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Experimental study of Redundant Snake Robot Based on Kinematic Model

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Abstract— **In this paper we consider modeling and control of a redundant snake robot with wheeled link mechanism based on kinematic model. We derive a kinematic model of a snake robot with introducing links without wheels and shape controllable points in the snake robot's body in order to make the system redundancy controllable. By using redundancy, it becomes possible to accomplish the main objective of controlling the position and the attitude of the snake robot head and the shape of the snake robot, and the sub-objective of the singular configuration avoidance. Simulations and experimental results show the validity of the control law and how snake robots move at the neighborhood of the singular configuration.**

I. INTRODUCTION

Unique and interesting gait of the snakes makes them able to crawl, climb a hill, climb a tree by winding and move on very slippery floor [1]. It is useful to consider and understand the mechanism of the gait of the snakes for mechanical design and control law of snake robots.

Hirose has long investigated snake robots and produced several snake robots, and he models the snake by a wheeled link mechanism (passive wheels are attached at the side of the snake robot body.) with no side slip [2]. Some other snake-like mechanisms are developed in [3] and [4]. Burdick and Chirikijian discuss the sidewinding locomotion of the snake robots based on the kinematic model [5]. Ostrowski and Burdick analyze the controllability of a class of nonholonomic systems that the snake robots are included on the basis of the geometric approach [6]. The feedback control law for the snake head's position using Lyapunov method has been developed by Prautesch et al. on the basis of the wheeled link model [7]. They point out the controller can stabilize the head position of the snake robot to its desired value, but the configuration of it converges to the singular configuration. Date et. al. proposed a new dynamic manipulability of the snake head which is realted to the constraint forces of the passive wheels and a controller which minimizes the cost function related to the constraint forces and the tracking error of the snake-head [8]. As a result of applying the controller the sigularity avoidance would be accomplished by trade-off between the singulatity avoidance and the tracking error. But this controller can not ensure the convergence of the trajectory tracking error.

We find that introduction of links without wheels and shape controllable points in the snake robot's body makes the system redundancy controllable based on the kinematic model [9]. The proposed control law can stabilize the head position and attitude of the snake robot to their desired value with avoiding the singular configuration. We demonstrate the validity of the control law by simulations[9]. But experiments have not carried out and we do not know how snake robots move at the singular configuration.

In this paper we carry out simulations and experiments in order to demonstrate the validity of the control law in [9]. We derive a kinematic model of a snake robot with introducing links without wheels and shape controllable points in the snake robot's body in order to make the system redundancy controllable. By applying the redundancy both the main objective of controlling the position and the attitude of the snake robot head and the shape of the snake robot, and the sub-objective of the singular configuration avoidance are accomplished. Simulation and experimental results show the validity of the control law and the movement of the snake robot at the neighborhood the singular configuration.

II. REDUNDANCY CONTROLLABLE SYSTEM

Let us define a redundancy controllable system. Let $\boldsymbol{q} \in R^{\bar{n}}$ be the state vector, $\boldsymbol{u} \in R^{\bar{p}}$ be the input vector, $\boldsymbol{w} \equiv S\boldsymbol{q} \in R^{\bar{q}}$ be the state vector to be controlled, S be a selection matrix whose row vectors are independent unit vectors related to generalized coordinates, $A(\boldsymbol{q}) \in R^{\bar{m}\times\bar{q}}, B(\boldsymbol{q}) \in R^{\bar{m}\times\bar{p}}$. We define that the system

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u}, \quad \boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_2 \tag{1}$$

is redundancy controllable if $\bar{p} > \bar{q}$ (redundancy I), $\bar{p} > \bar{m}$ (redundancy II),[1] the matrix A is full column rank, B is full row rank, and there exists an input $\boldsymbol{u}_1$ which accomplishes the main objective of the convergence of the vector $\boldsymbol{w}$ to the desired state $\boldsymbol{w}_d(\boldsymbol{w} \rightarrow \boldsymbol{w}_d, \dot{\boldsymbol{w}} \rightarrow \dot{\boldsymbol{w}}_d)$ and an input $\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_2$ which accomplishes the increase (or decrease) of a cost function $V(\boldsymbol{q})$, which is related to the sub-objective, compared to the input $\boldsymbol{u}_1$ and does not disturb the main objective. For a snake robot based on the wheeled link model we discuss a condition that the system is redundancy controllable.

