent image Jacobian which does not depend on the scale factors determined by the depths of feature points. A new Lyapunov function is employed to prove asymptotic convergence of the trajectory errors on the image plane. We have implemented the controller on a 3 degrees of freedom manipulator and demonstrated its good performance by experiments.

* Y.-H. Liu is also with Joint Center for Intelligent Sensing and Systems, National University of Defense Technology, Hunan, China. This work is partially supported by Hong Kong RGC under the grants 416703 and 414406, and the NSFC under the projects 60334010 and 60475029

II. KINEMATICS AND DYNAMICS

This section reviews the kinematics and dynamics of the visual servoing system using an uncalibrated fixed camera.

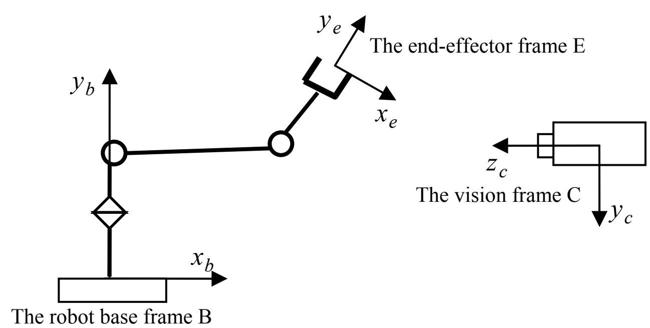


Fig. 1 A fixed camera setup for visual tracking.

As shown in Fig 1, there are *k* feature points marked on the robot manipulator, which are being traced by the vision system. Assume that the camera intrinsic and extrinsic parameters as well as the robot physical parameters are not calibrated. The problem addressed is defined as follows:

Problem 1: *Given desired time-varying trajectories of the feature points on the image plane of the fixed camera, design proper joint inputs for the robot manipulator under the aforementioned assumptions such that the feature points asymptotically trace the desired trajectories.*

Denote the joint angle of the manipulator by a $n \times 1$ vector $\mathbf{q}(t)$, where *n* is the number of degrees of freedom. Denote the *perspective projection matrix* of the camera by $\mathbf{M} \in \Re^{3\times 4}$. Denote the image coordinates of feature point *i* on the image plane by $\mathbf{y}_i(t)=(u_i, v_i)^T$ and its homogenous coordinates with respect to the robot base frame by a 4×1 vector $\mathbf{x}_i(t)$. From the robot kinematics,

$$\dot{\mathbf{x}}_i(t)=\mathbf{J}_i(\mathbf{q}(t))\dot{\mathbf{q}}(t) \quad (1)$$

where $\mathbf{J}_i(\mathbf{q}(t))$ is the Jacobian matrix of the robot manipulator associated with feature point i. Under the perspective projection model, the coordinates are related by

$$\mathbf{y}_i(t)=\frac{1}{{}^c z_i(t)}\mathbf{P}\mathbf{x}_i(t) \quad (2)$$

where $\mathbf{P}\in\Re^{2\times4}$ is the matrix consisting of the first two rows of the perspective projection matrix **M**; ${}^c z_i(t)$ denotes the depth of the feature point i with respect to the camera frame:

$${}^c z_i(\mathbf{q}(t))=\mathbf{m}_3^T\mathbf{x}_i(t) \quad (3)$$

where $\mathbf{m}_i^T$ denote the i-th row vector of the perspective projection matrix **M**. Differentiating eq. (2) gives rise to the velocity relationship:

$$\begin{aligned}\dot{\mathbf{y}}_i(t)&=\frac{1}{{}^c z_i(t)}(\mathbf{P}\dot{\mathbf{x}}_i(t)-\mathbf{y}_i(t){}^c\dot{z}_i(\mathbf{q}(t)))\\ &=\frac{1}{{}^c z_i(t)}\mathbf{A}_i(t)\mathbf{J}_i(\mathbf{q}(t))\dot{\mathbf{q}}(t)\end{aligned} \quad (4)$$

where $\mathbf{A}_i(t)$ represents the following 2×4 matrix:

$$\mathbf{A}_i(t)=\mathbf{P}-\mathbf{y}_i(t)\mathbf{m}_3^T=\begin{pmatrix}\mathbf{m}_1^T-u_i(t)\mathbf{m}_3^T\\ \mathbf{m}_2^T-v_i(t)\mathbf{m}_3^T\end{pmatrix} \quad (5)$$

The matrix $\mathbf{A}_i(t)$ is called *depth-independent interaction matrix* in [5], and $\mathbf{A}_i(t)\mathbf{J}_i(\mathbf{q}(t))$ is called *depth-independent image Jacobian matrix*. The components of the depth-independent interaction matrix are linear to the components of the perspective projection matrix. Define by $\mathbf{Q}(t)$ the combined depth-independent image Jacobian matrix:

$$\mathbf{Q}(t)=\begin{pmatrix}\mathbf{A}_1(\mathbf{y}_1(t))\mathbf{J}_1(\mathbf{q}(t))\\ \mathbf{A}_2(\mathbf{y}_2(t))\mathbf{J}_2(\mathbf{q}(t))\\ \dots\\ \mathbf{A}_k(\mathbf{y}_k(t))\mathbf{J}_k(\mathbf{q}(t))\end{pmatrix} \quad (6)$$

The dimension of the matrix $\mathbf{Q}(t)$ is $2k\times n$, From (4) and (6),

$$\begin{pmatrix}{}^c z_1(t)\dot{\mathbf{y}}_1(t)\\ {}^c z_2(t)\dot{\mathbf{y}}_2(t)\\ \dots\\ {}^c z_k(t)\dot{\mathbf{y}}_k(t)\end{pmatrix}=\mathbf{Q}(t)\dot{\mathbf{q}}(t) \quad (7)$$

It is well known in computer vision that the perspective projection matrix **M** has a rank of 3. Our early work [5] demonstrated that the depth-independent interaction matrix $\mathbf{A}_i(t)$ has a rank of 2.

