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A New Control Method Utilizing Stiffness Adjustment of Mechanical Elastic Elements for Serial Link Systems

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Abstract— **This paper proposes a tracking control method of sinusoidal motions utilizing stiffness adjustment of mechanical elastic elements for serial link systems. Although dynamics of the controlled objects is nonlinear, the stiffness adjustment realize a condition similar to a resonance of linear systems. We present a controller that adjusts stiffness of the elastic elements to reduce torque requirement of actuators while generating desired motions. The proposed controller works without using dynamics models nor parameters of the controlled objects. Stability of the controller is proved, and tracking errors are guaranteed to converge to a certain region. Simulation results demonstrate the validity of the proposed method. We also present an application of the proposed method to power assist systems.**

Index Terms— **Resonance, Robot Dynamics, Power Assist System**

I. Introduction

Recently, control methods and applications utilizing passive elements come under spotlight [1]-[4]. Elastic torque caused by gravity or springs can generate periodic motions without actuation. Passive walking robots can generate fundamental motions of walking by gravitational torque [1] [2]. Walsh *et al.* developed walking support systems utilizing springs and adjustable damping elements to reduce torque of actuators [3]. Fujimoto *et al.* proposed a control method based on iterative learning control for hopping robots [4]. This control method generates hopping motions, which require no actuation in steady states. In these studies, stiffness of the elastic torque is fixed, and motions are not controlled specifically.

However, in the case of applications to walking robots or human support systems such as prostheses, tracking control of periodical motions and specification of its properties such as frequency becomes important in some cases. For example, some power assist systems generate periodical desired motions based on information of its operator [5] [6]. Stiffness adjustment is one of the realistic ways to generate specified periodical motions while reducing actuation. Ozawa *et al.* proposed a control method to track specified sinusoidal motions utilizing elastic elements [7]. This method optimizes stiffness to realize an anti-resonance, which enables no energy consumption to track specified motions in steady states. However, in their study, controlled objects are restricted to linear systems containing two masses and two springs.

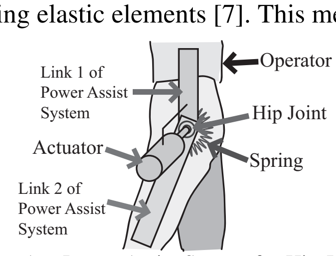

Fig. 1. Power Assist System for Hip Joint

We have proposed power assist systems like **Fig.1** that amplify the sinusoidal operator's torque while generating sinusoidal motions by minimum actuator's torque utilizing a resonance [5] [8] [9]. This method is assumed to be used as walking support systems of hip joints because there are some Clinical Gate Analysis (CGA) data show that torque and motion of hip joint are nearly sinusoidal [10]. However, even though the dynamics of the walking support systems of hip joints is nonlinear, our previous paper treated the dynamics of the controlled object as linear [5] [8] [9].

As stated above, stiffness adjustment has been considered for linear systems, because the resonance and the anti-resonance are concepts of linear systems originally.

This paper proposes a control method utilizing stiffness adjustment for serial link systems having nonlinear dynamics as shown in **Fig.2**. Mechanical elastic elements are installed in each joint of the model. We mathematically prove stability of a proposed controller and an effect of the stiffness adjustment. The proposed controller guarantees a region of tracking errors when time is consumed enough. The stiffness adjustment minimizes this region as if an optimal stiffness is realized even the controlled objects have nonlinear dynamics. As a result, inertial and gravity torque are largely compensated by torque of the adjusted stiffness like a resonance of linear systems.

In this paper, desired motions are assumed to be given firstly. In this case, dynamics models and all parameters of the controlled objects can be unknown, because we adopt a simple feedback controller. This controller guarantees tracking performance like a control method as stated in [11].

Secondary, the proposed method in this paper is combined with a method of our power assist systems [5] [8]. In this case, we propose a controller that simultaneously realize torque amplification and the stiffness adjustment. Due to a necessity of calculation of operator's torque, dynamics

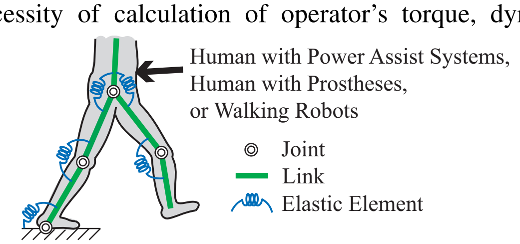

Fig. 2. Model of Serial Link System

model and all parameters of the model are assumed to be known. However, amplitudes, frequencies, and phases can be unknown to adapt change of operator's torque pattern. The model of Fig.2 is more precise than the linear model of our previous study to represent a model of walking support systems. Therefore, the method of this paper extends our previous power assist systems for support systems of whole walking motion.

II. System Including Nonlinear Stiffness

In this section, we consider a 1-dof pendulum system shown in **Fig.3** as an example of serial link systems. This system includes nonlinear elastic torque caused by gravity.

This section shows a basic concept of the proposed method.

