2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

Task Based Kinematical Robot Control in the Presence of Actuator Velocity Saturation and Its Application to Trajectory Tracking for an Omni-wheeled Mobile Robot

Giovanni Indiveri and Jan Paulus and Paul G. Plöger

Abstract— **Swedish wheeled mobile robots have remarkable mobility properties allowing them to rotate and translate at the same time. Being holonomic systems, their kinematics model results in the possibility of designing separate and independent position and heading trajectory tracking control laws. Nevertheless, if these control laws should be implemented in the presence of unaccounted actuator peak velocity limits, the resulting saturated linear and angular velocity commands could interfere with each other thus dramatically affecting the overall expected performance. Based on Lyapunov's direct method, a position and heading trajectory tracking control law for Swedish wheeled robots is developed. It explicitly accounts for actuator velocity saturation by using ideas from a prioritized task based control framework.**

I. Introduction

The basic idea of task based kinematical control of robots consists [1][2][3] in exploiting eventual kinematical redundancy to try to accomplish more than one motion task. For example in the simplest case there are only two tasks present and we assume for the moment actuators to be ideal (i.e. that any commanded velocity is instantly and perfectly implemented). Then according to the standard approach for redundant manipulators the two tasks would be accomplished as follows: denoting with $\mathbf{q}$ the n dimensional column vector of joint space variables, with $J(\mathbf{q}) \in R^{p\times n}$ the full rank, high priority, task Jacobian matrix and with $\mathbf{v}$ a p dimensional desired operational space, high priority task, velocity vector ($p \leq n$), the commanded joint space velocities $\dot{\mathbf{q}}_d$ would be split into:

$$\dot{\mathbf{q}}_d = J^{\dagger}(\mathbf{q})\,\mathbf{v} + P_J\dot{\mathbf{q}}_2 \tag{1}$$

being $J^{\dagger}$ a pseudo-inverse of J with $JJ^{\dagger} = I_{p\times p}$ and $P_J := \big(I_{n\times n} - J^{\dagger}(\mathbf{q})J(\mathbf{q})\big)$ the projection to the kernel, see e.g. [1], pg. 98. Being filtered by a projector in the null space of J, the joint velocity vector $\dot{\mathbf{q}}_2$ corresponds to the low priority task as opposed to $J^{\dagger}\,\mathbf{v}$ that corresponds to the high priority one. Indeed, assuming no actuator velocity saturation, $\dot{\mathbf{q}}_d$ given by equation (1) will always guarantee $\mathbf{v}$ to be perfectly realized whereas $\dot{\mathbf{q}}_2$ might even not be implemented at all if it should happen to be in the kernel P_J.

Since this idea of task based kinematics control has been developed, main stream research has focused in particular on extending the above formulation to the case of many concurrent tasks, on the definition of useful lower priority tasks (by example maximizing manipulability indexes in redundant manipulators) and on coping with singularities of $J(\mathbf{q})$ in case it should not be full rank for all admissible values of $\mathbf{q}$. Minor attention has been instead devoted to the problems that arise in the presence of actuator torque or velocity saturation. In the above described saturation free setting, as lower priority tasks are projected in the kernel of the Jacobians of higher priority ones, the former "do no harm" to the latter: this will no longer hold true when actuator velocity saturation should be present, in spite of the projection of lower priority tasks in the Jacobian kernel of higher priority ones. The paper addresses the issues relative to actuator velocity saturation based upon a kinematics robot description, whereas dynamic effects including eventual actuator torque saturations are not accounted for.

Should the low priority task cause even a single (of the n) actuators to saturate its velocity, the high priority task could be irremediably corrupted. As a simple example take a differential drive robot which travels on a straight line with constant linear velocity of both wheels being e.g. at some fraction q of the possible maximum wheel speeds v_{max}. Since the angular velocity will go with the difference of left and right wheel speeds, the maximum curvature of a feasible path in this situation is limited to the headroom left by the differences of the obtainable "left over" speeds of $1 - q$.

In some applications a simplistic and common way out of this problem is to uniformly scale $\dot{\mathbf{q}}_d$ in order to re-enter within the actuator limits, but this may cause serious performance degradation of the high priority task induced by the lower priority one. Rather than uniformly scaling $\dot{\mathbf{q}}_d$, the solution proposed in this paper builds on the idea of scaling the commanded joint velocities relative to lower priority tasks in favor of higher priority ones. The proposed method is tested on the trajectory tracking control problem for an omni-drive mobile robot. Omni-drive robots are equipped with so called Swedish (or Mecanum) wheels. A Swedish wheel differs from a common wheel in the fact that rollers are mounted on its perimeter. As reported in [4], the Swedish (or Mecanum) wheel was invented in 1973 by Bengt Ilon, an engineer working for the Swedish company Mecanum AB. The interest in such kind of wheels is related to the possibility of developing omni-directional robots in the sense of that the robot "have a full mobility in the plane which means that they can move at each instant in any direction

G. Indiveri is with Dipartimento Ingegneria Innovazione, University of Lecce, Italy giovanni.indiveri@unile.it

J. Paulus and P. G. Plöger are with Univ. Appl. Science Bonn-Rhein-Sieg and Fraunhofer IAIS Sankt Augustin, Germany jan.paulus@smail.inf.fh-bonn-rhein-sieg.de ploeger@ais.fraunhofer.de

