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An Analysis of Wheeled Mobile Robots in the Presence of skidding and slipping: Control Design Perspective

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***Abstract*— This paper presents an analysis on Wheeled Mobile Robots in the presence of wheel skidding and slipping from the perspective of control design. The analysis is based on the kinematic models that are recently developed from control perspective [1]. Four generic mobile robots are considered in this analysis. We relate the robot's maneuverability with its controllability which provides insights on the robot's ability to track a trajectory in the presence of wheel skidding and slipping. These findings lay a base for the deployments of various control design techniques to overcome mobile robot control problems in the presence of wheel skidding and slipping.**

I. INTRODUCTION

Wheeled Mobile Robots control problems have been intensively studied; and apparently, most problems have been properly addressed. However, most existing works assume that the robots satisfy the non-slipping and non-skidding conditions. In reality, these assumptions can't be met due to tire-deformation; hence, stability and control performance of these controllers are not guaranteed in real running.

Several controllers have been proposed for the popular Type (2,0) robot based on a kinematic model constructed in [9] to address the issue of the skidding effect represented by unknown bounded perturbation. Under the assumption of the unknown perturbation being state-vanishing, an exponentially stable robust stabilizing controller was proposed for the robot [9]. Later the model was used in [10][11] and uniform boundedness solutions were proposed for stabilization and tracking problems without assuming the perturbation to be constant or state-vanishing. In [12], Dixon *et al.* address the skidding by designing robust tracking and regulation controllers using the kinematic model. These controllers offer uniform boundedness solutions by robust control approach. However, if a high-precision control performance is desirable, these control laws would require high control input and fast switching actions. These methods can be constrained by implementation and mechanical issues.

In [7][14][15], another framework was proposed to address the tracking control problem in the presence of both skidding and slipping effects. A dynamic model was proposed to analyze the stability of a feedback linearization controller [6] using singular perturbation analysis theory [7]. The analysis shows that the control system remains stable for very mild skidding and slipping. Another linear time varying controller was proposed [14] to achieve uniformly locally exponential stability for the Type (2,0) unicycle based on the dynamic model. In [15], the model is applied to design a slow manifold controller to solve an output-tracking problem. It should be noted that these dynamic models rely on the accurate dynamic inertial parameters and a factor ϵ which are difficult to obtain in practice.

Based on the existing literature, we see that the available kinematic models do not provide sufficient insights for control design where the perturbations due to wheel skidding are regarded as general unknown terms. Recently, we presented kinematic models for four generic mobile robots that explicitly describe the perturbations due to wheel skidding and slipping using meaningful descriptions [1]. This explicit description allows these perturbations to be analyzed from control viewpoint. In this paper, these robots with wheel skidding and slipping are further examined based on the kinematic models. A unified framework in terms of maneuverability index is used to address the controllability of the four mobile robots. Tracking problem is also studied and new insights are revealed. These findings lay a foundation for the mobile robot control problem formulation and control design in the presence of wheel skidding and slipping.

II. KINEMATIC MODELS

The four mobile robots considered in this paper are the Type (2,0), (2,1), (1,1) and (1,2) robots. Each mobile robot has a body frame with a body axes $\{X_b, Y_b\}$ attached on it (Figure 1). We define the vector $\xi = (x, y)^T$ as the coordinates of the reference point P in global coordinates $\{X, Y\}$; θ denotes the orientation of the body frame with respect to the global axes. The posture of the robot q is defined as $q = (\xi, \theta)$. v denotes the velocity of the reference point P, v_y represents the velocity along Y_b and v_l denotes v along X_b direction. Geometrically, these velocities (v_l, v_y) are related with the slip angle δ by the geometric

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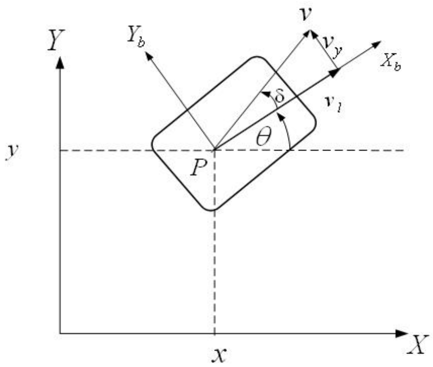

Fig. 1. A Wheeled Mobile Robot body frame

relationships

$$\tan\delta = \frac{v_y}{v_l}, \quad \sin\delta = \frac{v_y}{v}, \quad \cos\delta = \frac{v_l}{v}. \tag{1}$$