A. Dynamics

The dynamics of the 1-dof pendulum system can be described like this.

$$I\ddot{q} = -d\dot{q} - g\sin q - kq + \tau \qquad (1)$$

where q is an angle of the pendulum, I is an inertia, d is a viscosity, g is a constant of gravitational torque, k is an adjustable stiffness, and τ is torque of an actuator.

All physical parameters I, d, g are assumed to be unknown.

B. Control Objective

The control objective is to let the joint angle q track a sinusoidal desired trajectory $q_d = a\sin(\omega t+\phi)+a_p\pi$ and to reduce the torque of the actuator τ by adjusting the stiffness k, where a, ω, ϕ are an amplitude, an angular frequency, a phase of the desired trajectory respectively, and a_p is a constant, which is set to be 0 or 1. The amplitude a is assumed to satisfy the inequality $|a| < \pi - a_r$, where $a_r < \pi$ is a positive constant. Hence, the desired motion is a simple harmonic motion that the gravitational torque of its center is 0.

C. Optimal Stiffness

Here, let us consider necessary torque of the actuator τ_d to generate the desired motion. This necessary torque is calculated by substituting the desired trajectory q_d into the angle q of the dynamics (1).

$$\tau_d = I\ddot{q}_d + d\dot{q}_d + kq_d + g\sin q_d \qquad (2)$$

If the elasticity of the gravitational torque $g\sin q_d$ has a linear characteristic like gq_d, the right side of the inertial torque $I\ddot{q}_d$ and the elastic torque $kq_d + gq_d$ can vanish by an optimal stiffness of a resonant condition $k = I\omega^2 - g$ [8]. This means that an amplitude of the necessary torque τ_d is minimized.

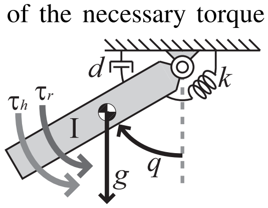

Fig. 3. Pendulum

The viscous torque $d\dot{q}_d$ can't be compensated by the torque of the stiffness kq_d because of requirement of energy supply.

Even though the vanishment can't be realized in the case of the nonlinear dynamics, we can define an optimal stiffness k_* that minimizes maximum of the value $|I\ddot{q}_d + k_* q_d + g\sin q_d|$. This means that the inertial and the elastic torque to generate the desired motion $I\ddot{q}_d + g\sin q_d$ are compensated by the torque of the optimal stiffness $k_* q_d$ as much as possible. This concept is similar to a resonance of linear systems.

D. Controller

1) Torque of Actuator τ: The torque of the actuator τ is designed using feedback of tracking errors with a viscosity compensation.

$$\tau = \hat{d}\dot{q}_d - k_v\Delta\dot{q} - k_p\Delta q \qquad (3)$$

where $\hat{d}$ is an estimated value of the viscosity d, k_p, k_v are error feedback gains of the angle and the angular velocity respectively, and $\Delta q = q - q_d$ is a tracking error of the angle.

The estimation of the viscosity is done by the adaptive technique.

$$\dot{\hat{d}} = -\gamma_d\dot{q}_d(\Delta\dot{q} + c\Delta q) \qquad (4)$$

where γ_d is an adaptive gain, and c is a constant selected to satisfy the inequality $d + k_v > cI$.

2) Adjustment Law of Stiffness k: To reduce the actuator's torque, an adjustment law of the stiffness is designed like this.

$$\dot{k} = \gamma_k q(\Delta\dot{q} + c\Delta q) \qquad (5)$$

where γ_k is an adaptive gain.

Similar adjustment law was proposed in our previous work for linear systems [5]. In the case of the linear systems, controlled systems become in a resonant condition by the stiffness adjustment, and an amplitude of actuator's torque is minimized in a steady state.

E. Stability

Stability of the proposed controller is proved by using a candidate of a Lyapnov function V.

$$\begin{aligned} 2V &= I\Delta\dot{q}^2 + (k_p + k_* + ck_v + cd)\Delta q^2 \\ &\quad +2cI\Delta\dot{q}\Delta q + \gamma_d^{-1}\Delta d^2 + \gamma_k^{-1}\Delta k^2 && (6) \\ \dot{V} &= -(d + k_v - cI)\Delta\dot{q}^2 - c(k_p + k_*)\Delta q^2 \\ &\quad +b(k)(\Delta\dot{q} + c\Delta q) && (7) \\ &\leq -b_1(b)\Delta\dot{q}^2 - b_2(b)\Delta q^2 + b_3 b^2 && (8) \\ b &= -g\sin q - k_* q_d - I\ddot{q}_d && (9) \end{aligned}$$

where $\Delta d = \hat{d} - d$, $\Delta k = k - k_*$, $b_1(b) = k_v - cI - \alpha_1 b^2$, $b_2(b) = c(k_p + k_* - \alpha_2 b^2)$, $b_3 = \frac{1}{\alpha_1} + \frac{c}{\alpha_2}$, and α_1, α_2 are positive constants introduced to prove the stability. Because the desired trajectory is a sinusoid and the gravitational torque $g\sin q$ is bounded $|g\sin q| < g$, there exists a constant b_m that satisfy the equation $b_m = \max|b|$. Then, the parameters $b_1(b_m), b_2(b_m)$ can be positive by adequate choices of the parameters α_1, α_2. Therefore, $\dot{V}$ is negative and V decreases while the following inequality is satisfied.