Property 1: *Given the world coordinates of a sufficient number of feature points and their projections on the image plane, the perspective projection matrix* **M** *can be determined only up to a scale.*

From Property 1, we can fix one component of the matrix **M**. List the remaining 11 components in a vector $\boldsymbol{\theta}$, which is called the parameter vector:

$$\boldsymbol{\theta}^T=(m_{11}, m_{12}, m_{13}, m_{14}, m_{21}, m_{22}, m_{23}, m_{24}, m_{31}, m_{32}, m_{33})$$

Property 2: *For any* 4×1 *vector* $\boldsymbol{\rho}$*, the product* $\mathbf{A}_i(t)\boldsymbol{\rho}$ *can be written in the following linear form:*

$$\mathbf{A}_i(t)\boldsymbol{\rho}=\mathbf{D}_i(\boldsymbol{\rho},\mathbf{y}_i(t))\boldsymbol{\theta} \quad (8)$$

where $\mathbf{D}_i(\boldsymbol{\rho},\mathbf{y}_i(t))$ *is a regression matrix without depending on the intrinsic and extrinsic parameters of the camera.*

It is well-known that the dynamic equation of the robot manipulator has the form:

$$\mathbf{H}(\mathbf{q}(t))\ddot{\mathbf{q}}(t)+\left[\frac{1}{2}\dot{\mathbf{H}}(\mathbf{q}(t))+\mathbf{C}(\mathbf{q}(t),\dot{\mathbf{q}}(t))\right]\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q}(t))=\boldsymbol{\tau} \quad (9)$$

where $\mathbf{H}(\mathbf{q}(t))$ is the $n\times n$ positive-definite and symmetric inertia matrix. $\mathbf{C}(\mathbf{q}(t),\dot{\mathbf{q}}(t))$ is a skew-symmetric matrix. The term $\mathbf{g}(\mathbf{q}(t))$ represents the gravitational force, and $\boldsymbol{\tau}$ is the $n\times1$ joint input of the robot manipulator.

The robot dynamical equation enjoys one property that will be of great use to us in parameter estimation:

Property 3: *The dynamics of the robots are linear in the physical parameters* φ *as*

$$\begin{aligned}&\mathbf{H}(\mathbf{q}(t))\ddot{\mathbf{q}}(t)+(\frac{1}{2}\dot{\mathbf{H}}(\mathbf{q}(t))+\mathbf{C}(\mathbf{q}(t),\dot{\mathbf{q}}(t)))\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q}(t))\\ &=\mathbf{F}(\mathbf{q}(t),\dot{\mathbf{q}}(t),\dot{\mathbf{q}}(t),\ddot{\mathbf{q}}(t))\varphi\end{aligned} \quad (10)$$

where $\mathbf{F}(\mathbf{q}(t),\dot{\mathbf{q}}(t),\dot{\mathbf{q}}(t),\ddot{\mathbf{q}}(t))$ *is the regressor matrix.*

III. Adaptive Image-based Visual Tracking

This section proposes a new controller that forces the feature points to trace their desired trajectories while estimating the unknown parameters on-line.

A. Definition of Nominal References

Denote the desired trajectory of the i-th feature point on the image plane by $(\mathbf{y}_{id}(t),\dot{\mathbf{y}}_{id}(t),\ddot{\mathbf{y}}_{id}(t))$, where $\mathbf{y}_{id}(t)$, $\dot{\mathbf{y}}_{id}(t)$ and $\ddot{\mathbf{y}}_{id}(t)$ represent the time-varying desired position, velocity and acceleration, respectively. For convenience, introduce the following nominal reference:

$$\dot{\mathbf{y}}_r(t)=\begin{pmatrix}\dot{\mathbf{y}}_{1r}(t)\\ \dot{\mathbf{y}}_{2r}(t)\\ \dots\\ \dot{\mathbf{y}}_{kr}(t)\end{pmatrix}=\begin{pmatrix}\dot{\mathbf{y}}_{1d}(t)-\lambda_1\Delta\mathbf{y}_1(t)\\ \dot{\mathbf{y}}_{2d}(t)-\lambda_2\Delta\mathbf{y}_2(t)\\ \dots\\ \dot{\mathbf{y}}_{kd}(t)-\lambda_k\Delta\mathbf{y}_k(t)\end{pmatrix} \quad (11)$$

where $\Delta\mathbf{y}_i(t)$ is the position error of the feature point on the image plane:

$$\Delta\mathbf{y}_i(t)=\mathbf{y}_i(t)-\mathbf{y}_{id}(t) \quad (12)$$

The λ_i is a positive constant. The error of the feature point to the defined nominal reference is given by

$$\mathbf{s}_{iy}(t)=\dot{\mathbf{y}}_i(t)-\dot{\mathbf{y}}_{ir}(t)=\Delta\dot{\mathbf{y}}_i(t)+\lambda_i\Delta\mathbf{y}_i(t) \quad (13)$$

Note that the image errors $\Delta\mathbf{y}_i(t)$ and $\Delta\dot{\mathbf{y}}_i(t)$ are convergent to zero if the error vector $\mathbf{s}_{iy}(t)$ is convergent to zero. For convenience, we define $\mathbf{h}_y(t)$ as follows:

$$\mathbf{h}_y(t) = \begin{pmatrix} {}^c z_1(t)\mathbf{s}_{1y}(t) \\ {}^c z_2(t)\mathbf{s}_{2y}(t) \\ ..... \\ {}^c z_k(t)\mathbf{s}_{ky}(t) \end{pmatrix} \quad (14)$$