III. KINEMATIC MODEL OF SNAKE ROBOTS

We consider an n-link snake robot with the wheeled link mechanism. Let n be the number of links, m be the number of wheeled links, $[x_h, y_h, \theta_h]^T$ be the vector of the position and the attitude of the snake head, $[\phi_1, \cdots, \phi_{n-1}]^T$ be the vector of relative joint angles and

[1] In the case of $\bar{m} = \bar{p}$, if the state vector to be controlled $\dot{\boldsymbol{w}}$ in (1) is given, the input $\boldsymbol{u}$ is determined uniquely. In this sense the system is not redundant, so we introduce the redundancy II.

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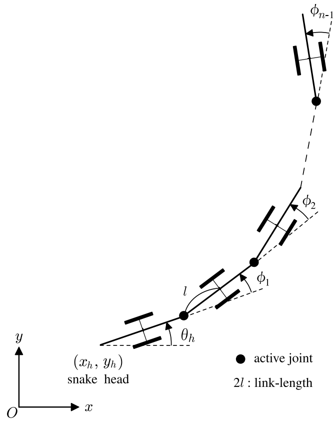

Fig. 1. An n-link snake robot.

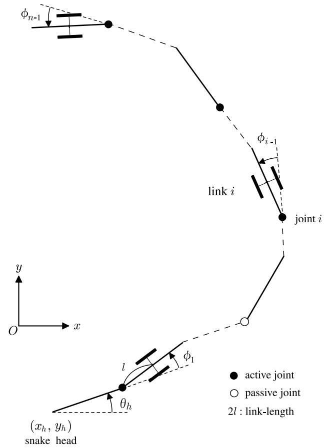

Fig. 2. A redundant n-link snake robot.

$\boldsymbol{q} = [x_h, y_h, \theta_h, \phi_1, \cdots, \phi_{n-1}]^T \in R^{n+2}$ be the generalized coordinates. The length of each link is $2l$. The wheels are located at the middle point of the wheeled link. As we introduce the assumption that the wheel does not slip to the side direction, the kinematic equation of snake robots is

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u}, \quad \boldsymbol{u} = \dot{\boldsymbol{\theta}} \tag{2}$$

where $\boldsymbol{w}$ is the state vector to be controlled, $\boldsymbol{\theta}$ is the vector of the active joint angles, $A \in R^{m\times q}, B \in R^{m\times p}$, and the angular velocity of the active joint is regarded as the input of the system.

As one wheeled link has one velocity constraint, the number m of the wheeled links is equal to the number of equations. We assume that at least the snake head's position and attitude are controlled.

IV. Condition for Redundancy Controllable System

We consider an n-link snake robot whose all links are wheeled as shown in Fig. 1. Let $\bar{\boldsymbol{w}} = [x_h, y_h, \theta_h]^T$ be the position and attitude of the snake head, $\bar{\boldsymbol{\theta}} = [\phi_1, \cdots, \phi_{n-1}]^T$ be relative angles of each link, $\boldsymbol{q} = [\bar{\boldsymbol{w}}^T, \bar{\boldsymbol{\theta}}^T]^T$ be generalized coordinates.

If all links are wheeled link, the system can be written as

$$\bar{A}(\boldsymbol{q})\dot{\bar{\boldsymbol{w}}} = \bar{B}(\boldsymbol{q})\bar{\boldsymbol{u}}, \quad \bar{\boldsymbol{u}} = \dot{\bar{\boldsymbol{\theta}}} \tag{3}$$

where $\bar{A} \in R^{n\times 3}, \bar{B} \in R^{n\times(n-1)}$. In the system (3), as the velocity constraint of the passive wheel of the head-link is expressed as

$$\dot{x}_h \sin\theta_h - \dot{y}_h \cos\theta_h - l\dot{\theta}_h = 0,$$

we find that the matrix $\bar{B}$ is not full row rank. It is necessary that the passive wheel of the first link is removed. [2]

Next, we consider the system that only the passive wheel of the head-link is removed. This system can be written as

$$\bar{A}'(\boldsymbol{q})\dot{\bar{\boldsymbol{w}}} = \bar{B}'(\boldsymbol{q})\bar{\boldsymbol{u}} \tag{4}$$

[2]Prautesch et al. consider the position control but not the attitude control of the snake head. In this case related B matrix is a square and nonsingular matrix, the system does not have the redundancy II.

where $\bar{A}' \in R^{(n-1)\times 3}, \bar{B}' \in R^{(n-1)\times(n-1)}$ and

$$\bar{B}' = \begin{bmatrix} l & & & & \\ b_{21} & l & & \mathbf{0} & \\ \vdots & & \ddots & & \\ \vdots & & & l & \\ b_{(n-1)1} & \cdots & \cdots & b_{(n-1)(n-2)} & l \end{bmatrix},$$

$b_{ij} = 2l\sum_{k=j}^{i-1}\cos(\sum_{s=k}^{i-1}\phi_s)$. We find that the matrix $\bar{B}'$ is invertible. As the velocity of the snake head $\dot{\bar{\boldsymbol{w}}}$ determines the input $\bar{\boldsymbol{u}}$ uniquely, the system (4) is not redundant (The redundancy II is not satisfied.).

