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Force/Position Tracking of a Robot in Compliant Contact with Unknown Stiffness and Surface Kinematics

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***Abstract*— This work deals with the problem of force/position trajectory tracking under uncertainties arising from surface position and orientation. A robotic finger with a soft hemispherical tip of uncertain compliance parameter is considered in contact with a rigid flat surface. A novel adaptive controller is designed using online estimates of the unknown parameters and is proved to achieve force and position tracking by ensuring the convergence of the estimated normal to the surface direction to its actual value. The performance of the proposed controller is demonstrated by a simulation example.**

I. INTRODUCTION

Many applications of robots involve tasks in which the robot end-effector is in contact with the environment. In such tasks, the end-effector position and the interaction force between the robot and the environment, have to be simultaneously controlled to achieve either setpoint targets, or desired trajectories. In the force and motion control problems, uncertainties may arise from both the robot model and the environment. Robot model uncertainties are mainly owing to the lack of knowledge of system parameters. Moreover, kinematic uncertainties affecting the robot and the contact kinematics or surface Jacobian may appear whenever the shape of the contacted surface and its position are unknown. For a compliant contact, there are further uncertainties regarding the stiffness parameter.

The majority of the works dealing with surface kinematic uncertainties consider rigid contact between the end effector and the environment [1]–[6]. In the rigid contact case, the end-effector can not move along the surface normal; hence, end-effector velocities lie on the plane tangent to the surface at the contact point. This fact allows the use of tip velocities in the identification of the constraint surface [1]. For a frictionless point contact, the contact force lies on the surface normal and can be directly used to determine the constraint Jacobian [2], [3], [5]. In the more practical case of contact with friction both measurements of velocities and force are used to calculate the surface normal direction [6]. In order to avoid calculation errors and the use of force derivatives in the control law, an adaptive force-motion controller with estimates of the constraint Jacobian is proposed in [4] exploiting the geometrical relationship between force and end-effector velocity. Regarding uncertainties on surface position, vision systems have been additionally used to identify the desired position [1], [5].

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In the case of compliant contact, the previous methodologies cannot be applied because the force error is coupled with the tip velocities. The problem of force/position regulation does not require an online adaptation of the stiffness parameter to guarantee system stability [7]–[9]. On the other hand, surface kinematic uncertainties affect the convergence of the position error [9]. Nevertheless, the problem of force/position regulation has been successfully solved under both stiffness and kinematic uncertainties by an online adaptation of the surface normal direction that converges to its actual value [8]. There are some works in force and position tracking under stiffness parameter uncertainties none of which however deals with surface kinematic uncertainties [10]–[13]. In fact in [10] adaptive force control is applied to estimate unknown parameters under the unrealistic assumption of a constant robot inertia matrix while in [11] adaptive control is applied for the unknown stiffness after fully linearizing and decoupling system dynamics. The works of Yao *et al.* [12] and Villani *et al.* [13] involve the use of the force derivative in the control law through the reference velocity derivative and hence they require extra control effort to overcome the problem of this noisy signal. At our knowledge there are no works that deal with surface kinematic uncertainties in compliant contact.

This work considers surface kinematic and stiffness uncertainties as in [8] but deals with the problem of force and position tracking as opposed to regulation [8]. An adaptive controller is proposed and is proved to achieve the control target by ensuring the convergence of the estimated surface orientation to its actual value.

II. PROBLEM DESCRIPTION

Consider a robot finger with soft hemispherical fingertip of radius r in contact with a rigid flat surface. Let $q \in \Re^{n_q}$ be the vector of the generalized joint variables and $\{B\}$ the inertia frame attached at the finger base (Fig. 1). Let the surface frame $\{s\}$ be attached at some point on the surface; its position is denoted by p_s and its orientation by matrix $R_s = [n_s \;\; o_s \;\; a_s]$ such that $n_s \in \Re^3$ is the unit vector normal to the contact surface pointing inwards. Consider also the frame $\{t\}$ at the finger rigid tip with position $p_t \in \Re^3$ and rotation matrix R_t that can be parameterized by three rotation angles $\varphi_t \in \Re^3$ around the axes of the inertia frame. Let the generalized rigid tip position be $p = \left[p_t^T \;\; \varphi_t^T\right]^T \in \Re^6$. The generalized velocity $\dot{p} = \left[\dot{p}_t^T \;\; \omega_t^T\right]^T \in \Re^6$ is related to the joint velocity $\dot{q}$ through the rigid tip Jacobian $J =$