Denote the time-varying estimation of the unknown parameters $\boldsymbol{\theta}$ by $\hat{\boldsymbol{\theta}}(t)$. Using the estimated parameters, we introduce the following nominal reference to map the image errors onto the joint space of the robot manipulator:

$$\dot{\mathbf{q}}_r(t) = \hat{\mathbf{Q}}^+(t)\begin{pmatrix} {}^c \hat{z}_1(t)\dot{\mathbf{y}}_{1r}(t) \\ {}^c \hat{z}_2(t)\dot{\mathbf{y}}_{2r}(t) \\ ... \\ {}^c \hat{z}_k(t)\dot{\mathbf{y}}_{kr}(t) \end{pmatrix} \quad (15)$$

where $\hat{\mathbf{Q}}^+(t)$ is the pseudo-inverse of the estimated matrix $\hat{\mathbf{Q}}(t)$. Assume that $2k > n$, and hence

$$\hat{\mathbf{Q}}^+(t) = (\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t))^{-1}\hat{\mathbf{Q}}^T(t) \quad (16)$$

From eq.(16), the estimated matrix $\hat{\mathbf{Q}}(t)$ must be full rank in order to calculate the pseudo-inverse. Then the error vector in the joint space is given by

$$\mathbf{s}_q(t) = \dot{\mathbf{q}}(t) - \dot{\mathbf{q}}_r(t) \quad (17)$$

B. *Estimation of the Unknown Parameters*

The proposed controller employs an on-line algorithm to estimate the unknown parameters.

As for the unknown camera parameters, which is designed based on the following three points. First, the nonlinear regression term in the closed loop dynamics is canceled. Second, the rank 3 condition of the estimated projection matrix is satisfied so that the pseudo-inverse $\hat{\mathbf{Q}}^+(t)$ in (16) is computable. Finally, we want to minimize on-line the error between the real image coordinates of the feature point and those calculated using the estimated projection matrix $\hat{\mathbf{M}}(t)$. First, to derive the regression term, we consider

$$\begin{aligned} &\mathbf{s}_q^T(t)\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) = \dot{\mathbf{q}}^T(t)\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\ &- \begin{pmatrix} {}^c \hat{z}_1(t)\dot{\mathbf{y}}_{1r}(t) \\ {}^c \hat{z}_2(t)\dot{\mathbf{y}}_{2r}(t) \\ .... \\ {}^c \hat{z}_k(t)\dot{\mathbf{y}}_{kr}(t) \end{pmatrix}^T \hat{\mathbf{Q}}(t)(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t))^{-1}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\ &= \dot{\mathbf{q}}^T(t)\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t)(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t))^{-1}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\ &- \begin{pmatrix} {}^c \hat{z}_1(t)\dot{\mathbf{y}}_{1r}(t) \\ {}^c \hat{z}_2(t)\dot{\mathbf{y}}_{2r}(t) \\ .... \\ {}^c \hat{z}_k(t)\dot{\mathbf{y}}_{kr}(t) \end{pmatrix}^T \hat{\mathbf{Q}}(t)(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t))^{-1}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \end{aligned} \quad (18)$$

where $\mathbf{K}_3$ is a positive definite matrix. As to be shown in the next subsection, this product is closely related to the regression term in the closed-loop dynamics. Note that

$$\begin{aligned} &\dot{\mathbf{q}}^T(t)\hat{\mathbf{Q}}^T(t) = \dot{\mathbf{q}}^T(t)\mathbf{Q}^T(t) + \dot{\mathbf{q}}^T(t)(\hat{\mathbf{Q}}^T(t) - \mathbf{Q}^T(t)) \\ &= \begin{pmatrix} {}^c z_1(t)\dot{\mathbf{y}}_1(t) \\ {}^c z_2(t)\dot{\mathbf{y}}_2(t) \\ ... \\ {}^c z_k(t)\dot{\mathbf{y}}_k(t) \end{pmatrix}^T + \dot{\mathbf{q}}^T(t)(\hat{\mathbf{Q}}^T(t) - \mathbf{Q}^T(t)) \end{aligned} \quad (19)$$

By submitting eq. (19) into eq. (18), we have

$$\begin{aligned} &\mathbf{s}_q^T(t)\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) = \\ &\mathbf{h}_y^T(t)\hat{\mathbf{Q}}(t)\left(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t)\right)^{-1}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\ &- \left\{ \dot{\mathbf{q}}^T(t)(\mathbf{Q}(t) - \hat{\mathbf{Q}}(t)) + \begin{pmatrix} ({}^c\hat{z}_1(t) - {}^c z_1(t))\dot{\mathbf{y}}_{1r}(t) \\ ({}^c\hat{z}_2(t) - {}^c z_2(t))\dot{\mathbf{y}}_{2r}(t) \\ .... \\ ({}^c\hat{z}_k(t) - {}^c z_k(t))\dot{\mathbf{y}}_{kr}(t) \end{pmatrix} \right\} \\ &\hat{\mathbf{Q}}(t)(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t))^{-1}\hat{\mathbf{Q}}(t)\mathbf{K}_3\mathbf{h}_y(t) \end{aligned} \quad (20)$$