In this paper, as we control the shape of the snake robot body in addition to the position and attitude of the snake head, some relative angles are included in the state vector to be controlled. This relative joint angles are defined as the shape controllable points and the number of the shape controllable points is called the shape controllability index. Let s be the shape controllability index.

To satisfy the condition (B is full row rank.) so that the system is redundancy controllable, we introduce the conditions.

[condition 1]: The head link is wheelless.
[condition 2]: The tail link is wheeled.
[condition 3]: If the i-th joint is the shape controllable point, the corresponding i-th link is wheelless.
[condition 4]: The passive joint angle is equivalent to the state variable to be controlled as the shape controllable point.

We remark that the joint of a wheeled link is a passive or an active joint and that the link which has the passive joint is wheelless. If the assumptions 1-4 are satisfied, the matrix B is full row rank. The assumptions 1-4 are the sufficient condition for the full row rankness of the matrix B. The result of the case that condition 4 changes for $4'$ "The passive joint angle is the state variable to be controlled as the shape controllable point. (not vice versa)" is omitted. In the case $4'$, we have the possibility that the shape controllable point is an active joint.

We consider a redundant n-link snake robot as shown in Fig. 2 with m wheeled links which satisfies the assumptions 1-4. In this case the system can be written as

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u} \tag{5}$$

where $A \in R^{m\times(3+s)}, B \in R^{m\times(n-1-s)}$, $\boldsymbol{w} = S\boldsymbol{q} \in R^{(3+s)}$, $\boldsymbol{\theta} = \bar{S}\boldsymbol{q} \in R^{(n-1-s)}$ and $\boldsymbol{u} = \dot{\boldsymbol{\theta}}$. We find that exclusion of $\boldsymbol{w}$ from $\boldsymbol{q}$ gives $\boldsymbol{\theta}$. $\{\boldsymbol{w}\} \cup \{\boldsymbol{\theta}\} = \{\boldsymbol{q}\}$, $\{\boldsymbol{w}\} \cap \{\boldsymbol{\theta}\} = \{\phi\}$. The necessary and sufficient condition for the existence of the solution of the system (5) is

$$\text{rank}[A, B\boldsymbol{u}] = \text{rank}A. \tag{6}$$

In the case of $m < \dim(\boldsymbol{w}) = q$ for the system (5), if an input $\boldsymbol{u}$ is given, then the solution $\dot{\boldsymbol{w}}$ does not determine uniquely. From the necessity of the uniqueness of the solution of the system (5), we introduce the condition $m \geq q = 3+s$. We set $m < p$ so as to satisfy the condition that the velocity vector $\dot{\boldsymbol{w}}$ does not determine the control input $\boldsymbol{u}$ uniquely. This condition means the redundancy of the input (redundancy II).

From two conditions $m \geq q$ and $m < p$ we find that the condition $p > q$ (redundancy I) is satisfied. The condition that the system (5) is redundancy controllable can be written as

$$3+s \ \leq \ m \ < \ (n-1)-s. \tag{7}$$

V. Controller Design

Let us define the control input as follows:

$$\boldsymbol{u} = B^{+}A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w}-\boldsymbol{w}_d)\} + (I - B^{+}B)\alpha\boldsymbol{\eta} \tag{8}$$

where B^{+} is a pseudo-inverse matrix of B, $\boldsymbol{\eta} = \nabla_{\boldsymbol{\theta}} V(\boldsymbol{q}) = [\partial V/\partial\theta_1, \cdots, \partial V/\partial\theta_{n-1-s}]^T$ is the gradient of the cost function $V(\boldsymbol{q})$ with respect to the vector $\boldsymbol{\theta}$ related to the input vector $\boldsymbol{u}$, and $\alpha \geq 0$, $K > 0$. The first term of the right side of (8) is the control input term to accomplish the main objective of the convergence of the state vector $\boldsymbol{w}$ to the desired value $\boldsymbol{w}_d$. As the second term $(I - B^{+}B)\alpha\boldsymbol{\eta}$ belongs to the null space of the matrix B, we obtain

$$B\boldsymbol{u} = A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w}-\boldsymbol{w}_d)\}. \tag{9}$$