1-4244-0602-1/07/$20.00 ©2007 IEEE.

without any reorientation" [5]. As opposed to traditional wheel car-like or differentially driven mobile robots, the translational velocity vector of a Swedish wheeled vehicle can point in an arbitrary direction at any time without reorienting the wheels. Otherwise stated, Swedish wheeled vehicles are not affected by non-holonomic constraints: as far as the structural properties of the kinematics model of a Swedish wheeled robot is concerned, angular and linear velocities are independent. As a consequence one can design separate and independent trajectory tracking guidance control laws for position and heading. Yet if these control laws are implemented in the presence of unaccounted actuator saturation, the resulting saturated linear and angular velocity commands could interfere with each other.

Our contribution is the application of the idea of task based scheduling to the non-redundant joint space decomposition as a direct sum. Furthermore we design a general schema to deal with velocity saturating actuators, where we proof overall convergence of tracking errors to zero in spite of the actuator saturations theoretically using Lyapunov methods. Finally we apply the kinematical control system to the specific problem of trajectory tracking for a Swedish wheeled mobile robot.

The outline of the paper is as follows: after describing the general schema to deal with actuator velocity saturation in Section II, the robot model on which the method is tested is reported in Section III. The trajectory tracking control law is designed and analyzed in Section IV while experimental results and conclusions are addressed in Sections V and VI respectively. The present paper generalizes recent preliminary results presented in [6].

II. Task Based Kinematical Control in the Presence of Actuator Saturation

Assume that applying standard task Jacobian projection methods (as, by example, in [7]) a commanded joint velocity vector should results in:

$$\dot{\mathbf{q}}_d = \sum_{l=1}^{N} \dot{\mathbf{q}}_l \tag{2}$$

being $\dot{\mathbf{q}}_l : l \in [1, N]$, N independent task inputs ordered by decreasing priority with increasing index ($\dot{\mathbf{q}}_1$ has highest priority). Any physical actuator will be able to produce only a bounded velocity: this implies that the commanded joint velocity vector should satisfy $\|\dot{\mathbf{q}}_d\|_\infty \leq \dot{q}_{\max}$ for some strictly positive threshold velocity $\dot{q}_{\max}$. In general, given also that task joint commands $\dot{\mathbf{q}}_l$ are often the output of feedback control laws, there is no guarantee that $\|\dot{\mathbf{q}}_d\|_\infty \leq \dot{q}_{\max}$ is actually satisfied at all times. To cope with this, the basic idea is to replace the plain sum in equation (2) with a weighted sum in which weights are dynamically and recursively computed so that $i)$ lower priority tasks are scaled at the expense of higher priority ones, $ii)$ the constraint $\|\dot{\mathbf{q}}_d\|_\infty \leq \dot{q}_{\max}$ is satisfied at all times and $iii)$ the overall resulting law still meets the requirements of the saturation free designed law (2). Consider the function $\sigma : R \times [0, \infty) \longrightarrow R$ such that

$$\sigma(x, c) = \begin{cases} 0 & \text{if} \quad x = 0 \\ 1 & \text{if} \quad 0 < |x| < c \\ c/|x| & \text{otherwise,} \end{cases} \tag{3}$$

where the non negative second argument c of $\sigma(x, c)$ will be called the *capacity* of x. By definition $\sigma(x, c)$ is simply a nonnegative scalar scaling factor such that $x\,\sigma(x, c)$ is "clipped" to $c\,\mathrm{sign}(x)$ whenever $|x|$ should exceed the capacity c and is equal to x otherwise, i.e. $x\,\sigma(x, c)$ is simply the saturated version of x in the range $[-c, c]$. Also notice that by its very definition

$$\sigma(x, 0) = 0 \;\; \forall \;\; x, \tag{4}$$

namely if x should be assigned zero capacity, then $x\,\sigma(x, 0) = 0$ for any value of x. Consider then the modified commanded joint velocity vector:

$$\dot{\mathbf{q}}_d = \sum_{j=1}^{N} \dot{\mathbf{q}}_j\, \sigma\left(\|\dot{\mathbf{q}}_j\|_\infty, c_j\right) \tag{5}$$

where each task capacity is recursively and dynamically computed as:

$$\begin{array}{rcl} c_1(t) & \leq & \dot{q}_{\max} \quad \text{(constant, i.e. } \dot{c}_1(t) = 0) \\ c_2(t) & = & c_1 - \|\dot{\mathbf{q}}_1\|_\infty\, \sigma\left(\|\dot{\mathbf{q}}_1\|_\infty, c_1\right) \\ c_3(t) & = & c_2(t) - \|\dot{\mathbf{q}}_2\|_\infty\, \sigma\left(\|\dot{\mathbf{q}}_2\|_\infty, c_2(t)\right) \\ \vdots & = & \vdots \\ c_N(t) & = & c_{N-1}(t) - \|\dot{\mathbf{q}}_{N-1}\|_\infty\, \sigma\left(\|\dot{\mathbf{q}}_{N-1}\|_\infty, c_{N-1}(t)\right). \end{array} \tag{6}$$