$$b_1(b_m)\Delta\dot{q}^2 + b_2(b_m)\Delta q^2 > b_3 b_m^2 \qquad (10)$$

This means that the tracking errors $\Delta\dot{q}^2, \Delta q^2$ decreases together with decrease of V until the inequality (10) is not satisfied. Therefore, the tracking errors converge to vicinity of a certain region $b_1(b_m)\Delta\dot{q}^2 + b_2(b_m)\Delta q^2 \leq b_3 b_m^2$ when $t \to \infty$. To avoid complex discussion, an accurate guaranteed region of the tracking errors is not shown in this paper, but the accurate region becomes smaller if the region $b_1(b_m)\Delta\dot{q}^2 + b_2(b_m)\Delta q^2 \leq b_3 b_m^2$ becomes smaller.

Using similar discussion to the paper [11], the region of tracking errors can be reduced by choices of larger error feedback gains k_p, k_v, because the parameters $b_1(b_m), b_2(b_m)$ becomes larger by the choices of larger gains k_p, k_v.

Therefore, the system is stabilized by the proposed controller (3), (4), (5).

F. Effect of Stiffness Adjustment

The purpose of the stiffness adjustment is to reduce the torque of actuator (3) while achieving the tracking control. The term $\hat{d}\dot{q}_d$ of the actuator's torque (3) compensates the viscous torque, and this term is needed to generate the desired motion as stated in the section II-C. Hence, the feedback terms of the controller (3) compensate the inertial and the gravitational torque to generate the desired motion. If the tracking errors $\Delta\dot{q}, \Delta q$ become smaller by the stiffness adjustment, the feedback terms become smaller. This means that the inertial and the elastic torque are largely compensated by the elastic torque of the adjusted stiffness kq. Therefore, the tracking errors are discussed here in detail to show the effect of the stiffness adjustment.

To discuss the tracking errors more precisely, we focus on the elastic torque $g\sin q + k_* q_d$ of the equation (9). The elastic torque $g\sin q + k_* q_d$ is merely treated as bounded values in the section II-E. However, this torque seems to contribute generation of sinusoidal motion partially, because elastic torque usually generate periodical motion. Hence, smaller region of the tracking errors may be calculated by focusing on the elastic torque.

Based on the discussion of the section II-E, if the third term of the (7) can be smaller, the guaranteed region of the tracking errors can be smaller. The parameter b of this term can be rewritten like $b = g(\sin q - \sin q_d) + (g\sin q_d - k_* q_d - I\ddot{q}_d)$. Hence, the third term $b(\Delta\dot{q} + c\Delta q)$ can be decomposed into the three terms $-gc(\sin q - \sin q_d)\Delta q$, $-g(\sin q - \sin q_d)\Delta\dot{q}$, $(-g\sin q_d - kq_d - I\ddot{q}_d)(\Delta\dot{q} + c\Delta q)$.

1) Terms of $b(\Delta\dot{q} + c\Delta q)$: In the following discussion, we consider a situation that the tracking error $|\Delta q|$ is smaller than a_r. This situation can be realized easily by choices of large feedback gains k_p, k_v as discussed in the section II-E.

The term $-gc(\sin q - \sin q_d)\Delta q$ is negative in the case of $a_p = 0$, because the sign of $\sin q - \sin q_d$ and Δq are the same in the situation $|\Delta q| < a_r$ and $|q_d| < \pi - a_r$. In the case of $a_p = 1$, this term becomes positive, and the inequality $-gc(\sin q - \sin q_d)\Delta q \leq g\Delta q^2$ is satisfied. Therefore, this term can be calculated as $-gc(\sin q - \sin q_d)\Delta q \leq a_p g\Delta q^2$

The term $-g(\sin q - \sin q_d)\Delta\dot{q}$ is calculated as a cross term of the tracking errors Δq^2 and $\Delta\dot{q}^2$ like this.

$$-2g(\sin q - \sin q_d)\Delta\dot{q} \leq g\left(\alpha_3\Delta q^2 + \alpha_3^{-1}\Delta\dot{q}^2\right) \quad (11)$$

where α_3 is a constant introduced to prove the effect of the stiffness adjustment.

On the other hand, if an elastic torque is linear like $g_l q$, the value $g_l(q - q_d)\Delta\dot{q}$ becomes a time derivative of the storage function $V_g = g_l\Delta q^2$, where the g_l is a stiffness. The nonlinear stiffness $g\sin q$ have similar characteristic if the angle q is small enough. Hence, a part of the term $-g(\sin q - \sin q_d)\Delta\dot{q}$ is composed of the time derivative of the storage function V_g, and the term $-g(\sin q - \sin q_d)\Delta\dot{q}$ can be calculated to reduce the effect of the term.

$$\begin{aligned} &-2g(\sin q - \sin q_d)\Delta\dot{q} + \dot{V}_g \\ &\leq (g - g_l)\left(\alpha_3\Delta\dot{q}^2 + \alpha_3^{-1}\Delta q^2\right) \end{aligned} \quad (12)$$

where g_l is a maximum constant satisfying the inequality (12).

