2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

A Hybrid Motion Model for Aiding State Estimation in Dynamic Quadrupedal Locomotion

Surya P. N. Singh[1] and Kenneth J. Waldron[2]

Abstract— **Trotting and galloping allow a quadruped to rapidly traverse rough terrain. Modeling this motion, which is only dynamically stable, is of importance for legged robot operation and for quadrupedal animal motion estimation. Derived from an eight-step galloping cycle, this study presents a kinetic hybrid model in which the states vary based on the principal forces present. As compared to foot contact, or kinematic, hybrid quadruped models this reduces the maximum number of possible states from 120 to 6 and provides an alternative to foot contact monitoring. This approach was tested on a trotting quadruped robot equipped with an inertial sensor aided by video. This was processed using an EKF estimator framework to give attitude estimates at rates of up to 250 Hz with 5° error.**

I. Introduction

Legged platforms offer unparalleled adaptation and obstacle traversal over rough terrain. Rapid field motion requires the adoption of dynamic gaits that, unlike walking, are statically unstable, but agile. For quadrupeds this is manifest in the trot and gallop, with the gallop achieving higher speeds through the asymmetric extension of the flight phase [1].

The introduction of a flight phase challenges the mechanical, control, and sensing systems as dynamic constraints, in particular those involving ground contact, become discontinuous. The leg mechanism and actuation must generate large power pulses that provide sufficient thrust to obtain flight. Controllers have discontinuous control authority as control may only be imparted during ground contact. Finally, onboard sensing must operate with sufficient fidelity to capture motion dynamics, yet be robust to landing shocks.

As long as there is a link to the ground (and its profile is known or assumed planar), it is possible to directly track pose and position by solving the kinematic chain(s) via instrumented legs [2] or ground range [3]. A flight phase disconnects this chain. Making the assumption that these intervals are completely ballistic, gives an approximate solution for forward position [4]. Therefore, to determine attitude it is necessary to measure the motion in a self-contained manner with respect to an inertial frame.

Compact, self-contained sensing with respect to a body-centered inertial frame is typically achieved using an inertial measurement unit (IMU) [5]. In the legged domain, this is complicated by footfall shocks and sensor misalignment leading to errors in the compensation of gravitational acceleration. This leads to the problems of saturation and drift.

Alternative sensing approaches are not ideal as they are limited in range or fidelity. Off-board tracking, such as that obtained using optical motion-capture systems, is limited to fixed workspaces [6]. Standard navigation solutions, such as global positioning (GPS), do not provide sufficient rates to fully capture motion dynamics and may be occluded in certain environments [7]. Recovery of motion (including pose) using vision, such as that calculated using structure from motion [8] or visual odometry [9], even in operating conditions compatible with a high-frame rate camera, is not ideal due to potential occlusions and the large computational loads associated with high-bandwidth, real-time processing.

Thus, for agile operations a dynamic legged robot requires rapid feedback controls based on an aided estimation of current state based on information from multiple measurement sources. In general, these estimators may be described as consisting of two parts: a forward predictor, and an updater based on weighted measurements. At the core of these estimators is a dynamic model describing system motion.

Given the aforementioned motion discontinuities, hybrid approaches have been advocated for both control [10] and estimation [11] of running robots. A central issue to these approaches is the mechanism, or state(s), that drive the transitions between the various hybrid models. For legged robots this has typically been based on leg location, and, in particular, leg contact [12]. Based on the notion that the topology of the forces is more important than particular foot contacts, the model presented uses the kinetic state as determined from inertial sensors (or even inferred from foot contact patterns) to switch model parameters.

The paper describes the galloping gait cycle and uses this to introduce the kinetic modes of the hybrid model. This model is then applied to the attitude estimation problem using an Extended Kalman Filter (EKF) framework. The paper then highlights the experimental setup and operation, for demonstration on a dyanamic quadrupedal robot trotting at speeds from 1.5–2.5 m/s.

II. Related Work

Dynamic legged locomotion, or legged motion balanced by kinetics, is an area of active interest. In analyzing the dynamics, a number of authors have mapped the discontinuities present to a hybrid system model. For instance, Raibert's hopping quadruped [13] transitioned from various modes (or finite states) of operation during the motion cycle.

This material is based upon work supported by the National Science Foundation under Grant No. IIS-0208664.

