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Dynamic Stable Manipulation via Soft-fingered Hand

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***Abstract*— This study clarifies that secure dynamic manipulation by a pair of soft fingertips can be achieved easily without any information of a grasped object, which is called *Blind Manipulation*. First, we describe two holonomic constraints and two nonholonomic constraints induced by minimum dof soft-fingered grasping. Next, we represent Lagrangian of the soft-fingered hand system that includes the elastic energy of the fingertip, the four constraints, and also gravity effect. Furthermore, we express equations of motion of the grasped object from the Lagrangian, and simulate the dynamic behavior of the object. Finally, we clarify that the position and posture of the object always converge to a corresponding point when a pair of fingers is freely activated.**

I. INTRODUCTION

In the case of dealing with soft-fingered manipulation, the modeling of elastic soft materials is much important. While a pretty accurate elastic model is able to describe a true behavior of an object grasped by soft fingers, it is extremely difficult to represent the exact model in an analytical procedure. Generally, nonlinear finite element analysis is used for computing the elastic force and showing the deformation process, whose model is based on experimental observations but not analytical ways. Hence, when we discuss the soft-fingered manipulation analytically, it is important how much we can get the model to be closer to an appropriate model that is able to express a real object motion through the soft fingertips.

Furthermore, modeling issue of soft fingers is intimately connected to not only modeling itself, but also stable grasping and robust manipulation in the robotic hand system as well. That is, position/posture controls of the grasped object without complicated control inputs that is usually designed for the conventional point-contact manipulation can be demonstrated by using the more suitable soft fingertip model.

In this paper, we first extend previous one-dimensional fingertip model [1], [2] to two-dimensional model by additionally applying the bending motion of the fingertip along the tangential direction of grasped object. Additionally, we formulate holonomic and nonholonomic constraints generated by two-fingered hand. By represent the Lagrangian for the handling system that includes the constraints and elastic potential energy induced by the deformation of the fingertip, we obtain the equations of motion of the grasped object during the soft-fingered manipulation. Finally, we simulate the dynamic behavior of the object in the case that both fingers arbitrary move according to an example motion. We clarify that the stable soft-fingered manipulation without any object information can readily be attained under the gravity force.

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II. TWO-DIMENSIONAL MODEL

In what follows, we assume that the slip motion between the object and the fingertip does not occur in all manipulation processes. Before formulating the elastic energy combined by both fingertips in actual two-fingered manipulation, we first describe the energy function in the case of a single contact between the object and a soft fingertip shown in Fig.1.

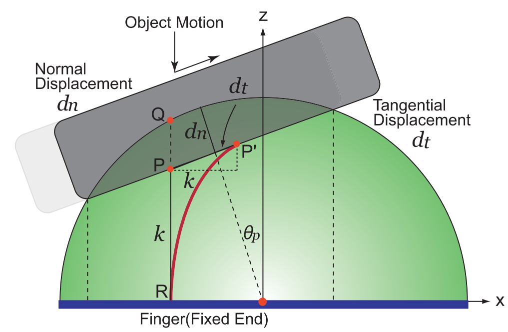

Fig. 1. Contact mechanism during the soft-fingered manipulation

Let Q and R be the opposite end points of a cylindrical virtual spring within the fingertip, and P be a point on the contacting surface. In addition, let k be the spring constant of the cylindrical component, and θ_p be the object orientation in this contact. When the point P shifts d_t from the original point to P' with constant normal displacement d_n, each force for vertical and parallel directions to the fixed end can be represented as follows:

$$\mathrm{d}F_v = k(PQ + d_t \sin\theta_p), \tag{1}$$

$$\mathrm{d}F_p = k d_t \cos\theta_p, \tag{2}$$

where we are assuming that the spring constant k is equivalent to that of bending motion. The elastic potential energy induced by the integrated deformation of the compression and bending is therefore expressed by

$$P = \frac{1}{2}\int\!\!\int_{ell} k\left\{(PQ + d_t \sin\theta_p)^2 + d_t^2\cos^2\theta_p\right\}, \tag{3}$$

where ell denotes an elliptical region obtained by projecting the contact surface onto the finger plane, as shown in Fig.1.

