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Statics as a Means to Assess the Sensitivity of a Manipulator to Kinematic Parameter Deviations

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Abstract—**The paper suggests a new method to determine the dependence of the pose of a manipulator end-effector on the dimensional variations of the manipulator rigid links. Based on the principle of virtual work, the method is applicable to all statically determinate manipulators, i.e., to the manipulators whose joint reactions can be found by resorting to equilibrium equations only. The distinctive feature of the proposed method over the previous ones is the ability to keep the complexity of the kinematic model of the analyzed manipulator to a minimum: no generalized kinematic model is necessary even in case a nominally special-geometry manipulator transforms into a general-geometry manipulator when geometric inaccuracies are taken into account. A numerical example shows application of the proposed method to a case study. Use of the method in kinematic calibration of manipulators is also outlined.**

I. Introduction

Assessing the sensitivity of a manipulator to the dimensional inaccuracies of its links is a preliminary step in choosing the manipulator architecture and dimensions, in specifying the manufacturing tolerances, and − depending on contingency − in the calibration process.

Even in case a given manipulator has a special geometry (i.e., parallelism or perpendicularity and/or intersection of revolute pair axes, etc.), it transforms into a general-geometry manipulator as soon as geometric deviations are taken into account. If a general-geometry kinematic model of the manipulator is available, the effects of geometric inaccuracies can be foreseen either rigorously [1], or − due to the smallness of the kinematic parameter deviations − by a first-order approximation of the general-geometry kinematic model in the neighborhood of the special-geometry model [2]-[5]. Needless to say, while a general-geometry kinematic model can be easily devised for a serial manipulator, this is seldom true for a multi-loop manipulator. Dispensing with the necessity to develop a general-geometry kinematic model for a special-geometry manipulator would be a valuable feature of any method devoted to evaluating the effects of the link geometric deviations on the end-effector pose.

This paper presents an original procedure to solve the kinematic sensitivity analysis of statically determinate manipulators, either serial or closed-loop. Applicable to both special and general-geometry manipulators, the proposed method is especially attractive in the former case, for it does not require any generalized-geometry kinematic model of the studied manipulator.

Similar to the method developed in [6] for investigating the sensitivity to joint clearances, the procedure here proposed has its underpinning in the principle of virtual work. Application on the proposed procedure requires solution of linear equations only. For any manipulator configuration, the change of the end-effector pose due to introduction of the link geometric deviations can be found by solving six static analyses of the nominal-geometry manipulator. The effectiveness of the suggested procedure is shown in a numerical example.

II. Geometric Deviations

A. Preliminary Considerations

The procedure explained in the paper will be presented with reference to a spatial robot manipulator, either serial or closed-loop. The mobility M [7] of the manipulator is immaterial and can be any (M equals the number of actuated kinematic pairs had them all exactly one degree of freedom).

The considered manipulator is composed of rigid links exclusively. The only immovable link is the manipulator base, whereas one of the remaining links is the manipulator end-effector. By changing the parameters of motion of the actuated kinematic pairs, any of ∞^C rigid-body positions of the end-effector relative to the base can be reached, where $C \leq \min\{6, M\}$ is the connectivity of the end-effector relative to the base [7].

It is assumed that the manipulator is statically determinate at the considered configuration, i.e., for an arbitrary external load applied to the end-effector, the (passive) reactions at all joints, together with all actuator forces (or torques), can be computed by resorting to equilibrium equations exclusively. Incidentally, it is precisely this assumption that allows assembling the manipulator even in presence of any geometric deviation of its links (see subsection II-C).

B. Defining the Geometry of a Link

As far as the problem of positioning the manipulator end-effector relative to the base is concerned, the geometry of a manipulator link is essentially defined by the relative placement of the pairing elements of the link. As is known [8], a pairing element of a given link is the locus of the points of the link that come into contact with another link. For instance, if two links are connected by a lower kinematic pair, the corresponding pairing elements on the two links are

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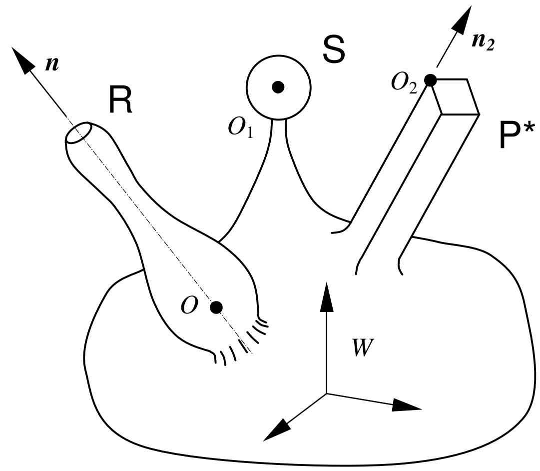


Fig. 1. Pairing elements on a ternary link.

two overlapping surfaces that can slide on each other.

