2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

Revising the robust control design for rigid robot manipulators

Luca Bascetta
Politecnico di Milano, Dip. Elettronica e Informazione
Piazza Leonardo da Vinci, 32 – 20133 Milano, Italia
bascetta@elet.polimi.it

Paolo Rocco
Politecnico di Milano, Dip. Elettronica e Informazione
Piazza Leonardo da Vinci, 32 – 20133 Milano, Italia
rocco@elet.polimi.it

Abstract— **Robust controllers for robot manipulators ensure stability properties of the closed loop system, even if only partial knowledge of the dynamic model of the manipulator is available. Existing derivations of robust control laws, while guaranteeing the stability result, present an undesired interaction between the gains of the controller of the nominal system and the robust control term.**

Based on a structured representation of the model uncertainty, this paper presents a derivation of the robust control law where these limitations are removed. A case study is discussed to show the benefits of the proposed approach. New insight in the robust control problem for more general mechanical systems might arise from structuring the model uncertainty as proposed in this paper.

I. INTRODUCTION

Model-based control of robotic manipulators has been a research issue for several decades. The equations of motion of the manipulator lend themselves to the application of advanced and elegant control laws. Practical applicability of the control laws based on the inverse dynamics of the manipulator has been however hampered in the past by limitations in the computing power of the available hardware. Nowadays computing power is no longer a significant issue and model based controller are being used even in industrial manipulators [1]. A renewed interest towards model based techniques is therefore justified, especially if this interest is motivated by the attempt to facilitate practical applicability of such advanced controllers.

In a realistic scenario perfect knowledge of the dynamic model of the manipulator can never be assumed. Robust controllers that integrate an inverse dynamics controller based on a nominal model of the system with an outer loop, suitably designed in order to robustify the closed loop system, have been proposed in the past and are now included in robotics textbooks [2], [3]. A comprehensive survey of robust control techniques developed until the beginning of the 90's can be found in [4]. Distinguished contributions in the field include [5], [6], [7].

The most well known robust control law, discussed in [2] and [3], is based on the derivation of the robust control action from the Lyapunov's second method. Although the derivation is elegant and based on clever arguments, we believe that the final result suffers from an inherent contradiction, where the robust term must be larger (in norm) the larger the control gains of the nominal PD controller are. This contradiction (already pointed out in [4]) is in turn the direct consequence of a representation of model uncertainty that does not account for the different nature of the uncertain terms. This entails that the bound on the uncertainty used in the derivation of the robust control law unexpectedly depends on the controller gains: the larger the controller gains are, the more uncertain the system looks as far as the design of the robust controller is concerned.

In this paper a different representation of the uncertainty in the dynamic model is proposed, in order to overcome these difficulties. The non nominal terms are in fact separated into a structural perturbation to the nominal error dynamics, due to modeling errors in the identification of the inertia matrix, and an additional uncertainty perturbation that is related to modeling errors in the description of the other nonlinear terms (centrifugal, Coriolis and gravitational ones). This structure of the uncertainty (already proposed in [8] for the stability analysis of decentralised PID controllers and here applied to the robust control problem) allows to formulate a different robust stability proof. While the proof might appear somewhat more involved (a Lyapunov argument is still used, but the Lyapunov function is based on the solution of a Riccati equation rather than a Lyapunov one) the result is neater. The design of the robust controller is clearly separated from the design of the PD controller for the nominal system. The proof is also constructive and yields a simple recipe to design a robust controller.

Setting the robust control problem for robot manipulators in the framework of the quadratic stability problem, as in the present paper, gives also insight in system structure and might be beneficial in studying different closed loop controlled mechanical systems.

The paper is organised as follows: Section II reviews the background in the design of the robust inverse dynamics control; Section III proposes the structure of the uncertainty suitable for a different outer loop design, which is dealt with in Section IV. A case study, based on a two-link planar manipulator, is discussed in Section V, after which some concluding remarks are proposed.

II. BACKGROUND ON THE ROBUST INVERSE DYNAMICS CONTROL

The robust inverse dynamics control as proposed in robotics textbooks [2], [3] will be reviewed here. The purpose is to identify the point where the mathematical development could be improved and to facilitate the comparison of the newly proposed stability proof with the present one.

1-4244-0602-1/07/$20.00 ©2007 IEEE.