Notice that by construction all the above task capacities are non negative, i.e. $c_j \geq 0 \;\; \forall \;\; j \in [1, N]$, and that

$$c_j \;\leq\; c_{j-1} \;\;\; \forall \;\;\; j \in [2, N]$$

$$c_i = 0 \implies c_j = 0 \;\;\; \forall \;\;\; j > i$$

namely if a given task is assigned zero capacity, all the lower priority tasks will also automatically have zero capacity and all their weights in the sum (5) will be zero. The capacity of task i can be viewed as the residual capacity after the higher priority task $i-1$ has been commanded; thus, by example, c_2 will be zero (and also $c_j : j > 2$) if the task 1 input $\dot{\mathbf{q}}_1$ is saturating all its capacity c_1. In words, each task will be commanded with a non null weight only if the higher priority task have not saturated. The fact that c_1 needs not to exceed $\dot{q}_{\max}$ is due to the fact that task 1 alone should not saturate the actuator capacity $\dot{q}_{\max}$; moreover given that $c_{j+1} \leq c_j \;\forall\; j \in [1, n-1]$ the condition $c_1 \leq \dot{q}_{\max}$ guarantees that *each* term in the sum equation (5) will have infinity norm smaller or equal to the threshold $\dot{q}_{\max}$. Most important, also the total control signal equation (5) has infinity norm smaller or equal than $\dot{q}_{\max}$. The proof can be obtained by summing the last $N-1$ equations in equation

(6) implying:

$$
\begin{aligned}
c_1 &= \sum_{k=1}^{N-1} \|\dot{\mathbf{q}}_k\|_\infty \, \sigma\left(\|\dot{\mathbf{q}}_k\|_\infty, c_k(t)\right) + c_N \\
&\geq \sum_{k=1}^{N} \|\dot{\mathbf{q}}_k\|_\infty \, \sigma\left(\|\dot{\mathbf{q}}_k\|_\infty, c_k(t)\right). \qquad (7)
\end{aligned}
$$

From equations (5, 6, 7)

$$
\|\dot{\mathbf{q}}_d\|_\infty \leq \sum_{k=1}^{N} \|\dot{\mathbf{q}}_k\|_\infty \, \sigma\left(\|\dot{\mathbf{q}}_k\|_\infty, c_k(t)\right) \leq c_1 \leq \dot{q}_{\max}. \quad (8)
$$

The proposed kinematical control law given by equations (5) and (6) will thus guarantee that the commanded joint velocity remains bounded while preventing lower priority tasks to corrupt higher priority ones by inducing actuator saturation. Remarkably experience has shown that many Lyapunov based task oriented control laws of the form of equation (2) can be quite easily shown to conserve their convergence properties when implemented in the form of equations (5 - 6). While a detailed analysis of the general conditions under which this can be proven is ongoing research, a significant experimental example of a trajectory tracking control law for an omni-directional robot is presented next.

III. Robot Kinematics Modeling

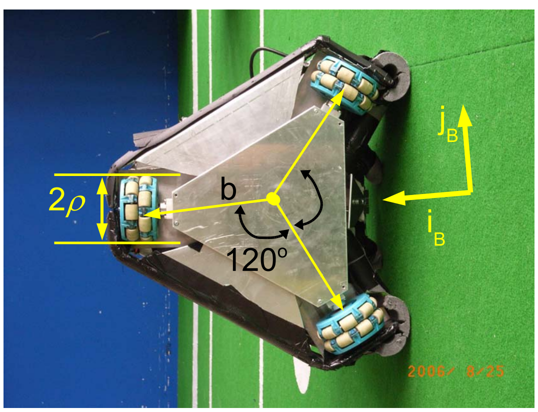

Fig. 1. Three wheel omni-drive robot used for the experiments: bottom view including body fixed frame (axis $\mathbf{k}_B$ goes inside the picture pointing towards the top of the robot)

With reference to Fig. 1, a three wheel omni-drive mobile robot is considered. The wheels have equal radius ρ and their main axis, i.e. hub axis, are assumed to always lie parallel to a fixed ground plane $\mathcal{P}$ having unit vector $\mathbf{k} \perp \mathcal{P}$. An orthonormal body fixed frame $< B >= \{\mathbf{i}_B, \mathbf{j}_B, \mathbf{k}_B\}$ is chosen such that $\mathbf{i}_B \times \mathbf{j}_B = \mathbf{k}_B = \mathbf{k}$. The three wheels are symmetrically located at 120^o degrees from each other at a same distance $b = \|\mathbf{b}_h\| \;\; \forall\, h = \{1, 2, 3\}$ from the robot's center. Calling $\mathbf{v}_c$ the linear velocity of the robots center and $\omega\,\mathbf{k}$ its angular velocity vector, the vehicle kinematics model can be expressed through the linear and angular velocity Jacobian matrices as:

$$
\begin{aligned}
{}^B\mathbf{v}_c &= J_{lv}\,\dot{\mathbf{q}} \;\;:\;\; J_{lv} \in R^{2\times 3} \qquad (9)\\
\omega &= J_\omega\,\dot{\mathbf{q}} \;\;:\;\; J_\omega \in R^{1\times 3}. \qquad (10)
\end{aligned}
$$

where the superscript B in ${}^B\mathbf{v}_c$ indicates that the components of vector $J_{lv}\,\dot{\mathbf{q}}$ are given in the body fixed frame $< B >$. In particular [8],

$$
\begin{aligned}
J_\omega &= -\frac{\rho}{3\,b}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \qquad (11)\\
J_{lv} &= \frac{\rho}{3}\begin{pmatrix} 0 & \sqrt{3} & -\sqrt{3} \\ -2 & 1 & 1 \end{pmatrix}. \qquad (12)
\end{aligned}
$$