The term $(-g\sin q_d - k_* q_d - I\ddot{q}_d)(\Delta\dot{q} + c\Delta q)$ satisfies the following inequality.

$$\begin{aligned} &2(-g\sin q_d - k_* q_d - I\ddot{q}_d)(\Delta\dot{q} + c\Delta q) \\ \leq\ &\left\{\alpha_4\Delta\dot{q}^2 + \alpha_5 c\Delta q^2 + \left(\alpha_4^{-1} + \alpha_5^{-1}c\right)n_1^2\right\} \end{aligned} \quad (13)$$

where n_1 is a constant that satisfies the equation $n_1 = \max| - g\sin q_d - k_* q_d - I\ddot{q}_d|$, and α_4, α_5 are constants introduced to prove the effect of the stiffness adjustment.

2) Region of Tracking Errors: As the result of the above discussion, the region of the tracking errors can be guaranteed as follows.

A candidate of a Lyapnov function V_2 is defined like this.

$$V_2 = V + V_g \quad (14)$$

$$\dot{V}_2 \leq -b_5\Delta\dot{q}^2 - b_6\Delta q^2 + b_7 n_1^2 \quad (15)$$

where $b_5 = d + k_v - cI - \frac{\alpha_3(g-g_l)}{2} - \frac{\alpha_4}{2}$, $b_6 = c(k_p + k_*) - a_p g - \frac{(g-g_l)}{2\alpha_3} - \frac{\alpha_5 c}{2}$, $b_7 = \frac{1}{2\alpha_4} + \frac{c}{2\alpha_5}$.

Therefore, the tracking errors decrease until the inequality $b_5\Delta\dot{q}^2 + b_6\Delta q^2 > b_7 n_1^2$ will not be satisfied, and the tracking errors converge to the vicinity of the region $b_5\Delta\dot{q}^2 + b_6\Delta q^2 \leq b_7 n_1^2$.

Then, the region of the tracking errors $b_5\Delta\dot{q}^2 + b_6\Delta q^2 \leq b_7 n_1^2$ are minimized, because k_* minimizes the equation $n_1 = |I\ddot{q}_d + k_* q_d + g\sin q_d|$ as stated in the section II-C.

Therefore, the region of the torque of the actuator is minimized by the stiffness adjustment. This means that the inertial torque $I\ddot{q}_d$ and the gravitational torque $g\sin q_d$ to generate the desired motion q_d are largely compensated by the torque of the stiffness kq_d.

G. Summary

For the system, which has the dynamics (1) including the nonlinear gravitational torque, the controller (3), (4), (5) guarantees the stability, and the stiffness adjustment (5) realizes the effect of the stiffness optimization. This means that the stiffness adjustment minimize the guaranteed region of the tracking errors, and the inertial and the gravitational torque to generate the desired motion are largely compensated by the torque of the adjusted stiffness. The proposed controller uses no dynamics models and no parameters of the dynamics owing to the simple structure of the controller.

III. Serial Link System

This section proposes a similar controller to that of the section II for serial link systems like Fig.2.

A. Dynamics

Dynamics of the serial link systems having n joints is described by the following equation.

$$R(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left\{\frac{1}{2}\dot{R}(\boldsymbol{q}) + S(\boldsymbol{q}, \dot{\boldsymbol{q}}) + D\right\}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = -K\boldsymbol{q} + \boldsymbol{\tau} \quad (16)$$

where $R(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \Re^{n\times n}$ is a positive definite inertia matrix, $S(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \Re^{n\times n}$ is a skew symmetric matrix, $D = \mathrm{diag}(d_1 \cdots d_n)$, $d_1 \cdots d_n$ are viscosity, $\boldsymbol{g}(\boldsymbol{q}) \in \Re^n$ is a vector of gravitational torque, $K = \mathrm{diag}(k_1 \cdots k_n)$ is a stiffness matrix, $k_1 \cdots k_n$ are adjustable stiffness of elastic elements installed in each joint, $\boldsymbol{q} = (q_1 \cdots q_n)^T$ is a vector of joint angles, and $\boldsymbol{\tau} = (\tau_1 \cdots \tau_2)^T$ is a vector of torque of actuators.

