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Remote Formation Control and Collision Avoidance for Multi-Agent Nonholonomic Systems

Silvia Mastellone, Dušan M. Stipanović and Mark W. Spong

Abstract— **This paper presents a novel decentralized control scheme that achieves dynamic formation control and collision avoidance for a group of nonholonomic robots. First, we derive a feedback law using Lyapunov-type analysis that guarantees collision avoidance and tracking of a reference trajectory for a single robot. Then, we extend this result to the case of multiple nonholonomic robots, and show how different classes of multi-agent problems involving an interacting group of nonholonomic robots such as formation control can be addressed in this framework. Finally, we combine the above results to address the problem of driving a group of robots according to a given trajectory while maintaining a specific formation.**

I. INTRODUCTION

The technological revolution that came along in the last century with the advent of wireless communication brought a breadth of innovation and provided ways to efficiently share information between systems. Interacting systems are no longer constrained to be physically connected. Thus, in several applications a single complex system can be replaced by interacting multi-agent systems with simpler structure. For example, automotive and aerospace applications range from assembling structures and carrying large objects to exploring unknown environments. In fact, a group of small robots (unicycles, car-like robots or unmanned aerial vehicles (UAVs)) with simple structure can achieve more complex tasks at a lesser cost than a single complex robot due to their modularity and flexibility.

In this framework a new set of problems needs to be addressed for groups of robots with nonholonomic dynamics such as coordination and formation control while guaranteeing collision avoidance. The problem of coordination of multiple agents has been addressed though different approaches, various stability criteria and control techniques. The recent literature on the subject shows a rich collection of results. Some of the existing approaches, as highlighted in [17], include the behavior based approach as in [1], the leader-follower approach adopted in [19]. Another approach focuses on maintaining a certain group configuration and forces each agent to behave as a particle in a rigid virtual structure [6], [4]. In a recent survey [11], several methods were identified to solve formation control problems using optimization based approach and potential fields approach.

When we consider systems with nonholonomic constraint the formation control problem becomes more challenging. Several techniques to control a single nonholonomic system, can be found in [3] and [12]. Some representative papers in the area of formation control for nonholonomic dynamics are [13] and [16] where using dynamic extension the model is linearized locally and a decentralized overlapping scheme is used to control the formation. Some of the existing results addressing tracking problems for nonholonomic systems are reported in [9], [2], [5]. In [20] tracking is achieved via adaptive control, and in [10] amplitude modulated sinusoids are used. Finally in [18] stabilization for nonholonomic dynamics is achieved as well as collision avoidance using global barrier functions.

In this paper we present a novel approach that addresses all of the above aspects. We present a twofold result: First, we design a controller that guarantees stable tracking and obstacle/vehicle collision avoidance for nonholonomic systems. We assume that a robot knows its position and can detect the presence of any object within a certain range. Second, we apply our result to formation control for multi-agent systems. Finally, we address the problem of driving a group of robots according to a trajectory provided by a remote supervisor while maintaining a specific formation. The main contribution is the design of a relatively simple controller which provably guarantees tracking and collision avoidance for systems subject to nonholonomic constraint. The collision avoidance control acts in real time and uses locally defined potential functions which can take different shapes and only require each agent to detect other objects in its neighborhood. This is a major advantage which distinguishes our method from other potential field approaches as in ([7]). Moreover, we show how this approach can be generalized to multiple robots to achieve formation control and mutual avoidance. Experiments implementing the controllers were conducted on a robotic testbed and the data shows that all these results are easily implementable and are robust with respect to communication unreliability, such as delays, communication dropout and bounded disturbances. The experimental results can be found in [8].

The paper is organized as follows: In Section II a controller is designed that guarantees tracking and collision avoidance for a single robot. In Section III we formulate the formation control problem as a tracking problem and solve it using our method. In Section IV we use the previous result to achieve velocity tracking for a group of nonholonomic robots while maintaining a desired formation. Finally, in Section V we draw some conclusions and future directions. At the end of each section we provide simulation data that illustrates the result in the section.

Silvia Mastellone, Dušan M. Stipanović and Mark W. Spong are with Coordinated Science Laboratory, University of Illinois, 1308 W. Main St., Urbana, IL 61801, USA {smastel2,dusan,mspong}@uiuc.edu.This research was partially supported by the Office of Naval Research grant N00014-05-1-0186.