A. Dynamic modeling

Consider thus the Euler-Lagrange equations of motion [2], [3] of a n-links rigid manipulator

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = u \tag{1}$$

where

- $q \in \mathbb{R}^n$ and $u \in \mathbb{R}^n$ are the vectors of joint variables and joint torques, respectively;
- $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix;
- $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is the vector of Coriolis and centrifugal terms;
- $F \in \mathbb{R}^{n\times n}$ is a diagonal matrix of viscous friction coefficients;
- $g(q) \in \mathbb{R}^n$ is the vector of gravitational torques.

Assuming that an exact inverse dynamics control cannot be achieved in practice, due to the uncertainties in the system parameters, a *realistic* inverse dynamics control input can be written as [2], [3]

$$u = \hat{M}(q)y + \hat{C}(q,\dot{q})\dot{q} + \hat{F}\dot{q} + \hat{g}(q) \tag{2}$$

where y is the new control input and the notation $\hat{(\cdot)}$ represents the estimated value of $(\cdot)$. Introducing then vector $n(q,\dot{q})$, defined as

$$n(q,\dot{q}) = C(q,\dot{q})\dot{q} + F\dot{q} + g(q)$$

the modeling error is represented by

$$\tilde{M}(q) = \hat{M}(q) - M(q) \qquad \tilde{n}(q,\dot{q}) = \hat{n}(q,\dot{q}) - n(q,\dot{q})$$

Substituting now equation (2) into the manipulator model (1) yields

$$\ddot{q} = y - \eta(q,\dot{q},y) \tag{3}$$

where

$$\begin{aligned}\eta(q,\dot{q},y) &= -M(q)^{-1}\left(\tilde{M}(q)y + \tilde{n}(q,\dot{q})\right)\\ &= \left(I - M(q)^{-1}\hat{M}(q)\right)y - M(q)^{-1}\tilde{n}(q,\dot{q})\end{aligned} \tag{4}$$

is called the uncertainty.
Theoretically, the application of the inverse dynamics control law, that perfectly cancels the nonlinearities in the robot equations of motion, turns the manipulator model into a set of double integrators. Instead, when a practical implementation of inverse dynamics control is considered, the resulting linearised model is more complicated (as reported in equation (3)), as the double integrators are now perturbed by the uncertainty, coming from a non perfect feedback linearisation.

B. Robust control

Consider now a control y as

$$y = \ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P(q_d - q) + w \tag{5}$$

where $q_d \in \mathbb{R}^n$ is the vector of desired joint trajectories, K_P and K_D are two diagonal positive definite matrices and w is a new control input. In terms of the tracking error

$$e = \begin{bmatrix}\tilde{q}\\ \dot{\tilde{q}}\end{bmatrix} = \begin{bmatrix}q_d - q\\ \dot{q}_d - \dot{q}\end{bmatrix}$$

the application of the control laws (2) and (5) to the robot equations of motion (1) yields

$$\dot{e} = Ae + B(\eta - w) \tag{6}$$

where

$$A = \begin{bmatrix}0 & I\\ -K_P & -K_D\end{bmatrix} \quad B = \begin{bmatrix}0\\ I\end{bmatrix}$$

In view of the error system here introduced, the robust PD control law (5) is formed by three different contributions that can be interpreted as follows:

- a PD linear feedback to stabilise the nominal system $\dot{e} = Ae$;
- a feedforward action ($\ddot{q}_d$) to compensate the accelerations of the desired trajectory;
- an additional control term w designed to overcome the potentially destabilising effect of the uncertainty η.

Matrix A is Hurwitz, if K_P and K_D are two diagonal positive definite matrices. Thus, picking K_P and K_D as follows

$$K_P = \mathrm{diag}\left(\omega_1^2,\ldots,\omega_n^2\right) \quad K_D = \mathrm{diag}\left(2\zeta_1\omega_1,\ldots,2\zeta_n\omega_n\right)$$

a nominal error dynamics characterised by a frequency ω_i and a damping factor ζ_i is established for each joint.
On the other hand, the additional control term w can be designed following the Lyapunov's second method [2], [3].