$\left[J_v^T \;\; J_\omega^T\right]^T \in \Re^{6\times n_q}$ as follows:

$$\dot{p} = J(q)\dot{q} \tag{1}$$

At the center point of the contact area, we attach the contact frame $\{c\}$, described by the position vector p_c and the orientation matrix $R_c = R_s$. For a soft hemispherical fingertip, the contact point identifies with the rigid tip projection on the surface i.e., $Q_s p_c = Q_s p_t$ where $Q_s = I - n_s n_s^T$ is the projection matrix. Moreover, for a flat surface the projections of p_c and p_s along the surface normal are equal ($n_s^T p_s = n_s^T p_c$) and hence we can derive the material deformation as a function of p_s, p_t :

$$\Delta x = r - n_s^T(p_s - p_t) \tag{2}$$

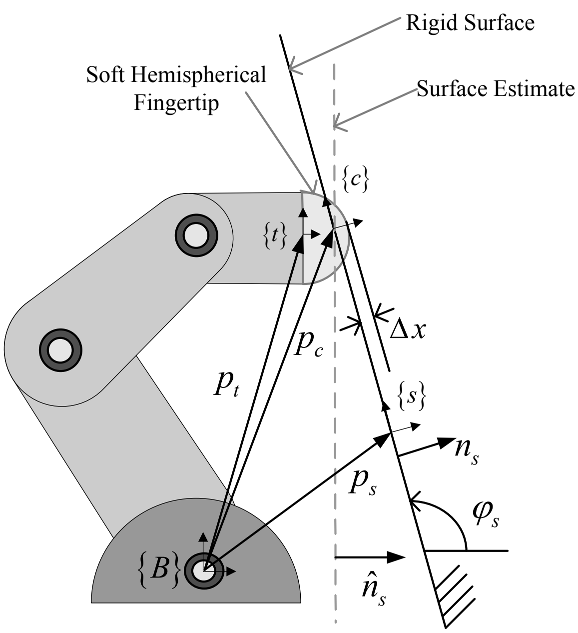

Fig. 1. A robot finger with a soft hemispherical fingertip in contact with a rigid surface

We reasonably assume that the integral of the stresses that are distributed nearly semi-circularly over the contact area of the deformed fingertip arises as an aggregated force perpendicular to the surface at the contact point. The concentrated force can be expressed as $F_c = n_s f$, where the force magnitude f is assumed to be measurable by a sensor attached at the fingertip [14] and is in general a monotonically increasing, continuously differentiable nonlinear function of $\Delta x : f(\Delta x)$ if $\Delta x > 0$, $f = 0$ if $\Delta x \leq 0$. A typical force-deformation relationship is $f(\Delta x) = k_s \Delta x^\mu$, $\mu \in \Re$, where k_s is the elasticity model constant. The contact force F_c is mapped to the rigid tip as a generalized force $F = nf \in \Re^6$, with $n = [n_s^T \;\; 0_3^T]^T \in \Re^6$ denoting the generalized normal direction.

In this work, we consider the force/position trajectory following problem under uncertainties arising from surface position and orientation. We further assume that although the force deformation relationship takes the typical structure given above the elasticity model constant maybe unknown. Uncertainties on the surface position and orientation affect the accurate definition of the desired position trajectory $p_{cd}(t)$ in the three dimensional space. Furthermore, measurements of the contact point position can not be easily obtained in the case of an area contact. These problems can be easily overcome if the surface orientation is known. Then, a trajectory defined in the three-dimensional space can be projected on the surface $Q_s p_{cd}$ to annihilate errors that may exist along the surface normal direction and since the rigid tip projection on the surface identifies with the contact point i.e. $Q_s p_c = Q_s p_t$, the error $Q_s(p_t - p_{cd})$ can be used for control purposes. Since the surface orientation is uncertain in this work, the basic idea is to design a controller to achieve both slope identification and force position tracking. In fact, the controller is designed using online estimates of the unknown parameters of surface orientation n_s (Q_s), surface distance $\theta_1 = n_s^T p_s$ and the inverse of elasticity model constant $\theta_0 = k_s^{-1}$ and achieves desired force $f_d(t) \in \Re^+$ and position $p_d(t) = [p_{cd}^T(t) \;\; \omega_{td}^T(t)]^T \in \Re^6$ tracking by ensuring the convergence of n_s estimate to its actual value.

We assume that joint positions and velocities are measured and that the rigid tip Jacobian J is known; hence, rigid tip position and velocity can be calculated by using the robot forward kinematic relationships. We further assume that the robot dynamic model is known. The dynamic model of the robot ignoring friction forces can be written as follows:

$$M(q)\ddot{q} + \left(\frac{1}{2}\dot{M}(q) + S(q,\dot{q})\right)\dot{q} + g(q) + J^T(q)nf = u \tag{3}$$

where $M(q) \in \Re^{n_q\times n_q}$ is the robot inertia matrix that is positive definite, $S(q,\dot{q}) \in \Re^{n_q\times n_q}$ is a skew symmetric matrix, $g(q) \in \Re^{n_q}$ denotes the gravity vector and u denotes the input control law.

