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A simple and efficient control scheme to reverse a tractor-trailer system on a trajectory

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Abstract— **Trailer reversing is a problem frequently considered in the literature, usually with fairly complex non-linear control theory based approaches. In this paper, we present a simple method for stabilizing a tractor-trailer system to a trajectory based on the notion of controlling the hitch-angle of the trailer rather than the steering angle of the tractor. The method is intuitive, provably stable, and shown to be viable through various experimental results conducted on our test platform, the CSIRO Autonomous Tractor.**

I. Introduction

Reversing of a tractor-trailer system is a common task in industrial and recreational activities and a task which can be arduous for an inexperienced driver. In Australia, the 'grey-nomad' phenomenom sees a growing number of retirees, a large proportion of whom are inexperienced in towing a caravan, embarking on a journey that takes them around the country. They are frequently required to reverse the caravan into parking spots throughout this journey, and, at least to start with, experience some difficulty in this process. This is one example in which a driver-aid or automation system for reversing a trailer could be of significant benefit.

This paper describes the development of a control algorithm for stabilizing a tractor-trailer system to a trajectory. The approach is relatively simple in comparison to existing algorithms in the literature, relying on an intuitive change of variables to simplify the control. Essentially, this involves a Proportional Integral controller on the tractor-trailer hitch-angle, from which a relatively straight-forward trajectory control algorithm can be applied.

The next section outlines the literature available in the field; Section III describes the platform and kinematics of the system considered in this paper; Section IV outlines our approach to hitch-angle stabilization; Section V describes the method used to stabilize the system to a trajectory; Section VI describes various experiments validating the approach; and Section VII concludes the paper.

II. Background

The control of tractor-trailer systems has received much attention in the scientific and patent literature. This problem has clear industrial applications but it also interesting due to its inherent non-linear nature. The following discussion is split between the scientific and patent literature and concludes with a section which highlights the difference in our approach to this problem. This section is in no way exhaustive but aims at giving a overview of the usual approaches to the problem.

A. Scientific literature

In the scientific literature, this problem has been approached in three ways: through a *change of variables* which brings the system into a standard form, *optimal control* methods, and *learning-based* approaches.

The *change of variables* approach relies on a transformation which brings the system into a chained (or similar) form, from which standard non-linear control theory can be applied, see for example [15], [13], [2]. These techniques have the advantage of being theoretically generalisable to any number of trailers. Their main disadvantage, with respect to the work presented in this paper, is their complexity and high sensitivity to numerical approximation. This complexity makes it difficult to implement these methods reliably, and also poses difficulties in the tuning of these systems — after the change of variables, many of the control parameters no longer have a physical meaning making the design of a tuning strategy challenging and somewhat non-intuitive. As for numerical stability, the chained-form-based conversion for instance requires several trigonometric operators and inversions, and the resulting control often requires inversion of matrices which can become singular at some point of the navigation. It is also worth mentionning controller based on the differential flatness property of our system[12], [8]. Although theoretically very interesting these approaches rely on evaluating several level of derivative of the system. In practice, it is hard to evaluate these derivative without introducing a lot of noise.

The *optimal control* based methods use optimisation schemes to derive a sequence of demands which will control the tractor-trailer system onto the desired trajectory or path, see for example [1], [6]. In simple terms, these methods use a vehicle model and a simulation process to compute the control commands that will lead to the best tracking of the trajectory. This requires accurate models of the vehicle's behaviour and a heavy reliance on computational resources. Deviations from the model, which in real-world implementations is inevitable, lead to errors, and these methods are also open-loop meaning that re-planning is necessary to deal with errors in localisation.

Finally, the *learning-based* controllers seek to ease the computational burdens of the previous methods by providing a mapping between the current vehicle state, the desired state and the required inputs to reach the desired state, see for example [9]. Essentially, a simulated model of the tractor-trailer system is used to try many different possible methods and parameters. 'Learning' occurs by searching the parameter space for the best set of methods/parameters which are then encoded into, for example, a neural network or look-up table, which maps the current to desired configuration. The main drawback of these techniques is the learning itself: if learning occurs from a model of the system, then errors in the model are clearly problematic; if learning occurs on the real vehicle, then there are clearly safety issues since it is not possible to predict the behaviour of the vehicle in the learning phase. Also, even if the best set of parameters performs well in practice, it is hard to guarantee its performance, which can be problematic in an application where reliability is an issue.