To define the geometry of a link, a Cartesian reference frame W is attached to it and the position of all the pairing elements of the link are specified with respect to W. By way of example, Fig. 1 shows a ternary link and its three pairing elements of revolute (R), spherical (S) and prismatic (P) kinematic pairs. The R and S kinematic pairs are passive, whereas the P pair is actuated (as denoted by the asterisk in Fig. 1).

By still referring to Fig. 1, specifying the position of the R pairing element relative to W might seem to require six independent parameters, as if the R pairing element were a rigid body that has to be placed in space. Although not wrong in principle, this line of reasoning is not the most efficient one for the case at hand. Since the R pairing element is a surface of revolution, any rotation about its axis does not change the revolute connection between the considered link and the adjacent link. Therefore the position of the R pairing element relative to W depends on five independent parameters only, which can be chosen as the direction of unit vector $\boldsymbol{n}$ along the pairing element axis (two independent parameters) and the coordinates of a point O fixed to the pairing element and laying on its axis (three additional parameters).

Similarly, the position with respect to W of the S pairing element of the considered link can be specified by the three coordinates of the center O_1 of the pairing element.

The P* pairing element in Fig. 1 deserves a special note. Should it belong to a passive pair, its position relative to W could be specified by five independent parameters – one less than a rigid body – because any translation relative to W of the pairing element along the direction of its generators (unit vector $\boldsymbol{n}_2$) would be compensated by a sliding movement in the prismatic pair, without altering the relative position of the connected links. Owing to the fact that the considered prismatic pair is actuated, the compensating sliding in the prismatic pair does not automatically take place. Therefore defining the position in space of a pairing element of an actuated P pair requires six independent parameters, as it were a rigid body.

Summing up, a rough count seems to assess at 5+3+6=14 the minimum number of parameters needed to define the geometry of the ternary link shown in Fig. 1. Actually, this is true only if the considered link is the base or the end-effector of the manipulator, because in such cases the link has additional geometric features for the rigid connection to the ground or end-effector tool respectively, and these features can be exploited to position the reference frame W.

Should the considered link be neither the base nor the end-effector of the manipulator, the choice of reference frame W would be arbitrary, which means that the minimum number of parameters defining the link geometry would be 14−6=8.

C. *Parameterization of Geometric Deviations*

Due to manufacturing tolerances, the dimensions of the manipulator links differ from their nominal values by quantities here collectively referred to as geometric deviations.

For a generic link, the minimum number of parameters needed to specify any possible deviation from the nominal geometry equals the number of independent parameters required to describe the link geometry.

For instance, if the link shown in Fig. 1 is the manipulator base or end-effector, its fourteen geometric deviations could be parameterized by:

a) two mutually-orthogonal rotations, perpendicular to $\boldsymbol{n}$, of the R pairing element relative to W, plus the three-component displacement of point O relative to W;
b) the three-component displacement of the center of the S pairing element, relative to W;
c) the six parameters needed to specify the rigid-body movement of the P* pairing element relative to W.

Should the link of Fig. 1 be neither the base nor the end-effector, its eight geometric deviations could be only those listed at points a) and b) above (this corresponds to selecting W always in the same position relative to the P* pairing element).

Devising a minimum set of geometric deviations for any manipulator link is not an essential requirement for exploiting the procedure presented in the sequel of this paper, but it is nevertheless the cleanest and most efficient way to take advantage of it.

III. The Effect of Geometric Deviations

This section explains the proposed procedure to relate the geometric deviations of the links of a manipulator to the corresponding deviation of the end-effector location. The consequences of geometric deviations of only one link are examined first.

A. *The Elemental Effect*

Let us consider a generic manipulator whose links are all dimensionally perfect but for one, henceforth referred to as inaccurate link, characterized by some geometric deviations with respect to the nominal geometric requirements. The inaccurate link can be indifferently the base, the end-effector, or any other link.