The maneuverability of a Type (m, s) robot is defined as the sum of the indexes (m, s) [5]. In this paper, all considered mobile robots have either maneuverability two (M2) or maneuverability three (M3). Without loss of generality, the kinematic model of a robot can be written in the form

$$\dot{\xi} = f^1_{m+s}(\theta, \mathbf{U}) \tag{2}$$
$$\dot{\theta} = f^2_{m,s}(\theta, \mathbf{U}) \tag{3}$$

$\mathbf{U}$ denotes the robot's control input and vectors $f^1_{m+s} \in R^2$ and $f^2_{m,s} \in R$ are given as follows depending on the robot's maneuverability index.

A. Maneuverability Two Robots

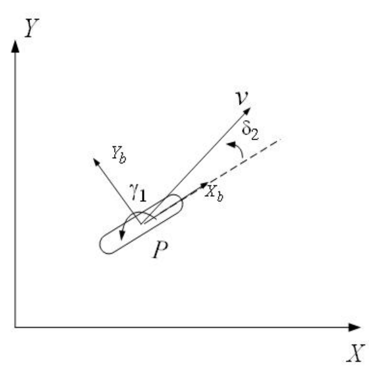

Fig. 2. Type (2,0) robot: In the presence of skidding and slipping effects

We present in Figures 2 and 3, pictures of Type (2,0) and Type (1,1) robots where control inputs of each robot are explicitly figured. For mobile robots with M2, i.e., for each

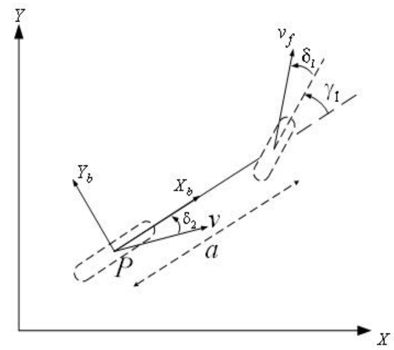

Fig. 3. Type (1,1) robot: In the presence of skidding and slipping effects

$(m, s) \in \{(2, 0), (1, 1)\}$, the vectors f^1_{m+s} and $f^2_{m,s}$ are

$$f^1_2 = \begin{bmatrix} v\cos(\theta + \delta_2) \\ v\sin(\theta + \delta_2) \end{bmatrix} \tag{4}$$
$$f^2_{2,0} = \gamma_1 + \delta_1 \tag{5}$$
$$f^2_{1,1} = \frac{v_l}{a}\tan(\gamma_1 + \delta_1) - \frac{v_y}{a} \tag{6}$$

where $v = \frac{v_l}{\cos\delta_2}$ and $v_l = r\dot{\varphi} - d$. d is the wheel's longitudinal slipping velocity, and r denotes the wheel's free-rolling radius. The robot's control input is $\mathbf{U} = (r\dot{\varphi}, \gamma_1)$ and the perturbations due to wheel skidding and slipping are $\{\delta_1, \delta_2, v_y, d\}$.

B. Maneuverability Three Robots

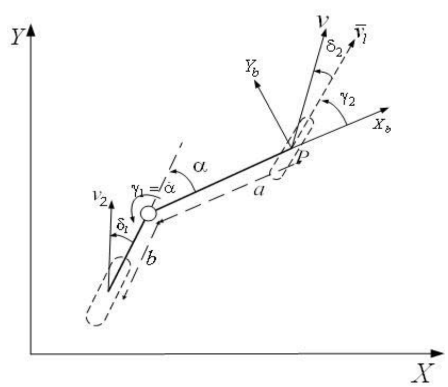

Fig. 4. Type (2,1) robot: In the presence of skidding and slipping effects

Similarly, Type (2,1) and Type (1,2) robots are shown in Figures 4 and 5 where the control inputs of each robots are explicitly figured. The vectors f^1_{m+s} and $f^2_{m,s}$ for maneuverability three robots are

$$f^1_3 = \begin{bmatrix} v\cos(\theta + \gamma_2 + \delta_2) \\ v\sin(\theta + \gamma_2 + \delta_2) \end{bmatrix} \tag{7}$$

$$f^2_{2,1} = \frac{v\sin(\gamma_2 + \delta_2 - \alpha - \delta_1) - \gamma_1 b\cos(\delta_1)}{a\cos(\alpha + \delta_1) + b\cos(\delta_1)} \tag{8}$$

$$f^2_{1,2} = \frac{v}{a}\left(\tan(\gamma_1 + \delta_1)\cos(\gamma_2 + \delta_2) - \sin(\gamma_2 + \delta_2)\right) \tag{9}$$

