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Robust Control Using Sliding Mode for a Class of Under-Actuated Systems With Mismatched Uncertainties

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Abstract—**Based on the methodology of sliding mode, this paper presents a robust controller for a class of under-actuated systems with mismatched uncertainties. Such a system consists of a nominal system and the mismatched uncertainties. The structural characteristic of the nominal system is that it is made up of several subsystems. Based on this characteristic, the hierarchical structure of the sliding mode surfaces is designed for the nominal system as follows. Firstly, the nominal system is divided into several subsystems and the sliding mode surface of every subsystem is defined. Secondly, the sliding mode surface of one subsystem is selected as the first layer sliding mode surface. The first layer sliding mode surface is then to construct the second layer sliding mode surface with the sliding mode surface of another subsystem. This process continues till the sliding mode surfaces of all the subsystems are included. For dealing with the mismatched uncertainties, a lumped sliding mode compensator is designed at the last layer sliding mode surface. The asymptotic stability of every layer sliding mode surface and the sliding mode surface of each subsystem is proven theoretically by Barbalat's lemma. Simulation results show the validity of this robust control method through stabilization control of a double inverted pendulums system with mismatched uncertainties.**

I. INTRODUCTION

MECHANICAL systems with fewer number of control inputs than the number of degrees of freedom to be controlled are called under-actuated systems. They arise in extensive applications. Some undesired properties of their dynamics, such as nonlinearities, non-holonomic constraints and couplings, make control design difficult. There have been increasing interests in the control problems of under-actuated systems in recent years.

In this paper, we focus on a class of under-actuated systems, including Acrobot [1], inverted pendulum(s) system [2], [11], [12], TORA [3], ball-beam system [11], etc. Such systems can be depicted by a canonical state space expression. They are often used for research on nonlinear control and education in various concepts, because they are simple enough to permit complete dynamic analyses and experiments, but there exist strong nonlinearities and dynamic couplings. In practice, uncertainties often exist because of external and internal disturbances. This will make the control problems of such systems further complex. Two methods are often used to handle the uncertainties. One is to design a robust controller for resisting the uncertainties [4], [5] and the other is to estimate the uncertainties thought artificial intelligence [6]–[8]. In this paper, we will work at designing a robust controller to deal with the uncertainties.

Sliding mode control (SMC) is a powerful and robust nonlinear feedback control method. It has been developed and applied to feedback control systems for the last three decades [9], which provides a good candidate. The sliding mode controller is insensitive to system parameter changes or external disturbances when system states keep sliding on sliding mode surface. Under matched condition, SMC can deal with matched uncertainties effectively [10]. There exist two crucial issues associated with the applications of SMC to the under-actuated systems with mismatched uncertainties. One is how to design a suitable sliding mode surface, because the parameters of the sliding mode surface of under-actuated systems can't be calculated directly by Hurwitz condition. The other is how to handle the mismatched uncertainties, because most physical systems do not satisfy the matched condition of SMC in practice.

As for the first issue, the hierarchical sliding mode surfaces can be designed for the class of under-actuated systems that can be divided into several subsystems, such as Pendubot, inverted pendulum(s) system and so on. Based on this idea, Lo and Kou [11] designed a decoupled fuzzy sliding-mode controller, which didn't prove the stability of the sliding mode surfaces strictly. Lin and Mon [12] presented a hierarchical fuzzy sliding mode controller, which only guaranteed the second level sliding mode surface was asymptotically stable. Wang, Yi, and Zhao [13] developed a hierarchical sliding mode controller whose sliding mode surfaces are asymptotically stable. However, [13] only considered such under-actuated systems with two subsystems. For the above second issue, the mismatched uncertainties have been considered by using sliding mode method in recent papers. [11] and [12] designed the distributed compensators and compensated the uncertainties at every layer sliding surface, but this made the controllers complex. [13] considered such a crucial issue in theoretical analysis, but didn't show it in simulation.