[1] S. Singh is presently at the Robotics and Mechatronics Laboratory of the School of Mechanical Engineering at the University of Western Australia, Perth, WA 6009, Australia. Email: spns@mech.uwa.edu.au

[2] K. Waldron heads the Robotics Locomotion Laboratory of the Mechanical Engineering Department at Stanford University, Stanford, CA 94305, USA. Email: kwaldron@stanford.edu

Even though it simplifies operation using an equivalent single "virtual" leg model, mode transitions are nonetheless determined by foot contact(s). Berkmeier's quadruped dynamics model [14] uses event sequences, which are also transitioned by the feet in contact with the ground.

Acceleration can also be used for model transition. As detailed in our previous work [1], flight and stance can be differentiated using an accelerometer with flight given by an approximately zero measurement (i.e., a body acceleration with respect to ground that is close to gravity ($-g$)). This concept has also been applied as part of an Interacting Multiple Model (IMM) estimator for the RHex robot [10]. While simple, this approach considers vertical accelerations of the body center, which limits the result to bouncing gaits, such as the bound and pronk.

One approach to extending this to faster gaits is to use an approximate dynamic model. The issue is that this has to be balanced against real-time computation limits. A convenient model for this form of locomotion is the Spring-loaded Inverted Pendulum (SLIP), which models the motion by assuming a single point mass connected to a sprung leg [15]. It is extended to quadrupeds via the use of "virtual legs" [4]; however, this is only applicable for symmetric gaits. Further, SLIP has no provision for capturing leg interactions. Even with these simplifications, including gravity results in a system that requires approximate numerical solution [16]. Berkmeier [14] provides a 2-DOF planar model that does not use "virtual legs"; but, this is limited to bound, pronk, and hopping. Palmer [17] extends this to control a trot. An impulse-based approach, as detailed in [18] and [19], provides an efficient gallop model. Even with an impulse model, a mechanism for switching models between the various gaits is still needed.

This work looks at directly extending the hybrid approach for dynamic gaits by switching between the major kinetic modes prevalent. Since foot contact provides a constraint on dynamics, it must be included. However, similarities present between particular foot contacts allow for reduction in the number of modes.

III. Galloping Gait

Biology suggests that the transverse gallop is the fastest and most efficient quadrupedal gait for endurance/distance running [20]. It has been suggested by Minetti [21] that the gallop is a "skipping" gait. This provides several unique characteristics, namely: that vertical energy fluctuations occur at half the rate of its forward ones; that potential energy is maximum during flight; and, that the pitch angular velocity between the stances of the two front feet is zero.

A second, intuitive reason for this is that the forward speed is given by a product of the effective stride length (L_s) and the stride frequency (f_s) as

$$S = \mathbf{v}_x = L_s \cdot f_s \tag{1}$$

where S is the forward speed. Variations in stride frequency are limited as the fundamental frequency is energetically favored. Thus, the strategy is to increase the stride length.

The gallop is a four-beat gait pattern in which there is typically one foot in contact with the ground with periods of flight and two feet contact [22]. It can be considered as consisting of eight phases based on which foot is in contact. In the transverse gallop, the transition from hind to front is across a diagonal set of foot contacts, which gives the eight phases as: right-hind, both-hind, left-hind, left-hind+right-fore, right-fore, both-fore, left-fore, and flight.

As illustrated in Figure 1, the gallop increases the stride through a longer flight phase achieved by adopting an asymmetric gait. The asymmetry present allows the foot contacts and subsequent thrusts to occur at uneven timings. Thus, there is no plane of symmetry for which the motions on one side are mirrored on the other [22]. This implies that the forward speeds and attitudes changes between strides. The consequence on modeling is that body accelerations are no longer odd functions that integrate to zero over symmetric limits [4], which prevents the use of simpler control strategies, especially those that simulate the motion as a bouncing ball [10].