III. Controller Design

For simplicity we consider the case of a linear elasticity model that implies $f_d = k_s \Delta x_d$, $\dot{f}_d = k_s \Delta \dot{x}_d$. Hence, estimates of the deformation trajectory and its derivative can be expressed with respect to the force trajectory f_d, $\dot{f}_d$ and the online estimate of parameter θ_0, $\hat{\theta}_0$ as follows:

$$\Delta \hat{x}_d = \hat{\theta}_0 f_d \tag{4}$$
$$\Delta \hat{\dot{x}}_d = \hat{\theta}_0 \dot{f}_d \tag{5}$$

The estimate of the deformation using (2) can be given by:

$$\Delta \hat{x} = r - \hat{\theta}_1 + \hat{n}_s^T p_t \tag{6}$$

where $\hat{\theta}_1$ and $\hat{n}_s$ are online estimates of θ_1, n_s respectively. The generalized estimate of normal direction denoted by $\hat{n}$ is defined according to the definition of n i.e. $\hat{n} = [\hat{n}_s^T \;\; 0_3^T]^T$. The Euclidean norm of $\hat{n}$ denoted by $\|\hat{n}\|$ depends on the adaptation law and can take non-unit values. However, if the adaptation law is such that $\|\hat{n}\| \neq 0$, the estimate of the generalized projection matrix $Q = I_6 - nn^T$ is defined as

$$\hat{Q} = I_6 - \frac{\hat{n}\hat{n}^T}{\|\hat{n}\|^2} \tag{7}$$

Notice that the estimate $\hat{Q}$ corresponds to a projection matrix that enjoys the basic properties: $\hat{Q} = \hat{Q}^T$, $\hat{Q} = \hat{Q}^2$ and $\hat{Q}\hat{n} = 0$.

Let us define the reference velocity vector $\dot{p}_r \in \Re^6$ in the rigid tip operational space:

$$\dot{p}_r = \hat{Q}\left(\dot{p}_d - \alpha\Delta p\right) + \frac{\hat{n}}{\|\hat{n}\|^2}\left(\Delta\dot{\hat{x}}_d - \beta\delta\hat{x} - \gamma\Delta F\right) \quad (8)$$

where α, β, γ are positive control gains, $\Delta p = p - p_d$ is the position error, $\delta\hat{x} = \Delta\hat{x} - \Delta\hat{x}_d$ is the estimated deformation error and $\Delta F = \int_0^t \Delta f d\tau$ is the integral of the force error. Using (4), (5) and (6), the reference velocity vector can be written as follows:

$$\begin{aligned}\dot{p}_r =& \hat{Q}\left(\dot{p}_d - \alpha\Delta p\right) + \frac{\hat{n}}{\|\hat{n}\|^2}\left(\Delta\dot{x}_d - \beta\delta x - \gamma\Delta F\right) \\ &+ \frac{\hat{n}}{\|\hat{n}\|^2}\left(-\zeta^T\tilde{\theta} + \beta p^T\tilde{n}\right)\end{aligned} \quad (9)$$

where $\delta x = \Delta x - \Delta x_d$ is the actual deformation error, $\tilde{n} = [\tilde{n}_s^T \;\; 0_3^T]^T$ with $\tilde{n}_s = n_s - \hat{n}_s$ is the normal direction error, $\zeta^T = \left[\begin{array}{cc}\dot{f}_d + \beta f_d & \beta\end{array}\right]$ is a regressor vector and $\tilde{\theta}^T = [\tilde{\theta}_0 \;\; \tilde{\theta}_1]$ is the parameter error vector with $\tilde{\theta}_0 = \theta_0 - \hat{\theta}_0$, $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$.