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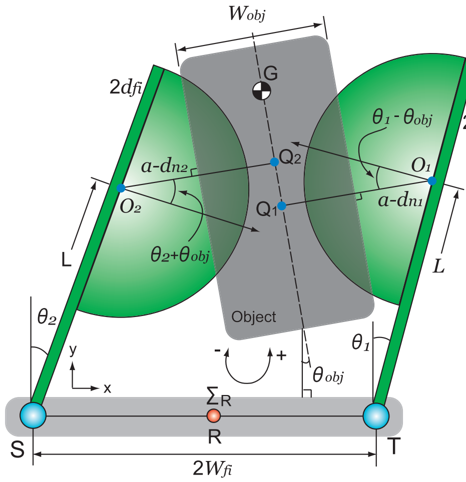


Fig. 2. Soft-fingered manipulation under the gravitational force

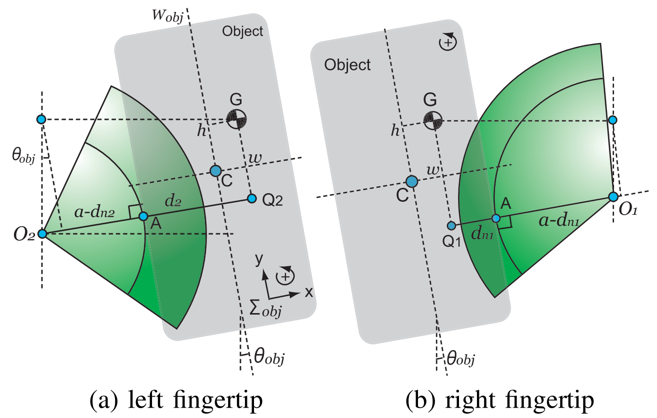


Fig. 3. Geometric relationship between grasped object and both fingertips

Developing Eq.(3) with the use of numerical analysis, the energy equation P can finally be represented as

$$P = \pi E \left\{ \frac{d_n^3}{3\cos^2\theta_p} + d_n^2 d_t \tan\theta_p + d_n d_t^2 \right\}, \tag{4}$$

where E denotes the Young's modulus of the material of the fingertips. Extending the above procedure to the two-fingered hand shown in Fig.2, P is then expressed by

$$P = \pi E \sum_{i=1}^{2} \left\{ \frac{d_{ni}^3}{3\cos^2\theta_{pi}} + d_{ni}^2 d_{ti} \tan\theta_{pi} + d_{ni} d_{ti}^2 \right\}, \tag{5}$$

where i means i-th finger of the hand in which 1 and 2 stand for right and left fingers, respectively, and

$$\theta_{pi} = \theta_i + (-1)^i \theta_{obj}. \tag{6}$$

III. HOLONOMIC AND NONHOLONOMIC CONSTRAINTS

A. Normal Constraints

As illustrated in Fig.2, let W_{obj} be the width of a grasped object, $2W_{fi}$ be the distance between both roots of the fingers, $2d_{fi}$ be the thickness of the finger, (θ_1, θ_2) be the rotational joint angles, L be the length of the finger, and G be the center of gravity of the object. Additionally, let $(x_{obj}, y_{obj}, \theta_{obj})$ be the position and orientation of the object. Considering the geometric relationship of the handling system shown in Fig.2, the coordinate of the fingertip center O_i is expressed with respect to $\sum_R$ as follows:

$$O_{ix} = (-1)^{i+1} W_{fi} + (-1)^i L \sin\theta_i + (-1)^i d_{fi} \cos\theta_i, \tag{7}$$

$$O_{iy} = L\cos\theta_i - d_{fi}\sin\theta_i. \tag{8}$$

The constraints along the normal direction to the object surface are holonomic equations, and these can be written by

$$\begin{aligned} C_i^H &= (-1)^i (x_{obj} - O_{ix}) \cos\theta_{obj} \\ &+ (-1)^i (y_{obj} - O_{iy}) \sin\theta_{obj} \\ &- (a - d_{ni}) + \frac{W_{obj}}{2} + (-1)^i w = 0. \end{aligned} \tag{9}$$

In this paper, we give essentric distances (w, h) of point G to be zero for ease of explanation, and θ_{obj} has a positive value in the counter-clockwise direction.

B. Tangential Constraints

Letting θ_i be positive when both fingers rotate inward as shown in Fig.2, the rolling velocity $\dot{s}_i$ of the object on the soft fingertip is expressed as

$$\dot{s}_i = -(a - d_i) \left\{ \dot{\theta}_i + (-1)^i \dot{\theta}_{obj} \right\}. \tag{10}$$

In addition, as illustrated in Fig.3, the distance GQ_i is represented as

$$GQ_i = -(x_{obj} - O_{ix}) \sin\theta_{obj} + (y_{obj} - O_{iy}) \cos\theta_{obj}. \tag{11}$$

Therefore, differentiating Eq.(11) with respect to time, a velocity constraint including the change of bending motion $\dot{d}_{ti}$ can be given as a nonholonomic constraint:

$$C_i^N = \dot{GQ}_i - \dot{s}_i + \dot{d}_{ti} = 0. \tag{12}$$

IV. GENERAL DESCRIPTION OF EQUATIONS OF MOTION AND ITS NUMERICAL ANALYSIS

In this study, we deal with the soft-fingered manipulation using a minimum degrees of freedom hand, and investigate the dynamic behavior of a grasped known object in the case of the presence of holonomic and nonholonomic constraints in the system.