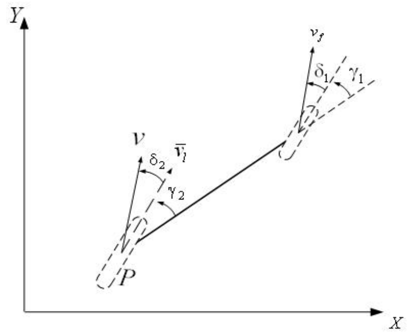

Fig. 5. Type (1,2) robot: In the presence of skidding and slipping effects

The longitudinal wheel velocity $\bar{v}_l$ is related with the wheel's slip velocity via $\bar{v}_l = r\dot{\varphi} - d$ where r denotes the wheel's free-rolling radius. Similarly, $v = \frac{\bar{v}_l}{\cos\delta_2}$. The robot's control input is $\mathbf{U} = (r\dot{\varphi}, \gamma_1, \gamma_2)$ and the perturbations due to wheel skidding and slipping are $\{\delta_1, \delta_2, d\}$.

III. Controllability

This section presents the controllability of the robots in the presence of wheel skidding and slipping effects. The assumption and definitions utilized in this paper are listed below.

Assumption 1: Perturbations δ_1, δ_2, d and $\dot{\delta}_2$ are bounded and measurable with $|\delta_2| < \frac{\pi}{2}$.

Assumption 1 implies the perturbations are bounded and measurable. Many theoretical and experimental works have been developed in measuring these perturbations for control using RTK-GPS and other aiding sensors [2], [3], [4]. For now, we proceed the analysis by assuming these perturbations are measurable. In the ideal case where *non-slipping* and *non-skidding* assumptions are satisfied, the controllability of a mobile robot is referred as the ability to steer it from an initial posture to a final posture in a finite time [8]. This definition can be stated as follows.

Definition 1: A wheeled mobile robot is said to be *posture controllable* if there exists a piecewise continuous input to steer the robot's reference point P from an initial posture $\{x(t_0),\ y(t_0),\ \theta(t_0)\}$ to a final posture $\{x(t_f),\ y(t_f),\ \theta(t_f)\}$ in a finite-time interval.

In the presence of wheel skidding, robots with M2 do not have posture controllability. Nevertheless, these robots should be able to achieve point control by performing appropriate steering action to compensate the skidding slipping perturbations.

Definition 2: A wheeled mobile robot is said to be *point controllable* if there exists a piecewise continuous input to steer the robot's reference point P from an initial point $\{x(t_0),\ y(t_0)\}$ to a final point $\{x(t_f),\ y(t_f)\}$ in a finite-time interval.

Next, we show that the kinematic model of a robot with M2 can be transformed into a form similar to the ideal kinematic model of a Type (2,0) [16] without wheel skidding and slipping.

Lemma 1: Consider a wheeled mobile robot with M2. Suppose that the perturbations (δ_1, δ_2, d) satisfy Assumption 1, then there exists an invertible coordinates transformation and a corresponding invertible input change,

$$\begin{aligned} \bar{q} &= \phi(q, \delta_2), && (10) \\ u &= \beta_1(q, \delta_1, \delta_2, d, \mathbf{U}), && (11) \end{aligned}$$

such that the transformed system becomes

$$\dot{\bar{q}} = G(\bar{q})u \tag{12}$$

where

$$\bar{q} = \begin{bmatrix} x \\ y \\ \bar{\theta} \end{bmatrix}, \quad G(\bar{q}) = \begin{bmatrix} \cos\bar{\theta} & 0 \\ \sin\bar{\theta} & 0 \\ 0 & 1 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{13}$$

with $\bar{\theta} = \theta + \delta_2$.