The reference acceleration $\ddot{p}_r$ can be calculated as follows:

$$\begin{aligned}\ddot{p}_r =& \frac{d}{dt}\big[\hat{Q}\big](\dot{p}_d - \alpha\Delta p) + \hat{Q}(\ddot{p}_d - \alpha\Delta\dot{p}) \\ &+ \frac{d}{dt}\left[\frac{\hat{n}}{\|\hat{n}\|^2}\right](\Delta\dot{\hat{x}}_d - \beta\delta\hat{x} - \gamma\Delta F) - \frac{\hat{n}}{\|\hat{n}\|^2}\gamma\Delta f \\ &+ \frac{\hat{n}}{\|\hat{n}\|^2}\left\{\tfrac{d}{dt}\big[\Delta\dot{\hat{x}}_d\big] - \beta\left(\tfrac{d}{dt}\big[\Delta\hat{x}\big] - \tfrac{d}{dt}\big[\Delta\hat{x}_d\big]\right)\right\}\end{aligned} \quad (10)$$

where $\frac{d}{dt}[\cdot]$ denotes the time derivative of a matrix, vector or scalar. The derivatives of the quantities appearing in (10) are calculated as follows:

$$\begin{aligned}\frac{d}{dt}\left[\hat{Q}\right] &= \frac{1}{\|\hat{n}\|^4}\left[2\hat{n}^T\dot{\hat{n}}\hat{n}\hat{n}^T - \|\hat{n}\|^2\left(\dot{\hat{n}}\hat{n}^T + \hat{n}\dot{\hat{n}}^T\right)\right] \\ \frac{d}{dt}\left[\frac{\hat{n}}{\|\hat{n}\|^2}\right] &= \frac{1}{\|\hat{n}\|^4}\left[\dot{\hat{n}}\|\hat{n}\|^2 - 2\hat{n}\hat{n}^T\dot{\hat{n}}\right] \\ \frac{d}{dt}\big[\Delta\dot{\hat{x}}_d\big] &= \hat{\theta}_0\ddot{f}_d + \dot{\hat{\theta}}_0\dot{f}_d \\ \frac{d}{dt}\big[\Delta\hat{x}_d\big] &= \hat{\theta}_0\dot{f}_d + \dot{\hat{\theta}}_0 f_d \\ \frac{d}{dt}\big[\Delta\hat{x}\big] &= -\dot{\hat{\theta}}_1 + \dot{\hat{n}}_s^T p_t + \hat{n}_s^T\dot{p}_t\end{aligned}$$

where $\dot{\hat{n}} \triangleq [\dot{\hat{n}}_s^T \;\; 0_3^T]^T$, $\dot{\hat{\theta}}_0$ and $\dot{\hat{\theta}}_1$ are given by update laws that will be defined through the subsequent stability analysis.

We also define the error:

$$s_p = \dot{p} - \dot{p}_r \quad (11)$$

that can be written as follows:

$$s_p = \hat{Q}\dot{p} + \frac{\hat{n}}{\|\hat{n}\|^2}\Delta\dot{x} - \frac{\hat{n}}{\|\hat{n}\|^2}\tilde{n}^T\dot{p} - \dot{p}_r \quad (12)$$

Substituting (9) in (12) we get:

$$s_p = \hat{s}_Q + \hat{s}_n \quad (13)$$

where

$$\hat{s}_Q = \hat{Q}(\Delta\dot{p} + \alpha\Delta p) \quad (14)$$

$$\hat{s}_n = \frac{\hat{n}}{\|\hat{n}\|^2}\left[\delta\dot{x} + \beta\delta x + \gamma\Delta F + \zeta^T\tilde{\theta} - (\dot{p} + \beta p)^T\tilde{n}\right] \quad (15)$$

On the other hand s_p can be expressed as follows:

$$s_p = \dot{p} + A(\hat{n})p - v(\dot{p}_d, \Delta\dot{x}_d, p_d, \Delta x_d, \tilde{\theta}, \hat{n}, \Delta F) \quad (16)$$

where $A(\hat{n})$ is a uniformly positive definite matrix for all $\hat{n}$ given by:

$$A(\hat{n}) = \alpha\hat{Q} + \beta\frac{\hat{n}\hat{n}^T}{\|\hat{n}\|^2} \quad (17)$$

and

$$\begin{aligned}v =& \frac{\hat{n}}{\|\hat{n}\|^2}\left\{\Delta\dot{x}_d + \beta\Delta x_d - \gamma\Delta F - \zeta^T\tilde{\theta} + \beta(\theta_1 - r)\right\} \\ &+ \hat{Q}(\dot{p}_d + \alpha p_d)\end{aligned} \quad (18)$$

In the non-redundant case, we can also define the reference joint velocity vector $\dot{q}_r$ as:

$$\dot{q}_r = J^{-1}\dot{p}_r \quad (19)$$

and the error s as follows:

$$s = \dot{q} - \dot{q}_r = J^{-1}s_p \quad (20)$$

The reference joint acceleration vector can be calculated by:

$$\ddot{q}_r = -J^{-1}\dot{J}\dot{q}_r + J^{-1}\ddot{p}_r \quad (21)$$

using (10) and (19).