First of all, the maximum number k of independent parameters able of describing any geometric deviation of the inaccurate link has to be assessed.

For instance, let us suppose that a manipulator has the inaccurate link shown in Fig. 2. Should only the revolute pairing element be affected by geometric inaccuracies, the maximum number k of independent geometric deviations would be five. They could be conveniently arranged into the five-component vector $\delta\boldsymbol{u}=(\delta O^{\mathrm{T}}, \delta\boldsymbol{\varphi}^{\mathrm{T}})^{\mathrm{T}}$, where $\delta O=(O'-O)$ is the displacement of the reference point on the revolute pairing element axis with respect to W, whereas $\delta\boldsymbol{\varphi}$ is a two-component vector generated as follows. If $\boldsymbol{l}$ and $\boldsymbol{m}$ are mutually-perpendicular unit vectors, both orthogonal to unit vector $\mathbf{n}$ parallel to the revolute pair axis and such that $\boldsymbol{l}$, $\boldsymbol{m}$, and $\mathbf{n}$ form a right-hand set, then the components of $\delta\boldsymbol{\varphi}$ are the projection of $\delta\boldsymbol{\vartheta}$ on $\boldsymbol{l}$ and $\boldsymbol{m}$, where $\delta\boldsymbol{\vartheta}$ is a vector that quantifies the rotation in space of the considered pairing element ($\delta\boldsymbol{n}=\boldsymbol{n}'-\boldsymbol{n}\cong\delta\boldsymbol{\vartheta}\times\boldsymbol{n}$; the $\cong$ sign is a reminder that this relation rigorously holds only if $\delta\boldsymbol{\vartheta}$ is infinitesimal).

A generic external load is now supposed as applied to the manipulator end-effector (which might incidentally happen to be the inaccurate link). Any other external load on the remaining links – such as weight – is intentionally neglected. Once a reference point A has been selected on the end-effector, the external load can be characterized by the resultant force $\boldsymbol{F}$ and the resultant moment $\boldsymbol{M}_{\mathrm{A}}$ with respect to A, so that the following six-component vector can be generated

$$\boldsymbol{H}=\left(\boldsymbol{F}^{T},\ \boldsymbol{M}_{A}^{T}\right)^{T} \tag{1}$$

Due to the external load $\boldsymbol{H}$ on the end-effector, and by purposely neglecting possible friction at the manipulator joints, the inaccurate link receives from the adjacent link(s) a reaction $\boldsymbol{U}$, which can be so expressed as to match the chosen parameterization of the geometric deviations of the inaccurate link.

By referring again to the explanatory case shown in Fig. 2, if the geometric deviation of the inaccurate link has been parameterized through the five-component vector $\delta\boldsymbol{u}=(\delta O^{\mathrm{T}}, \delta\boldsymbol{\varphi}^{\mathrm{T}})^{\mathrm{T}}$, then the reaction $\boldsymbol{U}$ is the five component vector $\boldsymbol{U}=(\boldsymbol{Q}^{\mathrm{T}}, \boldsymbol{N}_{\mathrm{O}}^{\mathrm{T}})^{\mathrm{T}}$, where $\boldsymbol{Q}$ is the resultant force applied to the revolute pairing element and $\boldsymbol{N}_{\mathrm{O}}$ is a two-component vector formed by the projections along unit vectors $\boldsymbol{l}$ and $\boldsymbol{m}$ (see Fig. 2) of the resultant moment of the reaction on the revolute pairing element, evaluated with respect to point O.

In general, $\delta\boldsymbol{u}$ is a k-component vector (in case all three pairing elements of the link shown in Fig. 2 are affected by geometric inaccuracies, k amounts to 14 if the link is the base or the end-effector, otherwise k equals 8).