It is important to notice that both J_ω and J_{lv} given in equations (11) and (12) are full rank and that

$$
J_{lv}\,J_\omega^T = \mathbf{0}. \qquad (13)
$$

As shown in the sequel, this last equation allows to design separate kinematics control laws for linear and angular velocities.

IV. Trajectory Tracking Control Law Design

Given an inertial (global) frame $< G >= (\mathbf{i}, \mathbf{j}, \mathbf{k})$ with $\mathbf{k} := (\mathbf{i} \times \mathbf{j}) \perp \mathcal{P}$ being $\mathcal{P}$ the floor plane, a reference (planar) trajectory is a $2D$ differentiable curve $\mathbf{r}_d(t)$ in $\mathcal{P}$ with curvilinear abscissa

$$
s(t) := \int_{t_0}^{t} \left\| \frac{d\,\mathbf{r}_d(\tau)}{d\tau} \right\| d\tau \qquad (14)
$$

and unit tangent vector

$$
\mathbf{t}_d = \frac{d\,\mathbf{r}_d}{ds}. \qquad (15)
$$

The kinematics trajectory tracking problem consists in finding a control law for the systems input $\dot{\mathbf{q}}$ such that the position and heading tracking errors

$$
\begin{aligned}
\mathbf{e}_r(t) &:= \mathbf{r}_d(t) - \mathbf{r}_c(t) \qquad (16)\\
e_\varphi(t) &:= \varphi_d(t) - \varphi(t) \qquad (17)
\end{aligned}
$$

converge to zero, namely such that:

$$
\begin{aligned}
\lim_{t\to\infty} \mathbf{e}_r(t) &= \lim_{t\to\infty} \left(\mathbf{r}_d(t) - \mathbf{r}_c(t)\right) = 0 \qquad (18)\\
\lim_{t\to\infty} e_\varphi(t) &= \lim_{t\to\infty} \left(\varphi_d(t) - \varphi(t)\right) = 0 \qquad (19)
\end{aligned}
$$

being $\mathbf{r}_c(t)$ the position in $< G >$ of a reference point (e.g. the geometrical center or the center of mass) of the robot, $\varphi(t)$ its heading, $\varphi_d(t)$ the desired reference heading, $\mathbf{e}_r(t) = (\mathbf{r}_d(t) - \mathbf{r}_c(t))$ the position tracking error and $e_\varphi(t) = (\varphi_d(t) - \varphi(t))$ the heading error. Notice that for non-holonomic vehicles having a unicycle or car-like kinematics model, the reference heading $\varphi_d(t)$ is *not* arbitrary, but needs to coincide with the heading of the trajectories unit tangent vector $\mathbf{t}_d$. To the contrary given any position reference trajectory $\mathbf{r}_d(t)$, a Swedish wheeled vehicle will be free to track any arbitrary heading $\varphi_d(t)$ that does not need to coincide with the heading of $\mathbf{t}_d$.

A. Trajectory tracking controller design

In accordance with the notation previously introduced, consider equations (9-10) being $\mathbf{v}_c = \dot{\mathbf{r}}_c(t)$ and $\omega = \dot{\varphi}(t)$ the time derivatives of the robots position $\mathbf{r}_c(t)$ and heading $\varphi(t)$. To solve the above stated trajectory tracking problem, consider the Lyapunov candidate function

$$V = \frac{1}{2}\mathbf{e}_r^T\, K_r\, \mathbf{e}_r + \frac{1}{2} e_\varphi^T\, K_\varphi\, e_\varphi \tag{20}$$

being $K_r \in R^{2\times 2}$ a symmetric positive definite ($K_r > 0$) matrix and K_φ a positive constant. The time derivative of V results in

$$\dot{V} = \mathbf{e}_r^T\, K_r\, (\dot{\mathbf{r}}_d(t) - J_{lv}\dot{\mathbf{q}}) + e_\varphi^T\, K_\varphi\, (\dot{\varphi}_d(t) - J_\omega \dot{\mathbf{q}})\,. \tag{21}$$