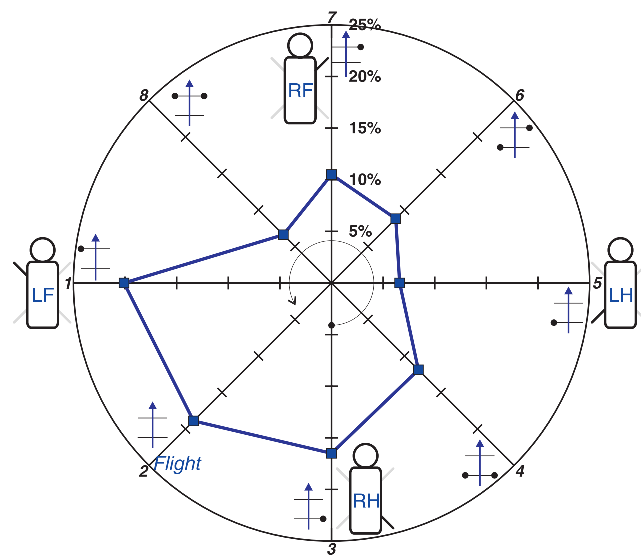

Fig. 1. Gait phase radar diagram for the transverse gallop of a horse clearly shows the asymmetry in leg timing. The axis shows the percent of cycle time for the major phases (as indicated by the icon where L/R is left/right and H/F is hind/fore). The sequence rotates counterclockwise. Superimposed at each of the eight are contact diagrams which show foot contact as dots at the end of the line (adapted from [23]). Gallop phases averaged from data in [24] for experiments at a speed of 13 m/s (f_s: 2.6 Hz, L_s: 5 m, t_{cycle}: $\sim$380 ms, and double support: 33%).

IV. Simplified Quadruped Kinetic Map

The approach taken to modeling the gallop and related dynamic gaits is to reduce the number of modes while capturing the salient dynamic properties for each gait. This is in keeping with earlier modeling approaches, such as SLIP and "virtual legs."

For a quadruped, the naive approach of modeling each contact gives 120 possible transitions between 16 foot contact states. Shifting to modeling the motion based on the dynamic ground-contact constraints (from 0 to 4 legs in contact) and the presence of a sprung leg (i.e., compliance or lack thereof (rigid leg)) gives a complete set of 10 states. This is further reduced to by considering the combinations affecting dynamic gait motions, in particular the gallop. For example,three and four legs in contact with the ground stability is determined statically. This gives 6 dynamic states as detailed in Table I and having 8 principal transitions as illustrated in Fig. 2.

Gait(s)	Contact(s)	$k_{\text{effective}}$	Modeling Strategy
Flight	0	—	Ballistic
Pace/Walk	1	High	Inv. pendulum
Trot/Bound/Pronk	1	Low	SLIP
Gallop	2	Low	Impulse
Slow walk	3	High	Alternating tripod
Standing	4	High	Static stability

TABLE I
REDUCED QUADRUPED DYNAMIC STATES

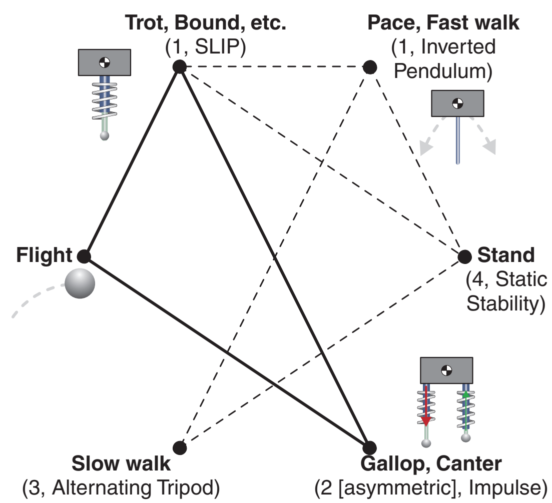

Fig. 2. Kinetic map for quadruped locomotion. The main kinetic states are shown as nodes. The numbers in parenthesis indicate the effective or equivalent number of ground contact constraints (i.e., including reductions made due to gait symmetry). The solid line indicates transitions occurring during the dynamic motion, such as the gallop. The dashed line indicates transitions for slower initial gaits. This model has a reduced number of states and transitions compared to a direct contact approach.

The particular gait employed and the transition between kinetic states within the gait is based on the ratio of kinetic to potential energy; that is, the Froude number, which can be expressed as:

$$Fr = \frac{\mathbf{v}^2}{gz} \tag{2}$$

where $\mathbf{v}$ is the magnitude of the velocity vector, g is gravitational acceleration, and z is the vertical position or height. The average (over the cycle) Froude number ($\overline{Fr}$) classifies the gait [25] with z defined as the standing height of the mass center. Hence, a $\overline{Fr} < 1$ indicates a walk, $1 < \overline{Fr} < 2.5$ indicates a trot, and $\overline{Fr} > 2.5$ indicates galloping. The Froude number computed from the current state can be used to switch between multiple models. For example, with $Fr \approx 1$ indicating flight and $Fr > 3$ indicating double support for the gallop.