In this paper, a robust controller based on the methodology

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of sliding mode is presented for a class of under-actuated systems with mismatched uncertainties. Such a system consists of a nominal system and the mismatched uncertainties. The structural characteristic of the nominal system is that it consists of several subsystems. Based on this idea, the hierarchical structure of the sliding mode surfaces is designed for the nominal system as follows. Firstly, the nominal system is divided into several subsystems and the sliding mode surface of every subsystem is defined. Next, the sliding mode surface of one subsystem is selected as the first layer sliding mode surface. The first layer sliding mode surface is then to construct the second layer sliding mode surface with the sliding mode surface of another subsystem. This process continues till the sliding mode surfaces of all the subsystems are included. For the mismatched uncertainties, a lumped sliding mode compensator is designed at the last layer sliding mode surface. This viewpoint can simplify the design. The asymptotic stability of the whole sliding mode surfaces is proven theoretically. Simulation results show the validity of this robust control method by stabilizing a double inverted pendulums system with mismatched uncertainties.

II. Design of Control Strategy

Considering a class of under-actuated systems with mismatched uncertainties, the state space expression of such a system can be depicted in the following form.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \mathrm{f}_1 + \mathrm{b}_1 u + \mathrm{d}_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \mathrm{f}_2 + \mathrm{b}_2 u + \mathrm{d}_2 \\ \qquad \vdots \\ \dot{x}_{2n-1} = x_{2n} \\ \dot{x}_{2n} = \mathrm{f}_n + \mathrm{b}_n u + \mathrm{d}_n \end{cases} \tag{1}$$

where $X = [x_1, x_2, \cdots, x_{2n}]^{\mathrm{T}}$ is defined as a state variable vector; f_i and b_i $(i = 1, 2, \cdots, n)$ are the nonlinear functions of the state variables; d_i is the term of lumped mismatched uncertainties, including system uncertainties and external disturbances; d_i is bounded by $|\mathrm{d}_i| \le \bar{\mathrm{d}}_i$ where $\bar{\mathrm{d}}_i$ is a known and positive constant; u is the single control input.

Equation (1) is the normal form of a class of SIMO under-actuated systems with mismatched uncertainties by different n, f_i, b_i and d_i. If n=2, it can represent Acrobot, TORA, single inverted pendulum system; if n=3, it can express double inverted pendulums system; if n=4, it can be considered as triple inverted pendulums system; etc.

Let $\mathrm{d}_i = 0$, (1) can be treated as the nominal system of such an under-actuated system. Therefore, it is considered that a hierarchical sliding mode controller can be designed for the nominal system and that a lumped sliding mode compensator can be designed for the mismatched uncertainties. The hierarchical sliding mode controller and the lumped sliding mode compensator work together and realize the robust control for the under-actuated systems with mismatched uncertainties. In the following two subsections, the robust controller using the methodology of sliding mode will be presented for such a system as (1) step by step.

A. Hierarchical Sliding Mode Control for Nominal System

The nominal system can represent n subsystems with second-order canonical form in terms of the physical structural characteristic of such under-actuated systems. The state variables (x_{2i-1}, x_{2i}) of the ith group can be treated as the states of the ith subsystem. And the state space expression of the ith subsystem is described by

$$\begin{cases} \dot{x}_{2i-1} = x_{2i} \\ \dot{x}_{2i} = \mathrm{f}_i + \mathrm{b}_i u \end{cases}. \tag{2}$$

In order to design the hierarchical sliding mode controller for the nominal system in this subsection, let us define the sliding mode surface of every subsystem at first.