Proof: The proof is straightforward and constructive. Define $\bar{q}$ and u as

$$\begin{aligned} \bar{q} &= [x \;\; y \;\; \theta + \delta_2]^T && (14) \\ u &= \left[\frac{1}{\cos\delta_2}(r\dot{\varphi} - d) \;\; f^2_{m,s}\right]^T && (15) \end{aligned}$$

By inspection, the coordinates transformation (14) is invertible. We show that the input change (15) is also invertible. It is clear from (15) that the auxiliary u_1 is invertible with respect to $r\dot{\varphi}$. As for u_2, we need to show that $f^2_{m,s}$ is invertible. For the Type (2,0) case, $f^1_{2,0} = \gamma_1 + \delta_1$ is invertible with respect to γ_1 since $f^2_{2,0}$ is a linear function. As for Type (1,1), $f^2_{1,1}$ is defined as

$$f^2_{1,1} = \frac{v}{a}\tan(\gamma_1 + \delta_1)\cos(\delta_2) - \frac{v}{a}\sin(\delta_2). \tag{16}$$

With some manipulations, we have

$$\gamma_1 = \tan^{-1}\left\{\frac{u_2 a}{v\cos\delta_2} + \tan\delta_2\right\} - \delta_1. \tag{17}$$

For any nonzero v, $f^2_{1,1}$ is invertible; hence, the kinematic model of a robot with M2 can be converted into (12).

□

With the help of Lemma 1, we can state the following result.

Theorem 1: Suppose that the perturbations $\{\delta_1, \delta_2, d\}$ of a wheeled mobile robot with M2 satisfy Assumption 1, then the robot is point controllable, but not posture controllable.

Proof: By Lemma 1, the kinematic model of a mobile robot with M2 can be transformed into (12) where input vector fields are

$$g_1 = \begin{bmatrix} \cos\bar{\theta} \\ \sin\bar{\theta} \\ 0 \end{bmatrix}, \quad g_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{18}$$

Let $\triangle_{\mathcal{C}}$ be the accessibility distribution generated by the vector fields $\{g_1, g_2\}$. The distribution has a rank of $rank\{\triangle_{\mathcal{C}}\} = 3$ for all $\bar{\theta}$. By applying controllability test condition [8], $rank\{\triangle_{\mathcal{C}}\} = 3$ indicates that the system is controllable in coordinates $\bar{q}$. Since ξ is a subvector of $\bar{q}$, controllability in $\bar{q}$ implies controllability in $[x,\ y]^T$ coordinates.

To show that a robot with M2 is not *posture controllable* in the presence of skidding and slipping, we express the point subsystem of a robot with M2,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = v \begin{bmatrix} \cos(\bar{u}) \\ \sin(\bar{u}) \end{bmatrix} \tag{19}$$

where we let $\bar{u} = \theta + \delta_2$ be a directional control input. If the reference point of the wheeled mobile robot ξ is to be steered from an initial point $\xi(0)$ to a final point $\xi(t_f)$, then the directional input control $\bar{u}$ is constrained along a feasible $\bar{u}_s(t)$ to achieve this goal. This implies that the orientation of the robot must satisfy $\theta(t) = \bar{u}_s(t) - \delta_s(t)$ to reach the desired point ξ_f. Since in general, the desired orientation $\theta_d(t)$ of a path that the robot is required to follow is not $\bar{u}_s(t) - \delta_s(t)$, we conclude that the orientation of the robot cannot approach $\theta_d(t)$ in order to maintain point controllability. Hence, the robot is not posture controllable.

□

Contrast to robots with M2, the additional orientation input γ_2 of a robot with M3 suggests that this class of wheeled mobile robot can handle the skidding perturbation more effectively. To confirm this intuition, we examine the controllability of the robots with M3.

Lemma 2: Consider a wheeled mobile robot with M3. Suppose that the perturbations $\{\delta_1, \delta_2, d\}$ satisfy Assumption 1, then there exists an invertible input change,

$$u = \beta_2(q, \mathbf{U}) \tag{20}$$

such that the kinematic model of a wheeled mobile robot with M3 becomes

$$\dot{q} = G(q)u, \tag{21}$$

where

$$\dot{q} = \begin{bmatrix} \dot{\xi} \\ \dot{\theta} \end{bmatrix}, \quad G(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{22}$$

Proof: By choosing $\gamma_2 = -\delta_2$ and auxiliary input

$$u_1 = \frac{1}{\cos\delta_2}(r\dot{\varphi} - d), \tag{23}$$

the point subsystem of a robot with M3 becomes

$$\dot{x} = u_1 \cos(\theta) \tag{24}$$

$$\dot{y} = u_1 \sin(\theta) \tag{25}$$

Similarly, the auxiliary input (23) is invertible. Consider the orientation subsystem for a wheeled mobile robot with M3. For Type (1,2) robot, the auxiliary inputs are defined as

$$u_2 = \frac{v}{a}\tan(\gamma_1 + \delta_1). \tag{26}$$

It is clear that auxiliary input (26) is invertible

$$\gamma_1 = \tan^{-1}(\frac{u_2 a}{v}) - \delta_1 \tag{27}$$

for $v \neq 0$.