II. Trajectory Tracking and Collision Avoidance

In this section we consider a nonholonomic mobile robot for which we want to design a controller that guarantees asymptotic tracking of a reference trajectory while avoiding collisions with objects in the plane. The robot is modeled by the following nonlinear ordinary differential equations (ODEs)

$$\begin{aligned} \dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= u \end{aligned} \tag{1}$$

where $x \in \mathbb{R}$ and $y \in \mathbb{R}$ are the Cartesian coordinates, $\theta \in [0, 2\pi)$ is the orientation of the robot with respect to a given frame and v, u are linear and angular velocity respectively. We are also given a reference trajectory for the robot to follow, denoted by x_d, y_d. Then, we define the position errors as $e_x = x - x_d$ and $e_y = y - y_d$. The coordinates of the object to be avoided are given by x_a, y_a and we can define

$$d_a = \sqrt{\left(\frac{x - x_a}{\alpha}\right)^2 + \left(\frac{y - y_a}{\beta}\right)^2} \tag{2}$$

for $\alpha, \beta > 0$. In this paper we will address the obstacle avoidance problem through the following potential function defined for $\alpha = \beta = 1$:

$$V_a = \left(\min\left\{0, \frac{d_a^2 - R^2}{d_a^2 - r^2}\right\}\right)^2 \tag{3}$$

defined in [15], where r is the radius of the avoidance region around the obstacle, and R is the radius of detection region around the obstacle, with $R > r > 0$. Thus, this function blows up whenever the robot approaches the avoidance region and is zero whenever the robot is outside the sensing region. To break the symmetry different shapes of the potential function, for example ellipsoids, can be obtained by choosing different values for the coefficients α, β. Upon taking the partial derivatives of V_a with respect to the x and y coordinates, we obtain

$$\frac{\partial V_a}{\partial x} = \begin{cases} 0, & \text{if } d_a \geq R \\ 4\frac{(R^2 - r^2)(d_a^2 - R^2)}{(d_a^2 - r^2)^3}(x - x_a), & \text{if } R > d_a > r \\ 0, & \text{if } d_a < r \end{cases} \tag{4}$$

and

$$\frac{\partial V_a}{\partial y} = \begin{cases} 0, & \text{if } d_a \geq R \\ 4\frac{(R^2 - r^2)(d_a^2 - R^2)}{(d_a^2 - r^2)^3}(y - y_a), & \text{if } R > d_a > r \\ 0, & \text{if } d_a < r \end{cases} \tag{5}$$

Let us define

$$E_x = e_x + \frac{\partial V_a}{\partial x}, \;\; E_y = e_y + \frac{\partial V_a}{\partial y},$$

for $(E_x, E_y) \neq (0, 0)$, the desired orientation as

$$\theta_d = \text{Atan2}(E_y, E_x), \tag{6}$$

and the orientation error: $e_\theta = \theta - \theta_d$. Note that θ_d defines a desired direction of motion that depends on the reference trajectory, the robot position and on the obstacle to avoid. Some configurations might lead to singular directions. In order to avoid singular cases, we will assume throughout the paper that the reference trajectory has the following characteristics:

Assumption 1: The reference trajectory is smooth and satisfies:

$$|e_\theta| \leq \arccos(\delta_\theta) \tag{7}$$

for some $\delta_\theta \in (0, 1]$.

Assumption 2: The reference trajectory remains constant inside the collision region, i.e. $\dot{x}_d = \dot{y}_d = 0$, for $r \leq d_a < R$.