For the state variables (x_{2i-1}, x_{2i}) of the ith subsystem, the sliding mode surface is defined as

$$s_i = c_i x_{2i-1} + x_{2i} . \tag{3}$$

Differentiating s_i with respect to time t, there exists

$$\dot{s}_i = c_i \dot{x}_{2i-1} + \dot{x}_{2i} = c_i x_{2i} + \mathrm{f}_i + \mathrm{b}_i u . \tag{4}$$

Let $\dot{s}_i = 0$, the equivalent control of the ith subsystem can be obtained as

$$u_{\mathrm{eq}i} = -(c_i x_{2i} + \mathrm{f}_i) / \mathrm{b}_i . \tag{5}$$

According to s_i and $u_{\mathrm{eq}i}$, the hierarchical sliding mode controller can be designed. The structure of the hierarchical sliding mode surfaces is shown in Fig. 1.

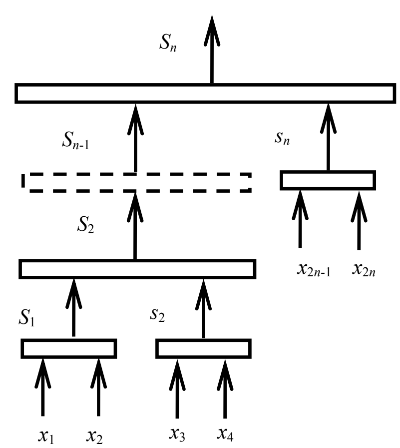

Fig. 1. Structure of hierarchical sliding mode surfaces

As the hierarchical structure has been shown in Fig. 1, the sliding mode surface of one subsystem is chosen as the first layer sliding mode surface. Then the first layer sliding mode surface is used to construct the second layer sliding mode surface with the sliding mode surface of another subsystem. This process continues till the sliding mode surfaces of the entire subsystems are included. This hierarchical structure makes the ith layer sliding mode controller include the information from the other i-1 layers. If the nth layer control law is gotten, it can control the nominal system of the uncertain under-actuated system with n subsystems.

Without loss of generality, the sliding mode surface of the first subsystem s_1 is defined as the first layer sliding mode surface S_1, thus we have

$$S_1 = s_1 \tag{6a}$$

The second layer sliding mode surface can be constructed as follow.

$$S_2 = a_1 S_1 + s_2$$

Similarly, the ith layer sliding mode surface can be defined as

$$S_i = a_{i-1} S_{i-1} + s_i \;\; (i = 2, \cdots, n)\,. \tag{6b}$$

For the first layer sliding mode surface, we define the control law and the Lyapunov function as

$$u_1 = u_{\mathrm{eq}1} + u_{\mathrm{sm}1} \tag{7a}$$

and

$$V_1(t) = S_1^2 / 2\,. \tag{7b}$$

Here $u_{\mathrm{sm}1}$ is the switch control of the first layer sliding mode controller.

Differentiate $V_1(t)$ with respect to time t and let $\dot{S}_1 = -k_1 S_1 - \eta_1 \operatorname{sgn} S_1$. Here k_1 and η_1 are positive constants. The following first layer sliding mode control law can be deduced from (3), (5), (6) and (7) at last.

$$u_1 = u_{\mathrm{eq}1} + \dot{S}_1 / \mathrm{b}_1 \tag{8}$$

Similarly, for the ith layer sliding mode surface, the control law and the Lyapunov function are defined as

$$u_i = u_{i-1} + u_{\mathrm{eq}i} + u_{\mathrm{sm}i} \tag{9a}$$

and

$$V_i(t) = S_i^2 / 2\,. \tag{9b}$$

Here $u_{\mathrm{sm}i}$ is the switch control of the ith layer sliding mode controller.