Consider a candidate Lyapunov function

$$V(e) = e^T Pe > 0 \quad \forall e \neq 0$$

Its time derivative, along the trajectories of the error system, will be

$$\begin{aligned}\dot{V} &= \dot{e}^T Pe + e^T P\dot{e}\\ &= e^T\left(A^T P + PA\right)e + 2e^T PB(\eta - w)\end{aligned} \tag{7}$$

Since A is Hurwitz, one can arbitrarily choose a positive definite matrix Q and let P be the unique symmetric positive definite matrix that satisfies the Lyapunov equation

$$A^T P + PA = -Q$$

Equation (7) can be thus rewritten as

$$\dot{V} = -e^T Qe + 2z^T(\eta - w) \tag{8}$$

where $z = B^T Pe$. If $z = 0$ the second term of (8) vanishes, otherwise w can be chosen as

$$w = \rho(\|e\|)\frac{z}{\|z\|}$$

Using the Cauchy-Schwartz inequality yields

$$\begin{aligned}z^T\left(\eta - \rho(\|e\|)\frac{z}{\|z\|}\right) &\leq \|z\|\|\eta\| - \rho(\|e\|)\|z\|\\ &= \|z\|\left(\|\eta\| - \rho(\|e\|)\right)\end{aligned}$$

Hence picking $\rho(\|e\|) \geq \|\eta\|$ the second term of (8) becomes negative and thus

$$\dot{V} \leq -e^T Qe < 0$$

The origin of the state space $\left(\tilde{q},\dot{\tilde{q}}\right)$ is therefore a globally asymptotically stable equilibrium point.

In conclusion, to accomplish the robust inverse dynamics control a suitable value for $\rho(\|e\|)$ have to be determined and to this end an upper bound to $\|\eta\|$ is required.

C. Determination of a suitable $\rho(\|e\|)$

Different approaches can be adopted (see e.g. [2], [3]) to determine a suitable gain $\rho(\|e\|)$ for the additional control term w. In the following, these approaches will be briefly analysed.

Firstly, the following assumptions are enforced

$$\begin{aligned} &0 < B_m \leq \|M^{-1}(q)\| \leq B_M \leq \infty && \forall q \\ &\|I - M^{-1}(q)\hat{M}(q)\| \leq \alpha \leq 1 && \forall q \\ &\sup_{t\geq 0} \|\ddot{q}_d\| < Q_M < \infty && \forall \ddot{q}_d \end{aligned}$$

(see [3] for more details and an interpretation of these assumptions).

From the definition of η the following upper bound is obtained

$$\begin{aligned} \|\eta\| &\leq \|I - M^{-1}\hat{M}\| \{\|\ddot{q}_d\| + \|K\|\|e\| + \|w\|\} + \|M^{-1}\|\|\tilde{n}\| \\ &\leq \alpha \{Q_M + \|K\|\|e\| + \rho(\|e\|)\} + B_M \Phi(\|e\|) \end{aligned} \quad (9)$$

where $K = \mathrm{diag}(K_P, K_D)$.
Hence, one can satisfy the inequality $\rho(\|e\|) \geq \|\eta\|$ assuming

$$\rho(\|e\|) \geq \frac{1}{1-\alpha} \{\alpha Q_M + \alpha \|K\|\|e\| + B_M \Phi(\|e\|)\} \quad (10)$$

A first approach, adopted in [3], lies in assuming a constant bound to $\|\tilde{n}\|$ and to $\|e\|$ and, consequently, a constant gain ρ. In fact, assuming that

$$\begin{aligned} &\|\tilde{n}\| \leq \Phi < \infty && \forall q, \dot{q} \\ &\|e\| \leq E_M && \forall q, \dot{q}, q_d, \dot{q}_d \end{aligned}$$

from equation (10) it follows

$$\rho \geq \frac{1}{1-\alpha} \{\alpha Q_M + \alpha \|K\| E_M + B_M \Phi\}$$

A different approach can be adopted assuming Φ and ρ dependent upon the error norm (see also [2]).
Given three positive scalars α_0, α_1 and α_2, one can assume that

$$\|\tilde{n}\| \leq \alpha_0 + \alpha_1 \|e\| + \alpha_2 \|e\|^2 \quad \forall q, \dot{q}, q_d, \dot{q}_d \quad (11)$$

and consequently

$$\rho(\|e\|) = \beta_0 + \beta_1 \|e\| + \beta_2 \|e\|^2 \quad (12)$$

where

$$\beta_0 \geq \frac{\alpha Q_M + \alpha_0 B_M}{1-\alpha}, \quad \beta_1 \geq \frac{\alpha \|K\| + \alpha_1 B_M}{1-\alpha}, \quad \beta_2 \geq \frac{\alpha_2 B_M}{1-\alpha} \quad (13)$$

satisfies the inequality $\rho(\|e\|) \geq \|\eta\|$.

III. STRUCTURING MODEL UNCERTAINTY

A closer analysis of the approaches presented in Section II-C reveals that whatever formulation is adopted to determine a suitable $\rho(\|e\|)$ it always depends on the PD gains K_P and K_D (this drawback was already pointed out in [4]).