Assumption 3: Define $\hat{\dot{\theta}}_d$ to be an estimate which entails some measurement error of

$$\dot{\theta}_d = \frac{E_x\dot{E}_y - \dot{E}_x E_y}{D^2} \tag{8}$$

where $D = \sqrt{E_x^2 + E_y^2}$. Then, we assume that

$$\left|\hat{\dot{\theta}}_d - \dot{\theta}_d\right| \leq \epsilon_\theta \tag{9}$$

for some small positive ϵ_θ. Note that most of the variables in $\dot{\theta}_d$ can be measured, in fact we have that $\left|\hat{\dot{\theta}}_d - \dot{\theta}_d\right| = \frac{E_x(\dot{E}_y - \hat{\dot{E}}_y) - E_y(\dot{E}_x - \hat{\dot{E}}_x)}{D^2}$, where the estimates can be chosen to as values $\hat{\dot{E}}_x = \frac{E_x(t+T) - E_x(t)}{T}$ and $\hat{\dot{E}}_y = \frac{E_y(t+T) - E_y(t)}{T}$, for some small T. Then, as E_x, E_y are smooth $a.e.$ we have that $(\dot{E}_x - \hat{\dot{E}}_x) \simeq (\dot{E}_x - \hat{\dot{E}}_x) \simeq o(T)$, and we can pick $\epsilon_\theta \simeq o(T)$. If we assume that $\dot{x}_d$, $\dot{y}_d$ can be measured then $\dot{\theta}_d$ can be exactly calculated and $\left|\hat{\dot{\theta}}_d - \dot{\theta}_d\right| = 0$.

Remark 1: Assumption 1 on the reference trajectory implies the following two conditions:

1) Outside the collision region ($d_a \geq R$) and for $(e_x, e_y) \neq (0, 0)$ we have $\theta_d = \text{Atan2}(e_y, e_x)$. The reference trajectory is such that it does not initiate sharp turns of angle greater than $\arccos(\delta_\theta)$ with respect to the current orientation of the robot. Note that this condition is not too restrictive since the robot can reorient itself in place if the condition is not satisfied.
2) Inside the safety region ($r \leq d_a < R$) the reference trajectory x_d, y_d must be such that $|e_\theta| \leq \arccos(\delta_\theta)$. The way to achieve this is to consider a perturbed reference trajectory instead of the real one whenever (7) is not satisfied. Given a reference trajectory x_d, y_d that enters the safety region around an obstacle at the point x_a, y_a, and do not satisfy Assumption 1, we can replace the reference trajectory with the following perturbed version:

$$\begin{aligned} \bar{x}_d &= x_d - \text{sign}(x - x_a)\epsilon_x \\ \bar{y}_d &= y_d - \text{sign}(y - y_a)\epsilon_y \end{aligned} \tag{10}$$

where $\epsilon_x \neq \epsilon_y$ are some small perturbation values, and $\text{sign}(y - y_a)$, $\text{sign}(x - x_a)$ define the direction opposite to the obstacle, i.e. we want to perturb the trajectory in the direction opposite to the obstacle. This condition guarantees that the system avoids the singularities and deadlock.

In Figure 1 two examples of non admissible (singular) directions are shown. In the top left is shown a nonholonomic agent outside in an obstacle free space; the direction of motion defined by the arrow is not admissible since violate the nonholonomic constraints. The top right shows a nonholonomic agent approaching a sensing region around an obstacle. D_t is the direction required by the reference trajectory, D_a is the avoidance direction, and D_r is the direction resulting from the other two, which is not admissible, since violate the nonholonomic constraints.

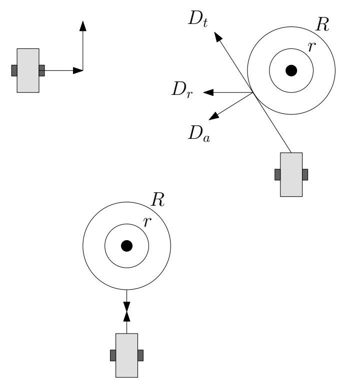

Fig. 1. Examples of non admissible trajectories which lead to violation of the nonholonomic constraints (top) and deadlock (bottom).

Note 1: The singularity condition $E_x = E_y = 0$ can occur in the following two cases: outside the collision region that gives $\frac{\partial V_a}{\partial y} = \frac{\partial V_a}{\partial x} = 0$, which corresponds to $e_x = e_y = 0$ and this case can easily be handled using zero controllers $u = v = 0$. Inside the collision region the condition corresponds to a singularity in which the reference direction for tracking is opposite to the direction for avoiding collision, this results in a deadlock as shown in Figure 1 (bottom). This case can be handled by changing the reference trajectory to drive the robot out of the singularity. We will not investigate this case further in this work.