A. Lagrangian

Let $\boldsymbol{q}$ be the generalized coordinate, (M_{obj}, I_{obj}) be the mass and moment of inertia of the grasped object respectively, and $\boldsymbol{I} = [I_1, I_2]^T$ be the moment of inertia of both fingers. In addition, let g be the acceleration of gravity, P_{gv} be the potential energy with respect to gravitational force, λ_i^H be the constraint force in terms of the holonomic constraint C_i^H

expressed in Eq.(9). The Lagrangian in the present handling system can then be described using Eq.(5) as

$$L = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}\dot{\boldsymbol{q}} - P - P_{gv} + \sum_{i=1}^{2} \lambda_i^H C_i^H, \tag{13}$$

where

$$\boldsymbol{M} = diag(M_{obj}, M_{obj}, I_{obj}, \boldsymbol{I}^T, M_{obj}, M_{obj}, M_{obj}, M_{obj}) \in R^{9\times 9}, \tag{14}$$

$$\boldsymbol{q} = [x_{obj}, y_{obj}, \theta_{obj}, \boldsymbol{\theta}^T, \boldsymbol{d}_n^T, \boldsymbol{d}_t^T]^T \in R^{9\times 1}, \tag{15}$$

$$P_{gv} = M_{obj} g y_{obj} + \sum_{i=1}^{2} M_i g L \cos\theta_i. \tag{16}$$

Note that the M_i denotes the mass of the i-th finger. In Eq.(13), the first term means the kinetic energy in the entire system, and the second and third terms stand for the elastic potential energy of the soft fingers and gravitational potential energy. In addition, the last term denotes a virtual energy due to the constraint forces that does not generate any energy, that is, it always be zero.

B. Equations of Motion

As expressed as Eqs.(9) and (12), this handling system has four constraint equations that is associated with normal and tangential directions to the grasped object. Note that while the holonomic constraints can be included into the Lagrangian directly, we are able to contain the nonholonomic constraints into the equations of motion for the first time [3].

We define the nonholonomic constraint matrix as $\boldsymbol{\Phi}^N \in R^{2\times 9}$ [4]. The element of the matrix is then expressed using Eq.(12) as

$$\Phi_{ij}^N = \frac{\partial C_i^N}{\partial \dot{q}_j} \quad (i = 1,2 : j = 1,\cdots,9). \tag{17}$$

Let $\boldsymbol{\lambda}^N = [\lambda_1^N, \lambda_2^N]^T$ be the constraint force vector directed to the tangential direction to the object surface. As long as $\boldsymbol{\Phi}^{NT}\boldsymbol{\lambda}^N$ is represented as a linear combination form relating to the time derivative of generalized coordinate, the equations of motion under nonholonomic constraints can be expressed as [3]

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = \boldsymbol{\Phi}^{NT}\boldsymbol{\lambda}^N \quad (j = 1,\cdots,9). \tag{18}$$

C. Constraint Stabilization Method including Nonholonomic Constraints

In order to investigate the dynamic analysis of a system under both kinds of constraints, we apply the *Constraint Stabilization Method* (CSM) [5] in this study.

Let $\boldsymbol{C}^H$, $\boldsymbol{C}^N$ be the vector description of Eqs.(9) and (12). Here, corrected constraint equations that is based on each constraint are described as

$$\ddot{\boldsymbol{C}}^H + 2\alpha\dot{\boldsymbol{C}}^H + \alpha^2\boldsymbol{C}^H = \boldsymbol{0} \in R^{2\times 1}, \tag{19}$$

$$\dot{\boldsymbol{C}}^N + \beta\boldsymbol{C}^N = \boldsymbol{0} \in R^{2\times 1}, \tag{20}$$

where α and β correspond to an arbitrary constant associated with the speed of asymptotical stablity of both equations. By using Eqs.(19) and (20) in the numerical analysis of corresponding equations of motion, we can obtain the solution of the system stably.