If δu_{i} is the i-th component of $\delta\boldsymbol{u}$ (i=1,..,k), then the i-th component U_{i} of the reaction $\boldsymbol{U}$ is the generalized force [9],

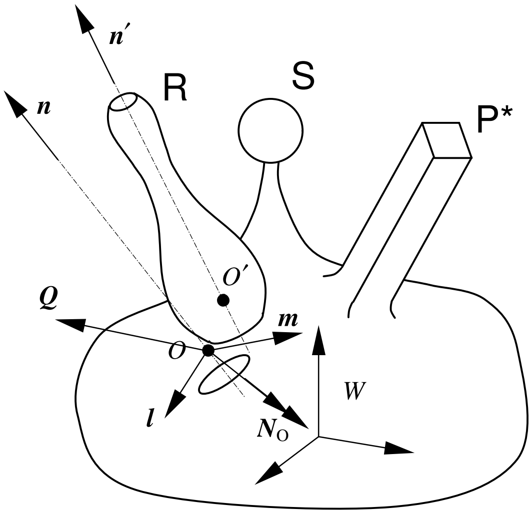


Fig. 2. Displacement of a revolute pairing element.

due to the external load $\boldsymbol{H}$, that corresponds to δu_{i} (therefore U_{i} is a force or a moment depending on whether δu_{i} is a linear or angular displacement respectively).

Let us momentarily suppose that the inaccurate link is not affected by geometric deviations, while the end-effector is exerted upon by the external load $\boldsymbol{H}$, so that the reaction of the adjacent link(s) on the inaccurate link is $\boldsymbol{U}$. Subsequently the geometry of the inaccurate link is slightly modified by progressively introducing the geometric deviations parameterized by vector $\delta\boldsymbol{u}$. During this process, all manipulator actuators are kept steady. The small dimensional changes of the inaccurate link cause a moderate rigid body displacement of the manipulator end-effector. Such a displacement can be parameterized by the following six-component vector $\delta\boldsymbol{h}$

$$\delta\boldsymbol{h}=\left(\delta\boldsymbol{A}^{T},\ \delta\boldsymbol{\alpha}^{\mathbf{T}}\right)^{T} \tag{2}$$

where $\delta\boldsymbol{A}$ is the displacement of the reference point $\boldsymbol{A}$ of the end-effector relative to the manipulator base, and $\delta\boldsymbol{\alpha}$ is the rotation of the end-effector relative to the manipulator base.

While the displacement $\delta\boldsymbol{h}$ is taking place, the reactions at all kinematic pairs – both passive and actuated – do not work, whereas the external load $\boldsymbol{H}$ does the work $\boldsymbol{H}^{\mathrm{T}}\delta\boldsymbol{h}$, and the inaccurate link does the work $-\boldsymbol{U}^{\mathrm{T}}\delta\boldsymbol{u}$ (for an observer fixed to the inaccurate link, the pairing elements exert on the remainder of the inaccurate link the reaction $\boldsymbol{U}$ – which does not work – whereas the remainder of the inaccurate link exerts on the pairing elements the reaction $-\boldsymbol{U}$, which does the work $-\boldsymbol{U}^{\mathrm{T}}\delta\boldsymbol{u}$).

The principle of virtual work [9] ensures that, for infinitesimal virtual displacements, the overall work is zero. For non-infinitesimal – though small – displacement vectors $\delta\boldsymbol{h}$ and $\delta\boldsymbol{u}$, the following condition can be written

$$\boldsymbol{H}^{T}\delta\boldsymbol{h}\cong\boldsymbol{U}^{T}\delta\boldsymbol{u} \tag{3}$$

Now the assumption that the manipulator is statically determinate at the considered configuration comes into play. Such an assumption makes it possible to determine the reaction $\boldsymbol{U}$ on the inaccurate link by solving a set of equilibrium equations that stem, for instance, from the free-body diagrams of the manipulator links. For a given external load $\boldsymbol{H}$ on the end-effector, without any external load on the remaining links, by also considering the no-friction postulate, the set of equilibrium equations that have to be solved in order to determine the reaction $\boldsymbol{U}$ on the inaccurate link is both linear and homogeneous in the components of vectors $\boldsymbol{H}$ and $\boldsymbol{U}$. Therefore the dependence of $\boldsymbol{U}$ on $\boldsymbol{H}$ can be synthesized by the ensuing condition

$$\boldsymbol{U} = \boldsymbol{D}\,\boldsymbol{H} \tag{4}$$

where the k×6 matrix $\boldsymbol{D}$ depends on the manipulator configuration only.

The elements of matrix $\boldsymbol{D}$ can be computed column after column by solving six static analyses of the manipulator, for six different external loads on the end-effector: for $\boldsymbol{H}$= $(1,0,0,0,0,0)^T$, the corresponding vector $\boldsymbol{U}$ is the leftmost column of $\boldsymbol{D}$, etc.