B. Patent literature

Several patents have been proposed for automatically reversing a tractor-trailer system [11], [3],*etc*. However, these patents are focused on reversing on straight trajectories. Several other patents have focussed on driver-assist type systems, e.g. [4],*etc*, providing methods to sense the position/hitch-angle of a trailer and to display this information to the driver. Some of these methods also address the sensing of potentially dangerous obstacles. There are also a number of patents which present ways of easing trailer reversing through the use of active steering on the trailer or by adding mechanical devices that hold the trailer straight.

C. Originality

The control scheme we propose in this paper has several advantages over the schemes proposed in the scientific and patent literature. It is able to control the tractor-trailer along straight and curved trajectories, and it does not require any mechanical modification of the tractor-trailer system other than the ability to sense the hitch-angle.

In control terms, the proposed scheme is simple to implement, numerically stable, and easy to predict and understand. Its main drawback is that, in its current state, it is not generalisable to any number of trailers.

III. Experimental platform

The platform used in these experiments is the CSIRO Autonomous Tractor (AT), as shown in Fig. 1. It is an Ackerman steered, ride-on mower which has been retro-fitted with an array of actuators, sensors, and a computer system enabling the implementation and testing of control and navigation algorithms. For full details of the vehicle's design, refer to [14].

The trailer hitch-angle is sensed using a set of string-pot encoders. Unlike much of the existing research, which presumes that the hitch-point lies over the centre of the rear axle, the hitch-point on this platform is offset from the rear-axle. This is kinematically more similar to the majority of tractor-trailer systems in general use and represents a further distinction of this work from the existing literature.

Fig. 1. The Autonomous Tractor with its trailer.

A. Kinematic model

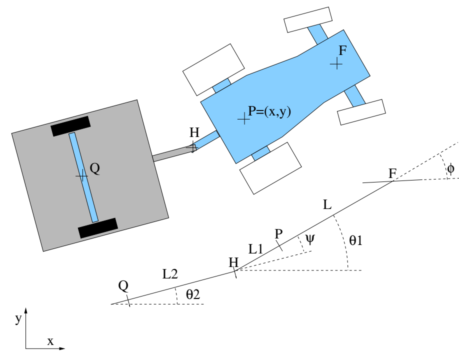


Fig. 2. Kinematic model of a tractor-trailer system

The geometry of our tractor-trailer system is depicted in Fig. 2. In this figure, point F is the middle of the front axle, point P is the middle of the tractor rear axle, point H is the hitch joint and point Q is the middle of the trailer axle. Three lengths play an important role in the system kinematics:

- L: the distance between the front and rear axles of the tractor $(1.2\,\mathrm{m})$;
- L_1: the distance between the rear axle of the trailer and the hitch joint $(0.45\,\mathrm{m})$;
- L_2: the distance between the trailer axle and the hitch joint $(1.2\,\mathrm{m})$.

The tractor-trailer system state is described by the following variables:

- (x, y): Cartesian coordinate of point P in the global frame;
- ϕ: steering angle of the front wheels;
- θ_1: heading of the tractor;
- ψ: hitch-angle, i.e. angle between the tractor heading and the trailer heading θ_2. Note that $\theta_2 = \theta_1 + \psi$.

This system is controlled via two inputs:

- v: the linear velocity of point P ($v = \|\dot{P}\|$);
- ϕ: the steering angle.

With this notation, the system can be described by the following equations:

$$\dot{x} = v\cos(\theta_1) \quad (1)$$
$$\dot{y} = v\sin(\theta_1) \quad (2)$$
$$\dot{\theta}_1 = \frac{\tan(\phi)}{L} \quad (3)$$
$$\dot{\psi} = -v[L_1\sin(\phi)\cos(\psi) + L_2\sin(\phi) + L\cos(\phi)\sin(\psi)]/(L_2 L\cos(\phi)) \quad (4)$$

The first three lines of this system are the standard kinematics for a car-like vehicle, see e.g. [5] for a derivation. Proving the expression for $\dot{\psi}$ requires consideration of the speed of point H V_H (resp. P or Q) in the global frame. Also, we define $\Omega_1 = [0, 0, \dot{\theta}_1]^T$ and $\Omega_2 = [0, 0, \dot{\theta}_2]^T$. Then,

$$V_H = V_P + \Omega_1 \times \vec{PH} \quad (5)$$
$$V_H = V_Q + \Omega_2 \times \vec{QH} \quad (6)$$

The expression for $\dot{\psi}$ is derived by solving

$$V_P + \Omega_1 \times \vec{PH} = V_Q + \Omega_2 \times \vec{QH} \quad (7)$$

with $\theta_2 = \theta_1 + \psi$.