Theorem 1: Consider system (1) and the reference trajectory described by (x_d, y_d) that satisfies Assumptions 1 to 3. Consider also an object to be avoided that is located at (x_a, y_a). Define the desired orientation as in (6). Then, stable tracking outside the collision region and collision avoidance are guaranteed if the following controller is applied,

$$u = -K_\theta e_\theta + \dot{\hat{\theta}}_d \tag{11}$$

$$v = -K \cos(e_\theta) D \tag{12}$$

for some gains K, $K_\theta > 0$ and $D = \sqrt{E_x^2 + E_y^2}$.

Proof: Consider the errors dynamics

$$\begin{aligned} \dot{e}_x &= u\left(\cos(e_\theta)\cos(\theta_d) - \sin(e_\theta)\sin(\theta_d)\right) - \dot{x}_d \\ \dot{e}_y &= u\left(\sin(e_\theta)\cos(\theta_d) + \cos(e_\theta)\sin(\theta_d)\right) - \dot{y}_d \\ \dot{e}_\theta &= v - \dot{\theta}_d \end{aligned} \tag{13}$$

Using the expressions $\cos(\theta_d) = \frac{E_x}{D}$ and $\sin(\theta_d) = \frac{E_y}{D}$ and applying the controller (11)-(12) we obtain

$$\begin{aligned} \dot{e}_x &= K\left(-E_x\cos^2(e_\theta) + E_y\cos(e_\theta)\sin(e_\theta)\right) - \dot{x}_d \\ \dot{e}_y &= K\left(-E_x\cos(e_\theta)\sin(e_\theta) - E_y\cos^2(e_\theta)\right) - \dot{y}_d \\ \dot{e}_\theta &= -K_\theta e_\theta + \dot{\hat{\theta}}_d - \dot{\theta}_d \end{aligned}$$

Let us pick Lyapunov-like function candidate as

$$\begin{aligned} V &= \frac{1}{2}(V_t + V_a) \\ &= \frac{1}{2}\left((e_x^2 + e_y^2 + e_\theta^2) + \left(\min\left\{0, \frac{d_a^2 - R^2}{d_a^2 - r^2}\right\}\right)^2\right) \end{aligned}$$

Then, the derivative along the trajectories of the error dynamics is

$$\begin{aligned} \dot{V} \leq\ & -K\cos^2(e_\theta)\left(E_x^2 + E_y^2\right) - \\ & e_x\dot{x}_d - e_y\dot{y}_d - |e_\theta|\left(K_\theta|e_\theta| - \epsilon_\theta\right) \end{aligned} \tag{14}$$

When the robot is outside the collision region ($d_a > R$), we have $\frac{\partial V_a}{\partial x} = \frac{\partial V_a}{\partial y} = 0$, and the previous inequality becomes

$$\begin{aligned} \dot{V} \leq\ & -K\cos^2(e_\theta)\left(e_x^2 + e_y^2\right) - e_x\dot{x}_d - e_y\dot{y}_d - \\ & |e_\theta|(K_\theta|e_\theta| - \epsilon_\theta) \end{aligned}$$

Hence $\dot{V} \leq 0$ whenever

$$|e_x| > \frac{|\dot{x}_d|}{K\delta_\theta^2}, \quad |e_y| > \frac{|\dot{y}_d|}{K\delta_\theta^2}, \quad |e_\theta| > \frac{\epsilon_\theta}{K_\theta}$$

Therefore stability of the error dynamics, and hence tracking, is guaranteed outside the collision region.

When the robot is inside the safety region ($r \leq d_a < R$), Assumption 2 implies $\dot{x}_d = \dot{y}_d = 0$; hence inequality (14) becomes

$$\dot{V} \leq -K\cos^2(e_\theta)D^2 - |e_\theta|(K_\theta|e_\theta| - \epsilon_\theta)$$

that is negative semidefinite for $|e_\theta| > \frac{\epsilon_\theta}{K_\theta}$. Hence as shown in [15] collision avoidance is guaranteed. ■