Insertion of the expression (4) for $\boldsymbol{U}$ into (3) leads to

$$\boldsymbol{H}^T\delta\boldsymbol{h} \cong \boldsymbol{H}^T\boldsymbol{D}^T\delta\boldsymbol{u} \tag{5}$$

Since (5) has to be satisfied by any external load $\boldsymbol{H}$ on the end-effector, the ensuing equations holds

$$\delta\boldsymbol{h} \cong \boldsymbol{D}^T\delta\boldsymbol{u} \tag{6}$$

This equation provides a simple and effective procedure to estimate how the location of the end-effector of a manipulator changes, following a moderate variation of the dimensions of one of its links. Implementation of the procedure requires the solution of six static analyses of the manipulator, at the considered configuration.

B. The Overall Effect

Extension of (6) to include all geometric deviations of a given manipulator is straightforward. If the manipulator has L links (inclusive of base and end-effector), then the overall deviation $\delta\boldsymbol{h}_t$ of the manipulator end-effector from the nominal rigid-body position, for a given set of actuator displacements (and for a given configuration, should the direct kinematics result in multiple solutions) is

$$\delta\boldsymbol{h}_t \cong \sum_{i=1}^{L}\boldsymbol{D}_i^T\delta\boldsymbol{u}_i \tag{7}$$

Matrices $\boldsymbol{D}_i$ (i=1,..,L) appearing in (7) are those relating the reaction $\boldsymbol{U}_i$ on link i to the external load $\boldsymbol{H}$ on the end-effector by a condition similar to (4)

$$\boldsymbol{U}_i = \boldsymbol{D}_i\,\boldsymbol{H} \tag{8}$$

The shape of matrix $\boldsymbol{D}_i$ is $k(i)$ ×6, where $k(i)$ is the number of components of the vector $\delta\boldsymbol{u}_i$ that parameterizes the geometric deviations of the i-th link of the manipulator.

It is worth noting that the whole set of matrices $\boldsymbol{D}_i$ (i=1,..,L) can be determined by an overall number of six static analyses of the manipulator, at the considered configuration.

Summing up, the proposed method – epitomized by (7) – makes it possible to estimate the sensitivity of a manipulator to the geometric inaccuracies of its links by suitably exploiting the results of a few linear problems (the six static analyses), in turn solvable by a simple custom-made computing procedure or off-the-shelf CAE software.

IV. NUMERICAL EXAMPLE

The proposed method is applied in this section to the closed-loop three-degree-of-freedom spatial manipulator shown in Fig. 3 and referred to as UP-3(UP*S) manipulator. For a given configuration of this manipulator, this section will determine the sensitivity of the end-effector location to the change of the geometry of one link.

With reference to Fig. 3, the considered manipulator has three variable-length legs B_iE_i (i=1,2,3), each of which is articulated to the base via a universal joint (U) centered at B_i and to the end-effector via a spherical pair (S) centered at E_i. The length of leg B_iE_i (i=1,2,3) can be varied by an actuated prismatic pair (P*). Triangles $B_1B_2B_3$ and $E_1E_2E_3$ are both equilateral. The center B of triangle $B_1B_2B_3$ coincides with the center of a fourth universal joint, which connects link Γ to the manipulator base. Link Γ itself is connected by a (passive) prismatic pair to the end-effector. With respect to the end-effector, point B moves along a straight line, which is also perpendicular to triangle $E_1E_2E_3$ and goes through its center E. The axis of the revolute pair that connects the base to the cross of the U-joint centered at vertex B_i (i=1,2,3) of triangle $B_1B_2B_3$ is parallel to the side of triangle $B_1B_2B_3$ opposite to B_i; the axis of the other revolute pair of the same U-joint is perpendicular to line B_iE_i, which is in turn parallel to the direction of relative motion in the actuated prismatic pair P* of leg B_iE_i. The axis of the revolute pair between the base and the cross of the U-joint centered at B is parallel to line B_3B_1, whereas the axis of the revolute pair between the same cross and link Γ is perpendicular to line BE. A last geometric peculiarity of the considered manipulator is the existence of a family of configurations characterized by having line BE orthogonal to the plane going through points B_1, B_2, and B_3 and, in addition, the sides E_1E_2, E_2E_3, E_3E_1 of triangle $E_1E_2E_3$ parallel, respectively, to the sides B_1B_2, B_2B_3, B_3B_1 of triangle $B_1B_2B_3$.