This situation is obviously quite unfortunate: increasing the gains of PD controller, in order to speed up the closed loop system and improve performance, has an adverse effect on the robustifying term, whose amplitude increases. However, this is a just a consequence of the way the perturbation to the nominal dynamics of the manipulator has been reproduced in the model. Once the uncertainty η has been expressed as in (4), the inequality (9) follows, expressing a bound over the norm of η. On the right hand side of this inequality the norm of matrix K appears. This means that increasing the controller gains actually increases the bound on the uncertainty, thus calling for a more energetic correction of the robust controller.

This unnecessary interaction between the linear controller for the nominal system and the robust controller can be removed if a more detailed representation of the uncertainty is considered. As a matter of fact the said undesired interaction stems from the fact that different uncertainty terms are dealt with in the same way. A different approach consists in separating the uncertainties that affect the error system (6) into two terms:

- a structural perturbation to the nominal error dynamics – described by matrix A – due to modeling errors in the identification of the inertia matrix $M(q)$;
- an additional uncertainty perturbation that is related to modeling errors in the description of the nonlinear term $n(q, \dot{q})$.

Equation (6) can in fact be rearranged as follows

$$\dot{e} = Ae - B\Delta B^T Ae + B\psi - Bv \quad (14)$$

where

$$\Delta = I - M^{-1}\hat{M} \quad \psi = \Delta \ddot{q}_d - M^{-1}\tilde{n} \quad v = M^{-1}\hat{M}w$$

The term Δ is related to the uncertainty in the estimation of the inertia matrix: if a perfect estimate of this matrix is available, matrix Δ vanishes. All the other terms of the model uncertainty have been gathered in vector ψ. Notice that in this alternative formulation the term ψ does not depend on the controller gains anymore. This has a clear consequence in the design of the outer loop controller, as detailed in next Section.

IV. A DIFFERENT OUTER LOOP DESIGN

Assuming that all the joints share the same error dynamics, i.e.

$$K_P = \omega_n^2 I \qquad K_D = 2\zeta\omega_n I$$

it can be easily concluded that system (14) is actually composed of n independent subsystems with state matrices

$$\mathsf{A} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \qquad \mathsf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Matrices (A, B) of the whole system can be rearranged, through a suitable change of coordinates, as follows

$$\mathscr{A} = \mathrm{bdiag}(\mathsf{A} \dots \mathsf{A}) \quad \mathscr{B} = \left(\mathscr{B}_1^T \dots \mathscr{B}_n^T\right)^T$$

where the symbol bdiag denotes a block diagonal matrix and

$$\mathscr{B}_i = \begin{bmatrix} 0_{2,i-1} & \mathsf{B} & 0_{2,n-i} \end{bmatrix}$$

Let ε be the state vector of the reformulated system with state matrices $(\mathscr{A},\mathscr{B})$, and consider the candidate Lyapunov function

$$V(\varepsilon)=\varepsilon^T P\varepsilon>0 \quad \forall\varepsilon\neq 0$$

Its time derivative, along the trajectories of the system, will be

$$\begin{aligned}\dot V &= \dot\varepsilon^T P\varepsilon+\varepsilon^T P\dot\varepsilon\\ &= \varepsilon^T\left(P\mathscr{A}+\mathscr{A}^T P-\mathscr{A}^T\mathscr{B}\Delta\mathscr{B}^T P-P\mathscr{B}\Delta\mathscr{B}^T\mathscr{A}\right)\varepsilon \\ &\quad +2\varepsilon^T P\mathscr{B}(\psi-v)\end{aligned} \tag{15}$$

Consider, now, a scalar $\delta>0$ and assume that parameters ω_n and ζ are such that

$$\omega_n^2=\delta^2\kappa_{01} \qquad 2\zeta\omega_n=\delta\kappa_{02}$$

with

$$\kappa_{02}>2 \qquad \kappa_{01}>\kappa_{02}-1 \tag{16}$$

Let

$$\mathsf{A}_0=\begin{bmatrix}0 & 1\\ -\kappa_{01} & -\kappa_{02}\end{bmatrix}$$

and P_0 the solution of the algebraic Riccati equation

$$(\mathsf{A}_0+I)^T P_0+P_0(\mathsf{A}_0+I)+\gamma^2 P_0\mathsf{B}\mathsf{B}^T P_0+\mathsf{A}_0^T\mathsf{B}\mathsf{B}^T\mathsf{A}_0=0 \tag{17}$$

The following preliminary results are first given, omitting the proofs.