Now, we can propose the following model based input control law:

$$u = J^T\hat{n}(f_d - k_f\Delta f) - Ds + M\ddot{q}_r + \left(\tfrac{1}{2}\dot{M}(q) + S(q,\dot{q})\right)\dot{q}_r + g \quad (22)$$

The controller consists of a feedforward term of the desired force trajectory magnitude and a proportional term of Δf through the positive control gain k_f, that lie on the estimated normal direction as well as a feedback of s through the positive definite gain matrix D. Substituting the input control law (22) into the robot dynamic model (3) we obtain the closed loop system:

$$M\dot{s} + \left(\tfrac{1}{2}\dot{M} + S\right)s + J^T\hat{n}k_f'\Delta f + J^T\tilde{n}f + Ds = 0 \quad (23)$$

where $k_f' = k_f + 1$. Taking the inner product of the closed loop system (23) with s we get:

$$\begin{aligned}&\frac{d}{dt}\left\{\frac{1}{2}s^T Ms + k_f' I(\delta x) + \frac{1}{2}k_f'\gamma\Delta F^2\right\} \\ &\quad + k_f'\Delta f\zeta^T\tilde{\theta} + \left[fs_p - k_f'\Delta f(\dot{p} + \beta p)\right]^T\tilde{n} \\ &\quad + \beta k_f'\delta x\Delta f + s^T Ds = 0\end{aligned} \quad (24)$$

where $I(\delta x) = \int_0^{\delta x}\{f(\xi + \Delta x_d) - f_d\}d\xi$ is the potential owing to deformation error with $I(\delta x) > 0$ for $\delta x \neq 0$. For the case of linear force-deformation relationship the potential is given by $I(\delta x) = \frac{1}{2}k_s \delta x^2$. The parameter update laws are chosen as follows:

$$\dot{\hat{\theta}} = -\Gamma \zeta k_f' \Delta f \tag{25}$$

$$\dot{\hat{n}}_s = -\mathcal{P}\left\{\Gamma_n \left[\begin{array}{cc} I_3 & O_{3\times 3}\end{array}\right]\left\{f s_p - k_f' \Delta f(\dot{p} + \beta p)\right\}\right\} \tag{26}$$

where $\Gamma = \text{diag}\left[\gamma_i\right]_{i=1}^{i=2}$, $\Gamma_n = \text{diag}\left[\gamma_{ni}\right]_{i=1}^{i=3}$ are diagonal matrices of parameter update gains and $\mathcal{P}$ is an appropriately designed projection operator [15] with respect to a convex set:

$$\mathcal{S} = \left\{\hat{n}_s \in \Re^3,\ \hat{n}_s^T(0)\hat{n}_s \geq \varepsilon\right\}, \quad \varepsilon \in \Re^+ \tag{27}$$

that contains the actual normal direction i.e. $\varepsilon < \cos\phi$, $|\phi| < 90°$ where ϕ denotes the angle of the initial estimate with the actual normal vector (Fig. 2). Hence, the positive constant ε is a measure of the allowed uncertainty. The projection operator ensures that $\|\hat{n}_s\| \neq 0$.

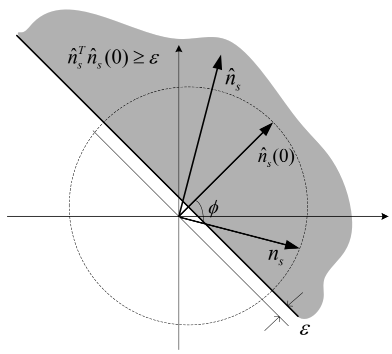

Fig. 2. The projection operator convex set

Notice that the control law given by (22) and the update laws (25), (26) do not require the use of force derivative.

Using (25) and (26) in (24) we get:

$$\frac{dV}{dt} + W = 0 \tag{28}$$

where

$$V = \frac{1}{2}s^T M s + k_f' I(\delta x) + \frac{1}{2}k_f' \gamma \Delta F^2 + \frac{1}{2}\tilde{n}_s^T \Gamma_n^{-1}\tilde{n}_s + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} \tag{29}$$

$$W = \beta k_f' \delta x \Delta f + s^T D s \tag{30}$$

Function V is positive definite with respect to s, δx, ΔF and parameter errors $\tilde{\theta}$, $\tilde{n}_s$ while function W is positive definite with respect to s, δx. Hence, function V has a negative derivative i.e. $\frac{dV}{dt} = -W \leq 0$ and can be regarded as a Lyapunov-like candidate function in order to prove the following theorem.