Differentiate $V_i(t)$ with respect to time t and let $\dot{S}_i = -k_i S_i - \eta_i \operatorname{sgn} S_i$, where k_i and η_i are positive constants. The following ith layer sliding mode control law can be obtained from (3), (5), (6) and (9).

$$u_i = \mathrm{num}(i) / \mathrm{den}(i) + \dot{S}_i / \mathrm{den}(i) \qquad i = 1, 2, \cdots, n \tag{10}$$

$$\mathrm{num}(i) = \begin{cases} \mathrm{b}_1 \, u_{\mathrm{eq}1} & i = 1 \\ a_{i-1}\, \mathrm{num}(i-1) + \mathrm{b}_i \, u_{\mathrm{eq}i} & 2 \le i \le n \end{cases}$$

$$\mathrm{den}(i) = \begin{cases} \mathrm{b}_1 & i = 1 \\ a_{i-1}\, \mathrm{den}(i-1) + \mathrm{b}_i & 2 \le i \le n \end{cases}$$

Remark: Equation (10) is a recursive formula. Let $i = n$, the total control law of the nominal system with n subsystems can be gotten from it. As (10) has shown, only the switch control of the last layer sliding mode controller works and the switch controls of the other n-1 layers are merged during the deduction.

Remark: In dynamic process, if any state deviates from its sliding mode surface, then the switch control of the last layer will drive it back to its own sliding mode surface. This makes the system states slide on the last layer sliding mode surface. Moreover, the states of every subsystem keep sliding on its own sliding mode surface.

B. Compensator for Mismatched Uncertainties

For the matched uncertainties, the above hierarchical sliding mode controller can resist them because of the invariant characteristic of the sliding mode. For the mismatched uncertainties, we will design a sliding mode compensator to resist them.

Generally speaking, there are two methods to design a compensator of this hierarchical sliding mode surfaces. One is to design a distributed compensator and compensate the mismatched uncertainties at every layer sliding mode surface [11], [12]. Two weak points of this idea are that this makes the controller structure complex and that if the compensator at a certain lower layer does not eliminate the uncertainties, it will affect the stabilities of the higher layers. The other method is to design a lumped compensator and compensate the mismatched uncertainties at the last layer. Its advantage is that this method simplifies the control design. Thus, we consider designing a lumped sliding mode compensator at the last layer in this subsection.

Based on the above viewpoints, the final control law of the uncertain under-actuated system with n subsystems can be defined as follow.

$$u = u_n + u_{cn} \tag{11}$$

where u_n is the hierarchical sliding mode control law of the nth layer; and u_{cn} is the sliding mode compensator at the last nth layer sliding mode surface. Here u_n and u_{cn} are given by

$$u_n = \mathrm{num}(n) / \mathrm{den}(n) + \dot{S}_n / \mathrm{den}(n)$$

and

$$u_{cn} = -\,\mathrm{num}_{\bar{c}}(n) \cdot \mathrm{sgn}(S_n) / \mathrm{den}(n)\,.$$

where

$$\mathrm{num}_{\bar{c}}(i) = \begin{cases} \bar{\mathrm{d}}_1 & i = 1 \\ a_{i-1}\, \mathrm{num}_{\bar{c}}(i-1) + \bar{\mathrm{d}}_i & 2 \le i \le n \end{cases}$$

Remark: From the design, the mismatched uncertainties are compensated by the lumped sliding mode compensator at the last layer sliding mode surface. Thus, it can be considered that all the mismatched uncertainties are added and eliminated at the last layer sliding mode surface. Moreover, the other lower n-1 layers can be considered as the sliding mode surfaces of the nominal system.

III. Stability Analysis

In this section, we mainly make use of Barbalat's lemma to prove the asymptotical stability of the entire sliding mode surfaces. Because of the lumped sliding mode compensator at the last layer sliding mode surface, its stability should be analyzed at first.

Theorem 1: Considering such an under-actuated system with mismatched uncertainties as (1), if the robust control law is adopted as (11) and the last layer sliding mode surface is defined as (6), then the last layer sliding mode surface is asymptotically stable.

Proof:

Let $\mathrm{num}_c(i) = \begin{cases} \mathrm{d}_1 & i=1 \\ a_{i-1}\,\mathrm{num}_c(i-1)+\mathrm{d}_i & 2 \le i \le n \end{cases}$.