Lemma 1. Matrix $\mathscr{A}+\delta I$ is Hurwitz $\forall\delta$ provided that parameters κ_{01} and κ_{02} satisfy conditions (16).

Lemma 2. Consider the transfer function

$$T_\delta(s)=-B^T A\left[sI-(A+\delta I)\right]^{-1}B$$

Then

$$\|T_\delta(s)\|_\infty=\|t(s)\|_\infty=\|\mathsf{B}^T\mathsf{A}_0\left[sI-(\mathsf{A}_0+I)\right]^{-1}\mathsf{B}\|_\infty$$

Lemma 3. The pair $(\mathscr{A}+\delta I,\mathscr{C})$, with $\mathscr{C}=\mathscr{B}^T\mathscr{A}$, is observable if and only if $\kappa_{01}\neq 0$.

Lemma 4. The solution of the algebraic Riccati equation

$$(\mathscr{A}+\delta I)^T P+P(\mathscr{A}+\delta I)+\gamma^2 P\mathscr{B}\mathscr{B}^T P+\mathscr{C}^T\mathscr{C}=0 \tag{18}$$

with $\mathscr{C}=\mathscr{B}^T\mathscr{A}$, may be written in the following form

$$P=\mathrm{bdiag}(\Pi,\ldots,\Pi) \qquad \Pi=\delta\mathbf{B}P_0\mathbf{B}$$

where $\mathbf{B}=\mathrm{diag}(\delta,1)$ and P_0 is a solution of the algebraic Riccati equation (17).

Lemma 5. If $\|t(s)\|_\infty\leq\gamma^{-1}$, there exists the positive semidefinite stabilising solution P_0 of the Riccati equation (17) and its eigenvalues are larger than 1, i.e. $\lambda_i(P_0)>1\,\forall i$. Moreover, if $(\mathscr{A}+\delta I,\mathscr{C})$ is observable the solution is positive definite.

With the previous assumptions, in view of Lemma 1, 2 and 3, it follows that matrix $\mathscr{A}+\delta I$ is Hurwitz and the pair $(\mathscr{A}+\delta I,\mathscr{C})$ is observable $\forall\delta$. These conditions are sufficient (see Lemma 5), being $\|T_\delta(s)\|_\infty<\gamma^{-1}$, to claim the existence of a positive definite solution P of the algebraic Riccati equation (18).
As a consequence, expression (15) can be rewritten as

$$\begin{aligned}\dot V=-\varepsilon^T&\left(2\delta P+\gamma^2 P\mathscr{B}\mathscr{B}^T P+\mathscr{A}^T\mathscr{B}\mathscr{B}^T\mathscr{A}\right.\\ &\left.+\mathscr{A}^T\mathscr{B}\Delta^T\mathscr{B}^T P+P\mathscr{B}\Delta\mathscr{B}^T\mathscr{A}\right)\varepsilon+2\varepsilon^T P\mathscr{B}(\psi-v)\end{aligned}$$

Finally, defining

$$\mathscr{L}_1=\varepsilon^T\Gamma\Gamma^T\varepsilon\geq 0,\ \forall\varepsilon \qquad \left(\Gamma=\gamma P\mathscr{B}+\frac{\mathscr{A}^T\mathscr{B}\Delta^T}{\gamma}\right)$$

$$\mathscr{L}_2=\left(\mathscr{B}^T\mathscr{A}\ \varepsilon\right)^T\left(I-\frac{\Delta^T\Delta}{\gamma^2}\right)\left(\mathscr{B}^T\mathscr{A}\ \varepsilon\right)\geq 0,\ \forall\varepsilon$$

being $\|\Delta\|\leq\gamma$ (and therefore $\|\Delta\|^2/\gamma^2<1$), and

$$\mathscr{R}=2\delta\varepsilon^T P\varepsilon \qquad \mathscr{D}=2\varepsilon^T P\mathscr{B}(\psi-v)$$

the time derivative of the Lyapunov function can be written as

$$\dot V=-\mathscr{L}_1-\mathscr{L}_2-\mathscr{R}+\mathscr{D}$$

From Lemma 4 it follows that P is a block diagonal matrix and $\Pi=\delta\mathbf{B}P_0\mathbf{B}$. Thus

$$\mathscr{R}=2\delta\varepsilon^T P\varepsilon=2\delta^2\left(\phi_1^T\phi_1+\cdots+\phi_n^T\phi_n\right)=2\delta^2\|\phi\|^2$$

where $\phi_i=P_0^{1/2}\mathbf{B}\varepsilon_i$ and $\phi=\left(\phi_1^T\ \phi_2^T\ldots\ \phi_n^T\right)^T$.
Finally, the term $\mathscr{D}$ can be analysed as in Section II. Defining a new variable $z=\mathscr{B}^T P\varepsilon$, $\mathscr{D}$ can be rewritten as

$$\begin{aligned}\mathscr{D}&=2z^T(\psi-v)\\ &=2z^T\left(\psi-\rho(\|e\|)M^{-1}\hat M\frac{z}{\|z\|}\right)\end{aligned}$$