On the other hand, let $\boldsymbol{\Phi}^H \in R^{2\times 9}$ be the holonomic constraint matrix of the system as well. Each element of the matrix is then expressed using Eq.(9) as

$$\Phi_{ij}^H = \frac{\partial C_i^H}{\partial q_j} \quad (i = 1,2 : j = 1,\cdots,9). \tag{21}$$

By developing Eqs.(19) and (20) with both constraint matrices $\boldsymbol{\Phi}^H$ and $\boldsymbol{\Phi}^N$, we can define γ^H and γ^N as follows:

$$\boldsymbol{\Phi}^H\dot{\boldsymbol{p}} = -\boldsymbol{b}^H(\boldsymbol{q},\boldsymbol{p}) - 2\alpha\dot{\boldsymbol{C}}^H - \alpha^2\boldsymbol{C}^H \triangleq -\gamma^H, \tag{22}$$

$$\boldsymbol{\Phi}^N\dot{\boldsymbol{p}} = -\boldsymbol{b}^N(\boldsymbol{q},\boldsymbol{p}) - \beta\boldsymbol{C}^N \triangleq -\gamma^N, \tag{23}$$

where $\boldsymbol{p}$ denotes the generalized velocity vector, and the relationship $\boldsymbol{p} = \dot{\boldsymbol{q}}$ is also satisfied.

Furthermore, let $\boldsymbol{f}_p$ be the potential force vector, $\boldsymbol{f}_{ext}$ be the vector of generalized external force, and $\boldsymbol{I}$ be the identity matrix. The state space description of the CSM including a control input vector $\boldsymbol{u}_{IN}$, which is able to deal with the holonomic and nonholonomic constraints simultaneously, is described as [6], [7]

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M} & -\boldsymbol{\Phi}^{HT} & -\boldsymbol{\Phi}^{NT} \\ \boldsymbol{0} & -\boldsymbol{\Phi}^H & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{\Phi}^N & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{p}} \\ \boldsymbol{\lambda}^H \\ \boldsymbol{\lambda}^N \end{bmatrix} = \begin{bmatrix} \boldsymbol{p} \\ -\boldsymbol{f}_p + \boldsymbol{f}_{ext} + \boldsymbol{u}_{IN} \\ \gamma^H \\ \gamma^N \end{bmatrix}. \tag{24}$$

By numerically integrating the above first-order differential equations, we compute the dynamic behavior for soft-fingered handling of a rigid object.

V. DERIVATION OF E.O.MS (EXAMPLE)

We show a set of equations of motion of the handling system illustrated in Fig.2 in detail. First, we show the constraint matrices that appear along the normal and tangential direction to the object surface, and nonlinear equations of motion of the system. In what follows, we describe the equations of motion with respect to the object position (x_{obj}, y_{obj}) and orientation θ_{obj}, and represent other equations with respect to the rotational angle of the finger θ_i and fingertip displacement (d_{ni}, d_{ti}) in Appendix I.

A. Constraint Matrix

The constraint matrix $\boldsymbol{\Phi} = [\boldsymbol{\Phi}^{HT}, \boldsymbol{\Phi}^{NT}]^T \in R^{4\times 9}$ including both holonomic and nonholonomic constraints in the present handling system can be expressed using Eqs.(9), (12), (17), and (21) as

$$\boldsymbol{\Phi} = \begin{bmatrix} -\mathrm{C}\theta_{obj} & -\mathrm{S}\theta_{obj} & A_{c1} & B_{c1} & 0 & 1 & 0 & 0 & 0 \\ \mathrm{C}\theta_{obj} & \mathrm{S}\theta_{obj} & A_{c2} & 0 & B_{c2} & 0 & 1 & 0 & 0 \\ -\mathrm{S}\theta_{obj} & \mathrm{C}\theta_{obj} & E_{c1} & F_{c1} & 0 & 0 & 0 & 1 & 0 \\ -\mathrm{S}\theta_{obj} & \mathrm{C}\theta_{obj} & E_{c2} & 0 & F_{c2} & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{25}$$

In Eq.(25), the symbols S and C denote the abbreviation of *sin* and *cos*, respectively. Additionally, $A_{ci}, B_{ci}, E_{ci}, F_{ci} (i = 1,2)$ correspond to the following equations.

$$
\begin{aligned}
A_{ci} &= (-1)^{i+1}(x_{obj} - O_{ix})\mathrm{S}\theta_{obj} \\
&+ (-1)^{i}(y_{obj} - O_{iy})\mathrm{C}\theta_{obj}, && (26)\\
B_{ci} &= -L\mathrm{C}\left\{\theta_i + (-1)^i\theta_{obj}\right\} \\
&+ d_{fi}\mathrm{S}\left\{\theta_i + (-1)^i\theta_{obj}\right\}, && (27)\\
E_{ci} &= -(x_{obj} - O_{ix})\mathrm{C}\theta_{obj} - (y_{obj} - O_{iy})\mathrm{S}\theta_{obj} \\
&+ (-1)^i(a - d_{ni}), && (28)\\
F_{ci} &= L\mathrm{S}\left\{\theta_r + (-1)^i\theta_{obj}\right\} \\
&+ d_{fi}\mathrm{C}\left\{\theta_i + (-1)^i\theta_{obj}\right\} + (a - d_{ni}), && (29)
\end{aligned}
$$