Owing to the existent mismatched uncertainties, the Lyapunov function of the actual system at the last layer sliding mode surface is defined as follow.

$$\bar{V}_n(t) = \bar{S}_n^2 / 2 .$$

The mismatched uncertainties make the dynamic process of the actual system different from the nominal system. Thus, let $\bar{S}_n = S_n$ and $\dot{\bar{S}}_n = [u_{cn}\,\mathrm{den}(n) + \mathrm{num}(n)] - k_n S_n - \eta_n \,\mathrm{sgn}\, S_n$. Differentiating $\bar{V}_n(t)$ with respect to time t, there exists

$$\begin{aligned} \dot{\bar{V}}_n &= \bar{S}_n \dot{\bar{S}}_n = S_n \dot{\bar{S}}_n \\ &= S_n[u_{cn}\,\mathrm{den}(n) + \mathrm{num}_c(n)] - \eta_n |S_n| - k_n S_n^2 \\ &= S_n\,\mathrm{num}_c(n) - |S_n|\,\mathrm{num}_{\bar{c}}(n) - \eta_n |S_n| - k_n S_n^2 \end{aligned} \quad (12)$$

Integrating both sides of (12) yields

$$\int_0^t \dot{\bar{V}}_n \,\mathrm{d}\tau = \int_0^t \left(S_n\,\mathrm{num}_c(n) - |S_n|\,\mathrm{num}_{\bar{c}}(n) - \eta_n |S_n| - k_n S_n^2 \right) \mathrm{d}\tau .$$

We can find

$$\begin{aligned} &\bar{V}_n(0) - \bar{V}_n(t) \\ &= \int_0^t \left(\eta_n |S_n| + k_n S_n^2 + |S_n|\,\mathrm{num}_{\bar{c}}(n) - S_n\,\mathrm{num}_c(n) \right) \mathrm{d}\tau . \\ &\ge \int_0^t \left(\eta_n |S_n| + k_n S_n^2 \right) \mathrm{d}\tau \end{aligned}$$

Further, we can obtain

$$\bar{V}_n(0) \ge \int_0^t \left(\eta_n |S_n| + k_n S_n^2 \right) \mathrm{d}\tau .$$

The following equation can be obtained at last.

$$\lim_{t\to\infty} \int_0^t \left(\eta_n |S_n| + k_n S_n^{\,2} \right) \mathrm{d}\tau \le \bar{V}_n(0) < \infty .$$

According to Barbalat's lemma, $\eta_n |S_n| + k_n S_n^{\,2} \to 0$ as $t \to \infty$, which means $\lim_{t\to\infty} S_n = 0$, namely, the last layer sliding mode surface is asymptotically stable. ■

Theorem 2: Consider an uncertain under-actuated system as (1), adopt the control law is as (11) and define every layer sliding mode surface as (6). Then the lower n-1 layers sliding mode surfaces are still asymptotically stable.

Proof:

As we have expressed, the lumped sliding mode compensator can compensate the mismatched uncertainties at the last layers. Thus, the lower n-1 layers sliding mode surfaces can be treated as the nominal system.

From (9), the Lyapunov function of the ith layer sliding mode surface is $V_i(t) = S_i^2/2$. Here $1 \le i \le n-1$.

Differentiating $V_i(t)$ with respect to time t, there exists

$$\dot{V}_i = S_i \dot{S}_i = -\eta_i |S_i| - k_i S_i^{\,2} . \quad (13)$$

Integrating both sides of (13) yields

$$\int_0^t \dot{V}_i \,\mathrm{d}\tau = \int_0^t \left(-\eta_i |S_i| - k_i S_i^2 \right) \mathrm{d}\tau .$$

Then, we have

$$V_i(0) - V_i(t) = \int_0^t \left(\eta_i |S_i| + k_i S_i^2 \right) \mathrm{d}\tau .$$

Further, we can get

$$V_i(0) = V_i(t) + \int_0^t \left(\eta_i |S_i| + k_i S_i^2 \right) \mathrm{d}\tau \ge \int_0^t \left(\eta_i |S_i| + k_i S_i^2 \right) \mathrm{d}\tau .$$