As for Type (2,1) robot, the auxiliary input for the robot's orientation subsystem is

$$u_2 = \frac{-v\sin(\alpha + \delta_1) - \gamma_1 b\cos(\delta_1)}{a\cos(\alpha + \delta_1) + b\cos(\delta_1)}. \tag{28}$$

We can see that the auxiliary input (28) is invertible since $|b\cos(\delta_1)| > 0$ for $|\delta_1| < \frac{\pi}{2}$ and the condition $|a\cos(\alpha + \delta_1) + b\cos(\delta_1)| > 0$ can be easily met in practice.

□

Similarly, Lemma 2 leads to the following result.

Theorem 2: Suppose that the perturbations $\{\delta_1, \delta_2, d\}$ of a wheeled mobile with M3 satisfy Assumption 1, then the robot is *posture controllable*.

Proof: By Lemma 2, the kinematic model of a mobile robot with M3 is input-equivalent to a nominal Type (2,0) kinematic model (22) which has a rank 3 accessibility distribution, i.e., $\dim \triangle_{\mathcal{C}} = 3$ for all θ; hence, the controllability test condition [8] shows the mobile robot is posture controllable.

□

IV. Wheeled Mobile Robot Tracking Problem

In this section, we study the tracking and path following control problems under the influence of the kinematic perturbations due to skidding and slipping. Path following is a special case of tracking control problem in the sense that path following problem only considers lateral and orientation errors; whereas tracking control problem encompasses lateral, longitudinal and orientation errors [8]. Hence, it is sufficient to focus on the tracking problem.

A tracking control problem is to maneuver the robot to follow a trajectory. In this paper, we consider the *posture tracking error*

$$\tilde{q} = \begin{bmatrix} \tilde{\xi} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \tag{29}$$

The reference trajectory (x_r, y_r, θ_r) represents the point coordinates and orientation of a reference trajectory which satisfies

$$\dot{x}_r = v_r \cos\theta_r \tag{30}$$

$$\dot{y}_r = v_r \sin\theta_r \tag{31}$$

$$\dot{\theta}_r = \omega_r \tag{32}$$

$\tilde{\xi}$ denotes the point tracking error and $\tilde{\theta}$ represents the orientation error. It can be shown that the error dynamics of $\tilde{q}$ can be described by

$$\dot{\tilde{\xi}} = f^3_{m+s} \tag{33}$$

$$\dot{\tilde{\theta}} = \omega_r - \omega \tag{34}$$

where

$$f^3_2 = \begin{bmatrix} v_r\cos\theta - v_l + \tilde{y}\omega \\ -\tilde{x}\omega + v_r\sin\tilde{\theta} - v_y \end{bmatrix} \tag{35}$$

$$f^3_3 = \begin{bmatrix} v_r\cos\theta - v\cos(\gamma_2 + \delta_2) + \tilde{y}\omega \\ -\tilde{x}\omega + v_r\sin\tilde{\theta} - v\sin(\gamma_2 + \delta_2) \end{bmatrix} \tag{36}$$

and $\omega = f_{m,s}^2$. One assumption that is usually imposed on the moving reference trajectory in the tracking control problem is stated below.

Assumption 2: The yaw rate $\omega_r(t)$, velocity $v_r(t)$, and $\dot{v}_r(t)$ of the reference trajectory are *piecewise continuous* and *bounded functions*. Furthermore, $v_r(t)$ is positive and $\inf\{v_r(t); \forall t \geq t_0\} > 0$.
The assumption is assumed in the following analysis.