All input data provided hereafter are exactly expressed by the shown number of digits, except when otherwise specified. All results will be approximated by numbers with five meaningful digits.

A reference frame fixed to the manipulator base is chosen with origin at point B, x-axis oriented and directed as vector (B_1–B_3), positive y-axis passing through B_2, and z-axis so chosen as to obtain a right-hand frame. The distance of points B_i (i=1,2,3) from point B is 400 mm, whereas the distance of points E_i (i=1,2,3) from point E is 300 mm. The manipulator is considered at a configuration characterized by the following set of actuated leg lengths: B_1E_1=600 mm, B_2E_2=B_3E_3=700 mm. At the chosen assembly configuration,

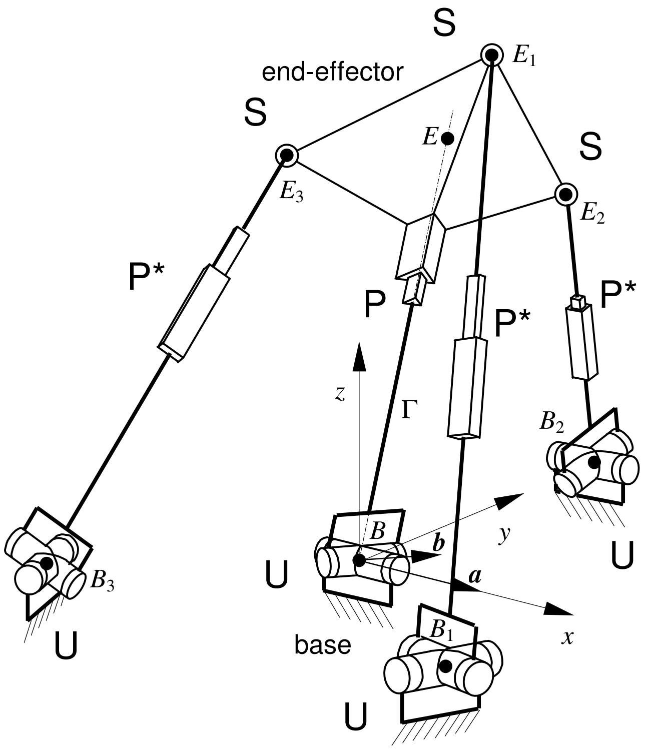


Fig. 3. The UP-3(UP*S) manipulator.

the coordinates in mm of points E_i (i=1,2,3) relative to frame $Bxyz$, approximated by eight decimal digits, are [10]

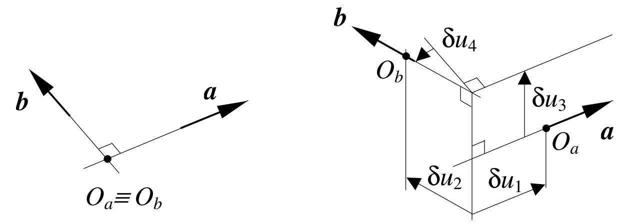
$$\begin{aligned} E_1 &= (\ \ 352.720522, -201.488430,\ \ 599.964969\ )^T \\ E_2 &= (\ \ \ 95.660363,\ \ 243.709147,\ \ 675.590308\ )^T \\ E_3 &= (-161.399796, -207.865648,\ \ 675.062439\ )^T \end{aligned} \tag{9}$$

Now the geometry of the cross of the U-joint centered at B is slightly altered. This link is neither the frame nor the end-effector, thus the maximum number k of independent parameters needed to quantify its geometry deviation is 5+5−6=4 (see section II). Figs. 4a and 4b schematically depict the cross before and after the geometric alterations. In Fig. 4a the axes of the two revolute pairing elements – oriented by unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ – are mutually perpendicular and the reference points O_a and O_b on these axes are superimposed one on the other ($\boldsymbol{a}$ and $\boldsymbol{b}$ are shown in Fig. 3 too). Fig. 4b shows a schematic of the altered cross: with respect to the common perpendicular to the two revolute pairing element axes, point O_a has been moved by δu_1 along $\boldsymbol{a}$, point O_b has been moved by δu_2 along $\boldsymbol{b}$, the two axes have been set apart by δu_3, and the angle between the same axes has been varied by δu_4. The ensuing vector $\delta\boldsymbol{u}$ is chosen

$$\delta\boldsymbol{u} = \left(0.30\,\text{mm} \quad 0.27\,\text{mm} \quad 0.14\,\text{mm} \quad 0.5\pi/180\,\text{rad}\right)^T \tag{10}$$

(To severely test the proposed procedure, the components of this vector are chosen one order of magnitude greater than the tolerances achievable in manufacturing.)