For the gallop, an additional simplification is to use the normalized potential energy state of the mass center ($p = \frac{z}{z_{\max}}$) as this may be measured directly using several methods. Based on observed energy fluctuations [21], a $p > 0.9$ indicates flight phase, $p < 0.2$ indicates double support, and $0.2 < p < 0.9$ indicates single support.

The advantage of an energy approach is its wider applicability and relative simplicity compared to other methods. While pitch rates are also unique to galloping, additional sensing is needed to track the cycle to determine flight phase robustly. Compared to foot contact sensing, the energy method is less ambiguous and nearly as convenient from a robot/machine implementation perspective. For example, it can distinguish between pronking or standing even though both gaits have a period with all feet in contact. Further, this does not preclude foot contact sensing, which can still be used to as an additional check.

V. METHOD

With a series of approximate models for describing particular sections of the gait and a mechanism for switching them, the paper now considers the state estimation problem in order to determine state from multiple measurements.

The use of camera motion alongside inertial sensors has been considered for aerial and ground applications [26], [27]. In previous work [5], the use of optical flow as a low-frequency complementary measure for aiding high-frequency inertial measurements was explored and found to estimate orientation as long as it was sufficiently initialized. Figure 3 illustrates the integrative approach that is used to limit drift. First optical flow is calculated from sparse features. Inertial data are used to determine the potential energy state, which comes from the vertical position. This is then used to select the mode. An EKF estimator is then used to calculate the state estimates.

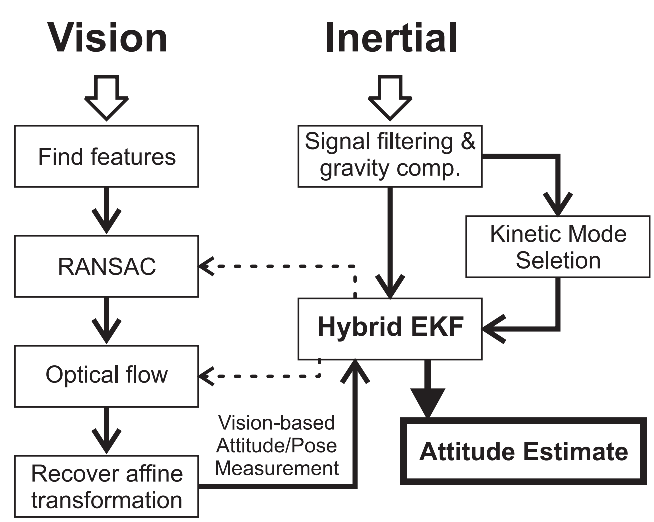

Fig. 3. Overview of the processing method in which inertial and video data are processed. The dashed lines indicate feedback paths for tuning vision algorithm parameters based on prior estimated motion.

A. Model

Based on the kinetic map, the trotting motion was modeled using two modes (flight and single sprung leg) with the gallop adding a third mode (double contact). In particular, these phases are a ballistic flight phase with a linear air friction model, a sprung leg having a stiffness equivalent to that for when leg or pair of legs in contact, and a lumped mass model for the periods of double support.

A full derivation of the models is presented in Ref. [4], [15], [18]. Approximate models are used to simplify the calculations and later linearizations needed by the EKF.

During flight phase, for instance, the measurement model covariances are tunned to account for the lack meaningful accelerometer measurements during free-fall. The system model, shown in Eq. 3 uses a linearized differential equation in height z with gravitational acceleration g, air density ρ, and ballistic coefficient β. As β is found empirically, a tuned flight phase coefficient k_{fp} can be used for the ratio of ρ to β. Note that if $\beta g \gg \rho \dot{z}$ air friction is negligible, giving the expected ballistic result.

$$\ddot{z} = \frac{\rho \dot{z}^2}{2\beta} - g \approx k_{fp}\dot{z} - g \tag{3}$$

During single contact the gait is modeled using SLIP conditions. This assumes that the effective (or "virtual") leg acts under the center of mass. When linearized, this becomes even more idealized as it approximates a lump mass with Hookean spring having an effective stiffness k_s as

$$\ddot{z} = \omega_0^2 (z_0 - z) - g \tag{4}$$

where ω_0^2 is k_s/m and z_0 is the height corresponding to unloaded ground contact.