A. Simulation Results

The controller can be used to achieve collision avoidance between robots, when multiple agents are operating in the same area. To illustrate the result we consider two unicycles in the $X - Y$ plane for which the objective is to track a trajectory which evolve in time around two circles of radius 5 centered in $(5, 10)$ and $(15.5, 10)$ respectively, while avoiding collision with each other. The initial conditions for the robot are $(x_1, y_1, \theta_1) = (-2, 10, 0)$, $(x_2, y_2, \theta_2) = (18, 10, 0)$. The detection and avoidance radii are $R = 3$, $r = 1$, respectively. Two sequential frames of the unicycles trajectories resulting by applying the controller (11)-(12) with gains $K_i = 1$, $K_{\theta i} = 10$, $i = 1, 2$ are represented in Figure 2. The robots track the desired trajectory while they are outside the collision region. When the reference trajectories intersect with the collision region, the robots deviate from their paths accordingly to (6) in order to avoid collisions.

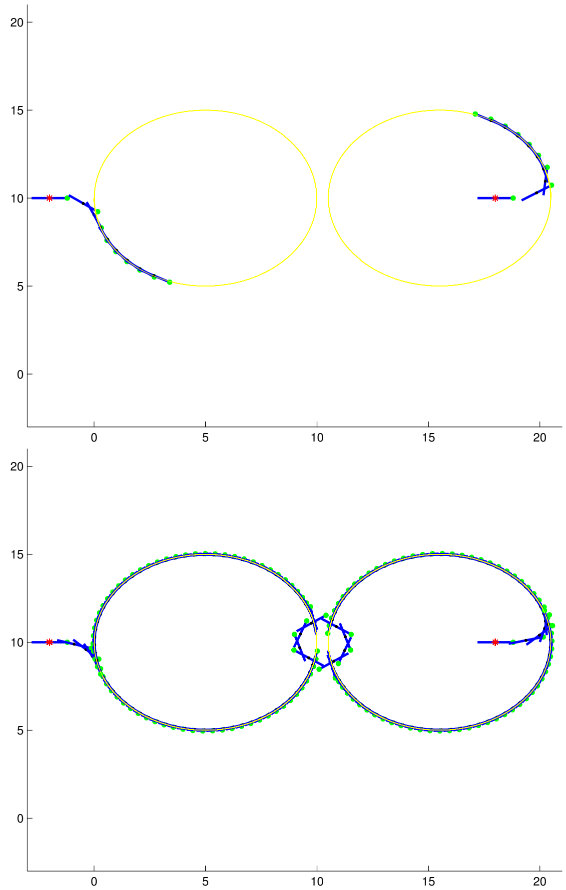

Fig. 2. Two unicycles tracking two circles while avoiding collision

III. Formation Control

In this section we study the problem of formation control for a group of nonholonomic agents. We consider a group of N nonholonomic robots whose kinematic model is described by

$$\begin{aligned} \dot{x}_i &= v_i \cos(\theta_i), & x_i(t_0) = x_{i0}, \\ \dot{y}_i &= v_i \sin(\theta_i), & y_i(t_0) = y_{i0} \\ \dot{\theta}_i &= u_i, & \theta_i(t_0) = \theta_{i0} \end{aligned} \quad (15)$$

where the quantities $x_i \in \mathbb{R}$, $y_i \in \mathbb{R}$, $\theta_i \in [0, 2\pi)$, v_i, u_i, $i = 1, \ldots, N$ are defined as above. Let us consider a formation described by a set of coordinates in the plane $\bar{x}_{fi}$, $\bar{y}_{fi}$, $i = 1, \ldots, N$. Define the formation center of mass as

$$\bar{x}_c = \frac{\sum_{i=1}^{N} \bar{x}_{fi}}{N}, \qquad \bar{y}_c = \frac{\sum_{i=1}^{N} \bar{y}_{fi}}{N}$$

and consider a desired trajectory for the center of mass $x_{dc}(t)$, $y_{dc}(t)$ starting at $x_{dc}(t_0) = \bar{x}_c$, $y_{dc}(t_0) = \bar{y}_c$. The formation control problem can be formulated as follows:

Problem 1: Given a desired formation for N robots and a desired trajectory for the center of mass of the formation, the objective is for the robots to converge to the formation and to follow the desired trajectory while keeping the formation. Moreover we would like to achieve such objective in a decentralized manner, i.e. the controller can be implemented locally on each agent.