The reaction $\boldsymbol{U}$ of the adjacent links on the considered cross (determined by referring to the unaltered cross) is a four-dimensional vector formed by: i) the component along $\boldsymbol{a}$ of the force that the frame applies to the cross (which is also



(a) (b)

Fig. 4. Geometric alteration of the cross of the U-joint.

the opposite of the component along $\boldsymbol{a}$ of the force that link Γ exerts on the cross); ii) the component along $\boldsymbol{b}$ of the force that link Γ applies to the cross; iii) the component along $\boldsymbol{a}\times\boldsymbol{b}$ of the force that link Γ applies to the cross; iv) the component parallel to $\boldsymbol{a}\times\boldsymbol{b}$ of the moment with respect to point B of the reaction exerted by link Γ on the cross.

The reference point A on the end-effector is chosen as superimposed on point E (see Fig. 3). The six-dimensional vector $\boldsymbol{H}$ that parameterizes the external load on the end-effector will have its first three components expressed in N and its last three components expressed in N mm. By first considering $\boldsymbol{H}$=(1,0,0,0,0,0)^T, then $\boldsymbol{H}$=(0,1,0,0,0,0)^T, etc., till $\boldsymbol{H}$=(0,0,0,0,0,1)^T, six static analyses of the nominal-geometry manipulator are solved. (For the case at hand, the static analyses can be easily carried out by noting that the manipulator base reacts on the U-joint at B_i (i=1,2,3) by a force parallel to leg B_iE_i. Moreover, the reaction of the base on the U-joint centered at B is a force orthogonal to segment BE, plus a moment orthogonal to the cross of the U-joint.)

As a result, the ensuing matrix $\boldsymbol{D}^T$ is obtained

$$\boldsymbol{D}^T = \begin{pmatrix} -0.73743 & 8.2470\ 10^{-3} & -0.10810 & 1.0265 \\ -8.0892\ 10^{-3} & 0.75444 & -1.1858\ 10^{-3} & 1.7882 \\ 0.10284 & 5.9765\ 10^{-2} & 1.5077\ 10^{-2} & -1.1481 \\ -9.8381\ 10^{-7} & 3.7271\ 10^{-4} & -1.4422\ 10^{-7} & 0.14401 \\ 3.6990\ 10^{-4} & 1.3267\ 10^{-5} & 5.4225\ 10^{-5} & -8.2796\ 10^{-2} \\ 3.1411\ 10^{-5} & 1.1266\ 10^{-6} & 4.6048\ 10^{-6} & 0.99657 \end{pmatrix} \tag{11}$$

In this expression, the elements D^T_{ij} of $\boldsymbol{D}^T$ are dimensionless for i=1,2,3 and j=1,2,3, and for i=4,5,6 and j=4; they are expressed in mm for i=1,2,3 and j=4, whereas their measurement unit is mm^{-1} for i=4,5,6 and j=1,2,3.

Equations (6), (10), and (11) straightforwardly yield the deviation of the location of the end effector consequent to the chosen geometric deviation vector $\delta\boldsymbol{u}$

$$\begin{aligned}\delta\boldsymbol{h} \cong \big(&-0.22518\,\text{mm},\quad 0.21671\,\text{mm},\quad 3.9082\,10^{-2}\,\text{mm}, \\ &1.3571\,10^{-3}\,\text{rad},\quad -6.0039\,10^{-4}\,\text{rad},\quad 8.7071\,10^{-3}\,\text{rad}\big)^T\end{aligned} \tag{12}$$

Results (12) are now checked by an independent procedure. Following introduction of the geometric inaccuracies (10), the considered UP-3(UP*S) manipulator has become a RRP-3(UP*S) manipulator. Solving the forward kinematics of these manipulators requires finding the roots of 14-th and 28-th univariate order algebraic equations respectively [10]. By resorting to the available forward kinematic procedure for the RRP-3(UP*S) manipulator [10], the change of rigid-body position of the end-effector caused by $\delta\boldsymbol{u}$ has been exactly computed in terms of the displacement vector δA_{ref} of reference point A, and the finite rotation vector $\delta\boldsymbol{\alpha}_{\text{ref}}$ of the end-effector

$$\begin{aligned}\delta A_{ref} &= \left(-0.22522\,\text{mm},\ 0.21860\,\text{mm},\ 3.1972\ 10^{-2}\ \text{mm}\right)^T \\ \delta\boldsymbol{\alpha}_{ref} &= \left(1.3555\ 10^{-3}\ \text{rad},\ -5.9557\ 10^{-4}\ \text{rad},\ 8.7076\ 10^{-3}\ \text{rad}\right)^T\end{aligned} \quad (13)$$