As the vector $B\boldsymbol{u}$ can be expressed as a linear combination of column vectors of the matrix A, the condition of the existence of the solution (5) is satisfied. The second term in (8) does not disturb the dynamics of the controlled vector $\boldsymbol{w}$. As there is no interaction between $\boldsymbol{w}$ and $\boldsymbol{\theta}$, we find that the control law (8) accomplishes the sub-objective. Actually we can derive

$$\begin{aligned}\dot{V}(\boldsymbol{q}) &= (\partial V/\partial\boldsymbol{w})\dot{\boldsymbol{w}} + (\partial V/\partial\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \\ &= (\partial V/\partial\boldsymbol{w})\dot{\boldsymbol{w}} + \boldsymbol{\eta}^T B^{+}A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w}-\boldsymbol{w}_d)\} \\ &\quad + \boldsymbol{\eta}^T(I - B^{+}B)\alpha\boldsymbol{\eta}\end{aligned} \tag{10}$$

As $I - B^{+}B \geq 0$ [10], we find that the second term of the input (8) accomplishes the increase of the cost function V.

The closed-loop system is expressed as

$$A\{(\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d) - K(\boldsymbol{w}-\boldsymbol{w}_d)\} = \boldsymbol{0}. \tag{11}$$

If the matrix A is full column rank, the uniqueness of the solution is guaranteed. The solution of (11) is given as

$$(\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d) - K(\boldsymbol{w}-\boldsymbol{w}_d) = \boldsymbol{0}$$

and we find that the controller ensures the convergence of the controlled state vector to the desired value ($\boldsymbol{w} \to \boldsymbol{w}_d$). A set of joint angles which satisfies rank$A < q$ (A is not full column rank.) means the singular configuration, for example a straight line ($\phi_i = 0, i = 1, \cdots, n-1$).

VI. Simulation

To demonstrate the validity of the proposed control law simulations have been carried out. In this simulation we set $B^{+} = B^T(BB^T)^{-1}$ and

$$V = a'(\det(A^TA)) + b'(\det(BB^T)) \tag{12}$$

where $a', b' > 0$. The first term of the right side of (12) implies the measure of the singular configuration. Concretely, if a snake robot becomes the singular configuration, $\det(A^TA)$ becomes zero because A is not full column rank. The second term of the right side of (12) is related to the manipulability of the system.

A. A 7-link model without shape controllable points

For simplicity we consider a 7-link snake robot with 5 wheeled links and does not introduce the shape controllable points ($s = 0$). As shown in Fig. 3, the first and the third links are unwheeled. In this case a set of joint angles which satisfy $\phi_2 + \phi_3 = \phi_4 = \phi_5 = \phi_6 = 0$ means the singular configuration. We can easily find it by checking the components of matrix A. We set the initial condition $\boldsymbol{w}(0), \boldsymbol{\theta}(0)$ and the desired trajectory $\boldsymbol{w}_d(t)$ as $\boldsymbol{w}(0) = [-0.01, -0.01, \pi - 0.05]^T$, $\boldsymbol{\theta}(0) = [\frac{\pi}{90}, \frac{\pi}{120}, \frac{\pi}{110}, \frac{\pi}{100}, \frac{\pi}{80}, \frac{\pi}{70}]^T$, $\boldsymbol{w}_d(t) = [-0.02t, 0, \pi]^T$, and $l = 0.0335$[m], $K = \mathrm{diag}(3,3,3)$. We set coefficients of the cost function V as $a' = a/l^2, b' = b/l^{10}$ in order to normalize with respect to the link length l. Figs. 5-6 show the time responses for $x_h, y_h, \theta_h, \phi_1, \cdots, \phi_6$, $\sqrt{\det(A^TA)}/l$, $\sqrt{\det(BB^T)}/l^5$ and $u_1, \cdots, u_6$.

Fig. 5 shows the responses for the case $\alpha = 0$ (case 1). In this case the controller does not use the redundancy. From the figure we find that the snake head tracks the desired state, but det$A$ converges to zero. In case 2 we set $\alpha = 0.005, a = 5, b = 1$. In this case, the controller utilizes the redundancy so as to accomplish the increase of the cost function V. From Fig. 6 we find that the snake head converges to the desired state and the avoidance of the singular point is accomplished. Fig. 7 shows the movement of the snake robot for each case. We find the crawling of the snake in the case 2. In the case 1 the snake robot converges to a singular configuration.