B. Controller

The controller is designed using similar strategy to that of the section II.

$$\boldsymbol{\tau} = \hat{D}\dot{\boldsymbol{q}}_{\boldsymbol{d}} - K_v\Delta\dot{\boldsymbol{q}} - K_p\boldsymbol{s}(\Delta\boldsymbol{q}) \quad (17)$$

$$\dot{\hat{\boldsymbol{d}}} = -\Gamma_d\dot{Q}_d(\Delta\dot{\boldsymbol{q}} + C\Delta\boldsymbol{q}) \quad (18)$$

$$\dot{\boldsymbol{k}} = \Gamma_k Q\left\{\Delta\dot{\boldsymbol{q}} + C\boldsymbol{s}(\Delta\boldsymbol{q})\right\} \quad (19)$$

where $\hat{D} = \mathrm{diag}(\hat{d}_1 \cdots \hat{d}_n)$ is an estimated matrix of D, $K_p = \mathrm{diag}(k_{p1} \cdots k_{pn})$, $K_v = \mathrm{diag}(k_{v1} \cdots k_{vn})$ are matrixes of feedback gains, $\Delta\boldsymbol{q} = (\Delta q_1 \cdots \Delta q_n)^T = \boldsymbol{q} - \boldsymbol{q_d}$, $\boldsymbol{q_d} = (q_{d1} \cdots q_{dn})^T$ is a vector of desired trajectories, $\boldsymbol{s}(\Delta\boldsymbol{q}) = (s_1(\Delta q_1) \cdots s_n(\Delta q_n))^T$, $s_1() \cdots s_n() \in \Re$ are saturated functions defined in a book [12], $\Gamma_d \in \Re^{n\times n}$ is a positive definite matrix of adaptive gains, $\hat{\boldsymbol{d}} = (\hat{d}_1 \cdots \hat{d}_n)^T$, $Q_d = \mathrm{diag}(q_{d1} \cdots q_{dn})$, $C = \mathrm{diag}(c_1 \cdots c_n)$, $c_1 \cdots c_n$ are positive constants, $\boldsymbol{k} = (k_1 \cdots k_n)^T$, $\Gamma_k \in \Re^{n\times n}$ is a positive definite matrix of adaptive gains, and $Q = \mathrm{diag}(q_1 \cdots q_n)$.

The desired motion is set to be $q_{di} = a_i \sin(\omega_i t + \phi_i) + a_{pi}\pi \quad (i = 1 \cdots n)$, where a_i, ω_i, ϕ_i are amplitudes, angular frequencies, phases, and a_{pi} are set to be 0 or 1. In order for the links not to exceed the vertical direction of gravitational force, the desired motion has to satisfy the condition $|\sum_{j=1}^{i} q_{dj}| < \pi - a_{ri} \quad (i = 1 \cdots n)$, where a_{ri} are positive constants. Most walking motions satisfy this condition.

C. Stability

The candidate of a Lyapnov function V_r can be defined using passivity of error dynamics [12] and similar discussion to the section II-F.

$$\begin{aligned} 2V_r &= \Delta\dot{\boldsymbol{q}}^T R(\boldsymbol{q})\Delta\dot{\boldsymbol{q}} + \Delta\boldsymbol{q}^T(K + CK_v + CD + G_l)\Delta\boldsymbol{q} \\ &\quad + \boldsymbol{s}(\Delta\boldsymbol{q})^T K_p \boldsymbol{s}(\Delta\boldsymbol{q}) + 2\boldsymbol{s}(\Delta\boldsymbol{q})^T CR(\boldsymbol{q})\Delta\dot{\boldsymbol{q}} \\ &\quad + \Delta\boldsymbol{d}^T\Gamma_d^{-1}\Delta\boldsymbol{d} + \Delta\boldsymbol{k}^T\Gamma_k^{-1}\Delta\boldsymbol{k} \end{aligned} \quad (20)$$

$$\dot{V}_r \leq -b_{10}||\Delta\dot{\boldsymbol{q}}|| - b_{11}||\boldsymbol{s}(\Delta\boldsymbol{q})|| + b_{12}(\boldsymbol{k}_*) \quad (21)$$

where $\Delta\boldsymbol{d} = \hat{\boldsymbol{d}} - \boldsymbol{d}$, $\boldsymbol{d} = (d_1 \cdots d_n)^T$, $\Delta\boldsymbol{k} = \boldsymbol{k} - \boldsymbol{k}_*$, $\boldsymbol{k}_* = (k_{*1} \cdots k_{*n})$ are constants, $b_8, b_9, b_{10}, b_{11}, b_{12}(\boldsymbol{k}_*)$ are positive constants, $G_l = \mathrm{diag}(g_{l1} \cdots g_{ln})$, and $g_{l1} \cdots g_{ln}$ are constants playing the same role as the g_l in the section II-F.1.

Therefore, the tracking errors $\Delta\dot{\boldsymbol{q}}, \Delta\boldsymbol{q}$ converges to a vicinity of the region $b_{10}||\Delta\dot{\boldsymbol{q}}|| + b_{11}||\boldsymbol{s}(\Delta\boldsymbol{q})|| \leq b_{12}(\boldsymbol{k}_*)$.

D. Effect of Stiffness Adjustment

An effect of the stiffness adjustment is also proved using similar discussion to the section II-F.2. We can define the stiffness $\boldsymbol{k}_*$ as a vector that minimizes the constant $b_{12}(\boldsymbol{k}_*)$. The optimal stiffness K_* can be defined as a matrix that minimizes maximum value of $||R(\boldsymbol{q}_d)\ddot{\boldsymbol{q}}_d + \left\{\frac{1}{2}\dot{R}(\boldsymbol{q}_d) + S(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d)\right\} + \boldsymbol{g}(\boldsymbol{q}_d) + K_* q_d||$. This concept is also similar to the resonance of linear systems. Therefore, the proposed controller (17), (18), (19) minimizes the region of the tracking errors as if the optimal stiffness is realized.