From Property 2, the last term in eq. (20) can be represented as a linear form of the estimation errors of the parameters, and hence eq. (18) can be re-written as

$$\begin{aligned} &\mathbf{s}_q^T(t)\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) = \mathbf{h}_y^T(t)\hat{\mathbf{Q}}(t)\left(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t)\right)^{-1}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\ &-\Delta\hat{\boldsymbol{\theta}}^T(t)\mathbf{Y}(\dot{\mathbf{q}}(t), \mathbf{y}(t), \dot{\mathbf{y}}(t), \hat{\boldsymbol{\theta}}(t))\mathbf{h}_y(t) \end{aligned} \quad (21)$$

Consider another term to derive regression term:

$$-\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\begin{pmatrix} ({}^c\hat{z}_1(t) - {}^c z_1(t))\mathbf{s}_{1y}(t) \\ ({}^c\hat{z}_2(t) - {}^c z_2(t))\mathbf{s}_{2y}(t) \\ ..... \\ ({}^c\hat{z}_k(t) - {}^c z_k(t))\mathbf{s}_{ky}(t) \end{pmatrix} = \mathbf{L}(\mathbf{x}_i(t), \mathbf{y}_i(t), \hat{\boldsymbol{\theta}}(t))\Delta\boldsymbol{\theta}(t) \quad (22)$$

Next, we consider how to guarantee the existence of the pseudo-inverse of $\hat{\mathbf{Q}}(t)$. It is important to note:

Proposition 1: *The estimated depth-independent interaction matrix* $\hat{\mathbf{A}}_i(t)$ *is of rank 2 if* $\hat{\mathbf{M}}(t)$ *has a rank of 3.*

Proposition 2: *Assume that the manipulator is not at the singular configuration and the end-effector is rigid. If the estimated projection matrix* $\hat{\mathbf{M}}(t)$ *has a rank 3, the pseudo-inverse of the matrix* $\hat{\mathbf{Q}}(t)$ *can be always calculated.*

The proofs of the two propositions are given in [11]. To guarantee the matrix $\hat{\mathbf{M}}(t)$ has a rank of 3, we introduced the following the potential force ([11]):

$$\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial \hat{\boldsymbol{\theta}}(t)} = \begin{cases} -\dfrac{2\eta p(t)e^{\eta p^2(t)}}{(e^{\eta p^2(t)} - 1 + \gamma)^2}\dfrac{\partial p(t)}{\partial \hat{\boldsymbol{\theta}}(t)} & |p(t)| < \varepsilon - \phi \\ -\dfrac{a}{\phi}(\varepsilon - |p(t)|)p(t)\dfrac{\partial p(t)}{\partial \hat{\boldsymbol{\theta}}(t)} & \varepsilon - \phi \le |p(t)| \le \varepsilon \\ 0 & |p(t)| > \varepsilon \end{cases} \quad (26)$$

where

$$a = \frac{2\eta e^{\eta(\varepsilon - \phi)^2}}{(e^{\eta(\varepsilon - \phi)^2} - 1 + \gamma)^2} \quad (27)$$

Finally, we consider the error between the real image coordinates of the feature point and those calculated using the estimated projection matrix:

$$\mathbf{e}_i(t) = {}^c\hat{z}_i(t)\mathbf{y}_i(t) - \hat{\mathbf{P}}(t)\mathbf{x}_i(t) \tag{28}$$

$\mathbf{e}_i(t)$ is called *estimated projection error* of the feature point, considering eq.(2), we can re-write the equation as follows:

$$\begin{aligned}\mathbf{e}_i(t) &= {}^c\hat{z}_i(t)\mathbf{y}_i(t) - {}^c z_i(t)\mathbf{y}_i(t) - \hat{\mathbf{P}}(t)\mathbf{x}_i(t) + \mathbf{P}(t)\mathbf{x}_i(t) \\ &= [{}^c\hat{z}_i(t) - {}^c z_i(t)]\mathbf{y}_i(t) - [\hat{\mathbf{P}}(t) - \mathbf{P}]\mathbf{x}_i(t)\end{aligned} \tag{29}$$

Noting the relation (3), we have

$$\mathbf{e}_i(t) = (\hat{\mathbf{m}}_3^T(t) - \mathbf{m}_3^T)\mathbf{x}_i(t)\mathbf{y}_i(t) - (\hat{\mathbf{P}}(t) - \mathbf{P})\mathbf{x}_i(t) \tag{30}$$

From Property 2, eq. (30) can be represented in a linear form of the parameter estimation errors:

$$\mathbf{e}_i(t) = \mathbf{W}(\mathbf{x}_i(t), \mathbf{y}_i(t))\Delta\boldsymbol{\theta}(t) \tag{31}$$

Note that the error $\mathbf{e}_i(t)$ changes with time. The matrix $\mathbf{W}(\mathbf{x}_i(t), \mathbf{y}_i(t))$ does not depend on the parameters.

Based on the above discussions, the following adaptive rule is proposed for updating the estimation of the parameters:

$$\begin{aligned}\frac{d}{dt}\hat{\boldsymbol{\theta}}(t) = -\boldsymbol{\Gamma}^{-1}\{&\mathbf{Y}(\dot{\mathbf{q}}(t), \mathbf{y}_i(t), \dot{\mathbf{y}}_r(t), \hat{\boldsymbol{\theta}}(t))\mathbf{h}_y(t) \\ &+ \mathbf{L}^T(\mathbf{x}_i(t), \mathbf{y}_i(t), \hat{\boldsymbol{\theta}}(t))\mathbf{s}_q(t) \\ &+ \sum_{j=1}^{k}\mathbf{W}^T(\mathbf{x}_j(t), \mathbf{y}_j(t))\mathbf{B}_1\mathbf{e}_j(t) + \mathbf{B}_2\left\|\mathbf{s}_q(t)\right\|^2\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}\}\end{aligned} \tag{32}$$

where $\boldsymbol{\Gamma}$, $\mathbf{B}_1$ and $\mathbf{B}_2$ are positive-definite and diagonal gain matrices, and their dimensions are, respectively, 11×11, 3×3, and 11×11. Note that the first two terms are regressive terms and the third term represents on-line minimization of the estimated projection error $\mathbf{e}(t)$ in the gradient descending direction. The fourth term generates a repulsive force from values of the parameters that result in singular estimated projection matrix.