B. An 8-link model with one shape controllable point

Next, we consider an 8-link snake robot with 5 wheeled links and introduce one shape controllable point ($s = 1$). As shown in Fig. 4 the first, third and fifth links are unwheeled, and the second joint is passive. By controlling the shape controllable point, we test the effectiveness of introducing it. In this case $\boldsymbol{w} = [x_h, y_h, \theta_h, \phi_2]^T$ and a set of joint angles which satisfy $\phi_2 + \phi_3 = \phi_4 + \phi_5 = \phi_6 = \phi_7 = 0$ means the singular configuration. We set the initial condition and the desired trajectory $\boldsymbol{w}(0) = [0.02, 0.02, \pi + 0.1, \frac{\pi}{90}]^T$, $\boldsymbol{\theta}(0) = [\frac{\pi}{120}, \frac{\pi}{110}, \frac{\pi}{100}, \frac{\pi}{80}, \frac{\pi}{70}, \frac{\pi}{60}]^T$, $\boldsymbol{w}_d = [0.02t, 0, \pi, \phi_{2d}]^T$ and $l = 0.0335$[m], $K = \mathrm{diag}(3,3,3,3)$.

The case of $\alpha \neq 0$ is omitted because the same result as case 2 is obtained.

Fig. 8 shows the time responses for $x_h, y_h, \theta_h, \phi_1, \cdots, \phi_7$, $\sqrt{\det(A^TA)}/l^2$, $\sqrt{\det(BB^T)}/l^5$ and $u_1, \cdots, u_6$.

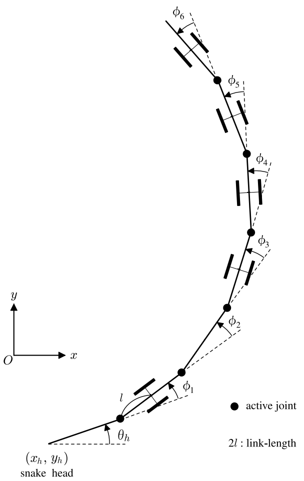

Fig. 3. A 7-link snake robot.

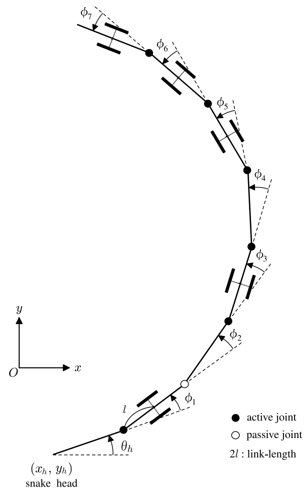

Fig. 4. A 8-link snake robot.

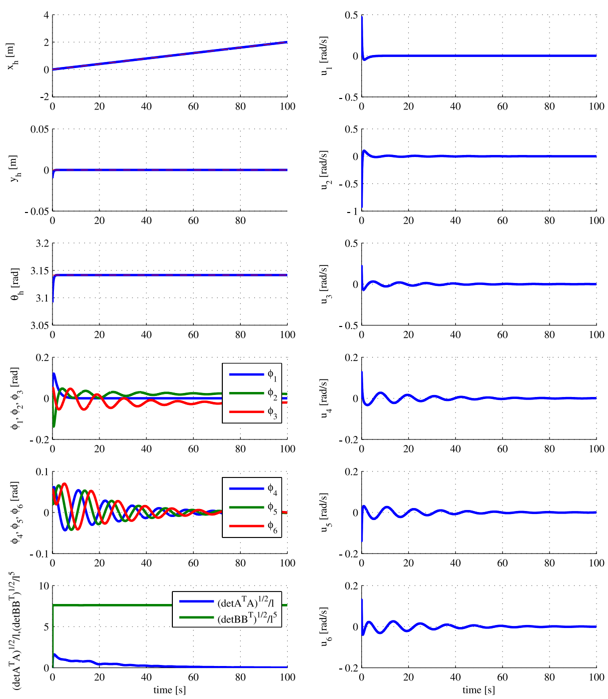

Fig. 5. Time responses for the controller without considering the redundancy (case 1 : $\alpha = 0$).

In the case of $\alpha = 0, \phi_{2d} = 0$ (case 3), the controller does not use the redundancy and the desired value of the shape controllable point is set zero. From the result of simulation, we find that the snake head tracks the desired trajectory, but $\det(A^T A)$ converges to zero. It means that the snake robot converges to a singular configuration.

Fig. 8 shows the responses for the case $\alpha = 0, \phi_{2d} = 0.5 \sin(\frac{\pi t}{2})$ (case 4). From Fig. 8 we find that the snake head tracks the desired trajectory and the snake robot crawls without converging to the singular configuration.