Denoting with $J_{lv}^\dagger$ and $J_\omega^\dagger$ the right pseudo-inverse matrices of full rank J_{lv} and J_ω respectively (J_{lv} and J_ω are full rank by hypothesis),

$$J_{lv}^\dagger = J_{lv}^T \left(J_{lv}\, J_{lv}^T\right)^{-1} \quad \text{and} \quad J_\omega^\dagger = J_\omega^T \left(J_\omega\, J_\omega^T\right)^{-1} \tag{22}$$

a possible value for $\dot{\mathbf{q}}$ making $\dot{V}$ in equation (21) negative definite is:

$$\begin{aligned}
\dot{\mathbf{q}}_d(t) &= \dot{\mathbf{q}}_{lvd}(t) + \dot{\mathbf{q}}_{\varphi d}(t) && (23)\\
\dot{\mathbf{q}}_{lvd}(t) &= J_{lv}^\dagger\, (\, \dot{\mathbf{r}}_d(t) + K_r\, \mathbf{e}_r(t)\,) && (24)\\
\dot{\mathbf{q}}_{\varphi d}(t) &= J_\omega^\dagger\, (\, \dot{\varphi}_d(t) + K_\varphi\, e_\varphi(t)\,) && (25)
\end{aligned}$$

implying in closed loop

$$\dot{V} = -\mathbf{e}_r^T\, K_r\, K_r\, \mathbf{e}_r - (K_\varphi\, e_\varphi)^2 < 0. \tag{26}$$

As for standard tracking controllers, the solution in equation (23) is a combination of feedforward terms proportional to the reference linear and angular velocities and a feedback term. The proposed solution guarantees global exponential stability of equilibrium $\mathbf{e}_r = \mathbf{0}, e_\varphi = 0$ of the error dynamics, thus (robustly) solving the trajectory tracking problem. Control law (23) is the sum of two contributions: the first (24) relative to position tracking and the second (25) to heading tracking. In the light of property (13), it should be noticed that the two contributions do not interfere with each other, namely the contribution of $\dot{\mathbf{q}}_{lvd}$ to the robots angular velocity and the contribution of $\dot{\mathbf{q}}_{\varphi d}$ to the robots linear velocity are both null, i.e.

$$J_\omega \dot{\mathbf{q}}_{lvd} = J_\omega \left(J_{lv}^T \left(J_{lv}\, J_{lv}^T\right)^{-1}\right) (\, \dot{\mathbf{r}}_d + K_r\, \mathbf{e}_r) = 0$$

$$J_{lv} \dot{\mathbf{q}}_{\varphi d}(t) = J_{lv} \left(J_\omega^T \left(J_\omega\, J_\omega^T\right)^{-1}\right) (\, \dot{\varphi}_d + K_\varphi\, e_\varphi) = \mathbf{0}$$

due to 13). When designing vehicle kinematics guidance laws it must be assumed that the lower level (actuator) dynamics should be much faster than the kinematics. This requirement is reflected on design choices such as actuator power and desired reference trajectories: the former needs to be sufficiently large for the given inertial properties of the vehicle so that maximum vehicle accelerations can be much larger than the maximum reference accelerations $\ddot{\varphi}_d(t)$ and $\ddot{\mathbf{r}}_d(t)$. As far as the ratio of maximum vehicle acceleration over maximum reference acceleration is sufficiently large the dynamic behavior of the kinematics guidance law will be fine. Thus, as for any other kinematics designed guidance solution, the proposed control law should be implemented on Swedish wheeled vehicles with sufficiently powerful actuators with respect to the maximum reference accelerations $\ddot{\varphi}_d(t)$ and $\ddot{\mathbf{r}}_d(t)$. As for actuator saturation, the situation is slightly more complex. Given the proportional nature of the control law (23), the tracking error (either in position or heading) or the desired reference velocities can always happen to be large enough for the actuators to saturate. If we choose $\dot{q}_{\max} > 0$ the maximum absolute angular velocity that the vehicles actuators are able to generate, whatever the gains K_r and K_φ should be, depending on $\dot{\varphi}_d(t)$, $\dot{\mathbf{r}}_d(t)$, $\mathbf{e}_r(t)$ or $e_\varphi(t)$ the saturation condition

$$\|\dot{\mathbf{q}}_d\|_\infty \;\le\; \dot{q}_{\max} \tag{27}$$

may always be violated. Notice that while the feedforward signals $\dot{\varphi}_d(t)$ and $\dot{\mathbf{r}}_d(t)$ can eventually always be bounded, the tracking error's initial conditions are not design parameters. Hence a commanded $\dot{\mathbf{q}}_d$ with exceeding infinity-norm due to odd initial conditions cannot be *a priori* excluded.

B. Actuator Saturation

In order to implement the above described trajectory tracking law in the presence of actuator saturation, assume that the reference feedforward linear and angular velocities are sufficiently small, namely that

$$\left\|J_{lv}^\dagger\, \dot{\mathbf{r}}_d(t)\right\|_\infty < \frac{1}{2}\; \dot{q}_{\max} \quad \forall\, t \tag{28}$$

$$\left\|J_\omega^\dagger\, \dot{\varphi}_d(t)\right\|_\infty < \frac{1}{2}\; \dot{q}_{\max} \quad \forall\, t. \tag{29}$$

These conditions are necessary to guarantee that the tracking task is asymptotically feasible, namely that when the position and heading tracking errors are null the control effort of the control law (23) is compatible with the actuator saturation limit, i.e.