where A_{ci} corresponds to $(-1)^i GQ_i$ as shown in Eq.(11) and Fig.3, and B_{ci} stands for the tangential component of position vector from each origin T,S of the i-th finger to each origin O_i of the i-th fingertip, as shown in Fig.2. Furthermore, F_{ci} means the normal component of position vector from each origin T,S of the i-th finger to the center of the contacting circle on the fingertip. E_{ci} can be transformed using Eq.(9) into a constant expression:

$$E_{ci} = -(-1)^i \frac{W_{obj}}{2} - w. \quad (30)$$

B. Equations of Motion

By developing Eq.(18), the equations of motion relating to the object position and orientation $(x_{obj}, y_{obj}, \theta_{obj})$ can be represented as

$$
\begin{aligned}
M_{obj}\ddot{x}_{obj} + \boldsymbol{\lambda}^T \boldsymbol{s}_x &= 0, && (31)\\
M_{obj}\ddot{y}_{obj} + \boldsymbol{\lambda}^T \boldsymbol{s}_y &= 0, && (32)\\
I_{obj}\ddot{\theta}_{obj} + A + B - \boldsymbol{\lambda}^T \boldsymbol{s}_\theta &= 0, && (33)
\end{aligned}
$$

where $A, B, \boldsymbol{s}_x, \boldsymbol{s}_y, \boldsymbol{s}_\theta$ correspond to the following equations:

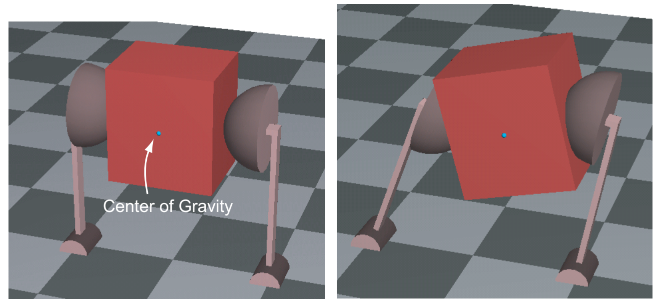
$$A = \frac{2\pi E}{3}\left\{\frac{d_{n1}^3 \mathrm{S}(\theta_1 - \theta_{obj})}{\mathrm{C}^3(\theta_1 - \theta_{obj})} - \frac{d_{n2}^3 \mathrm{S}(\theta_2 + \theta_{obj})}{\mathrm{C}^3(\theta_2 + \theta_{obj})}\right\}, \quad (34)$$

$$B = \pi E\left\{\frac{-d_{n1}^2 d_{t1}}{\mathrm{C}^2(\theta_1 - \theta_{obj})} + \frac{d_{n2}^2 d_{t2}}{\mathrm{C}^2(\theta_2 + \theta_{obj})}\right\}, \quad (35)$$

$$
\begin{aligned}
\boldsymbol{s}_x &= [\mathrm{C}\theta_{obj}, -\mathrm{C}\theta_{obj}, \mathrm{S}\theta_{obj}, \mathrm{S}\theta_{obj}]^T, && (36)\\
\boldsymbol{s}_y &= [\mathrm{S}\theta_{obj}, -\mathrm{S}\theta_{obj}, -\mathrm{C}\theta_{obj}, -\mathrm{C}\theta_{obj}]^T, && (37)\\
\boldsymbol{s}_\theta &= [A_{c1}, A_{c2}, E_{c1}, E_{c2}]^T. && (38)
\end{aligned}
$$

VI. SIMULATION

In this paper, we clarify that the stable grasping and manipulation using the soft-fingered hand can consistently be achieved steadily. In what follows, we consider an example motion of the hand, and simulate the dynamic behavior of the grasped object expressed as Eqs.(31), (32), and (33) during the given manipulating motion.



(a) initial state (b) operation 5

Fig. 4. Snapshots on simulation

TABLE I
SIMULATION PARAMETERS

Parameters	Values
Rungekutta sampling time	0.1 msec
α	20000
β	10000
P-Gain : K_P	300
D-Gain : K_D	14
I-Gain : K_I	0.1
Viscosity for d_{ni} : c_n	300 Ns/m
Viscosity for d_{ti} : c_t	300 Ns/m

A. Example Motion

The example motion of the fingers, which is dealt with in this study, is as follows:

1. Initial state: Both fingers grasp an object in parallel (Fig.4-(a))
2. Operation 1: $(\theta_1^d, \theta_2^d) = (6\text{ deg}, 6\text{ deg})$
3. Operation 2: $(\theta_1^d, \theta_2^d) = (20\text{ deg}, -10\text{ deg})$
4. Operation 3: $(\theta_1^d, \theta_2^d) = (-2\text{ deg}, 13\text{ deg})$
5. Operation 4: $(\theta_1^d, \theta_2^d) = (-10\text{ deg}, 20\text{ deg})$
6. Operation 5: $(\theta_1^d, \theta_2^d) = (-7\text{ deg}, 17\text{ deg})$(Fig.4-(b))
7. Operation 6: $(\theta_1^d, \theta_2^d) = (17\text{ deg}, -7\text{ deg})$
8. Operation 7: $(\theta_1^d, \theta_2^d) = (-15\text{ deg}, 25\text{ deg})$
9. Operation 8: $(\theta_1^d, \theta_2^d) = (5\text{ deg}, 5\text{ deg})$

As shown in Fig.4-(a), the fingers are positioned in the initial state so that geometric point-contact between the object and soft fingers is maintained. After the operation 1, we perform a feedback control with respect to the rotational angle θ_i according to the above desired angle of the finger. A PID control law is applied to the present system, and it is described as

$$u_{INi} = -K_P(\theta_i - \theta_i^d) - K_D\dot{\theta}_i - K_I \int_0^t (\theta_i - \theta_i^d)\mathrm{d}\tau. \quad (39)$$

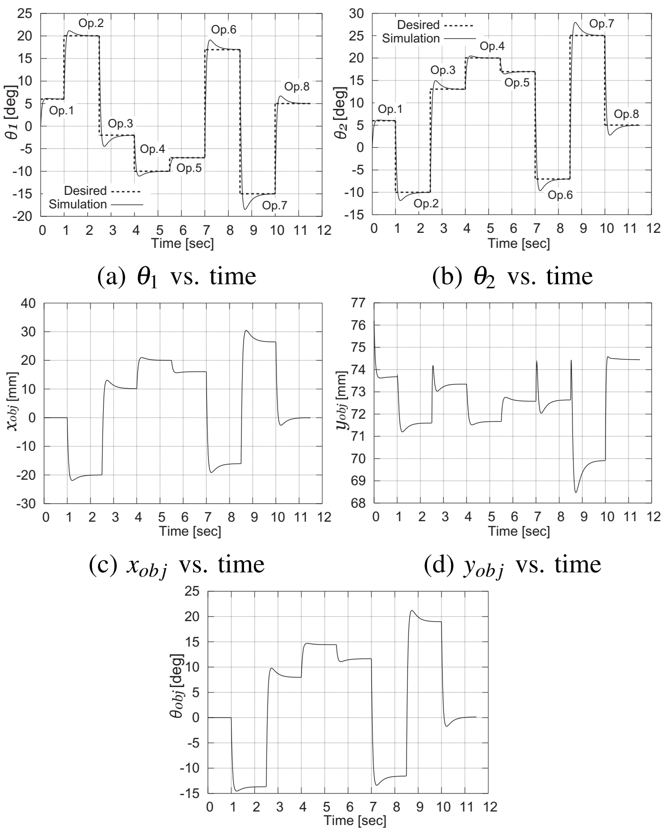
In this system, we do not consider any disturbance, and also set that the external force is zero in Eq.(24) such that $\boldsymbol{f}_{ext} = \boldsymbol{0}$. We input Eq.(39) into $\boldsymbol{u}_{IN}$ expressed in Eq.(24) as a torque command. Parameters in the numerical analysis are given in Table I. Also, Mechanical parameters used for the two-fingered hand are given in Table II.

B. Results

1) Object Motion: Fig.5-(a) and (b) show the results of trajectory of both fingers, and Fig.5-(c), (d), and (e) show

TABLE II
SYSTEM PARAMETERS

Parameters	Values
L	76.2 mm
$2W_{fi}$	97 mm
a	20 mm
W_{obj}	49 mm
M_{obj}	0.3 kg
I_{obj}, I_1, I_2	125 kg·mm^2
d_{fi}	4 mm
Young's modulus : E	0.232 MPa