As for the robot dynamics parameters, we design the following adaptive rule:

$$\frac{d}{dt}\hat{\varphi}(t) = -\frac{1}{\mathbf{K}_4}\mathbf{F}^T(\mathbf{q}(t), \dot{\mathbf{q}}(t), \dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t))\mathbf{s}_q(t) \tag{33}$$

C. Controller Design

Based on the nominal references and the adaptive rule in the previous section, we propose the following controller:

$$\boldsymbol{\tau} = \mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \dot{\mathbf{q}}_r(t), \ddot{\mathbf{q}}_r(t))\hat{\varphi}(t) - (\mathbf{K}_1 + \mathbf{K}_2\left\|\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}\right\|)\mathbf{s}_q(t) - \hat{\mathbf{Q}}^T(t)\mathbf{K}_3\begin{pmatrix}{}^c\hat{z}_1(t)\mathbf{s}_{1y}(t) \\ {}^c\hat{z}_2(t)\mathbf{s}_{2y}(t) \\ \ldots\ldots \\ {}^c\hat{z}_k(t)\mathbf{s}_{ky}(t)\end{pmatrix} \tag{34}$$

where $\mathbf{K}_i\,(i=1,2,3)$ are positive-definite gain matrices. The first terms use the estimated robot physical parameters to cancel the inertia forces, the nonlinear centrifugal and Coriolis forces, and the gravitational force. The second term represents a feedback in the joint space. The last term is the image error feedback.

It is important to note that by using the depth-independent interaction matrix in [5], the depth factor $1/{}^c z(\mathbf{q})$ does not appear in control law. From Property 3, the robot dynamics can be revised as

$$\begin{aligned}&\mathbf{H}(\mathbf{q}(t))\dot{\mathbf{s}}_q(t) + [\frac{1}{2}\dot{\mathbf{H}}(\mathbf{q}(t)) + \mathbf{C}(\mathbf{q}(t), \dot{\mathbf{q}}(t))]\mathbf{s}_q(t) \\ &+ \mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \dot{\mathbf{q}}_r(t), \ddot{\mathbf{q}}_r(t))\varphi(t) = \boldsymbol{\tau}\end{aligned} \tag{35}$$

By substituting the control law into the robot dynamics, we obtain the following closed loop dynamics:

$$\begin{aligned}&\mathbf{H}(\mathbf{q}(t))\dot{\mathbf{s}}_q(t) + [\frac{1}{2}\dot{\mathbf{H}}(\mathbf{q}(t)) + \mathbf{C}(\mathbf{q}(t), \dot{\mathbf{q}}(t))]\mathbf{s}_q(t) = \\ &\mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \dot{\mathbf{q}}_r(t), \ddot{\mathbf{q}}_r(t))\Delta\varphi(t) - (\mathbf{K}_1 + \mathbf{K}_2\left\|\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}\right\|)\mathbf{s}_q(t) \\ &- \hat{\mathbf{Q}}^T(t)\mathbf{K}_3\left[\begin{pmatrix}{}^c z_1(t)\mathbf{s}_{1y}(t) \\ {}^c z_2(t)\mathbf{s}_{2y}(t) \\ \ldots\ldots \\ {}^c z_k(t)\mathbf{s}_{ky}(t)\end{pmatrix} + \begin{pmatrix}({}^c\hat{z}_1(t) - {}^c z_1(t))\mathbf{s}_{1y}(t) \\ ({}^c\hat{z}_2(t) - {}^c z_2(t))\mathbf{s}_{2y}(t) \\ \ldots\ldots \\ ({}^c\hat{z}_k(t) - {}^c z_k(t))\mathbf{s}_{ky}(t)\end{pmatrix}\right] \\ &= -(\mathbf{K}_1 + \mathbf{K}_2\left\|\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}\right\|)\mathbf{s}_q(t) - \hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\ &+ \mathbf{L}(\mathbf{x}_i(t), \mathbf{y}_i(t), \hat{\boldsymbol{\theta}}(t))\Delta\boldsymbol{\theta}(t) + \mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \dot{\mathbf{q}}_r(t), \ddot{\mathbf{q}}_r(t))\Delta\varphi(t)\end{aligned} \tag{36}$$

where $\Delta\varphi(t) = \hat{\varphi}(t) - \varphi(t)$ is the robot physical parameters estimation errors.

D. Stability Analysis

We here analyze the stability of the robot manipulator under the control of the proposed controller and adaptive algorithm. For simplicity, we assume that the feature points are visible during the motion so that their depths with respect to the camera frame are always positive. Following is the main result of this paper:

Theorem 1: *Under the control of controller (34) and the adaptive rule (32)(33), the feature points are convergent on the image plane in the following way:*

$$\lim_{t\to\infty}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) = \mathbf{0} \tag{37}$$

Proof: Introduce the following non-negative function:

$$V(t) = \frac{1}{2}\{\mathbf{s}_q^T(t)\mathbf{H}(\mathbf{q}(t))\mathbf{s}_q(t) + \Delta\boldsymbol{\theta}^T(t)\boldsymbol{\Gamma}\Delta\boldsymbol{\theta}(t) + \Delta\varphi^T(t)\mathbf{K}_4\Delta\varphi(t)\} \tag{38}$$