The following equation can be obtained at last.

$$\lim_{t\to\infty} \int_0^t \left(\eta |S_i| + k S_i^{\,2} \right) \mathrm{d}\tau \le V_i(0) < \infty$$

According to Barbalat's lemma, $\eta_i |S_i| + k_i S_i^{\,2} \to 0$ as $t \to \infty$, which means $\lim_{t\to\infty} S_i = 0$, namely, the ith layer sliding mode surface ($1 \le i \le n$-1) is asymptotically stable. ■

Theorem 3: Consider an uncertain under-actuated system as (1), adopt the control law as (11) and define the sliding mode surfaces of all the subsystems as (3). Then the sliding mode surfaces of all the subsystems are asymptotically stable.

Proof:

From theorem 1 and theorem 2, there exists $\lim_{t\to\infty} S_i = 0$. Here $1 \le i \le n$.

1) As has been defined that $S_1 = s_1$, the sliding mode surface of the first subsystem is asymptotically stable.

2) Let us prove that the sliding mode surfaces of the other n-1 subsystems are asymptotically stable by contradiction.

We assume that s_i ($2 \le i \le n$) is not asymptotically stable, namely

$$\lim_{t\to\infty} s_i \ne 0 . \quad (14)$$

From (6b), we can obtain that

$$S_i = a_{i-1} S_{i-1} + s_i \quad (2 \le i \le n) .$$

Calculating the limit of both sides of (6b) yields

$$\begin{aligned} \lim_{t\to\infty} S_i &= \lim_{t\to\infty} (a_{i-1} S_{i-1} + s_i) \\ &= \lim_{t\to\infty} a_{i-1} S_{i-1} + \lim_{t\to\infty} s_i \\ &= \lim_{t\to\infty} s_i \ne 0 \end{aligned} .$$

This case contradicts the case $\lim_{t\to\infty} S_i = 0 \quad (2 \le i \le n)$ that we have gotten from theorem 1and theorem 2. Therefore, our initial assumption (14) is false and the opposite case of (14) that $\lim_{t\to\infty} s_i = 0 \quad (2 \le i \le n)$ comes to existence.

In a word, the sliding mode surfaces of all the subsystems are asymptotically stable from part 1 and part 2. ■

IV. Simulation Results

In this section, we shall demonstrate this robust control strategy is applicable to stabilize a double inverted pendulums system. The structure of such a system is shown in Fig. 2. It consists of three subsystems: the lower pendulum, the upper pendulum and the cart. Control objective of stabilizing the system is to balance both of the pendulums upright and put the cart to the rail origin by moving the cart.

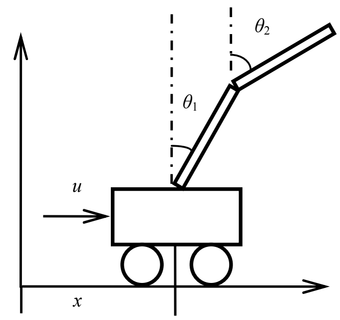

Fig. 2. Structure of double inverted pendulums system

From (1), let n=3, the state space expression of the double inverted pendulums system can be described by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \mathrm{f}_1 + \mathrm{b}_1 u + \mathrm{d}_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \mathrm{f}_2 + \mathrm{b}_2 u + \mathrm{d}_2 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = \mathrm{f}_3 + \mathrm{b}_3 u + \mathrm{d}_3 \end{cases} \tag{15}$$

here $x_1 = \theta_1$ is the lower pendulum angle with respect to the vertical line; $x_3 = \theta_2$ is the upper pendulum angle with respect to the vertical line; $x_5 = x$ is the cart position with respect to the origin; x_2 is the angular velocity of the lower pendulum; x_4 is the angular velocity of the upper pendulum; x_6 is the velocity of the cart; u is the single control input; f_i and b_i (i=1, 2, 3) are given in [11] and d_i is the mismatched uncertain term whose bound is known.