IV. Hitch-angle stabilisation

This section describes the method used to reverse the system while controlling the hitch-angle via the tractor steering angle. In short, a simple proportional-integral controller is enough to achieve this task:

$$\phi = K_p(\psi^\star - \psi) + K_I \int_0^t (\psi^\star - \psi)du \quad (8)$$

where $\psi^\star$ is the demanded hitch-angle, and K_p and K_I are the proportional and integral gains.

A. Stability analysis

1) Proportional-only control law: Let us first consider a proportional-only control law (i.e. $K_I = 0$). In this case, substitution of the control law into the trailer hitch-angle state equation gives:

$$\dot{\psi} = -v \frac{\left[\begin{array}{r} \sin(K_p(\psi^\star - \psi))(L_1\cos(\psi) + L_2) \\ +L\cos(K_p(\psi^\star - \psi))\sin(\psi) \end{array} \right]}{L_2 L\cos(K_p(\psi^\star - \psi))} \quad (9)$$

Once linearised around $\psi^\star$, the above equation becomes:

$$\dot{\psi} = A + B\psi + O((\psi^\star - \psi)^2) \quad (10)$$

with,

$$A = -v\left(\frac{\sin(\psi^\star)L + \psi^\star C}{LL_2}\right)$$
$$B = v\left(\frac{C}{LL_2}\right)$$
$$C = K_P\cos(\psi^\star)L_1 + K_P L_2 - \cos(\psi^\star)L$$

The linearised ODE converges if and only if $B < 0$, and if so it converges to:

$$\psi^\infty = -\frac{A}{B} = \psi^\star + \frac{\sin(\psi^\star)L}{C} \quad (11)$$

The conditions for convergence are satisfied when

- $v < 0$, i.e. it is a reversing manouver[1] and
- $C > 0$, which gives a condition on the minimum value of K_P:

$$K_P > \frac{L}{\cos(\psi^\star)L_1 + L2} > \frac{L}{L_1 + L_2} \quad (12)$$

These conditions are easily verified on a normal system. Their intuitive meaning is that it is harder to stabilise a very small trailer with a very long tractor.

2) Compensation of steady-state error: Equation 11 indicates that when stabilising to a non-zero hitch-angle, a proportional only control law will lead to a steady state error. The limit hitch-angle ψ^∞ can be expressed as a function of the demand $\psi^\star$. If we could invert this relation, we could compute the required $\psi^\star$ to demand in order to achieve a given ψ^∞. Equation 11 cannot be inverted, but is reasonably approximated (within a 1% error) by its linearisation in the range of angles we are considering. This linearisation (around $\psi^\star = 0$) is given by:

$$\psi^\infty = \left(1 + \frac{L}{K_P L_1 - L + K_P L_2}\right)\psi^\star + O(\psi^{\star 2}) \quad (13)$$

This expression can be inverted to compute the demanded hitch-angle ψ^d that will make the system converge to a desired angle $\psi^\star$:

$$\psi^d(\psi^\star) = \frac{K_P L_1 - L + K_P L_2}{K_P(L_1 + L_2)}\psi^\star \quad (14)$$

Using the above relations, we can modify our control law to obtain a theoretical convergence on $\psi^\star$:

$$\phi = K_p(\psi^d(\psi^\star) - \psi) + K_I \int_0^t (\psi^\star - \psi)du \quad (15)$$

In this equation, the integral term is not essential for the stability of the control. It is added to account for minor inaccuracies remaining after the proportional control. Such inaccuracies can result from the linearisation leading to $\psi^d(\psi^\star)$, or from errors in the vehicle model (L, L_1, and L_2).