Using the Cauchy-Schwartz inequality this term can be bounded as follows

$$\mathscr{D}\leq 2\|z\|\left(\|\psi\|-\rho(\|e\|)\|I-\Delta\|\right)$$

and exploiting relation (11)

$$\rho(\|e\|)=\beta_0+\beta_1\|e\|+\beta_2\|e\|^2 \tag{19}$$

where

$$\beta_0\geq\frac{\alpha Q_M+B_M\alpha_0}{1-\alpha},\quad \beta_1\geq\frac{B_M\alpha_1}{1-\alpha}\quad \beta_2\geq\frac{B_M\alpha_2}{1-\alpha} \tag{20}$$

In conclusion, the proof proposed here is articulated in two steps. Firstly, the global asymptotic stability of the origin of the error system (14) for every admissible uncertain matrix Δ, without the additional uncertainty ψ, is analysed in the context of quadratic stability [9]. Then, to counteract a possible reduction of the stability region, due to the effect of the nonlinear term ψ, an appropriate additional control v is designed.
This approach has an undoubted advantage over the traditional robust inverse dynamics control introduced in Section II. In fact, separating the uncertainty that affects the nominal error dynamics from the one due to errors in modeling the nonlinear term $n(q,\dot q)$, i.e. giving a structure to the uncertainty, results into a bound on the gain $\rho(\|e\|)$ of the addition control w that is independent of the PD gains K_P and K_D.

V. A CASE STUDY

As a case study for the proposed approach to the design of a robust inverse dynamics control law we will consider a two d.o.f. planar manipulator moving in the gravity plain. Suppose that the two links have the same length (1 m), the same mass (50 Kg) and are connected by rotational joints. Moreover, the arms are driven by two motors with the same mass (5 Kg) and moment of inertia (0.01 $Kg \cdot m^2$). For a detailed description of the robot see the example reported in [3].

The inverse dynamics controller has been designed adopting the following parameters:

- a diagonal matrix $\hat{M}$, obtained evaluating the inertia of the nominal model of the manipulator in the position $q = (0, 0)^T$;
- an estimate $\hat{C}(q, \dot{q})$, $\hat{g}(q)$ of the vector of Coriolis and centrifugal terms and of gravitational torques, respectively, derived from a model of a manipulator obtained perturbing the mechanical parameters of the nominal one.

A comparison between two robust inverse dynamics control laws, that make use of different relations to calculate the gain $\rho(\|e\|)$ of the additional control w, will be discussed. The first control law (indicated in the following as *Case A*) is based on the classical method, described in Section II, and $\rho(\|e\|)$ is obtained from relations (12) and (13) (remember that in this case ρ also depends on the PD gains).
The second one (indicated in the following as *Case B*), instead, is based on the method described in Section IV and uses a gain $\rho(\|e\|)$, obtained from relations (19) and (20), that is independent of the PD gains.
Finally, in both the situations the same closed loop nominal error dynamics are forced, using the following PD gains

$$K_P = \text{diag}(25,\ 25) \qquad K_D = \text{diag}(5,\ 5)$$

The manipulator moves on a line following a typical joint trajectory based on trapezoidal velocity profiles, as depicted in Figs. 1 and 2.

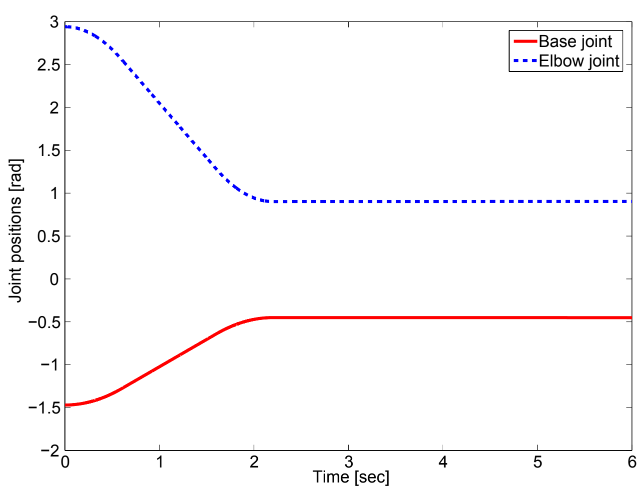

Fig. 1. Joint space trajectory.