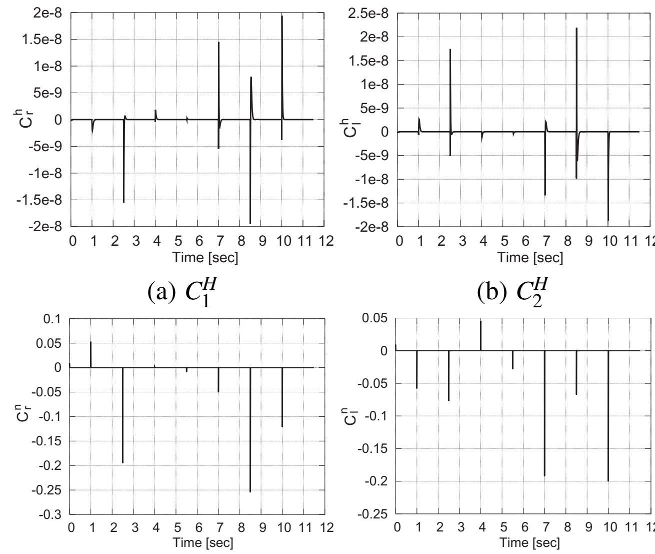


(a) θ_1 vs. time (b) θ_2 vs. time

(c) x_{obj} vs. time (d) y_{obj} vs. time

(e) θ_{obj} vs. time

Fig. 5. Simulation results of object position and orientation



(a) C_1^H (b) C_2^H

(c) C_1^N (d) C_2^N

Fig. 6. Convergence of four constraints

the object position and orientation with respect to time, respectively. As shown in all the figures in Fig.5, the resultant force and moment induced by the elastic deformation of the soft fingers consistently keep the manipulating motion stable. In other words, we can find that the position and orientation converge to a certain state determined by the force-moment equilibrium on the soft fingertips. In addition, we evaluate whether the CSM containing both constraints works well in the numerical analysis. That is, we verify that each constraint equation expressed as Eqs.(9) and (12) converges to zero during the computation. Fig.6-(a) and (b) show the error value of normal constraint on the i-th fingertip, and Fig.6-(c) and (d) show that of tangential constraint.

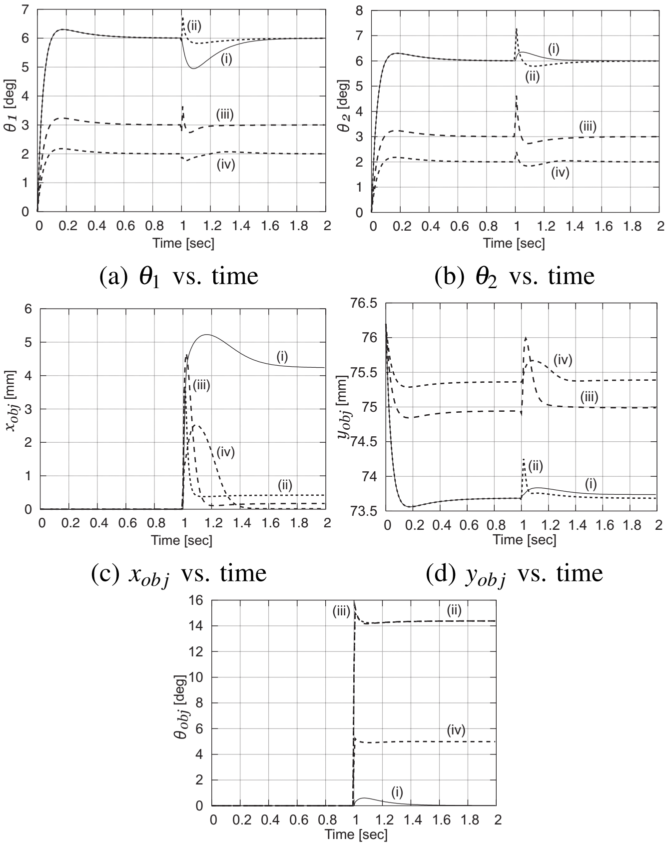
In the result of holonomic constraint C_i^H, we find that the numerical order plotted on y-axis becomes approximately 10^{-8}, and as a result, Eq.(9) is satisfied in the numerical computation. In the case of nonholonomic constraint, we know that the value on each switching point of the individual operation relatively increases at one point. This results from the fact that the time derivative of rotational angle $\dot{\theta}_i$ becomes substantially large value in the simulation due to the step inputs in all operations, as shown in Fig.5-(a) and (b). At the same time, the numerical order along y-axis exhibits 10^{-6} except the switching points. As a result, we can conclude that the CSM including holonomic and nonholonomic constraints works well in the numerical simulation.

2) Robustness for Disturbances: We input three sorts of disturbances represented as Eqs. (40), (41), and (42) to the grasped object when the desired angle of each finger is given as $\theta_1^d = \theta_2^d = 6$ deg, $\theta_1^d = \theta_2^d = 3$ deg, and $\theta_1^d = \theta_2^d = 2$ deg, respectively.

$$(F_x^{dst}, F_y^{dst}, M^{dst}) = (130 \text{ N}, 130 \text{ N}, 5 \text{ Nm}), \quad (40)$$
$$(F_x^{dst}, F_y^{dst}, M^{dst}) = \delta(t) \cdot (0.9 \text{ N}, 0, 0.327 \text{ Nm}), (41)$$
$$(F_x^{dst}, F_y^{dst}, M^{dst}) = \delta(t) \cdot (0.3 \text{ N}, 0, 0.109 \text{ Nm}). (42)$$

Eq. (40) is given as a step disturbance and Eqs. (41) and (42) are given as an impulse disturbance, and these are imposed to the object after operation 1 indicated in Fig.5-(a).