During double support the attitude of the body becomes important. Double support can be modeled using the impulse method, but is non-linear [18]. Using a small angle assumption and linearizing Newtonian mechanics gives the simplest governing equations as:

$$\ddot{z} = \frac{(f_1 + f_2)}{m} - g \tag{5}$$

$$\ddot{\theta} = b f_2 - a f_1 \tag{6}$$

where f_1 and f_2 are the vertical contact forces at the legs and a and b are the moment arms from the shoulder to the mass center. The contact forces can be approximated by measuring the leg compression or stroke. Sideways movement (y) is small because an no-slip condition is assumed. For the cycle, forward speed ($\dot{x}$) is bounded by the constraint in Eq. 1.

B. Hybrid EKF Estimation Techniques

Estimation is a process for calculating system variables from measurement source(s). The Hybrid EKF is a state-space approach that is optimal in a least-squares sense under the (strict) assumption of white, mutually independent linear environments [28]. Alternative estimation algorithms, such as the Unscented KF or Particle filter, provide better linearization; however, their use has to be balanced against computational resources and the update rates required.

Using the notation adopted in previous work [3], we define $\mathbf{x}$ as the target state vector, $\mathbf{F}$ as the system dynamics matrix, $\mathbf{H}$ as the measurement matrix, and $\mathbf{v}$ and $\mathbf{w}$ as the process and measurement noise vectors respectively. Thus, the system can be modeled as $\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{v}$, and the measurement as $\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{w}$. Hybrid models can be implemented as a function that smoothly varies $\mathbf{F}$.

VI. Implementation

The goals of this method are to obtain state estimates robust to the eccentricities present in legged locomotion. To do this, the technique was extended to the KOLT robot, where the principal task is to estimate attitude (especially pitch) for use in its trot and galloping controllers. The implemented estimator operates at peak rates of 250 Hz. In practice, this was often run at half the rate to free resources.

A. KOLT

Pictured in Fig. 4, the Kinetically Ordered Locomotive Tetrapod (KOLT) robot is a testbed for dynamic legged locomotion theory with application to high-capacity legged robots. Its four identical 3-DOF legs are fully actuated. Its speed presents a significant challenge as control simulations indicate the need for rapid pitch feedback at rates >50 Hz.

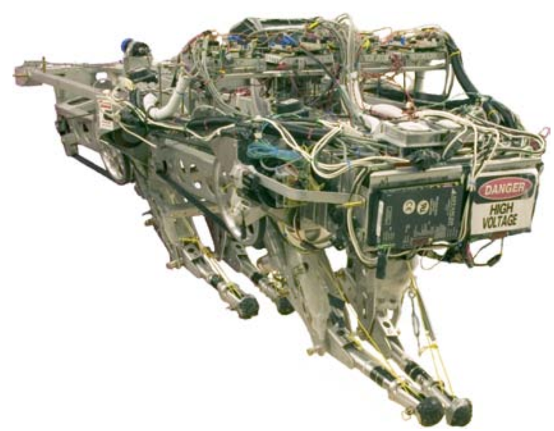

Fig. 4. The KOLT robot is ~2 m long and 75 kg in weight.

B. Configuration

KOLT performance is measured using a custom inertial sensor suite consisting of commercial, micromachined accelerometers (Kionix KMX52-1050 and ADXL210 respectively for tracking translational motion and determining phase transitions through impulse shocks) and gyros (three Silicon Sensing CRS03-11). Data sets are captured using a Kteam Kameleon board. Video is recorded from a TV-format camera (Pulnix TMC with a Pentax 4.8 mm lens) using a Bt848a video capture card (chipset) at 320×240 pixels resolution and 30 fps.

As the Kameleon has no provision for video, it was processed using a separate PC. The original video is deinterlaced and converted to grayscale. The feature detector and RANSAC selection algorithm were tuned to typically net 10 to 15 features. One advantage is that flow from

RANSAC points can be computer with stricter criteria, with little penalty in computation time. The features are then tracked using a pyramidal implementation of the Lucas-Kanade algorithm with a nominal depth of three. The optical flow is used to define feature pairs that are accumulated over time to give the ego-motion.