But, this result is obtained by simulations, not theoretical. We need to research about the desired value of the shape controllable point in the future.

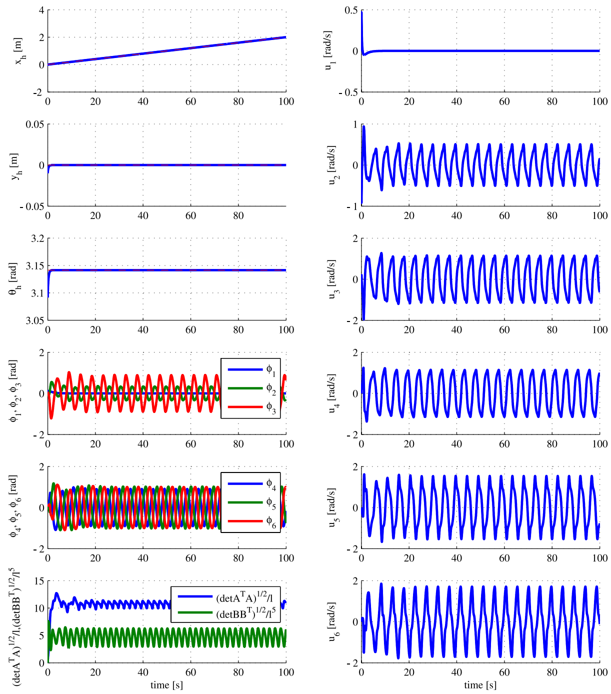

Fig. 6. Time responses with considering the redundancy (case 2 : $\alpha = 0.005, a = 5, b = 1$).

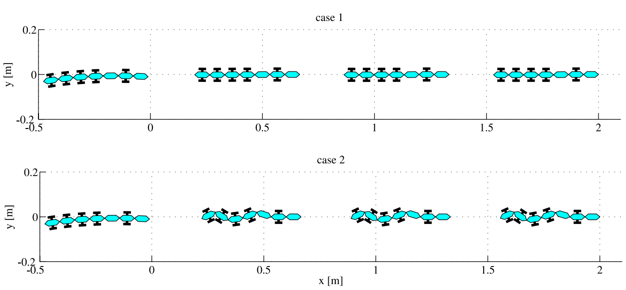

Fig. 7. Movement of the snake robot (case1, 2).

From simulation results we find that the second term of the control law (8) can ensure the singularity avoidance and the vibratory motion of the shape controllable point can avoid convergence of the singular configuration.

VII. Experiment

To demonstrate the validity of the control law and simulations, experiments have been carried out. In this experiment we set l, B^+, V, a', b' the same as simulations. Fig. 9 shows the experimental system of a redundant snake robot. This robot is composed of velocity controlled motors with potentiometers and passive wheels. A stereo vision system (Quick MAG IV) is used for measuring the position and attitude of the robot.

A. A 7-link model without shape controllable points

First, we consider a 7-link snake robot with 5 wheeled links and does not introduce the shape controllable points like

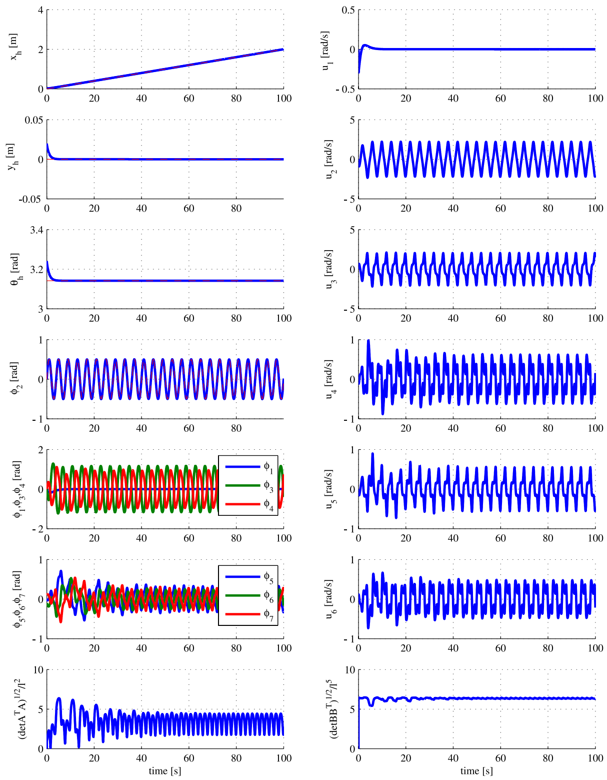

Fig. 8. Time responses for the controller without considering the redundancy (case 4 : $\alpha = 0, \phi_{2d} = 0.5\sin(\frac{\pi t}{2})$).