The formation control problem can be addressed in a trajectory tracking framework. Given a desired formation for N robots and a desired trajectory for the center of mass of the formation, we can define a desired trajectory for each robot to follow as $x_{di}(t) = x_{dc}(t) + \delta_{xi}$, $y_{di}(t) = x_{dc}(t) + \delta_{yi}$ for $i = 1, \ldots, N$, where δ_{xi}, δ_{yi} are the components of the distance of each robot from the center of mass of the formation. Then, as $x_i(t) \to x_{di}(t)$, $y_i(t) \to y_{di}(t)$, $i = 1, \ldots, N$, and collision avoidance between robots is guaranteed, the robots will converge to the formation and follow the desired trajectory while keeping the formation without colliding.

Theorem 2: Consider a group of nonholonomic agents and a desired formation for the group so that the desired position of each robot is outside the collision regions. Consider also a desired trajectory for the center of mass of the formation that satisfies Assumptions 1 to 3. Then, the robots converge to the desired formation and track the desired trajectory for the center of mass while avoiding collisions, if the following controllers are applied,

$$\begin{aligned} u_i &= -K_{\theta_i}(e_{\theta i}) + \dot{\hat{\theta}}_{di} \\ v_i &= -K_i \cos(e_{\theta i}) \sqrt{E_{x_i}^2 + E_{y_i}^2} \end{aligned} \quad (16)$$

where the desired orientation is defined for $(|E_{x_i}|, |E_{y_i}|) \neq (0, 0)$ as $\theta_{di} = \mathrm{Atan2}(E_{y_i}, E_{x_i})$, where we define E_{x_i} and E_{y_i}, as for the case of single robot. The potential functions for collision avoidance are defined as

$$V_{ai} = \sum_{j=1,\, j\neq i}^{N-1} V_{aij}$$

where $V_{aij} = \left(\min\left\{0, \frac{d_{aij}^2 - R^2}{d_{aij}^2 - r^2}\right\}\right)^2$ and $d_{aij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, $j \neq i$, $j = 1, \ldots, N-1$.

Proof: Consider the following Lyapunov-like function candidate

$$V = \sum_{i=1}^{N} \left[\left(e_{xi}^2 + e_{yi}^2 + e_{\theta i}^2\right) + \sum_{j\neq i, j=1}^{N-1} \frac{1}{2} V_{aij} \right]$$

If we calculate the derivative of the function V along the trajectories of the system of robots we obtain

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^{N} \Big[-K_i \cos^2(e_{\theta_i}) \left((2E_{xi})^2 + (2E_{yi})^2 \right) \\ & + 2e_{xi}|\dot{x}_{di}| + 2e_{yi}|\dot{y}_{di}| - 2|e_{\theta_i}| \left(K_{\theta_i}|e_{\theta i}| - \epsilon_{\theta i}\right) \Big] \end{aligned} \quad (17)$$

Assuming that the robots are outside the safety region, if

$$|e_{xi}| > \frac{|\dot{x}_{di}|}{K_i \delta_{\theta i}^2}, \quad |e_{yi}| > \frac{|\dot{y}_{di}|}{K_i \delta_{\theta i}^2}, \quad |e_{\theta i}| > \frac{\epsilon_{\theta i}}{K_{\theta_i}}$$

then we have $\dot{V} < 0$. Therefore stability of the error dynamics, and hence formation tracking, is guaranteed outside the collision region.

When the robots approach each other and enter their safety region ($r \leq d_a < R$), Assumption 2 implies $\dot{x}_{di} = \dot{y}_{di} = 0$; hence inequality (17) becomes

$$\dot{V} \leq \sum_{i=1}^{N} -K_i cos^2(\theta_i) D_i^2 - |e_{\theta_i}| \left(|e_{\theta i}| - \frac{\epsilon_{\theta i}}{K_{\theta_i}} \right)$$

where $D_i^2 = \left[|2E_{xi}|^2 + |2E_{yi}|^2 \right]$ that is negative semidefinite for $|e_{\theta i}| > \frac{\epsilon_{\theta i}}{K_{\theta i}}$. Hence as shown in [15] collision avoidance is guaranteed. ∎