VII. Experiments and Results

Experiments were performed on KOLT to evaluate the performance of the HEKF method. To facilitate comparison and to ensure safe robot operation, the robot was connected to an instrumented boom arm. The arm is 2.75 *m* long has has 3DOF (pitch along the axis plus roll and yaw about the center post). For these experiments, data from precision encoders (6,000 count) on the boom arm were considered to be the control values (i.e., arm and KOLT coupling flexion are assumed to be negligible). The large boom arm radius resulted in small pose changes per sample, especially for both yaw. Synchronization of the control was made by having KOLT record boom encoder data.

The experiments were performed for bound and trot gaits that were programed using the symmetric and virtual leg methodologies. As current research is refining a sustained galloping controller, data for the gallop are not presented. To increase resources available for control, the estimator was simplified for KOLT operation as only to track attitude values. Thus, the estimator's state space had of 9-terms (all three DOF for 3D motion, plus their first derivatives, plus their biases).

The results of the HEKF estimator for a typical trotting experiment are shown in Figs. 5 and 6. For comparison with the encoders the estimated state values were transformed to the boom arm origin frame. The gyro covariance and initial bias value was found through a calibration procedure. Many experiments were performed for short periods of time (∼1 minute), yet the inertial drift, if unchecked, would have exceeded practical limits (i.e., greater than 90 deg.).

For dynamic trotting motion, such as that shown in Fig. 6, the HEKF estimator has an error of approximately 5 deg. RMS. This large an error might seem surprising, but can be attributed to errors in the inertial measurements which lead to biases in the estimate. Further, when the inertial data are significantly in error, the HEKF is not able to adapt rapidly to become more reliant on the aided (visual) data. Tuning the HEKF for this would lead to a case where the estimator over weights the importance of the visual data, which will lead to the estimates lagging (due to the delay of the visual measurements). At an extreme, this is equivalent to operating without inertial measurements.

The use of optical flow does not need an *a priori* ground model assumption as would be associated with leg or range pose recovery methods. However, it adds delay, which limits its use in control applications. Perhaps with emerging high-speed, low-light, low-noise cameras and dedicated visual processors, such processing will soon be available in an integrated package. The optical flow is not a complete observer of all motion. For example, it is prone to errors resulting from

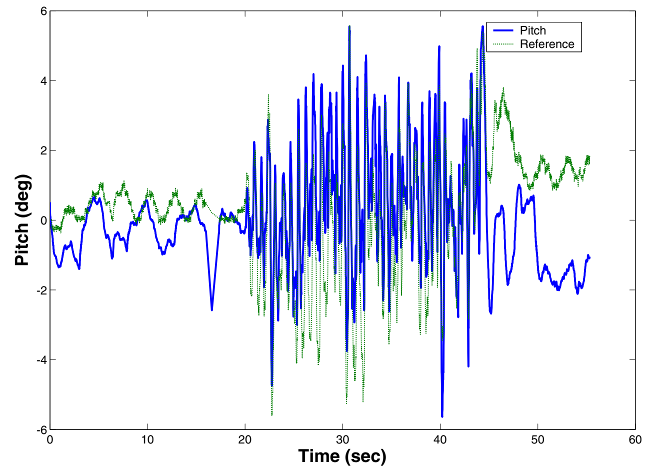

Fig. 5. The pitch data from the HEKF inertial estimator and from the reference encoder on the boom arm show stable estimator performance over 36 cycles (during ∼20 seconds of running). The loss of balance, as seen during the final landing, is poorly compensated by the estimator.

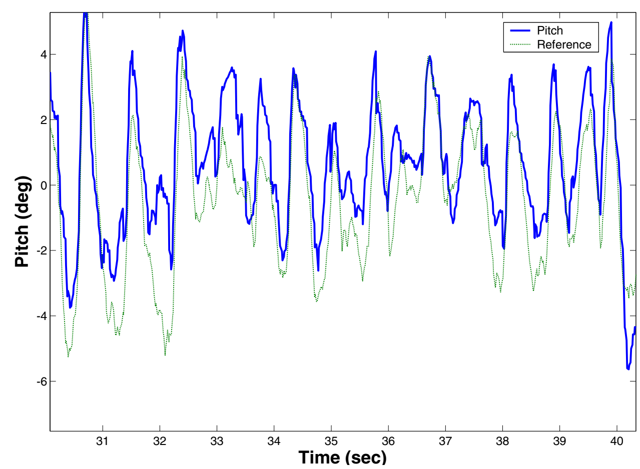

Fig. 6. A sub-section of the trotting pitch data. The tracking performance is improved compared to general motion, especially for rapid positive (nose upwards) pitch motions.

the "aperture problem" and, in a monocular arrangement, is not able to disambiguate small changes in attitude from small translations. Further, the current implementation makes use of a brightness constancy assumption which limits operation to areas where light levels are constant.