Multiplying the $\dot{\mathbf{q}}^T(t)$ from the left to the closed loop dynamics (36) results in

$$\begin{aligned}&\mathbf{s}_q^T(t)\mathbf{H}(\mathbf{q}(t))\dot{\mathbf{s}}_q(t) + \frac{1}{2}\mathbf{s}_q^T(t)\dot{\mathbf{H}}(\mathbf{q}(t))\mathbf{s}_q(t) = \\ &-\mathbf{s}_q^T(t)(\mathbf{K}_1 + \mathbf{K}_2\left\|\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}\right\|)\mathbf{s}_q(t) \\ &-\mathbf{s}_q^T(t)\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) + \mathbf{s}_q^T(t)\mathbf{L}(\mathbf{x}_i(t), \mathbf{y}_i(t), \hat{\boldsymbol{\theta}}(t))\Delta\boldsymbol{\theta}(t) \\ &+ \mathbf{s}_q^T(t)\mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \dot{\mathbf{q}}_r(t), \ddot{\mathbf{q}}_r(t))\Delta\varphi(t)\end{aligned} \tag{39}$$

From equation (18), we have

$$
\begin{aligned}
&\mathbf{s}_q^T(t)\mathbf{H}(\mathbf{q}(t))\dot{\mathbf{s}}_q(t)+\frac{1}{2}\mathbf{s}_q^T(t)\dot{\mathbf{H}}(\mathbf{q}(t))\mathbf{s}_q(t)= \\
&\quad -\mathbf{s}_q^T(t)(\mathbf{K}_1+\mathbf{K}_2\left\|\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}\right\|)\mathbf{s}_q(t) \\
&\quad +\mathbf{h}_y^T(t)\hat{\mathbf{Q}}(t)\left(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t)\right)^{-1}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\
&\quad -\Delta\hat{\boldsymbol{\theta}}^T(t)\mathbf{Y}(\dot{\mathbf{q}}(t),\mathbf{y}(t),\dot{\mathbf{y}}(t),\hat{\boldsymbol{\theta}}(t))\mathbf{h}_y(t) \\
&\quad +\mathbf{s}_q^T(t)\mathbf{L}(\mathbf{x}_i(t),\mathbf{y}_i(t),\hat{\boldsymbol{\theta}}(t))\Delta\boldsymbol{\theta}(t) \\
&\quad +\mathbf{s}_q^T(t)\mathbf{F}(\mathbf{q}(t),\dot{\mathbf{q}}(t),\dot{\mathbf{q}}_r(t),\ddot{\mathbf{q}}_r(t))\Delta\varphi(t)
\end{aligned} \tag{40}
$$

By multiplying the $\Delta\boldsymbol{\theta}^T(t)$ from the left to the adaptive rule of camera parameters (32), we obtain

$$
\begin{aligned}
&\Delta\boldsymbol{\theta}^T(t)\boldsymbol{\Gamma}\Delta\dot{\boldsymbol{\theta}}(t)=-\Delta\boldsymbol{\theta}^T(t)\mathbf{Y}^T(\dot{\mathbf{q}}(t),\mathbf{y}_i(t),\dot{\mathbf{y}}_r(t),\hat{\boldsymbol{\theta}}(t))\mathbf{h}_y(t) \\
&-\Delta\boldsymbol{\theta}^T(t)\mathbf{L}^T(\mathbf{x}_i(t),\mathbf{y}_i(t),\hat{\boldsymbol{\theta}}(t))\mathbf{s}_q(t) \\
&-\sum_{j=1}^{k}\Delta\boldsymbol{\theta}^T(t)\mathbf{W}^T(\mathbf{x}_j(t),\mathbf{y}_j(t))\mathbf{B}_1\mathbf{W}^T(\mathbf{x}_j(t),\mathbf{y}_j(t))\Delta\boldsymbol{\theta}(t) \\
&-\Delta\boldsymbol{\theta}^T(t)\mathbf{B}_2\left\|\mathbf{s}_q(t)\right\|^2\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}
\end{aligned} \tag{41}
$$

By multiplying the $\Delta\varphi^T(t)$ from the left to the adaptive rule of robot physical parameters (33), we obtain

$$
\Delta\varphi^T(t)\mathbf{K}_4\Delta\dot{\varphi}(t)=-\Delta\varphi^T(t)\mathbf{F}(\mathbf{q}(t),\dot{\mathbf{q}}(t),\dot{\mathbf{q}}(t),\ddot{\mathbf{q}}(t))\mathbf{s}_q(t) \tag{42}
$$

By combining the equations (39)-(42), we have,

$$
\begin{aligned}
&\dot{V}(t)=-\mathbf{h}_y^T(t)\hat{\mathbf{Q}}(t)(\hat{\mathbf{Q}}^T(t)\hat{\mathbf{Q}}(t))^{-1}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t) \\
&-\sum_{i=1}^{k}\Delta\boldsymbol{\theta}^T(t)\mathbf{W}^T(\mathbf{x}_i(t),\mathbf{y}_i(t))\mathbf{B}_1\mathbf{W}(\mathbf{x}_i(t),\mathbf{y}_i(t))\Delta\boldsymbol{\theta}(t) \\
&-\mathbf{s}_q^T(t)\mathbf{K}_1\mathbf{s}_q(t)-\mathbf{s}_q^T(t)(\mathbf{K}_2-\left\|\Delta\boldsymbol{\theta}^T(t)\mathbf{B}_2\right\|)\left\|\frac{\partial U(\hat{\boldsymbol{\theta}}(t))}{\partial\hat{\boldsymbol{\theta}}(t)}\right\|\mathbf{s}_q(t)
\end{aligned} \tag{43}
$$