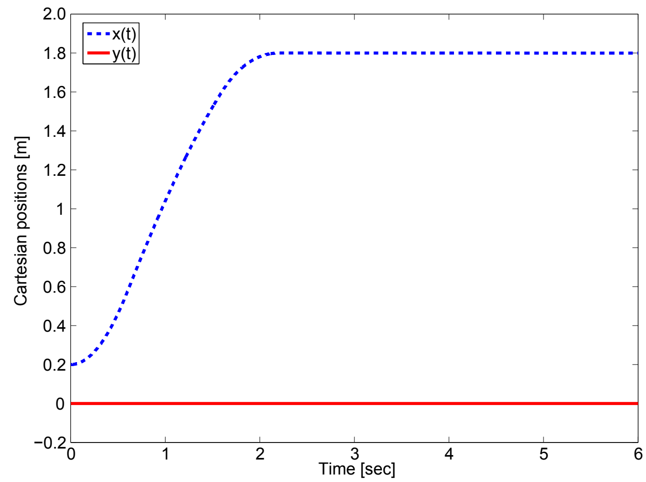

Fig. 2. Cartesian space trajectory.

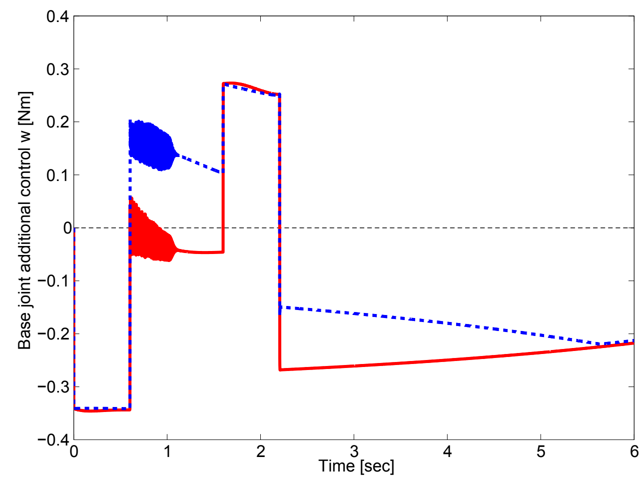

Fig. 3. Additional control w for *Case A* (red solid line) and *Case B* (blue dotted line): base joint.

A comparison between relations (13) and (20) reveals, as one can expect, that the outer loop design here proposed results into a lower value of $\rho(\|e\|)$ and, consequently, the energy related to the additional control w decreases.
This fact is shown in Figs. 3 and 4 (for the base and for the elbow joints, respectively) and in Fig. 5. Furthermore, note that the actual energy of the additional control w (Figs. 3 and 4), i.e. the 2-norm of the signal, in *Case B* is 11 % lower than the one in *Case A*.

Finally, the last conclusion that can be drawn from the outer loop design proposed in Section IV concerns the effect of the PD gains on the bound of $\rho(\|e\|)$. From relations (13) and (20) it is evident that increasing the bandwidth of the nominal error system, β_1 increases in *Case A* whereas it remains constant in *Case B*.
Fig. 6 compares the time history of the gain $\rho(\|e\|)$ for *Case A* and *Case B* when the following PD regulator has been considered

$$K_P = \text{diag}(100,\ 100) \qquad K_D = \text{diag}(10,\ 10)$$

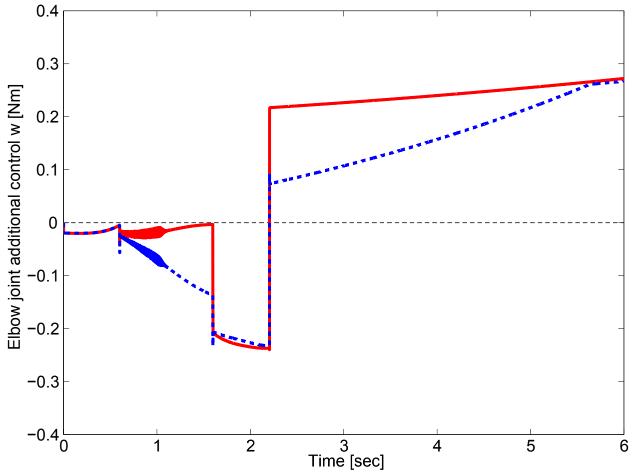

Fig. 4. Additional control w for *Case A* (red solid line) and *Case B* (blue dotted line): elbow joint.