As shown in Fig.7-(a) and (b), we first find that the both fingers (θ_1, θ_2) stably converge to the corresponding desired angles due to the PID control law, which is represented in Eq. (39), in spite of the large disturbances. Additionally, Fig.7-(c) and (d) show that the response of (iii) oscillates more largely than that of (ii). It is because that the handling motion of (iii) corresponds to *light grasping* due to $\theta_1^d = \theta_2^d = 3$ deg, but the handling motion of (ii) becomes *strong grasping* due to $\theta_1^d = \theta_2^d = 6$ deg. This phenomenon can easily be seen in human finger's manipulation.



(a) θ_1 vs. time (b) θ_2 vs. time

(c) x_{obj} vs. time (d) y_{obj} vs. time

(e) θ_{obj} vs. time

Fig. 7. Responses for disturbances: (i) Eq. (40) step response in $\theta_1^d = \theta_2^d = 6$ deg, (ii) Eq. (41) impulse response in $\theta_1^d = \theta_2^d = 6$ deg, (iii) Eq. (41) impulse response in $\theta_1^d = \theta_2^d = 3$ deg, (iv) Eq. (42) impulse response in $\theta_1^d = \theta_2^d = 2$ deg.

VII. CONCLUDING REMARKS

In this study, we have first formulated a two-dimensional fingertip model that contains the compression and bending motion simultaneously. Also we have simulated the dynamic behavior of a parallel-rigid object grasped by a minimum dof soft-fingered hand. In this process, we have applied an extended CSM that includes holonomic and nonholonomic constraints induced by the soft-fingered manipulation.

This study indicates that the soft-fingered manipulation is able to simply achieve secure grasping and robust manipulation even when we actuate two fingers freely without any object information, which is called ”*Blind Manipulation*”. Note that the concept of Blind manipulation using the soft fingers needs to be distinguished from that of *Blind Grasping* first mentioned in related works [8], [9]. The cause of the problem is that any force feedforward term is not required for the stable grasping in the soft-fingered handling. In addition any feedback term is not necessary for robust manipulation, which is verified in Eq.(39), while the both control inputs are absolutely imperative in the secure handling discussed in the conventional study [10]. We have clarified that the flexibility of soft fingertips greatly contributes to stable grasping and manipulation, and conclude that the LMEE [1], [2] plays an important role in robust soft-fingered manipulation.

APPENDIX I
EQUATIONS OF MOTION WITH RESPECT TO θ_i AND (d_{ni}, d_{ti})

Developing Eq.(18) as well as Eqs.(31), (32), and (33), the equations of motion with respect to θ_i and (d_{ni}, d_{ti}) are described using Eqs.(27) and (29) as follows:

$$
\begin{aligned}
& I_i\ddot{\theta}_i - \frac{2\pi E d_{ni}^3 \mathrm{S}\left\{\theta_i + (-1)^i\theta_{obj}\right\}}{3\mathrm{C}^3\left\{\theta_i + (-1)^i\theta_{obj}\right\}} \\
& + \frac{\pi E d_{ni}^2 d_{ti}}{\mathrm{C}^2\left\{\theta_i + (-1)^i\theta_{obj}\right\}} - \lambda_i^H B_{ci} - \lambda_i^N F_{ci} \\
& - M_i g L \sin\theta_i = u_{INi}, \qquad (43) \\
& M_{obj}\ddot{d}_{ni} + \frac{\pi E d_{ni}^2}{\mathrm{C}^2\left\{\theta_i + (-1)^i\theta_{obj}\right\}} \\
& + 2\pi E d_{ni} d_{ti} \tan\left\{\theta_i + (-1)^i\theta_{obj}\right\} \\
& + \pi E d_{ti}^2 - \lambda_i^H + c_n\dot{d}_{ni} = 0, \qquad (44) \\
& M_{obj}\ddot{d}_{ti} + \pi E d_{ni}^2 \tan\left\{\theta_i + (-1)^i\theta_{obj}\right\} \\
& + 2\pi E d_{ni} d_{ti} - \lambda_i^N + c_t\dot{d}_{ti} = 0, \qquad (45)
\end{aligned}
$$

where c_n and c_t denote the coefficient of viscosity of the soft fingertip along the normal and tangential directions to the object surface, as shown in Table II. By substituting the viscosity into Eqs.(44) and (45), a Voigt model associated with fingertip displacements (d_{ni}, d_{ti}) is formed.

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