We properly select a gain $\mathbf{K}_2$ such that

$$
\mathbf{K}_2\geq\sigma\xi\sqrt{\frac{V(0)}{\|\Gamma\|}}\mathbf{I} \tag{43}
$$

where ξ is the maximum eigenvalue of gain matrix $\mathbf{B}_2$ and σ is a positive constant greater than 1. The gain $\mathbf{K}_3$ is so selected that

$$
\mathbf{K}_3=\alpha\mathbf{I} \tag{45}
$$

where $\mathbf{I}$ is the identity matrix. From (43), the $\dot{V}(t)$ is non-positive and the function *V(t)* never increases its value so that it is upper bounded. From the definition (38), bounded *V(t)* directly implies that the joint velocity, the image errors, and the estimation errors are all bounded. Then, we can claim the boundedness of the $\dot{\mathbf{s}}_q(t)$ from the closed-loop dynamics (36) and that of $\dot{\hat{\boldsymbol{\theta}}}(t)$ from the adaptive algorithm (32). Therefore, the joint velocity $\dot{\mathbf{q}}(t)$ and the estimated parameters are uniformly continuous. Therefore, from the Barbalat's Lemma, we can conclude that

$$
\lim_{t\to\infty}\hat{\mathbf{Q}}^T(t)\mathbf{K}_3\mathbf{h}_y(t)=\mathbf{0} \tag{46}
$$

$$
\lim_{t\to\infty}\mathbf{s}_q(t)=\mathbf{0} \tag{47}
$$

$$
\lim_{t\to\infty}\mathbf{W}(\mathbf{x}_i(t),\mathbf{y}_i(t))\Delta\hat{\boldsymbol{\theta}}(t)=\mathbf{0} \tag{48}
$$

From (48), in computer vision, it is well known that seven non-coplanar positions of a feature point can be used to determine the perspective projection matrix up to a scale. Therefore, the estimated camera parameters will be convergent to the true values up to a scale. ■

Generally speaking, it is not possible to conclude the convergence of the feature points to the desired trajectories from eq. (46). The image errors are convergent to zero only when the number of the joints of the manipulator is larger than or equal to the dimension *2k* of the image errors, i.e., $2k\leq n$.

We further consider the condition for the convergence of the image error $\Delta\mathbf{y}_i(t)$ when $2k\geq n$. Note that all the feature points are on the rigid end-effector. Denote the position and orientation of the end-effector with respect to the robot base frame by a homogeneous transformation matrix $\mathbf{T}_e(t)$, which changes with time during motion. The homogenous coordinates of the feature point *i* is given by

$$
\mathbf{x}_i(t)=\mathbf{T}_e(t)\,{}^e\mathbf{x}_i \tag{49}
$$

where ${}^e\mathbf{x}_i$ denotes the position of the feature point with respect to the end-effector frame. Note that ${}^e\mathbf{x}_i$ is a constant vector. Then, the image coordinates can be related as follows:

$$
\mathbf{y}_i(t)=\frac{1}{\mathbf{m}_3^T\mathbf{T}_e(t)\,{}^e\mathbf{x}_i}\mathbf{P}\mathbf{T}_e(t)\,{}^e\mathbf{x}_i \tag{50}
$$

By substituting this equation into eq. (46), all the image coordinates are replaced by $\mathbf{T}_e(t)$. Note that $\mathbf{T}_e(t)$ can be parameterized by 6 unknowns, i.e. three translational and three rotational variables. When the manipulator has more than six degrees of freedom and the manipulator Jacobian matrix is not singular, eq. (46) implies the existence of more than six nonlinear equations. When the nonlinear equations admit only one solution of the position and orientation of the rigid end-effector, only one set of image coordinates $(\mathbf{y}_1(t),\mathbf{y}_2(t),...,\mathbf{y}_k(t))$ satisfy the equation. Since $\Delta\mathbf{y}_i(t)=\mathbf{0}$ for all *i* are obviously a solution of the equation (46), we can conclude the convergence of the image errors of the feature points to zero. If the six nonlinear equations admit more than one solution, the image errors may not be convergent to zero.

IV. EXPERIMENTS

To verify the performance of the proposed controller, we have implemented the controller in a 3 DOF robot manipulator in the Chinese University of Hong Kong. Figure 2 shows the experiment setup system. The robot manipulator designed by the Networked Sensors and Robotics Laboratory has three revolute joints driven by Maxon brushed DC motors. High-precision encoders with a resolution of 2000 pulses/turn are used to measure the joint angles. The joint velocities are obtained by differentiating the joint angles. A Ptgrey camera connecting to an IEEE 1394 card installed in a PC with Intel Pentium IV CPU acquires the video signal with 120fps frame rate. This PC processes the image and extracts the image features. The sampling time used in the experiments is 13 ms.