Theorem: The input control law (22) with the update laws (25), (26), applied in (3) achieves the convergence to zero of force, position and velocity tracking errors as well as slope identification i.e. $\Delta f \to 0$, $\Delta \dot{f} \to 0$, $Q\Delta p \to 0$, $Q\Delta \dot{p} \to 0$ and $\tilde{n}_s \to 0$.

Proof: $\frac{dV}{dt} = -W \leq 0$ implies $V(t) \leq V(0)$ and hence s (s_p, $\hat{s}_Q$, $\hat{s}_n$), δx (Δf), ΔF, $\tilde{\theta}$, $\tilde{n}_s$ ($\tilde{n}$) $\in \mathcal{L}_\infty$. Hence, given p_d, $\dot{p}_d$, f_d, $\dot{f}_d$ are bounded trajectories, $v \in \mathcal{L}_\infty$ and consequently (16) implies $\dot{p} + A(\hat{n})p \in \mathcal{L}_\infty$. Since A is bounded and positive definite, it can be easily proved that $\dot{p}, p \in \mathcal{L}_\infty$ and in turn $\delta\dot{x} \in \mathcal{L}_\infty$. If the robot moves away from singular positions, the boundedness of $\dot{p}$ implies that $\dot{q}$ is bounded. Hence, $\dot{\hat{n}}_s$, $\dot{\hat{\theta}}$ are bounded. From (23), $\dot{s}$ can be expressed as a sum of bounded quantities and hence $\dot{s}$ ($\dot{s}_p$) $\in \mathcal{L}_\infty$. From (28)-(30), δx, $s \in \mathcal{L}_2$. Further, δx, s are uniformly continuous because of the boundedness of δx, s and their derivatives. Hence, it follows that $\delta x \to 0$ ($\Delta f \to 0$), $s \to 0$ (s_p, $\hat{s}_n$, $\hat{s}_Q \to 0$). From (16), the boundedness of $\dot{s}_p$ implies that $\ddot{p}$ is bounded as a function of bounded quantities and hence $\dot{p}$ ($\delta\dot{x}$) is uniformly continuous. Furthermore, (23) implies that $\dot{s}$ is uniformly continuous as a function of uniformly continuous quantities. The uniform continuity of $\delta\dot{x}$, $\dot{s}$ as well as the convergence of δx, s to zero implies $\delta\dot{x}$, $\dot{s} \to 0$ and in turn $J^T(t)f_d(t)\tilde{n} \to 0$. Using the fact that $f_d(t)$ is strictly positive and the assumption that J is non-singular we get

$$\int_t^{t+T_0} J^T(\tau)J(\tau)f_d^2(\tau)d\tau \geq \alpha_0 T_0, \qquad \forall t \geq 0$$

for some α_0, $T_0 > 0$ that implies $\tilde{n} \to 0_6$ ($\hat{n}_s \to n_s$, $\hat{Q}_s \to Q_s$). The convergence of $\hat{n}_s$ to the actual normal direction as combined with $\hat{s}_Q \to 0$ implies $Q(\Delta p + \alpha\Delta\dot{p}) \to 0$ and consequently $Q\Delta p$, $Q\Delta\dot{p} \to 0$. Notice that the convergence of $\tilde{\theta}$ is not required. ■

Remark 1: The theorem holds even when $k_f = 0$ and $\gamma = 0$. However, the use of a proportional and integral term for the force error in (22) and (8) respectively provides an extra degree of freedom to the designer for improving the performance of the system response through the appropriate tuning of gains k_f and γ.

Remark 2: For the case of the nonlinear force-deformation relationship $f = k_s \Delta x^\mu$, the estimated parameter is $\theta_0 = \sqrt[\mu]{k_s^{-1}}$ and the regressor ζ becomes:

$$\zeta^T = \left[\begin{array}{cc} \sqrt[\mu]{f_d}(\frac{\dot{f}_d}{\mu f_d} + \beta) & \beta \end{array}\right]$$

Remark 3: In the presence of unknown friction and robot dynamics the proposed controller can be extended to include a regressor term for the unknown parameters. In this case the convergence of the normal vector to its actual value would require persistent excitation of the involved signals.

Remark 4: The above analysis is valid even if the source of compliance is the surface or both the fingertip and the surface. Furthermore, the controller will operate satisfactorily even in surfaces with small curvature.