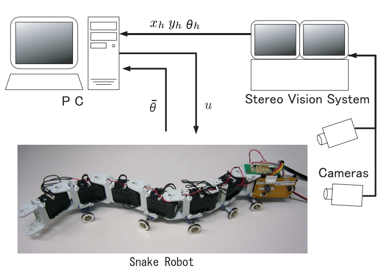

Fig. 9. Experimental system.

the case 1 and 2 of the simulations. We set the desired trajectory $\boldsymbol{w}_d(t) = [0.5, 1.0 - 0.01t, \frac{\pi}{2}]^T$ and $K = \text{diag}(2, 2, 2)$.

Fig. 10 shows the responses for $\alpha = 0$ (case 1). From Fig. 10 we find as follows. First, the snake head tracks the desired trajectory with converging to the straight line of the singular configuration, and stops moving at 15 [s]. Then, the snake robot restarts moving after the snake robot vibrates at 33[s] and the snake head tracks again with converging to the singular configuration. We can guess that these movements are caused by singularity, the position error with respect to the desired trajectory, the sliding of the passive wheels to the side direction. If the snake robot converges to the straight line of the singular configuration, the robot can not move because the orientations of all passive wheels are parallel to the desired trajectory. So the position error with respect to the desired trajectory increases and the input also increases. The sliding of the passive wheels to the side direction happens because of the large input and the snake robot recedes from the neighborhood of the singular configuration.

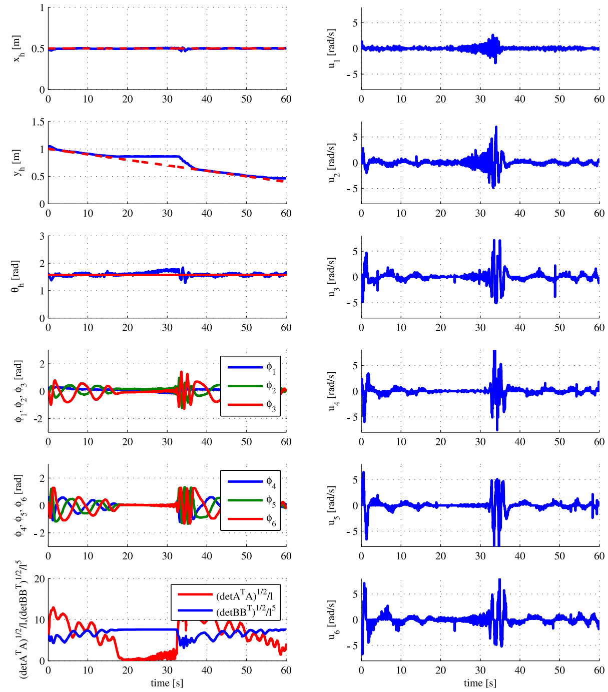

Fig. 10. Time responses for the experiment of case 1.

Fig. 11 and 12 show the responses with utilizing the redundancy for $\alpha = 0.005, a = 5, b = 1$ (case 2) and the movement of the experimental snake robot, respectively. From Fig. 11 and 12 we find that the snake head converges to the desired trajectory and the avoidance of the singular point is accomplished.

B. An 8-link model with one shape controllable point

Next, we consider an 8-link snake robot with 5 wheeled links and introduce one shape controllable point ($s = 1$) which is used in simulations. We set the desired trajectory $\boldsymbol{w}_d(t) = [0.5, 1.0 - 0.01t, \frac{\pi}{2}, \phi_{2d}]^T$ and $K = \text{diag}(4, 4, 4, 4)$.

The case of $\alpha \neq 0$ and the case of $\alpha = 0, \phi_{2d} = 0$ are omitted because the same results as case 2 of experiments and as case 1 of experiments are obtained.

Fig. 13 shows the responses for $\phi_{2d} = 0.5\sin(\frac{\pi t}{2})$ (case 4). From Fig. 13 we find that the snake head converges to the desired trajectory in a measure with avoiding the singular configuration. But vibrations are found in the response of θ_h. We can guess that this error is affected by the modeling error caused by sliding of the passive wheeles to the side direction and measurement errors of the stereo vision system which measures the position and the attitude of the snake robot.

From experimental results we demonstrated the validity of the control law and we found the movement of a snake robot at the neighborhood the singular configuration.