VIII. Conclusions

Due to the discontinuities, dynamic legged locomotion is a unique domain separate from aerial or wheeled vehicles. This is treated using a hybrid model that models the trot as consisting of two dynamic modes and the gallop as three. Modes are transitioned using an energy based metric. This builds on and is consistent with prior work in this field, though adopts a kinetic instead of kinematic framework for hybrid transitions.

As measured by pitch excursion changes, the performance of the estimator on the quadruped is sound with the estimator converging. In fairness, the trot gait is a more stable motion that is compatible with the constant angular velocity (no torque) flight phase assumption.

To an extent, this result is somewhat expected as the hybrid model incorporates more information and is a prudent means of capturing the discrete dynamics in an implementation. The interesting result is this also suggests that an efficient cycle may be constructed with only three instead of the five states suggested by Raibert or the 120 possible transitions present.

The experiment contributes a quantification of KOLT performance. It also shows that modifying HEKF estimation techniques to include characteristics unique to trotting or galloping legged movement results in stable self-contained attitude tracking with low latency and fast updates, which would not be possible with one sensing modality alone.

IX. Future Work

A limiting issue with this and other estimation approaches is the need for several tuning values for the stiffness, damping, estimator covariances, optical flow feature finder, and RANSAC amongst others. Future work is considering the extension of the estimator logic to calibrate these values robustly against ground truth values.

The models describing quadruped performance are simplistic by design so as to enable faster computation. Obviously, off-line applications can afford more processing. Thus, future work is considering the extension of the impulse methods to yield more complete, yet efficient, estimation solutions. Even with a more involved model, the estimator could act as a smoother instead of a filter, updating the state history when resources are free.

X. Acknowledgments

This research and paper are supported, in part, through a National Science Foundation Grant (No. IIS-0208664) and through a National Defense Science and Engineering Graduate (NDSEG) program fellowship. The co-author (S. Singh) also acknowledges the Fulbright Fellowship program for sponsoring his exchange to the University of Western Australia. The authors acknowledge the numerous contributions of the KOLT team members including Paul Csonka, Joaquin Estremera, J. Gordon Nichol, Prof. David Orin, and Luther Palmer.

References

[1] J. G. Nichol, S. P. N. Singh, K. J. Waldron, L. R. Palmer, and D. E. Orin, "System design of a quadrupedal galloping machine," *The International Journal of Robotics Research*, vol. 23, no. 10-11, pp. 1013–1027, 2004.

[2] P.-C. Lin, H. Komsuoglu, and D. E. Koditschek, "A leg configuration measurement system for full body pose estimates in a hexapod robot," *IEEE Transaction on Robotics*, vol. 21, no. 3, pp. 411–422, June 2005.

[3] S. P. N. Singh and K. J. Waldron, "Attitude estimation for dynamic legged locomotion using range and inertial sensors," in *Proceedings of the International Conference on Robotics and Automation (ICRA 2005)*, April 2005, pp. 1675–1680.

[4] M. H. Raibert, "Running with symmetry," *The International Journal of Robotics Research*, vol. 5, no. 4, pp. 3–19, 1986.

[5] S. P. N. Singh, P. J. Csonka, and K. J. Waldron, "Optical Flow Aided Motion Estimation for Legged Locomotion," in *Proceedings of the International Conference on Intelligent Robots and Systems (IROS)*, Oct 2006, pp. 1738–1743.

[6] T. B. Moeslund and E. Granum, "A survey of computer vision-based human motion capture," *Computer Vision and Image Understanding*, vol. 81, no. 3, pp. 231–268, 2001. [Online]. Available: citeseer.ist.psu.edu/moeslund01survey.html

[7] B. Parkinson, J. J. Spilker, Jr., P. Axelrad, and P. Enge, Eds., *Global positioning system : theory and applications*, ser. Progress in Astronautics and Aeronautics. Washington: American Institute of Aeronautics and Astronautics, 1996, vol. 163-164.