B. Experimental results

Fig. 3 depicts the tracking of a sine-shaped reference hitch-angle profile while reversing at a constant speed of $0.3\,\mathrm{m/s}$. Good tracking accuracy is achieved (within $0.02\,\mathrm{rad}$) but the hitch-angle oscillates around the reference trajectory. This oscillation is mainly due to the poor mechanical response of the system: the maximum steering angle acceleration and rate are very low, and there is also play in the tow ball mechanism.

Interestingly, when reversing the same trailer, a human driver puts much more stress on the mechanics by making much larger and faster steering movements than the automation system is capable of.

[1] This requirement can be eliminated by introducing the velocity into the control law.

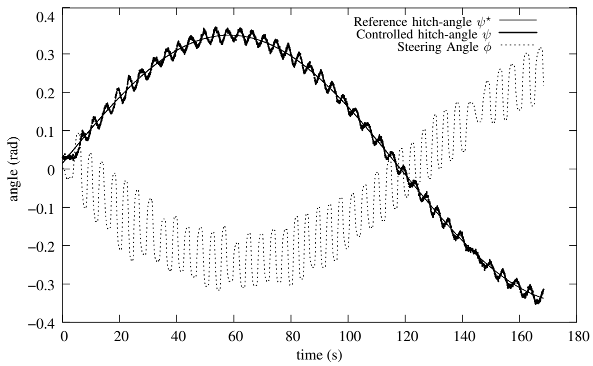


Fig. 3. Closed-loop control of hitch-angle while reversing at $0.3\,\mathrm{m/s}$.

V. Stabilisation of an articulated vehicle to a trajectory

This section describes a method for stabilising an articulated vehicle to a trajectory. In this context a trajectory $\mathcal{T}$ is a sequence of vehicle states indexed by time:

$$\begin{array}{rcccc} \mathcal{T}: & [0 \ldots t_{\max}] & \longrightarrow & \mathbb{R}^4 \\ & t & \longmapsto & (x, y, \theta_1, \psi) \end{array} \tag{16}$$

A. Path control

The first important aspect of a tractor-trailer system is that its dynamics are asymmetric. When driving forward, the trailer angle is naturally exponentially stable. When reversing, it is naturally unstable. Consequently we use different control laws for both situations.

When driving forward, we use a standard trajectory-tracking control law, such as *pure pursuit*, and we ignore the trailer. When reversing, we use a trajectory control law which is largely inspired from [10] where a Load Haul Dump (LHD) vehicle is considered. These vehicles are used in underground metalliferous (non-coal) mining operations for the transport of ore. They are four-wheel, centre-articulated vehicles whose steering is achieved by control of a centered hitch joint angle. In principle, these vehicles are similar to the tractor-trailer system considered here differing only in that the hitch angle is controlled directly rather than via steered wheels.

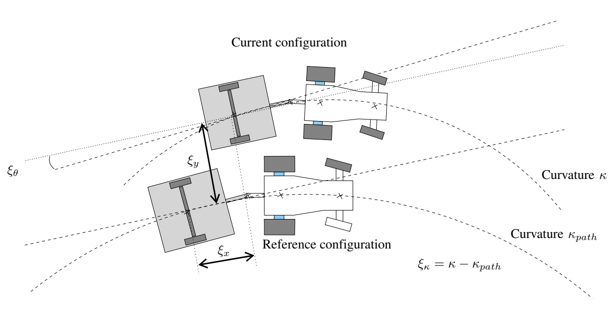


Fig. 4. Error measurement used in the stabilizing control law

The control law presented in [10] aims at stabilizing the vehicle on a path, i.e. a 2D curve in the plane. It relies on three error measurements, as depicted in Fig. 4:

- ξ_y: the lateral error, i.e. the distance between the vehicle reference point and the path.
- ξ_θ: the heading error, i.e. the difference between the heading of the vehicle θ_1 and the heading of the tangent vector to the path.
- ξ_κ: the curvature error, i.e. the difference between the curvature resulting from the vehicle hitch-angle and the curvature of the reference path.

In the original work, the authors controlled the hitch-angle rate of the LHD using these measurements. Here we control the hitch-angle and the control law is defined as:

$$\psi = K_y \xi_y + K_\theta \xi_\theta + K_\kappa \xi_\kappa \tag{17}$$

where K_y, K_θ and K_κ are tuning parameters. The resulting hitch-angle is then fed to the hitch-angle stabilisation law described in Section IV. The path-tracking system is described in block diagram form in Fig. 5.