The reader can verify the satisfactory agreement of (13) with the first three and, respectively, last three components of $\delta\boldsymbol{h}$ in (12).

V. Concluding Remarks

The results presented in this paper can be regarded as a generalization of the dualism between kinematics and statics that is epitomized, for manipulators, by the Jacobian $\boldsymbol{J}$. As is known [11], $\boldsymbol{J}$ relates the infinitesimal displacements of the actuators to the infinitesimal rigid-body displacement of the end-effector, whereas $\boldsymbol{J}^{\text{T}}$ relates the external forces on the end-effector to the active reactions of the actuators. This paper has enucleated that the relationship between the infinitesimal change of the geometry of the manipulator links and the consequent rigid-body displacement of the end effector is mirrored by the relationship between the external forces on the end-effector and the corresponding passive reactions on the manipulator links (matrix $\boldsymbol{D}^{\text{T}}$ in (6) corresponds to $\boldsymbol{J}$). Finding by statics means the matrix $\boldsymbol{D}$ behind the statics relationship (4) has been the key to discovering the kinematics relationship, i.e., to solving the kinematic sensitivity analysis.

The suggested procedure is conducive, for any manipulator configuration, to a set of linear equations between the manipulator geometric deviations on one side, and the end-effector pose change on the other. If some information about the end-effector pose change is known for a number of manipulator configurations, a set of linear equations in the supposedly unknown manipulator geometric deviations can be laid down and subsequently solved, thus working out a calibration problem.

By still referring to manipulator calibration, it is worth observing that not all geometric deviations can be expected to be identified. Let us suppose, for instance, that a revolute pair connects two manipulator links. If the revolute pairing elements of both links are displaced by the same amount along their axes, the revolute connection between the two links does not change for any manipulator configuration, despite introduction of one geometric inaccuracy on each link. Therefore the original manipulator behaves exactly as the altered one, as would result by applying the sensitivity analysis procedure presented in this paper. Consequently, the end-effector pose is sensitive to the difference between the considered geometric inaccuracies. Conversely, any geometric calibration procedure can provide information about such a difference, but not on the magnitude of each geometric deviation.

The proposed procedure is also applicable to the kinematic sensitivity analysis and calibration of manipulators with uncontrolled extra mobility, provided that they prove statically determinate when an external load is applied to the end-effector (extra mobility might stem, for instance, from a linear actuator with S joints at the extremities, for the actuator can freely turn about the line through the centers of the S joints). By restricting the reasoning to manipulators that have lower kinematic pairs only, the maximum number K of independent unknowns of the calibration problem – inclusive of the offsets of the actuators – is given by

$$\begin{aligned}K = {}& 4\left(N_{\text{H}} + N_{\text{R}} + N_{\text{C}}\right) + 3N_{\text{S}} + 2\left(N_{\text{P}} + N_{\text{E}}\right) \\ & - N_{\text{L}} + V_{SS} + V_{PP} + 6\end{aligned} \quad (14)$$

where N_{H}, N_{R}, N_{C}, N_{S}, N_{P}, and N_{E}, are the number of helical, revolute, cylindrical, spherical, prismatic and plane-on-plane kinematic pairs; N_{L} is the number of instances of extra mobility; V_{SS} and V_{PP} are, respectively, the number of binary S-S and P-P links (equation (14) is a generalization of the expression reported in [12]). Quantity K is lower than the collective maximum number G of independent geometric deviations of the manipulator links, in turn provided by

$$G = K + N_{\text{H}} + N_{\text{R}} + 3N_{\text{P}} + N_{\text{E}} - V_{SS} - V_{PP} \quad (15)$$

Finally, by keeping to the hypothesis of rigid links, the presented procedure can both be specialized to planar and spherical statically determinate manipulators, and applied to the kinematic sensitivity analysis of statically determinate mechanisms (spatial, spherical, or planar).

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