$$\begin{aligned}
&\mathbf{e}_r = \mathbf{0}, e_\varphi = 0 \quad \Longrightarrow\\
&\|\dot{\mathbf{q}}_d(t)\|_\infty = \left\|J_{lv}^\dagger\, \dot{\mathbf{r}}_d(t) + J_\omega^\dagger\, \dot{\varphi}_d(t)\right\|_\infty \;\le\\
&\le\; \left\|J_{lv}^\dagger\, \dot{\mathbf{r}}_d(t)\right\|_\infty + \left\|J_\omega^\dagger\, \dot{\varphi}_d(t)\right\|_\infty \;<\; \dot{q}_{\max}.
\end{aligned}$$

As a first example, assume that position tracking is assigned highest priority with respect to heading tracking. Then define:

$$\begin{aligned}
\dot{\mathbf{q}}_1 &:= J_{lv}^\dagger\, \dot{\mathbf{r}}_d(t) && (30)\\
\dot{\mathbf{q}}_2 &:= J_{lv}^\dagger\, K_r\, \mathbf{e}_r(t) && (31)\\
\dot{\mathbf{q}}_3 &:= J_\omega^\dagger\, \dot{\varphi}_d(t) && (32)\\
\dot{\mathbf{q}}_4 &:= J_\omega^\dagger\, K_\varphi\, e_\varphi(t). && (33)
\end{aligned}$$

With these definitions consider the control law (5-6) with

$$c_1(t) = \dot{q}_{\max} \;>\; 0 \;\;\forall\;\; t$$

that together with the feasibility condition (28) implies

$$0 \;<\; \frac{1}{2}\,\dot{q}_{\max} \;\;\le\;\; c_2 \;\;\le\;\; \dot{q}_{\max},$$

i.e. the tasks 1 and 2 have always non null capacity. Moreover as by hypothesis $\|\dot{\mathbf{q}}_1\|_\infty < 0.5\,\dot{q}_{\max}$ (equation (28)) and $c_1 = \dot{q}_{\max}$, it follows that

$$\dot{\mathbf{q}}_1\,\sigma(\|\dot{\mathbf{q}}_1\|_\infty, c_1) = \dot{\mathbf{q}}_1 \quad \forall \quad t.$$

Consequently

$$\begin{aligned} V_1 &= \frac{1}{2}\mathbf{e}_r^T\,K_r\,\mathbf{e}_r \quad \Longrightarrow \qquad (34)\\ \dot{V}_1 &= \mathbf{e}_r^T\,K_r\,\Big(\dot{\mathbf{r}}_d(t) - J_{lv}\,\Big[J_{lv}^\dagger\,\dot{\mathbf{r}}_d(t) + \\ &\quad +J_{lv}^\dagger\,K_r\,\mathbf{e}_r(t)\,\sigma\left(\left\|J_{lv}^\dagger\,K_r\,\mathbf{e}_r(t)\right\|_\infty, c_2\right)\Big]\Big) = \\ &= -\mathbf{e}_r^T\,K_r\,K_r\,\mathbf{e}_r(t)\,\sigma\left(\left\|J_{lv}^\dagger\,K_r\,\mathbf{e}_r(t)\right\|_\infty, c_2\right) < 0 \end{aligned}$$

i.e. $\dot{V}_1$ is negative definite that proves asymptotic global Lyapunov stability of $\mathbf{e}_r = \mathbf{0}$. Notice that $\dot{\mathbf{q}}_3$ and $\dot{\mathbf{q}}_4$ do not contribute to $\dot{V}_1$ as they belong to the null space of J_{lv} (equation 13). As far as the secondary (heading) task is concerned, convergence can also be proven through a Lyapunov argument. The global asymptotic stability of $\mathbf{e}_r = \mathbf{0}$ guarantees that

$$\lim_{t\to\infty}\dot{\mathbf{q}}_2(t) = \mathbf{0} \quad \Longrightarrow \quad \lim_{t\to\infty} c_3 = c_2 \;\geq\; \frac{1}{2}\dot{q}_{\max}.$$

Given the feasibility condition (29), this means that

$$\exists\ t^* \ :\ \dot{\mathbf{q}}_3\,\sigma(\|\dot{\mathbf{q}}_3\|_\infty, c_3) = \dot{\mathbf{q}}_3 \ \text{ and } \ c_4 > 0 \ \forall\ t \geq t^*.$$

It follows that

$$\begin{aligned} &V_2 = \frac{1}{2}e_\varphi^T\,K_\varphi\,e_\varphi \quad \Longrightarrow \qquad (35)\\ &\dot{V}_2(t)\Big|_{t\geq t^*} = e_\varphi^T\,K_\varphi\,\left(\dot{\varphi}_d(t) - J_\omega\dot{\mathbf{q}}_d\right) = \\ &= e_\varphi^T\,K_\varphi\,\left[\dot{\varphi}_d(t) - J_\omega(J_\omega^\dagger\,\dot{\varphi}_d(t) + \right.\\ &\left. +J_\omega^\dagger\,K_\varphi\,e_\varphi(t)\,\sigma\left(\left\|J_\omega^\dagger\,K_\varphi\,e_\varphi(t)\right\|_\infty, c_4\right))\right] = \\ &= -e_\varphi^T\,K_\varphi^2\,e_\varphi(t)\,\sigma\left(\left\|J_\omega^\dagger\,K_\varphi\,e_\varphi(t)\right\|_\infty, c_4\right) < 0 \end{aligned}$$

namely there exists a finite time t^* after which the time derivative of V_2 is always negative, thus proving convergence to zero of the heading error $e_\varphi(t)$. Prior to t^* the heading error $e_\varphi(t)$ is not guaranteed to be decreasing. Notice that $\dot{\mathbf{q}}_1$ and $\dot{\mathbf{q}}_2$ do not contribute to $\dot{V}_2$ as they belong to the null space of J_ω (equation 13).