We mark a feature point on the end-effector of the robot manipulator. The desired trajectory of the feature point is to trace the following circular trajectory:

$$\mathbf{y}_d(t) = \begin{pmatrix} 60\times\sin(0.2t)+265 \\ -60\times\cos(0.2t)+190 \end{pmatrix} \text{pixel}$$

The control gains used are $\Gamma = 0.0002$, $\mathbf{K}_1 = 10$, $\mathbf{K}_2 = 0.0001$, $\mathbf{K}_3 = 0.00075$, $\mathbf{B}_1 = 0.1$ and $\mathbf{B}_2 = 0.0001$. The initial estimation of the camera extrinsic parameters is $\hat{\mathbf{T}} = \begin{bmatrix} 1 & 0 & 0 & 0.6 \\ 0 & 0 & -1 & -0.1 \\ 0 & 1 & 0 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. The real values of the intrinsic parameters are $a_u = 1806$ pixels, $a_v = 1812$ pixels, $u_0 = 282$ pixels and $v_0 = 249$ pixels. The initial estimations are $\hat{a}_u(0) = 2000$ pixels, $\hat{a}_v(0) = 2000$ pixels, $\hat{u}_0(t) = 300$ pixels and $\hat{v}_0(0) = 300$ pixels. The initial estimations of the robot physical parameters are: $\hat{\varphi}(0) = (0.0022 \quad 0 \quad 0.0032 \quad 0 \quad 0.0005 \quad 0.0009 \quad -0.0027$

$0.0005 \quad 0.0035 \quad 0.0005 \quad 0.0012 \quad -0.0001 \quad 0.002$

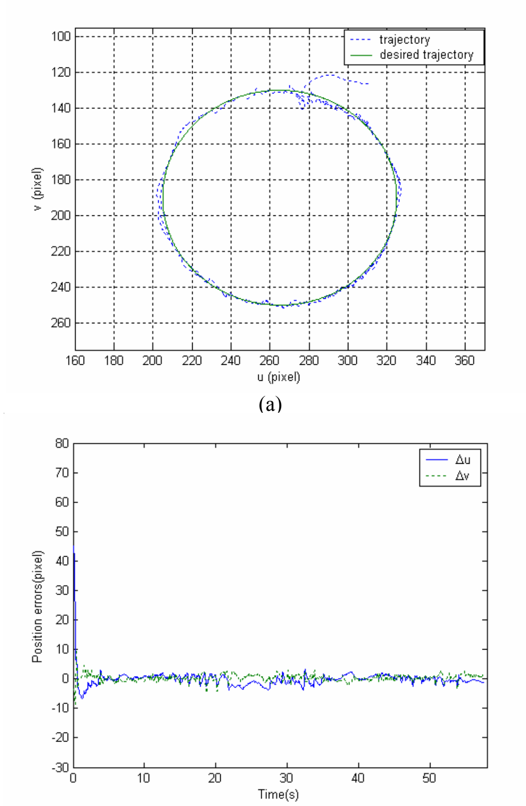
$0.0004 \quad 0.0259 \quad 0.0061)^T$

The trajectory of the feature point and the position errors on the image plane are shown in Figure 3, which confirmed expected asymptotic convergence of the trajectory error to the zero under the control of the proposed method.

V. Conclusions

This paper proposed a new adaptive controller for trajectory tracking of a number of feature points on a robot manipulator using an uncalibrated fixed camera. The controller employs the pseudo inverse of the depth-independent interaction matrix to map the image errors onto the joint space of the manipulator. A potential function is introduced to guarantee the existence of the pseudo inverse of the estimated depth-independent interaction matrix. A novel adaptive algorithm has been developed to estimate the unknown camera and robot physical parameters. It is rigorously proved by the Lyapunov method based on the nonlinear robot dynamics that the trajectories of the feature points are asymptotically convergent to the desired ones. Experimental results validated the proposed method.

Fig. 2 The experiment setup



(a)

(b)

Fig. 3 Experimental results: (a) the desired and real trajectories on the image plane. (b) the position errors on the image plane

References

[1] F. Chaumette, “Potential problems of stability and convergence in image based and position based visual servoing.” *In Conflux of vision and control LNCIS*, vol. 237, pp. 66–78, 1998.

[2] S. Hutchinson, G. D. Hager, and P. I. Corke, “A tutorial on visual servo control,” *IEEE Trans. on Robotics and Automation,* vol. 12, no. 5, pp. 651-670, 1996.

[3] L. Hsu and P. L. S. Aquino, “Adaptive visual tracking with uncertain manipulator dynamics and uncalibrated camera,” *Proc. of the 38th IEEE Int.l Conf. on Decision and Control*, pp. 1248-1253, 1999.

[4] Y. H. Liu, H. Wang and K. Lam, “Dynamic visual servoing of robots in uncalibrated environments”, *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 3142-3148, 2005.

[5] Y. H. Liu, H. Wang, C. Wang, and K. Lam, “Uncalibrated visual servoing of robots using a depth-independent interaction matrix”, *IEEE Tran. on Robotics*, Vol. 2, no. 4, pp. 804-817, 2006

[6] N. P. Papanikolopoulos and P. K. Khosla, “Adaptive robotic visual tracking: theory and experiments,” *IEEE Trans. on Automatic Control,* vol. 38, no. 3, pp. 429-445, 1993.

[7] E. Zergeroglu, D. M. Dawson, M.S. de Queiroz, and A. Behal, “Vision-Based Nonlinear Tracking Controllers with Uncertain Robot-Camera Parameters”, *IEEE/ASME Trans. On Mechatronics,* Vol. 6, No 3, September 2001.

[8] D. Xiao, B. Ghosh, N. Xi, and T. J. Tarn, “Intelligent robotic manipulation with hybrid position/force control in an uncalibrated workspace,” *Proc. of 1998 IEEE Int. Conf. on Robotics and Automation,* pp. 1671-1676, 1998.

[9] H. Wang and Y. H. Liu, "Adaptive image-based trajectory tracking of robots," *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 2564-2569, 2005.

[10] Y. H. Liu, H. Wang and D. Zhou, "Dynamic tracking of manipulators using visual feedback from an nncalibrated fixed camera," *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp.4124-4129, 2006.

[11] H. Wang, Y. H. Liu and D. Zhou, “Dynamic visual tracking for manipulators using an uncalibrated fixed camera,” accepted by *IEEE Trans. on Robotics*, 2007.