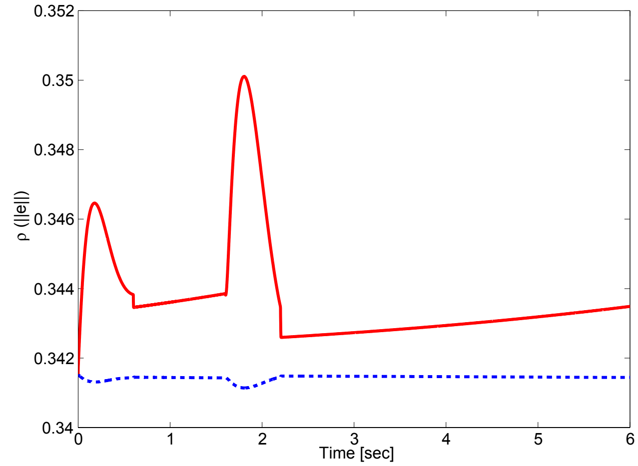

Fig. 5. Gain $\rho(\|e\|)$ for *Case A* (red solid line) and *Case B* (blue dotted line).

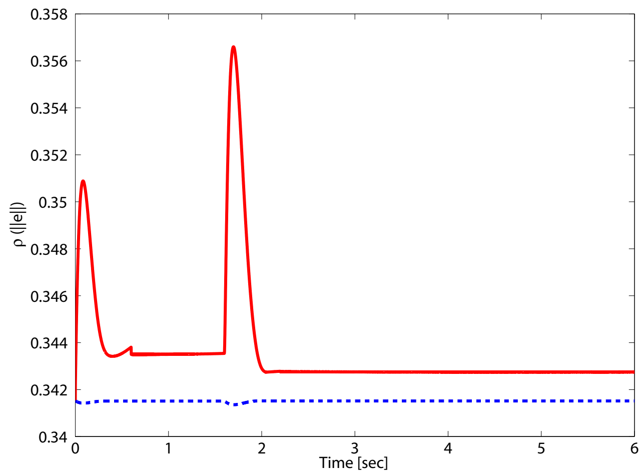

Fig. 6. Gain $\rho(\|e\|)$ for *Case A* (red solid line) and *Case B* (blue dotted line) with a higher bandwidth PD controller.

The figure clearly shows that in *Case B* the value of $\rho(\|e\|)$ remains exactly the same as the one shown in Fig. 5, while in *Case A* it is clearly increased.

VI. CONCLUSIONS

The ongoing interest for model based control laws (including robust control), due to the increase in computing power of control hardware, motivated the revision of the robust control law presented in this paper. Framing the model uncertainty into a more rational structure, naturally leads to a new design of the outer controller, where the undesired interaction among the linear controller gains and the robust control term is removed.

Extension of the proposed approach to more general closed loop mechanical system is currently under study.

REFERENCES

[1] T. Brogårdh, "Present and future robot control development: an industrial perspective," in *Preprints of the 8th IFAC Symposium on Robot Control*, 2006.

[2] W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot modeling and control*. USA: John Wiley & Sons, Inc., 2006.

[3] L. Sciavicco and B. Siciliano, *Modeling and control of robot manipulators*, 2nd ed. London: Springer-Verlag, 2000.

[4] C. Abdallah, D. Dawson, P. Dorato, and M. Jamshidi, "Survey of robust control for rigid robots," *IEEE Control Systems Magazine*, vol. 11, no. 2, pp. 24–30, February 1991.

[5] J.-J. E. Slotine, "Robust control of robot manipulators," *International Journal of Robotics Research*, vol. 4, pp. 49–64, 1987.

[6] M. W. Spong and M. Vidyasagar, "Robust linear compensator design for nonlinear robotic control," *IEEE Journal on Robotics and Automation*, vol. 3, pp. 345–351, 1987.

[7] M. W. Spong, "On the robust control of robot manipulators," *IEEE Transactions on Automatic Control*, vol. 37, pp. 1782–1786, 1992.

[8] P. Rocco, "Stability of pid control for industrial robot arms," *IEEE Transactions on Robotics and Automation*, vol. 12, pp. 606–614, 1996.

[9] J. Doyle, K. Glover, P. Khargonekar, and B. Francis, "State-space solutions to standard h_2 and h_∞ control problems," *IEEE Transactions on Automatic Control*, vol. AC-34, pp. 831–847, 1989.