For simulative comparison, the structural parameters are selected as the cart mass M=1kg, the lower pendulum mass m_1=1kg, the upper pendulum mass m_2=1kg, the lower pendulum length l_1=0.1m, the upper pendulum length l_2=0.1m, the gravitational acceleration g=9.81m·s^{-2}, which have appeared in [12]. The mismatched uncertain terms of the system are assumed as follows.

$$\mathrm{d}_1 = 0.0872 + 0.5\rho,\ \mathrm{d}_2 = 0.0872 + 0.5\rho,\ \text{and}\ \mathrm{d}_3 = 0.5\rho$$

Here ρ is a random number whose range is from -1 to 1.
Thus, the bounds of the mismatched uncertain terms d_1, d_2 and d_3 can be obtained as 0.5872, 0.5872 and 0.5. The lumped sliding mode compensator can be gotten. Further, the parameters of the hierarchical sliding mode controller are selected as c_1=184.26, c_2=15.96, c_3=0.72, a_1= -0.06, a_2=0.45, k=1.50, and η=0.02 after trial and error. Control objective is from the initial states (π/6, 0, π/18, 0, 0, 0) to the desired states (0, 0, 0, 0, 0, 0). Simulation results are shown as follows.

Fig. 3 shows the angular curves and the positional curves. The solid curves are with a compensator and the dashed curves are without a compensator. Although both controllers can make the double pendulums upright, the positional curves show the presented robust controller can resist the mismatched uncertainties effectively and realize the control objective.

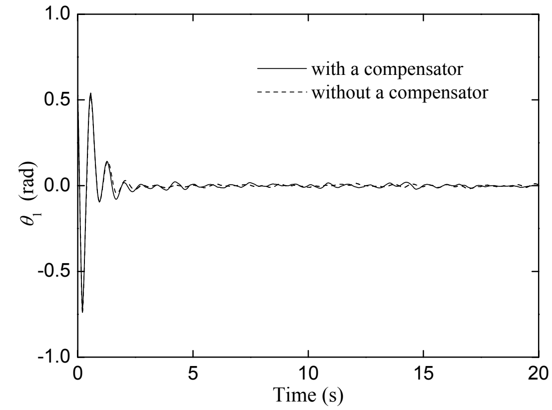

Fig. 3(a). Angle of the lower pendulum

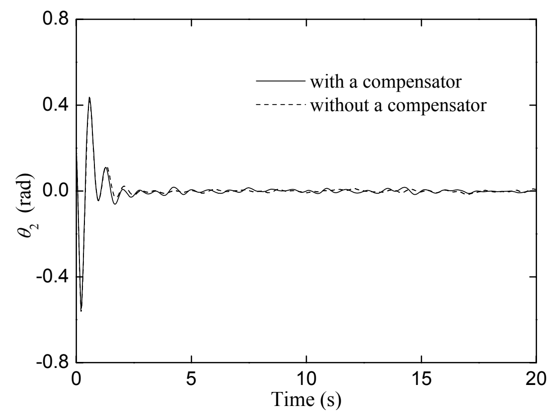

Fig. 3(b). Angle of the upper pendulum

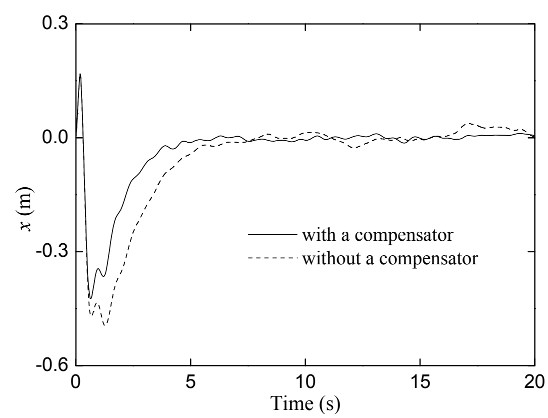

Fig. 3(c). Cart position

Fig. 4 is the control force added to the cart. As the curve shows, this hierarchical structure can decrease chattering phenomenon effectively.