As a second example, heading can be selected to be the highest priority task, it is then sufficient to select $\dot{\mathbf{q}}_1, \ldots, \dot{\mathbf{q}}_4$ as

$$\dot{\mathbf{q}}_1 \ := \ J_\omega^\dagger\,\dot{\varphi}_d(t) \qquad (36)$$
$$\dot{\mathbf{q}}_2 \ := \ J_\omega^\dagger\,K_\varphi\,e_\varphi(t) \qquad (37)$$
$$\dot{\mathbf{q}}_3 \ := \ J_{lv}^\dagger\,\dot{\mathbf{r}}_d(t) \qquad (38)$$
$$\dot{\mathbf{q}}_4 \ := \ J_{lv}^\dagger\,K_r\,\mathbf{e}_r(t) \qquad (39)$$

in equations (5-6); Lyapunov stability of the heading error and asymptotic convergence of the position error could be proven accordingly.

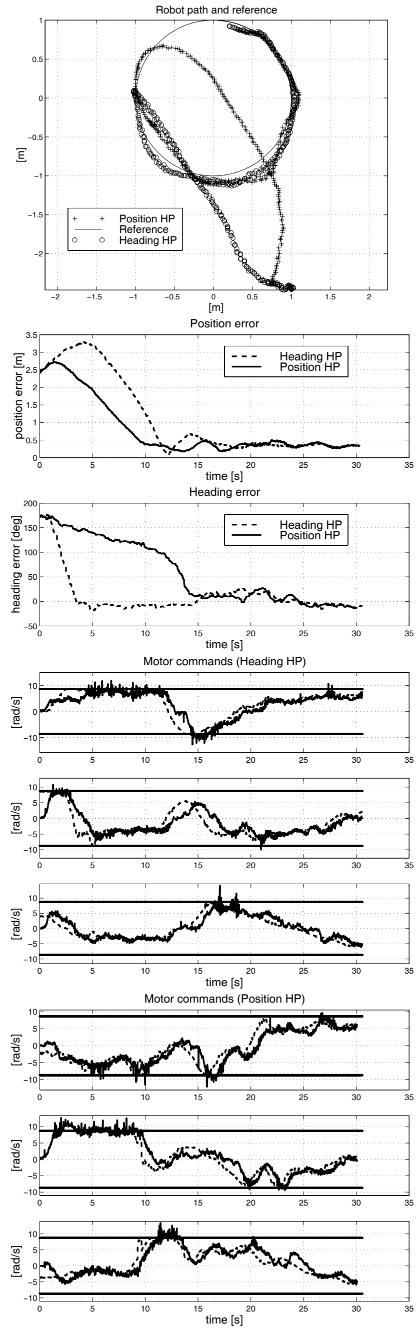

Fig. 2. Experimental results. HP stands for High Priority. Refer to text for details.

V. Experimental Results

The proposed control law has been experimentally tested on an omni-wheeled variant of the Volksbot platform (*www.volksbot.de*) [9] developed at the Fraunhofer IAIS Institute of Sankt Augustin, Germany. The robot is about 8Kg in weight and is actuated by three 90 Watts, 24 volt DC motors with a $1\ :\ 5,6$ gear ratio. Low level wheel speed control is achieved through a three channel PID motor driver (the IAIS TMC200 board) interfaced to an on-board laptop via a serial RS232 line, with wheel speeds delivered back. The presented kinematics trajectory tracking control law is implemented on the on-board laptop. Motor power is supplied through NiMH batteries with $3,5$ Ah capacity. The three omni-directional wheels have a 5cm radius, are made of lightweight plastic and are mounted at an angle of 120^o from each other. The robot is equipped with an omni-vision system made by a 30Hz, 640×480 pixels YUV color FireWire camera pointing towards a 70mm diameter hyperbolic mirror. Such systems are used for map based Monte Carlo self localization [10] [11]. Details can be found in [12]. In order to evaluate the performance of the proposed control solution the position and heading of the robot must be measured reliably and compared with the desired reference values. To this extent a test bench has been designed. We measured the exact pose of the robot using a combination of an external fixed laser range finder pointing on the robot and its heading was measured by the robot itself using its omni-vision system. These two independent devices delivered the highest accuracy under all feasible setups, e.g. the fixed SICK laser scanner delivers a systematic error of $+/-15mm$ with a random error of $1\sigma = 5mm$.We recorded (x, y) position of the robot, direction and wheel speeds under the two different highest priority tasks (HP) of either correcting heading or pose first. All collected data was suitably synchronized with the desired references. Extensive experimental trials with several different references have shown the effectiveness of the proposed solution: the case of a circular reference trajectory with constant (with respect to a fixed frame) heading is reported in the top three subplots of figure (2). The position and heading error plots with respect to time clearly confirm the effectiveness of the priority assignment policy. The growth of the position errors in the first few seconds of the experiment (second subplot) are due to the robots dynamics that was neglected in the control law design. As expected, as long as the actuators guarantee large enough accelerations with respect to the reference accelerations, the kinematics designed control law exhibits good dynamic performance, i.e. there is only a small lag with respect to the ideal purely kinematics case. In the motor command plots (lower three subplots of figure (2)), the commanded ($\dot{q}_d$, dashed lines) and encoder measured wheel speeds (solid lines) are reported with respect to time. Notice that for the sake of performance measurement accuracy, the saturation threshold was artificially set to the value of $\pm$8.7rad/s (thick solid lines) via software in order to achieve saturation at acceptable linear speeds. The gains control law gains K_r and K_φ were empirically selected based upon an estimate of the maximum tracking errors. The asymptotic position tracking error of about 0.5[m] displayed in figure (2) is believed to be related to (i) the finite precision with which the robot geometrical parameters were measured to compute the jacobian matrices and to (ii) the limited size of the integral gain in the low level PID motor drivers.