Wheeled mobile robot *point tracking problem* is said to be solvable if for a small initial tracking error $\tilde{q}(0)$, there exists a control input $\mathbf{U}(t)$ such that the point error $\tilde{\xi}$ converges to zero; and the *posture tracking problem* is said to be solvable if for a small initial tracking error $\tilde{q}(0)$, these exists a control input $\mathbf{U}(t)$ such that $\tilde{q}$ converges to zero. Many controllers have been proposed to solve the posture tracking problem for the Type (2,0) configuration under the non-skidding and non-slipping assumptions, e.g., [13], [16]. We will show that there exists a control input such that a robot with M2 in the presence of skidding and slipping is able to track the trajectory with the point tracking error converges to zero. Similarly, we show that there exists a control law such that the posture tracking error of a robot with M3 in the presence of skidding and slipping converges to zero. The following result summarizes this finding.

Theorem 3: Assume that the perturbations $\{\delta_1, \delta_2, d\}$ satisfy Assumption 1. Then in the presence of skidding and slipping,

1) a robot with M2 is point tracking solvable, but not posture tracking solvable.
2) a robot with M3 is posture tracking solvable.

Proof: We first consider a robot with M2. For the nominal case (in the absence of skidding and slipping), the tracking error dynamics $\dot{\tilde{q}}$ is as follows:

$$\begin{aligned} \dot{\tilde{x}} &= v_r \cos\tilde{\theta} - v_l + \omega\tilde{y} && (37)\\ \dot{\tilde{y}} &= v_r \sin\tilde{\theta} - \omega\tilde{x} && (38)\\ \dot{\tilde{\theta}} &= \omega_r - \omega && (39) \end{aligned}$$

Integrator backstepping has been applied for tracking control problem in the absence of wheel skidding and slipping [13]. Here, we show that integrator backstepping can also be applied to solve the point tracking problem in the presence of skidding and slipping.

In the absence of skidding and slipping effects, it has been shown in [13] that by choosing $\tilde{\theta}$ as a virtual control input for point error subsystem (37), (38), there exists a velocity control $v_l = \alpha_1$, a continuously differentiable feedback control law

$$\tilde{\theta} = \alpha_2(\tilde{\xi}), \quad (40)$$

and a positive definite function $V_1(\tilde{\xi})$ such that its derivative satisfies

$$\dot{V}_1 \leq -W(\tilde{\xi}) \leq 0 \quad (41)$$

where $W(\tilde{\xi})$ is a positive definite function. Let $z = \tilde{\theta} - \alpha_2$. Then by integrator backstepping, there exists a positive definite function

$$V_2(\tilde{\xi}, z) = V_1 + \frac{1}{2}z^2 \quad (42)$$

and a feedback control law ω which renders $(\tilde{\xi}, z) \to 0$ as $t \to \infty$.

In the skidding and slipping case, the following invertible auxiliary inputs

$$\begin{aligned} v_l &= r\dot{\varphi} - d && (43)\\ \omega &= f_{m,s}^2 && (44) \end{aligned}$$

can eliminate $\{\delta_1, d\}$ of a wheeled mobile robot with M2. As a result, the tracking error dynamics of a robot with M2 can be simplified as

$$\begin{aligned} \dot{\tilde{x}} &= v_r \cos\tilde{\theta} - v_l + \omega\tilde{y} && (45)\\ \dot{\tilde{y}} &= v_r \sin\tilde{\theta} - \omega\tilde{x} - v_y && (46)\\ \dot{\tilde{\theta}} &= \omega_r - \omega && (47) \end{aligned}$$

where v_l and ω can be converted to original control input $(r\dot{\varphi}, \gamma_1)$. Note that an additional perturbation term v_y appears in the lateral error dynamic $\dot{\tilde{y}}$. This observation suggests the choice of $v_l = \alpha_1$ and a differentiable feedback control law

$$\begin{aligned} \tilde{\theta} &= \sin^{-1}\{\sin\alpha_2 + \frac{v_y}{v_r}\} && (48)\\ &= \alpha_3 \end{aligned}$$

renders the derivative of $\dot{V}_1$ to satisfy condition (41). Similarly, by defining $z = \tilde{\theta} - \alpha_3$, we show that there exists a positive definite function

$$V_2(\tilde{\xi}, z) = V_1 + \frac{1}{2}z^2 \quad (49)$$

and a control law ω such that $(\tilde{\xi}, z) \to 0$ as $t \to \infty$ based on integrator backstepping. On the other hand, it is easy to show that the robot is not posture tracking solvable. Since Theorem 1 indicates that a robot with M2 cannot control its position without compromising the robot's orientation θ if δ_2 is nonzero; we conclude that the orientation of the mobile robot cannot approach θ_r when the robot's reference point converges to the desired reference point trajectory (x_r, y_r).