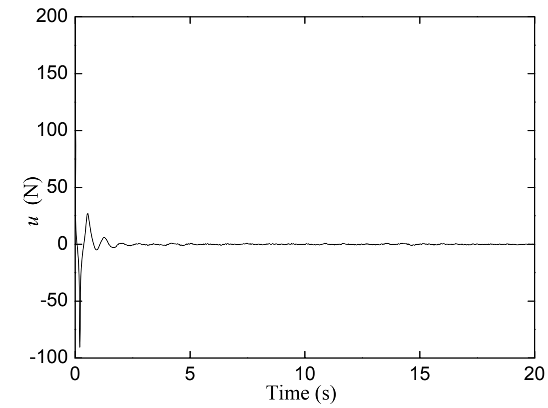

Fig. 4. Control force

Remark: In [11], the control objective was only to make the double pendulums upright but did not consider the cart position. Compared with [11], our objective is more difficult. Compared with [12], our curves are smoother and the response time is shorter. Further, the presented controller is robust although it needs a large control force.

Fig. 5 shows the entire sliding mode surfaces. By this robust control method, not only every layer sliding mode surface is asymptotically stable, but also the sliding mode surfaces of all the subsystems possess the asymptotic stability, as we have proven in theorem 1, theorem 2 and theorem 3.

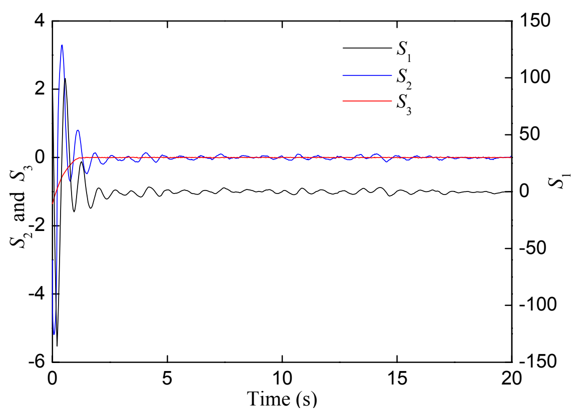

Fig. 5(a). Every layer sliding mode surface

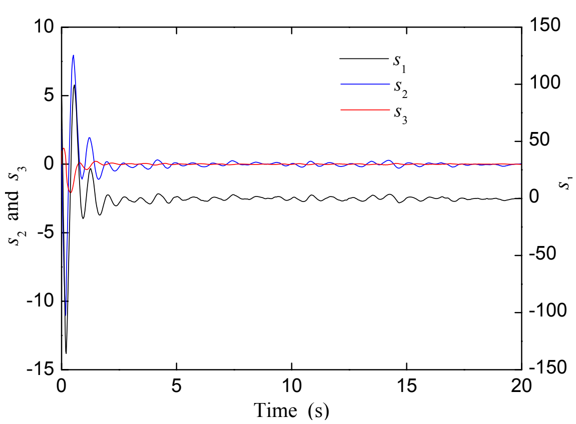

Fig. 5(b). Subsystem sliding mode surfaces

V. Conclusions

In this paper, a robust controller using the methodology of sliding mode has been presented for a class of under-actuated systems with mismatched uncertainties. Such a system consists of a nominal system and the mismatched uncertainties. The nominal system is made up of several subsystems. Based on this structural characteristic, a hierarchical sliding mode controller has been designed for the nominal system. For the mismatched uncertainties, a lumped sliding mode compensator has been designed to deal with them. The asymptotic stability of the entire sliding mode surfaces has been proven theoretically. In the simulation, the proposed control method is applied to stabilization control of a double inverted pendulums system. The simulation results have shown the validity of the control strategy. This provides a robust control strategy for a class of under-actuated systems with mismatched uncertainties.

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