E. Simulation

We conducted a numerical simulation of a 2-link serial link system as shown in **Fig.4** to demonstrate validity of the proposed controller. This model is similar to a simplest walking model [2]. Hence, the q_1 is assumed to be an angle of an ankle joint, and the q_2 is assumed to be an angle of a hip joint.

The desired motion $\boldsymbol{q_d}$ was set to represent a half cycle of a walking motion like taking a step forward as shown in **Fig.5(a)**, **Fig.5(b)**. The equation (16) was adopted as a dynamics. The equations (17), (18), (19) were adopted as a controller.

Fig.5(a), Fig.5(b), **Fig.5(c)**, **Fig.5(d)** shows that angles and angular velocities nearly converged to the desired ones. The stiffness nearly converged to constant values with a little oscillation as shown in **Fig.5(e)**. The estimation of viscosity were almost achieved as shown in **Fig.5(f)**.

The necessary torque to generate the desired motion without viscous torque τ_{d1}, τ_{d2} were almost compensated by the torque of the stiffness $k_1 q_1, k_2 q_2$ after the convergence of the other variables as shown in **Fig.5(g)**, **Fig.5(h)**, where $(\tau_{d1}\tau_{d2})^T = R(\boldsymbol{q_d})\ddot{\boldsymbol{q}}_{\boldsymbol{d}} + \left\{\frac{1}{2}\dot{R}(\boldsymbol{q_d}) + S(\boldsymbol{q_d}, \dot{\boldsymbol{q}}_{\boldsymbol{d}})\right\}\dot{\boldsymbol{q}}_{\boldsymbol{d}} + \boldsymbol{g}(\boldsymbol{q_d})$. Hence, the actuator's torque τ_1, τ_2 were nearly necessary to compensate the viscous torque to generate the desired motion $d_1\dot{q}_{d1}, d_2\dot{q}_{d2}$, like the resonant condition of linear systems as shown in **Fig.5(i)**, **Fig.5(j)**. Therefore, the effect of the stiffness adjustment was verified through the simulation results.

F. Summary

The controller (17), (18), (19) guarantees the convergence of the tracking errors to the certain region for serial link

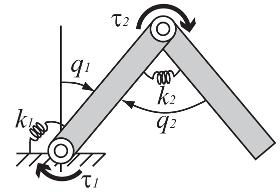

Fig. 4. Controlled Object of Simulation

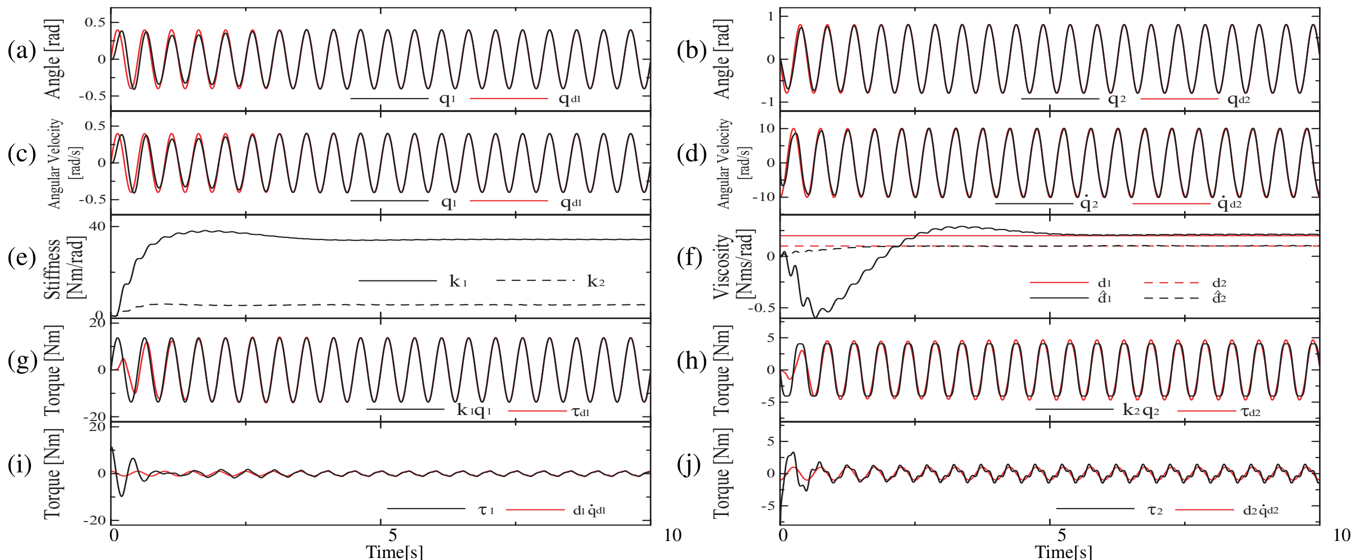

Fig. 5. Simulation Results

systems as shown in Fig.4. Therefore, the torque of the actuators converges to the certain region, and the stiffness adjustment (19) minimizes the region. The proposed controller uses no dynamics models and no parameters of the dynamics. The effectiveness of the controller is demonstrated by the simulation results.