As for the robots with M3, the proof is straightforward after applying Lemma 2. Since the robot is input equivalent to a nominal Type (2,0) robot, there exists tracking controllers that were designed based on non-skidding and non-slipping assumptions [13], [16]; therefore, a mobile robot with M3 is posture tracking solvable and this completes the proof.

□

Theorem 3 implies that there exists a control input for a robot with M2 such that the point error converges to zero if the v_y satisfies equality (48). In addition, the results also indicate the robot's orientation has to be "compromised" to achieve zero point tracking error if the skidding perturbation δ_2 is nonzero. Achieving good point tracking performance is desirable and in many practical cases is sufficient. In some cases, orientation error is equally important as the point tracking error. The following results provide a measure on the orientation error while the point tracking error converges to zero.

Theorem 4: Suppose that there exists a continuously differentiable control input **U**(t) such that the point tracking error $\tilde{\xi}$ converges to zero. Then,

1) the steady-state orientation error $\tilde{\theta}$ of a wheeled mobile robot with M2 is

$$\lim_{t\to\infty}\left(\tilde{\theta}(t)-\delta_2(t)\right)=0. \tag{50}$$

2) the steady-state orientation error $\tilde{\theta}$ of a wheeled mobile robot with M3 is

$$\lim_{t\to\infty}\left(\tilde{\theta}(t)-\gamma_2(t)-\delta_2(t)\right)=0. \tag{51}$$

Proof: We first consider the case of a robot with M2. By assumption, the point tracking error $\tilde{\xi}$ approaches zero and $v_r, \dot{v}_r, \omega_r$ of the reference trajectory are bounded. The differentiability of the control input **U** guarantees the boundedness of $\ddot{\tilde{\xi}}$; hence $\dot{\tilde{\xi}}$ is uniformly continuous. By Barbalat lemma [17], we conclude that $\dot{\tilde{\xi}} \to 0$ as $t \to \infty$. One essential observation is that for any well-defined control input, the error dynamics of $\tilde{y}$ is governed by $\dot{\tilde{y}} = -\tilde{x}\omega + v_r \sin\tilde{\theta} - v_y$, and $(\tilde{\xi}, \dot{\tilde{\xi}}) \to 0$ leads to

$$\lim_{t\to\infty}\left(\sin\tilde{\theta}-\frac{v_y}{v_r}\right)=0. \tag{52}$$

Since $\dot{\tilde{\xi}} \to 0$ implies $v \to v_r$ as $t \to \infty$, geometric relation (1) leads (52) to (50).

We can also show that $(\tilde{\xi}, \dot{\tilde{\xi}}) \to 0$ as $t \to \infty$ for a robot with M3. The error dynamic equation $\dot{\tilde{y}} = -\tilde{x}\omega + v_r \sin\tilde{\theta} - v\sin(\gamma_2+\delta_2)$ of a M3 robot implies

$$\lim_{t\to\infty}\left(\sin\tilde{\theta}-\frac{v\sin(\gamma_2+\delta_2)}{v_r}\right)=0. \tag{53}$$

Similarly, $v \to v_r$ as $t \to \infty$. Hence, (53) leads to (51), and this completes the proof.

□

Theorem 4 indicates the steady-state orientation error of a robot with M2 is the slip angle δ_2. On the other hand, a robot with M3 does not have this limitation if the steerable wheel γ_2 is properly designed to compensate against perturbation δ_2. (51) suggests that the steady-state orientation error can be eliminated if the steering angle γ_2 is chosen such that the steady-state orientation error approaches zero. These findings show that it is impossible for a robot with M2 to solve the posture tracking problem if the slip angle δ_2 is nonzero. On the other hand, the result shows that there exists a control input $\mathbf{U}(t)$ such that the posture tracking error of a robot with M3 converges to zero.

V. Conclusions

In this paper, several useful results and properties regarding mobile robots in the presence of wheel skidding and slipping are developed. We show that a higher maneuverability robot is more controllable than a lower maneuverability robot. This result has important implications of formulating control objectives for a mobile robot in the presence of wheel skidding and slipping. Additionally, these findings shed light on the possible control strategy for precision control in the presences of skidding and slipping. In fact, control schemes with positive experimental results have been established based on the kinematic models and the analysis presented in this paper [2], [4].

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