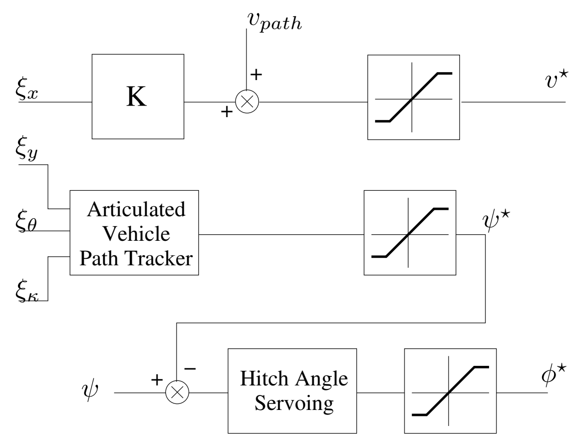


Fig. 5. Complete block diagram for the tractor-trailer reversing control law

B. Convergence and implementation

Ridley, in [10], presents an analysis of the properties of the above control law. An important result is that the convergence properties of this control law depend on the speed of the LHD. For a given tuning of K_y, K_θ and K_κ, there exists a maximum speed above which the feedback control loop will become unstable. Consequently, the use of dynamic gains is required.

C. Trajectory tracking

The control presented in [10] deals with path tracking and does not consider time. In order to follow a trajectory, the speed of the vehicle also has to be controlled. This control corrects for the longitudinal error $\xi_x(t)$, defined as the distance between the reference $\mathcal{T}(t)$ and current positions, along the tangent to the path (see Fig. 4). The desired velocity $V^\star$ is computed as:

$$V^\star = V_{ref}(t) + K_x \xi_x(t) \tag{18}$$

where $V_{ref}(t)$ is the velocity of the vehicle on the trajectory, and K_x a constant tuning gain.

D. Switching logic

Two pieces of switching logic are needed to implement the trajectory tracker in a robust manner.

First, we have to switch from driving forward to reversing, according to the direction of motion in the reference trajectory. This is straightforward to implement, but one must pay attention that the switching is made when the velocity is null.

Second, even if the reversing control law is theoretically convergent, numerous uncertainties such as control saturation, inaccurate models, loose mechanical parts, noise in localisation, *etc.* can bring the tractor-trailer system to a jack-knife situation from which it is impossible to recover without forward motion. In this situation, the hitch-angle diverges toward $\pi/2$ and the range of steering angle does not offer enough control to re-stabilise the articulation. The only solution is to start driving forward until the system gets back on track.

In our current implementation, we have determined that a jack-knife event is likely if:

- the hitch-angle $\psi > 0.6\,\mathrm{rad}$ **or**
- the heading error $\xi_\theta > 0.6\,\mathrm{rad}$ **or**
- the trailer heading error $\xi_{\theta_2} > 0.6\,\mathrm{rad}$.

If any of these conditions are met, motion is switched from reverse to forward motion. Corrective motion is deemed to be complete when:

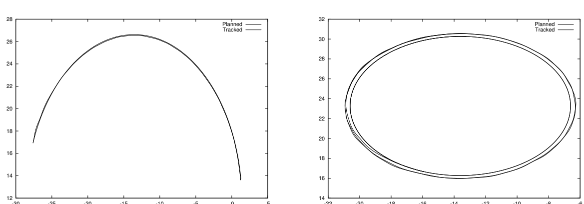
- the trailer heading error $\xi_{\theta_2} < 0.1\,\mathrm{rad}$ **and**
- the hitch-angle error ξ_ψ (difference between current hitch angle and reference one) $< 0.05\,\mathrm{rad}$.

Finally, trajectory tracking is aborted if a corrective forward motion reaches a segment of the reference trajectory where the system was driving forward (effectively a cusp point of the trajectory). In effect, if the system did not reach a suitable state by driving forward, it is unlikely to achieve it by adding more reversing.

VI. Experimental results

This section presents results obtained when tracking various reversing trajectories, first using an odometry based localisation, then using an external localisation.