IV. Application to Power Assist System

This section shows an application of the control method in this paper to power assist systems.

Walking motions of human have been studied a lot recently [10], and some assistive systems are proposed utilizing characteristics of the walking motion [5] [8] [3]. One of the characteristics is that some parts of walking motions and torque are composed of sinusoids [10] [3]. Hence, we have proposed power assist systems that amplify sinusoidal operator's torque while generating sinusoidal motions by minimum actuator's torque utilizing stiffness adjustment [5] [8]. Effectiveness of the power assist systems have been verified through some simulations and experiments [9]. However, our previous method is restricted to linear systems assisting 1 joint of its operator. The control method of this paper extends our previous power assist systems to nonlinear systems assisting some joints of its operator.

A. Dynamics

Dynamics of an exoskeleton type power assist system with its operator having n joints is similar to the serial link systems (16).

$$R(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left\{\frac{1}{2}\dot{R}(\boldsymbol{q}) + S(\boldsymbol{q},\dot{\boldsymbol{q}}) + D\right\}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = -K\boldsymbol{q} + \boldsymbol{\tau} + \boldsymbol{\tau_h} \quad (22)$$

where $\boldsymbol{\tau_h}$ is a vector of torque of the operator, and other vectors and matrixes are the same as the section III-A.

B. Assumption

The torque of the operator assumed to be sinusoidal.

$$\begin{aligned}\boldsymbol{\tau_h} &= (\tau_{h1}\cdots\tau_{hn})^T \\ &= (a_1\sin(\omega_1 t+\phi_1)\cdots a_n\sin(\omega_n t+\phi_n))^T\end{aligned} \quad (23)$$

where $a_1\cdots a_n, \omega_1\cdots\omega_n, \phi_1\cdots\phi_n$ are amplitudes, angular frequencies, phases of the torque of the operator $\boldsymbol{\tau_h}$,

In the case of the application to the power assist systems, the dynamics model and the all parameters are assumed to be known because of necessity of calculation of the operator's torque $\boldsymbol{\tau_h}$. However, to adapt change of pattern of the operator's torque $\boldsymbol{\tau_h}$, the amplitudes, the angular frequencies, the phases $a_i, \omega_i, \phi_i (i = 1, 2\cdots n)$ are assumed to be unknown. Therefore, the torque of the operator $\boldsymbol{\tau_h}$ can be calculated from the dynamics model (22), but the calculated values can not be used directly as the torque of the actuators $\boldsymbol{\tau}$ in view of causality, because dimension of the calculated values is torque.

C. Control Objective

Under above the dynamics and the assumptions, control objective is to amplify the torque of the operator $\boldsymbol{\tau_h}$ by the torque of the actuators $\boldsymbol{\tau}$ and to reduce the torque of the actuators $\boldsymbol{\tau}$ by adjustment of the stiffness K.

D. Controller

1) Torque of Actuators: To amplify the torque of the operator $\boldsymbol{\tau_h}$ by factors of amplification gains $k_{pa1}\cdots k_{pan}$, the torque of the actuators are designed using estimated values of operator's torque $\hat{\boldsymbol{\tau}}_{\boldsymbol{h}}$.

$$\boldsymbol{\tau} = K_{pa}\hat{\boldsymbol{\tau}}_{\boldsymbol{h}} - K_v\Delta\dot{\boldsymbol{q}} - K_p\boldsymbol{s}(\Delta\boldsymbol{q}) \quad (24)$$

$$\boldsymbol{q_d} = (E + K_{pa})D^{-1}\hat{\boldsymbol{\tau}}_{\boldsymbol{hi}} \quad (25)$$

where $\Delta\boldsymbol{q} = \boldsymbol{q} - \boldsymbol{q_d}$, $K_{pa} = \mathrm{diag}(k_{pa1}\cdots k_{pan})$, $E = \mathrm{diag}(1\cdots 1) \in \Re^{n\times n}$, $\hat{\boldsymbol{\tau}}_{\boldsymbol{hi}} = \int_0^t \hat{\boldsymbol{\tau}}_{\boldsymbol{h}} dt$ and the others are the same as the equation (17). The desired trajectories $\boldsymbol{q_d}$ are assumed to be satisfied the condition of the section III-B.

If the estimated values $\hat{\boldsymbol{\tau}}_{\boldsymbol{h}}$ converges to the true values $\boldsymbol{\tau}_{\boldsymbol{h}}$, the first term of the controller (24) becomes amplified torque of the operator's torque $\boldsymbol{\tau}_{\boldsymbol{h}}$.