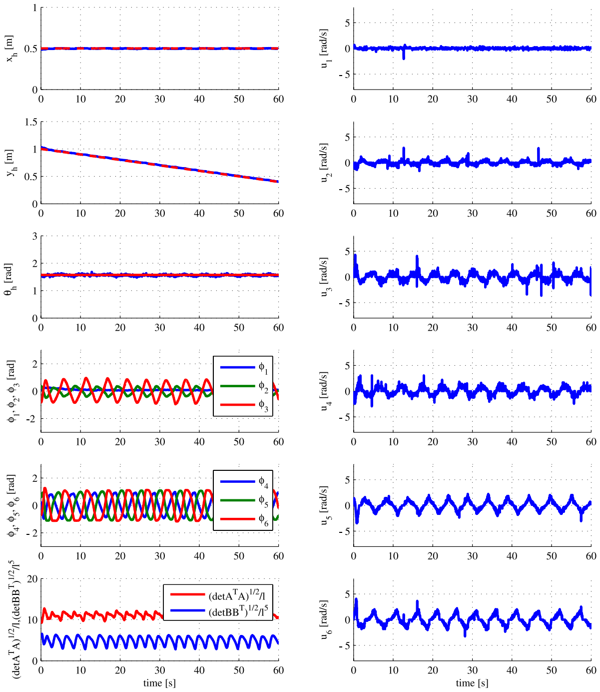

Fig. 11. Time responses for the experiment of case 2.

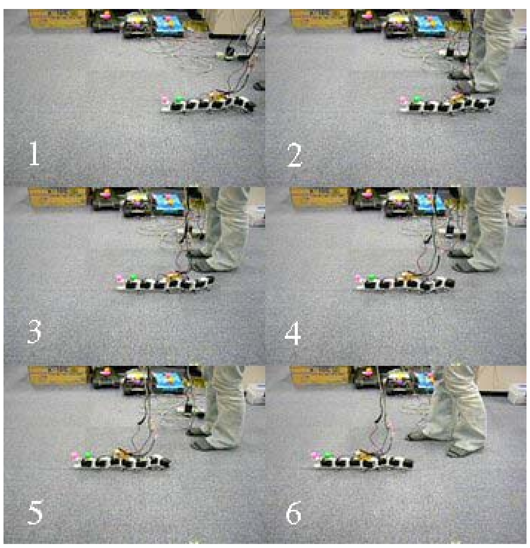

Fig. 12. The movement of the experimental snake robot (case 2).

VIII. Conclusion

We derive a kinematic model of a snake robot with introducing links without wheels and shape controllable points in the snake robot's body in order to make the system redundancy controllable. Using redundancy, we can accomplish both the main objective of controlling the position and the attitude of the snake robot head and the shape of the snake robot, and the sub-objective of the singular configuration avoidance. Simulation and experimental results showed the validity of the control law and how a snake robot moves at the neighborhood of the singular configuration.

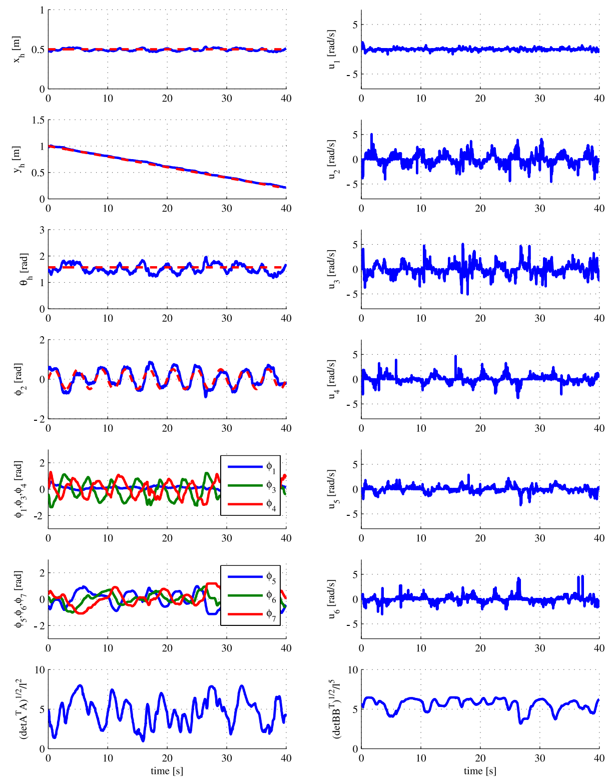

Fig. 13. Time responses for the experiment of case 4.

Though the desired values for the shape controllable points should determine so as to accomplish the given tasks, the relation between them is not clear. As the future works, we should discuss how to determine the desired values for the shape controllable points related to the various given tasks.

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