A. Reversing on a circle with fixed radius



(a) radius = $15\,\mathrm{m}$ (b) radius = $7.5\,\mathrm{m}$

Fig. 6. Trajectory tracking with odometry based localisation

In this first set of experiments, we define reference trajectories as arcs of circle of various radii. The pose of the vehicle is computed using odometry information. The advantage of this setting is that the vehicle pose is a very clean and smooth signal. The disadvantage is that odometry localisation is known to drift over time, and consequently, is not suitable for real-life, long range, robotic operations. The next section will present results using an external localisation system.

Fig. 6 shows tracking results for a circular path, with radius varying from $7.5\,\mathrm{m}$ to $15\,\mathrm{m}$. As can be observed, the tracking is very accurate for radii of $15\,\mathrm{m}$. For a radius of $7.5\,\mathrm{m}$, the maximum achievable curvature is reached and the system cannot converge to the required trajectory. However, one should note that after the initial transient phase, the system converges to a stable orbit, which is the best it can do to follow the required curvature. In this particular test, three complete revolutions where completed.

B. Reversing on a pre-planned trajectory

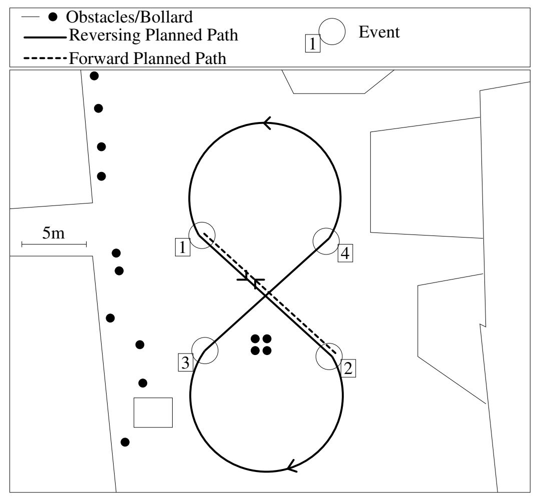


Fig. 7. Trajectory tracking with external localisation: planned trajectory. *events* are denoted by the boxed numbers.

In this second set of experiments, we want to track a more complex trajectory, as depicted in Fig. 7. This trajectory is composed of 5 segments:

- From event 2 to event 1 (dashed), a forward motion phase aimed at aligning the vehicle correctly before reversing;
- From 1 to 2 (plain), a straight line reversing motion;
- From 2 to 3 (plain), a circular reversing;
- From 3 to 4 (plain), another straight line reversing motion;
- From 4 to 1 (plain), another circular reversing;
- From 1 to 2 (plain), a final straight line.

Several aspects of this plan deserve comment: first the circular paths have a curvature greater than the maximum trackable curvature. We chose our circles as big as reasonably feasible in our environment. Second, the junction between the straight line and circular segments (events 1 to 4 in Fig. 7) show a discontinuity in curvature. This makes the paths more challenging for the control law. Nevertheless, in order to present results of our control at the limit of its performance, we decided not to use continuous curvature paths, such as clothoids[7].

In this experiment, we also use an external localisation system. This is implemented using a front-facing laser range-finder and reflective landmarks setup sparsely in our experimental site. This setting offers localisation without drift at the cost of a more noisy signal. In particular, a discontinuity in position results from a localisation correction when a new landmark is observed after a long period of dead-reckoning. This kind of discontinuity is especially challenging for the trajectory tracking system.

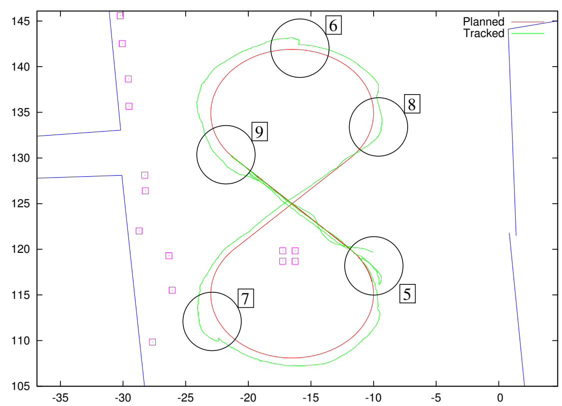


Fig. 8. Trajectory tracking with external localisation: tracked trajectory. *events* are denoted by the boxed numbers.