2) Estimation of Operator's Torque: Estimation of the operator's torque is done using the adaptive observer [9].

$$\hat{\boldsymbol{\tau}}_{\boldsymbol{h}} = -K_o \Delta\boldsymbol{\tau}_{\boldsymbol{h}} - \Theta\hat{\boldsymbol{\tau}}_{\boldsymbol{hi}} \tag{26}$$

where the matrix $K_o > 0$ is a matrix of observer gains, $\Delta\boldsymbol{\tau}_{\boldsymbol{h}} = \hat{\boldsymbol{\tau}}_{\boldsymbol{h}} - \boldsymbol{\tau}_{\boldsymbol{h}}$, $\Theta = \mathrm{diag}(\theta_1 \cdots \theta_n)$, and $\theta_1 \cdots \theta_n$ are estimated values of $\omega_1^2 \cdots \omega_n^2$.

The estimation of the $\theta_1 \cdots \theta_n$ is done by the update law.

$$\dot{\boldsymbol{\theta}} = \Gamma_\theta \hat{T}_h \Delta\boldsymbol{\tau}_{\boldsymbol{h}} \tag{27}$$

where $\boldsymbol{\theta} = (\theta_1 \cdots \theta_n)^T$, Γ_θ are adaptive gains, and $\hat{T}_h = \mathrm{diag}(\hat{\tau}_{h1} \cdots \hat{\tau}_{hn})$. Therefore, the torque of the actuators (24) don't include signals whose dimension is torque, and the proposed controller satisfy causality. Detail of the adaptive observer is written in our previous paper [9].

3) Adjustment Law of Stiffness: The same adjustment law of the stiffness described in (19) is adopted for the power assist systems.

E. Convergence of Torque Estimation

Convergence of the estimation (26) is proved using a candidate of a Lyapnov function V_e.

$$2V_e = \Delta\boldsymbol{\tau}_{\boldsymbol{h}}^T \Delta\boldsymbol{\tau}_{\boldsymbol{h}} + \Delta\boldsymbol{\tau}_{\boldsymbol{hi}}^T W \Delta\boldsymbol{\tau}_{\boldsymbol{hi}} + \Delta\boldsymbol{\theta}^T \Gamma_\theta^{-1} \Delta\boldsymbol{\theta} \tag{28}$$

$$\dot{V}_e = -\Delta\boldsymbol{\tau}_{\boldsymbol{h}}^T K_o \Delta\boldsymbol{\tau}_{\boldsymbol{h}} \tag{29}$$

where $\boldsymbol{\tau}_{\boldsymbol{hi}} = \hat{\boldsymbol{\tau}}_{\boldsymbol{hi}} - \int_0^t \boldsymbol{\tau}_{\boldsymbol{h}} dt$, $W = \mathrm{diag}(\omega_1^2 \cdots \omega_n^2)$. Using LaSalle's invariance theorem, $\hat{\boldsymbol{\tau}}_{\boldsymbol{h}} \rightarrow \boldsymbol{\tau}_{\boldsymbol{h}}, \theta_i \rightarrow \omega_i^2 (i = 1 \cdots n)$ are guaranteed when $t \rightarrow \infty$. Therefore, the torque $K_{pa}\hat{\boldsymbol{\tau}}_{\boldsymbol{h}}$ converges to amplified torque of the operator $K_{pa}\boldsymbol{\tau}_{\boldsymbol{h}}$, and the torque amplification is realize by the controller (24).

F. Stability

Stability is guaranteed by a candidate of a Lyapnov function $V_{pa} = V_e + V_r$ composed of V_e and V_r of the section III-C.

G. Effect of Stiffness Adjustment

Discussion of an effect of the stiffness adjustment is almost the same as that of the section II-F. Therefore, the inertial and the elastic torque are largely compensated by the torque of the adjusted stiffness.

H. Summary

For the power assist system, which has the nonlinear dynamics (22), the controller (24) (26) (27) (19) realize the amplification of its sinusoidal operator's torque and the stiffness adjustment under some assumptions of the section IV-B. Therefore, the inertial and the gravitational torque to generate the desired motions are largely compensated by the torque of the adjusted stiffness. The proposed controller uses the dynamics model and all the parameters of the dynamics owing to calculation of the operator's torque. However, the amplitudes, the frequencies, the phases of the operator's torque can be unknown to adapt change of torque pattern of the operator.

V. Conclusion

This paper has presented a control method utilizing stiffness adjustment of mechanical elastic elements for serial link systems. The proposed controller guarantees convergence of tracking errors to a certain region, and minimizes a region of actuator torque by the stiffness adjustment. Thus, inertial and gravitational torque to generate desired motions is largely compensated by torque of the adjusted stiffness. The controller uses neither dynamics models nor parameters of the controlled objects owing to simplicity of the controller.

An application of the control method to power assist systems was also presented. In this case, a controller was proposed to amplify sinusoidal torque of its operator, and to adjust stiffness to compensate inertial and gravitational torque by torque of the adjusted stiffness. The controller of the application uses information of dynamics model and all parameters of the dynamics, but amplitudes, frequencies, phases of the operator's torque can be unknown in order to adapt changes of torque pattern of its operator.

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