A. Simulation Results

We consider a group of three robots in the $X - Y$ plane. The objective is to reach a triangular formation whose center of mass is at $x_c = 6, y_c = 14.5$ and from there the center of mass must follow a trajectory $x_d = \frac{t}{5};, y_d = 30\sin(2\pi 0.01 x_d)$. We apply the controllers (16) and simulated the robots dynamics. The resulting trajectory for the group are shown in Figure 3. The three robots, starting at initial conditions $(x_1, y_1, \theta_1) = (0, 0, \frac{2}{3}\pi)$, $(x_2, y_2, \theta_2) = (5, 0, \frac{\pi}{2})$ and $(x_3, y_3, \theta_3) = (10, 0, \frac{\pi}{4})$, converge to the desired formation and then follow the desired trajectory while keeping the formation stable.

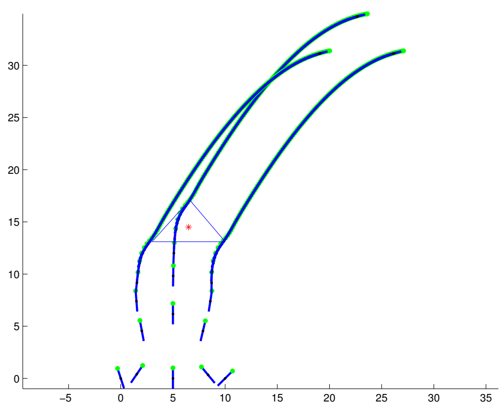

Fig. 3. Formation control for a group of 3 robots

IV. Remote Tracking and Formation Control

Consider a group of N nonholonomic robots whose kinematic model is described by (15) with $i = 1, \ldots, N$. The set of N robots is composed of a leader robot and a group of $N - 1$ followers. A supervisor specify a desired velocity and a desired shape (or formation), for the group. The desired velocity, characterized by amplitude V_d and directional angle ψ_d, is communicated to the leader from a remote supervisor through a human driven master device. The desired formation can be described by relative angle φ_i and relative distance d_i from each follower to the leader (see for example Figure 4). Hence a desired configuration for N robots is described by $2(N-1)$ parameters d_i, φ_i, $i = 1, \ldots, N-1$. Each follower receives the parameters (d_i, φ_i) from the supervisor, and the leader position x_0, y_0 from the leader. Perfect communication is assumed. In order to guarantee collision avoidance we also assume that each robot can sense the others in its

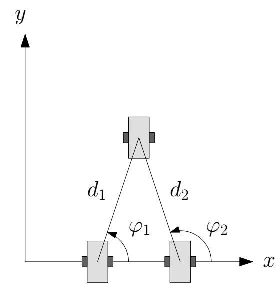

Fig. 4. Formation shape described as relative distance and orientation of each follower with respect to the leader.

proximity. In summary the supervisor provides the following set of desired parameters: V_d, ψ_d, d_i, φ_i, $i = 1, \ldots, N-1$.

We need to guarantee that the leader will track the velocity commands V_d, ψ_d given by the supervisor, and that the followers will keep the desired formation specified by the parameters d_i, φ_i, $i = 1, \ldots, N-1$. The formation problem and collision avoidance can be addressed as in the previous section using the control laws (16). We address the velocity tracking in the following section.

A. Velocity Tracking

Recall the kinematic model (1). Our objective in this section is to drive the velocity of the center of mass of the robot according to specified values of velocity magnitude and direction, which are given by V_d and ψ_d in polar coordinates or equivalently $\dot{x}_d$, $\dot{y}_d$ in Cartesian coordinates:

$$\begin{aligned} \dot{x}_d &= V_d \cos(\psi_d) \\ \dot{y}_d &= V_d \sin(\psi_d) \end{aligned}$$

Since the velocity input u directly affects the output that we aim to drive $(\dot{x}, \dot{y})$ but not all the directions are feasible due to the nonholonomic constraints, we need to guarantee that the system is aligned with the direction of motion that is $\theta_d = \psi_d$ and define the error dynamics as

$$\begin{aligned} \dot{e}_\theta &= u - \dot{\theta}_d \\ \dot{e}_x &= v\cos(\theta) - V_d \cos(\theta_d) \\ \dot{e}_y &= v\sin(\theta) - V_d \sin(\theta_d) \end{aligned} \quad (18)$$