VI. Conclusions

A trajectory tracking control law for Swedish wheeled robots has been derived that takes explicitly into account motor saturations. Motor saturation is always present and may have a severe impact on motion control performances of mobile robots. This is particularly relevant for omni-directional mobile robots equipped with Swedish wheels: these offer a lower grip with the floor with respect to traditional wheels resulting in a higher probability of exhibiting skidding and/or sliding when high velocity commands are issued. As a consequence the possibility of commanding motor speeds always compatible with the saturation limits is extremely important for omni-directional mobile robots. Moreover the introduction of a task based prioritization of heading and position tracking may have a relevant impact on the behavior control level. The selection of heading or position tracking tasks as higher priority ones will generally depend on the (dynamic) role assignment: by using the described lower level control solution the highest priority tasks errors are guaranteed to converge faster to zero without ever commanding motor speeds exceeding the maximum HW allowed values. Future work directions should include on one hand studies on how the behavior system should take advantage of a guaranteed prioritized convergence of the tracking errors and also an extension of the presented technique to the dynamical model (actuator torque saturation) of the robot.

References

[1] L. Sciavicco and B. Siciliano, "Modelling and Control of Robot Manipulators", Springer Verlag (ISBN:1852332212), 2000

[2] Y. Nakamura, "Advanced Robotics: Redundancy and Optimization", Addison-Wesley Longman Publishing Co., Inc. Boston, MA, USA, 1990

[3] C. Samson, M. Le Borgne and B. Espiau, "Robot Control: The Task Function Approach. Oxford Engineering Science Series No. 22. Clarendon Press, 1991.

[4] O. Diegel, A. Badve, G. Bright, J. Potgieter and S. Tlale, "Improved Mecanum Wheel Design for Omni-directional Robots", Proc. 2002 Australian Conference on Robotics and Automation, Auckland, 27-29 November 2002.

[5] G. Campion, G. Bastin and B. D'Andréa-Novel, "Structural Properties and Classification of Kinematic and Dynamic Models of Wheeled Mobile Robots", IEEE Transactions on Robotics and Automation, Vol. 12, No. 1, February 1996, pp. 47 - 62.

[6] G. Indiveri, J. Paulus, and P.G. Plöger, "Motion Control of Swedish Wheeled Mobile Robots in the Presence of Actuator Saturation", In Proceedings of the RoboCup International Symposium 2006 Bremen, Germany, 19th - 20th June 2006.

[7] B. Siciliano, J-J. E. Slotine, "A General Framework for Managing Multiple Tasks in Highly Redundant Robotic Systems", Proceedings of the IEEE ICAR 1991, International Conference on Advanced Robotics, pages 1211 1216, Pisa, Italy, June 1991.

[8] J. Agulló, S. Cardona and J. Vivancos, "Kinematics of Vehicles with Directional Sliding Wheels", Mech. Mach. Theory, Vol. 22, No. 4, 1987, pp. 295 - 301.

[9] T. Wisspeintner, W. Nowak and Ansgar Bredenfeld, "Volksbot: A flexible component based mobile Robot system", in: A. Bredenfeld et al. (Eds), RoboCup 2005, Springer LNAI 4020, pp. 716-723.

[10] E. Menegatti, A. Pretto, and E. Pagello, "A new omnidirectional vision sensor for monte-carlo localization", In Int. RoboCup Symposium CD-ROM, Lisbon, Portugal, 2004.

[11] D. Schulz, W. Burgard, D. Fox, and A. Cremers, "People tracking with a mobile robot using sample-based joint probabilistic data association filters", International Journal of Robotics Research (IJRR). Springer, 2003.

[12] Seven Olufs, "Vision-Based Probabilistic State Estimation using Omnidirectional Cameras", Master Thesis FH-BRS, St. Augustin, 2005.