Fig. 8 illustrates how the planned path is tracked by our tractor-trailer system. We note from this graph that:

- On straight line segments the tracking is accurate and converges reasonably fast.
- On circular segments, the curvature is too high for accurate tracking, so the system converges to the closest feasible orbit.
- Localisation discontinuities (event 6 and 7) are handled gracefully.
- Curvature discontinuities (event 5, 8 and 9) are more problematic, and the tracking system can only achieve the new curvature by taking the vehicle reference point far from the path.
- Event 5 illustrates the results of our switching logic. At this point, when trying to pass the curvature discontinuity, the system brought itself into a jack-knife situation. Once detected, a short forward motion is initiated to realign the system with the trajectory. Then the system switches back to reversing and successfully passes the discontinuity.

From these and the previous experiments, it is clear that in order to obtain very accurate tracking, the path planner needs to take into account the strong constraints of this system, in particular, the small maximum trackable curvature and the need for smooth curvature profiles. However, the trajectory control performed extremely well within these constraints.

VII. Conclusion

This article introduced a new control scheme for a tractor-trailer system. This scheme is based on a two layer control loop: first a hitch-angle stabilisation loop controls the angle between tractor and trailer, then a path tracking control loop, initially designed for an articulated mining vehicle, is adapted to our tractor-trailer system.

The main advantage of this approach over traditional methods is its simplicity of implementation. Only a few parameters need to be tuned, and due to the linearity of the system and the clear physical meaning of the parameters, they are relatively easy to tune. Although simple, this control scheme has been proved locally stable around its intended operating point.

Finally, this control law has been implemented on a real vehicle and experiments were conducted on challenging trajectories. Given the limited dynamic performance of our platform (slow response time, loose components, low speed actuation), the control law exhibited excellent convergence and stability properties. Furthermore, compared with methods such as [15], it demonstrated very smooth response in all our experiments.

Acknowledgments

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References

[1] C. Altafini, A. Speranzon, and B. Wahlberg. A feedback control scheme for reversing a truck and trailer vehicle. *IEEE Transactions on Robotics and Automation*, 2001.

[2] P. Bolzern, R.M. DeSantis, and A. Locatelli. An input-output linearization approach to the control of an n-body articulated vehicle. *Journal of Dynamic Systems, Measurement, and Control*, 2001.

[3] D.N. Carter and J.R. Lormor. Vehicle steering aid system, 2004. GB patent GB2398050.

[4] R.A. Cook and R.A. Blasius. Apparatus for vehicle/trailer hitch guidance device, 2006. US patent US7057498.

[5] Alessandro De Luca and Giuseppe Oriolo. Modelling and control of nonholonomic mechanical systems. In *Kinematics and Dynamics of Multi-Body Systems*. 1995.

[6] A.W. Divelbiss and J.T. Wen. Trajectory tracking control of a car-trailer system. *IEEE Transactions on Control Systems Technology*, 1997.

[7] Th. Fraichard and A. Scheuer. From reeds and shepp's to continuous-curvature paths. *IEEE Trans. on Robotics*, 2004.

[8] J. Hermosillo and S. Sekhavat. Feedback control of a bi-steerable car using flatness: Application to trajectory tracking. In *Proc. of the American Control Conference*, 2003.

[9] J.R. Koza. A genetic approach to finding a controller to back up a tractor-trailer truck. In *Proceedings of the 1992 American Control Conference*, 1992.

[10] P. Ridley and P. Corke. Load haul dump vehicle kinematics and control. *Journal of Dyn. Sys, Measurement and Control*, 2003.

[11] S.D. Robert. Trailer backing up device and method, 2004. US patent US2004215374.

[12] P. Rouchon, M. Fliess, J. Lvine, and Ph. Martin. Flatness and motion planning: the car with n-trailers. In *Proc. of European Control Conference*, 1993. Groningen, pages 1518–1522, 1993.

[13] M. Sampei, T. Itoh Tamura, and M. T. Nakamichi. Path tracking control of trailer-like mobile robot. In *Proc. of IEEE/RSJ In. Workshop on Intelligent Robots and Systems '91*, 1991.

[14] K. Usher. *Visual homing for a car-like vehicle*. PhD thesis, Queensland University of Technology, Brisbane, Australia, June 2005.

[15] G. Walsh, D. Tilbury, S. Sastry, R. Murray, and J.P. Laumond. Stabilization of trajectories for a systems with nonholonomic constraints. *IEEE Transaction onf Robotic and Automation*, 1994.