Notice that the theorem is valid provided that the contact between the robotic finger and the surface is maintained i.e. $\delta x > -\Delta x_d$, $\forall t \geq 0$. This corresponds to an upper bound for the potential owing to deformation error, i.e. $I(\delta x) < f_d \Delta x_d - \int_0^{\Delta x_d} f(\xi)d\xi$, $\forall t \geq 0$ (Fig. 3) and in turn to $V < k_f' \left(f_d \Delta x_d - \int_0^{\Delta x_d} f(\xi)d\xi\right)$, $\forall t \geq 0$. Since

V is a decreasing function i.e. $V(0) \geq V$, $\forall t \geq 0$, starting within the region $V(0) < k'_f \left(f_d \Delta x_d - \int_0^{\Delta x_d} f(\xi) d\xi \right)$ ensures contact maintenance. For the case of linear force-deformation relationship the condition can be written as: $V(0) < \frac{1}{2} k'_f f_d \Delta x_d$ and ensures both contact maintenance and less than 100% force overshoot.

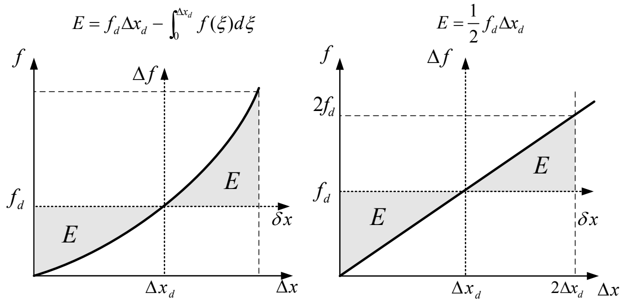

Fig. 3. Contact maintenance and potential owing to deformation error

IV. Simulation Results

Consider a two-dof planar manipulator with link lengths $l_1 = 0.3$ m, $l_2 = 0.2$ m masses $m_1 = 0.8$ kg, $m_2 = 0.6$ kg and inertias $I_i = \frac{m_i l_i^2}{12}$. The surface is modeled by a line with slope $\varphi_s = 105°$ with a normal vector $n_s = \left[\begin{array}{cc} \sin\varphi_s & -\cos\varphi_s \end{array} \right]^T$. Normal force magnitude is simulated by $f = k_s \Delta x$ where $k_s = 10,000$. The end-effector initial contact point position is $p_c(0) = \left[\begin{array}{cc} 0.3615 & 0.1701 \end{array} \right]^T$ (m) and exerts a normal force of $f(0) = 4$ N. The control purpose is to exert a time-variant normal force with magnitude $f_d(t) = 9 + 3\cos(t)$ (N) that corresponds to an initial step of 8 N and to track the desired position trajectory $p_d(t) = P_d + A_d \sin(t)$ where $P_d = \left[\begin{array}{cc} 0.3915 & 0.2001 \end{array} \right]^T$ (m) and $A_d = \left[\begin{array}{cc} 0 & 0.1 \end{array} \right]^T$ (m) that corresponds to an initial error of $|o^T \Delta p| = 0.021$ m, where o is the tangential unit vector, without loosing contact. The initial estimate of the line slope given in the adaptive law is $\hat{\varphi}_s(0) = 90°$ that corresponds to an initial angle error of $15°$.