[8] C. Tomasi and T. Kanade, "Shape and Motion from Image Streams under Orthography: a Factorization Method," *International Journal of Computer Vision*, vol. 9, no. 2, pp. 137–154, November 1992.

[9] D. Strelow and S. Singh, "Optimal motion estimation from visual and inertial measurements," in *Proceedings of the Sixth IEEE Workshop on Applications of Computer Vision (WACV 2002)*, 2002, p. 314.

[10] S. Skaff, A. Rizzi, H. Choset, and P.-C. Lin, "A context-based state estimation technique for hybrid systems," in *Proceedings of the International Conference on Robotics and Automation (ICRA 2005)*, April 2005, pp. 3935–3940.

[11] H. Rehbinder and X. Hu, "Drift-free attitude estimation for accelerated rigid bodies," *Automatica*, vol. 40, no. 4, p. 183, 2004.

[12] P.-C. Lin, H. Komsuoglu, and D. E. Koditschek, "Sensor data fusion for body state estimation for a hexapod robot with dynamical gaits," in *Proceedings of the International Conference on Robotics and Automation (ICRA 2005)*, April 2005, pp. 4744–4749.

[13] M. H. Raibert, "Trotting, pacing and bounding by a quadruped robot." *Journal of biomechanics*, vol. 23, pp. 79–98, 1990.

[14] M. D. Berkemeier, "Modeling the Dynamics of Quadrupedal Running," *The International Journal of Robotics Research*, vol. 17, no. 9, pp. 971–985, 1998.

[15] J. E. Seipel and P. Holmes, "Running in three dimensions: Analysis of a point-mass sprung-leg model," *The International Journal of Robotics Research*, vol. 24, no. 8, pp. 657–674, 2005.

[16] P. Holmes, "Poincare, Celestial mechanics, Dynamical-Systems Theory and 'Chaos'," *Physics Reports*, vol. 193, no. 3, pp. 137–63, Sep 1990.

[17] L. Palmer and D. Orin, "Control of a 3D quadruped trot," in *Proceedings of the 8th International Conference on Climbing and Walking Robots (CLAWAR)*, 2005.

[18] K. J. Waldron and V. Kallem, "Control modes for a three-dimensional galloping machine," in *ASME Design Automation Conference (DETC 2004)*, no. DETC2004-57587, 2004, design Engineering Technical Conferences.

[19] J. G. Nichol, "Design for energy loss and energy control in a galloping artificial quadruped," Ph.D. dissertation, Stanford University, 2005.

[20] D. Hoyt and C. Taylor, "Gait and the energetics of locomotion in horses," *Nature*, vol. 292, no. 5820, pp. 239–240, 1981.

[21] A. E. Minetti, "The biomechanics of skipping gaits: a third locomotion paradigm?" *Proceedings of the Royal Society of London. Series B*, vol. 265, no. 1402, pp. 1227–1235, July 1998.

[22] M. Hildebrand, "Analysis of asymmetrical gaits," *Journal of Mammalogy*, vol. 58, no. 2, pp. 131–156, May 1977.

[23] J. Gray, *Animal locomotion*. New York: Norton, 1968.

[24] N. R. Deuel and L. M. Lawrence, "Kinematics of the equine transverse gallop," *Journal of Equine Veterinary Science*, vol. 7, no. 6, pp. 375–382, 1987.

[25] R. M. Alexander, "Terrestrial locomotion," in *Mechanics and Energetics of Animal Locomotion*, R. M. Alexander and G. Goldspink, Eds. London: Chapman and Hall, 1977, ch. 5, pp. 168–203.

[26] P. Corke, "An inertial and visual sensing system for a small autonomous helicopter," *Journal of Robotic Systems*, vol. 21, no. 2, pp. 43–51, Feb 2004.

[27] J. Kim and S. Sukkarieh, "Complementary SLAM aided GPS/INS navigation in GNSS denied and unknown environments," in *Proceedings of the International Symposium on GNSS/GPS*, Dec 2004, pp. 1–6.

[28] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman filter: Particle Filters for Tracking Applications*. Boston: Artech House, 2004.