By applying the simple proportional controller

$$\begin{aligned} v &= V_d \\ u &= -K(\theta - \theta_d) \end{aligned} \quad (19)$$

with $\theta_d = \psi_d$ and some gain $K > 0$, we have the closed loop system

$$\begin{aligned} \dot{e}_x &= V_d(\cos(e_\theta)\cos(\theta_d) - \sin(e_\theta)\sin(\theta_d) - \cos(\theta_d)) \\ \dot{e}_y &= V_d(\sin(e_\theta)\cos(\theta_d) + \cos(e_\theta)\sin(\theta_d) - \sin(\theta_d)) \\ \dot{e}_\theta &= -K(\theta - \theta_d) - \dot{\theta}_d \end{aligned} \quad (20)$$

It can be easily shown, using the ISS-Lyapunov function $V = \frac{1}{2}e_\theta^2$, that this system is input-to-state stable (ISS) [14] with respect to the input $\dot{\theta}_d$ and that the output is bounded for bounded values of the input V_d. This guarantee that we can drive the velocity of the leader according to some desired bounded value using controllers (19).

Note 2: In [17] a result was proven that establishes the ISS property for nonholonomic systems of the form (1) with a dynamic extension, when tracking a given trajectory. However, since we are only interested in velocity tracking we only need ISS with respect to the orientation.

B. Simulation Results

We consider a group of unicycles composed of one leader and two followers. The leader is driven by the following velocity command

$$\begin{cases} V_d = 1, & \psi_d = \pi/2, & t \leq 5 \\ V_d = 1, & \psi_d = \pi/4, & 5 < t \leq 7 \\ V_d = 1, & \psi_d = 0, & t > 7 \end{cases} \tag{21}$$

A formation is specified so that in the first part of the trajectory ($t \leq 7$) the group moves in a "V" shape, while in the second part ($t > 7$) the robots are on a straight line. The leader uses the control law (19) while the followers use the controllers in (16). In Figure 5, the resulting trajectories of the unicycles are depicted, where the arrow represents the velocity command.

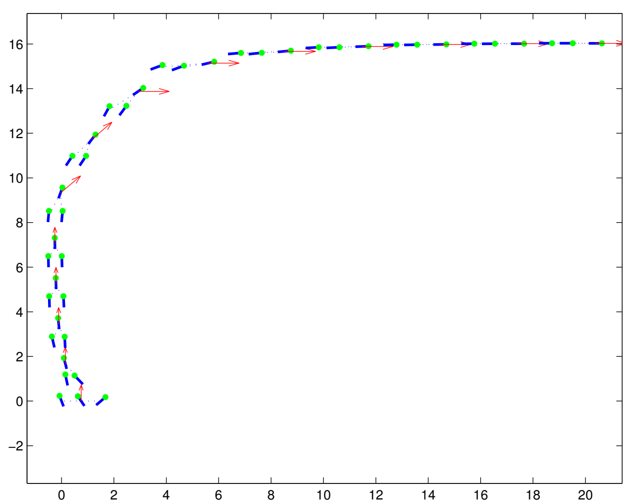

Fig. 5. Group of unicycles driven by velocity commands while changing formation.

V. Conclusion

We studied the problem of trajectory tracking and collision avoidance for nonholonomic systems. We designed a controller that guarantees collision avoidance and stable tracking outside the collision region. The design is based on Lyapunov approach and potential functions. We used this basic framework to solve leader-follower and formation control problems for multi-agent nonholonomic systems. Finally we address the problem of remote control of the fleet velocity and shape formation, using the tracking plus collision avoidance approach.

We showed how most of the results that require cooperation and coordination of agents can be restated in terms of tracking. This allows to have a decentralized control scheme that guarantees robustness and reliability while minimizing the communication between the agents. Future directions include considering a more complex kinematic model that allows extensions to a larger class of physical systems. One of the problems is that the potential function considered has infinite values in the proximity of the object resulting in possible high values of the inputs. To overcome this problem we will investigate alternative collision avoidance methods.

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