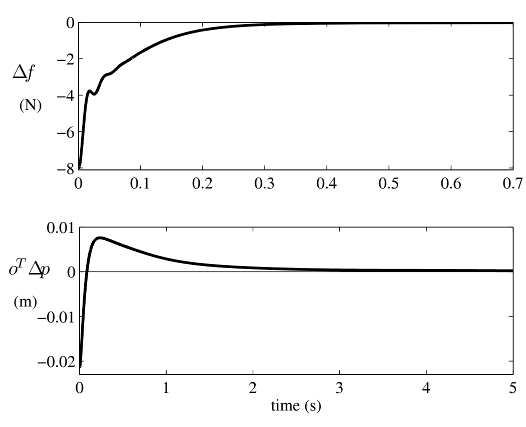

Fig. 4. Force and position error response

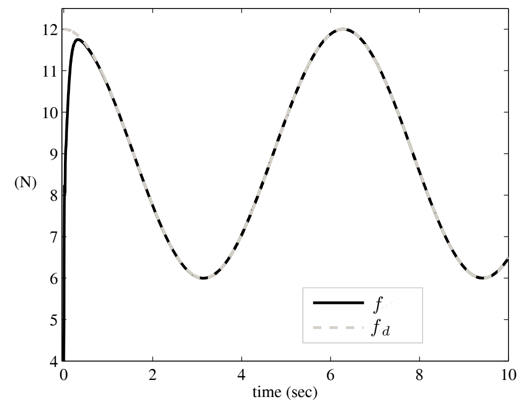

Fig. 5. Desired and actual force trajectories

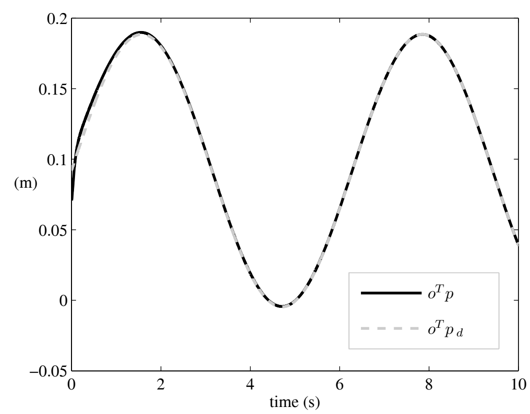

Fig. 6. Desired and actual tangent position trajectories

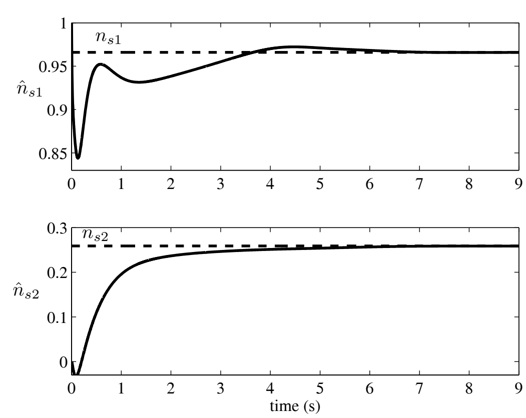

Fig. 7. Normal vector estimate response

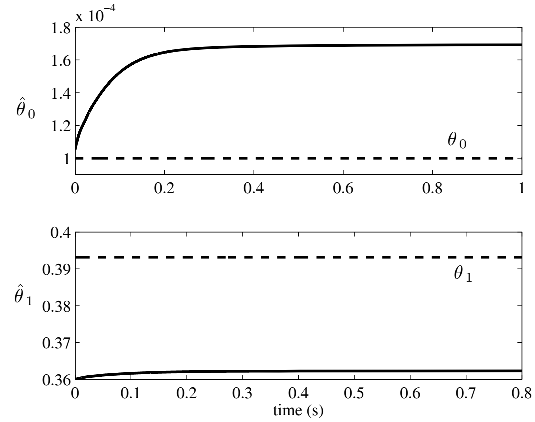

Fig. 8. Parameters' response $\hat{\theta}(t)$

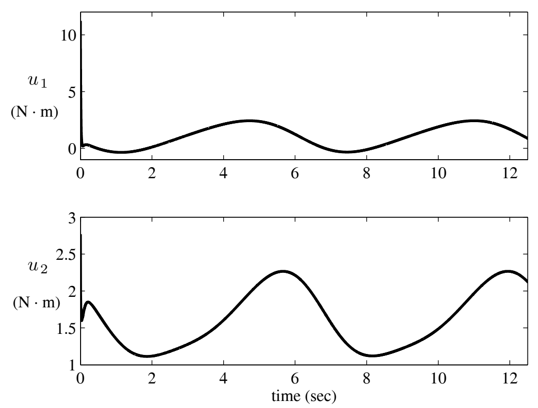

Fig. 9. Control input response

The estimate of elasticity model constant is $\hat{k}_s = 9,500$, that is 5% smaller than the actual, and corresponds to $\hat{\theta}_0(0) = \hat{k}_s^{-1}$. The initial estimate of θ_1 is $\hat{\theta}_1(0) = 0.36$ that is 8% smaller than the actual. The force integrator is initially set to 0.45. The gains of the controller have the following values: $\alpha = 12$, $\beta = 0.3$, $\gamma = 0.6$, $k_f = 1.1$, $D = \text{diag}[7.5, 0.9]$, $\Gamma = \text{diag}[2 \cdot 10^{-5}, 8 \cdot 10^{-3}]$, $\Gamma_n = \text{diag}[2.5, 0.25]$.

Fig. 4 shows the transients of force and position error while Figs. 5, 6 show the desired and actual force and tangent position trajectories. Fig. 7 shows the convergence of the estimated normal vector coordinates to their actual values. Force error converges in 0.5 s while the position error convergence follows the convergence of the normal vector parameters and is achieved in 4 s. Fig. 9 shows the response of the estimated elasticity and surface position parameters $\hat{\theta}_0$, $\hat{\theta}_1$ together with their actual values. Note that despite the lack of convergence of the estimated parameters to their actual values, the control target is achieved. Input torques, shown in Fig. 9, remain small although the initial force and position values do not belong to the desired trajectories.

V. Conclusions

This work proposes an adaptive controller for the problem of force/position tracking to cope with parametric uncertainties in surface kinematics and fingertip compliance. The control scheme requires measurements of joint variables and force magnitude. Future works will investigate the effect of friction and dynamic model uncertainties.

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