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Wide Band Suppression of Motion-Induced Vibration *

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***Abstract*— Motion profiles such as *Swing Free* and *Input Shaping* are intended to eliminate residual vibrations. This paper demonstrates a simple generalized method of creating and understanding pulse-based profiles, and presents an approach to suppress a wider band of induced vibration than previously attainable. The current approach of *Input Shaping* ignores the potential contributions the base profile and pulse shape can have on the profile. When examined from a signal processing point of view these pulse-based profiles can be thought of as sampled low pass filters. The sampling of the filter kernel defines the magnitude of the pulses, but as in any digital system it also produces aliasing. An interpolating filter can then be used to both remove the aliasing and to smooth and shape the base profile. This dual purpose of the interpolating filter allows the use of the *Swing Free* step input in conjunction with a rate-limit as a base profile. A generalized methodology is proposed which can create smooth motion profiles with continuous derivatives, while keeping the computational efficient of traditional *Input Shaping*. Experimental validation of two multi-DOF systems is shown.**

I. Introduction

This paper describes a new method to create smooth pulse-based profiles with control of the smoothness and frequency spectrum of the profile. With this method, pulse trains can be generated that act as low-pass filters to suppress a wider range of frequencies than previous pulse profiles. The method is based on both exploiting the frequency spectrum of the base profile and viewing pulse trains as sampled profiles that create aliasing in the frequency domain. The building of profiles from simple components creates profiles that allow position and velocity commands to be modified in real-time.

We will first examine existing pulse-based and low-pass profiles then introduce the rate-limit as an underlying foundation of the new profiles. With the use of Boxcar filters the rate-limit will be smoothed in the time domain to create continuous velocity and acceleration profiles. Pulse-based *Input Shaping* will then be generalized into sampled alias-based profiles. Finally, the simple alias profiles will be expanded to IFIR filters by giving the time domain smoothing of the Boxcar a dual functionality. The Boxcar filters will both smooth in the time domain and act as anti-alias filters in the frequency domain. This new method will create smooth profiles with an improved frequency spectrum while maintaining the computational efficiency of the original *Swing Free* and *Input Shaping* methods.

A. Background of Pulse-Based Profiles

Posicast control was the first use of pulses to eliminated unwanted oscillations in lightly-damped systems [1], [2]. Posicast control was developed as a compensator to be used in conjugation with an existing control system. Posicast control was then expanded to the realm of profile generation by creating profiles that eliminate or reduce the unwanted residual vibrations starting with the *Swing Free* motion work of Starr [3]. Alici, et al. [5] expanded Starr's work by creating a cycloid position profile that also created *Swing Free* motion. In more recent work, Singer and Seering focused on creating profiles that, act like notch filters eliminating a small set of predetermined frequency ranges. Their first paper on *Input Shaping* [6] inspired numerous other papers on the subject including research by Singer, Seering, Singhose, Derezinski, Chuang, Pao, Crain, Porter, Tuttle, and Lau [7]–[15], among others. Whereas Smith solved the problem in the s-domain by canceling poles with zeros, Starr solved the problem from an energy perspective, and Singer and Seering's *Input Shaping* extended Starr's work by approaching the problem from a time-frequency domain perspective. Both approaches created profiles that first excite then cancel out a range of frequencies in the system that is being driven. Starr's original work solved the problem by ramping up the velocity in two steps separated by half the period of the frequency of the system. Singer and Seering approached the problem by convolving a double pulse kernel with a position profile. Their pulses were also separated by half the period of the natural frequency of the system. This is effectively identical to Starr's work, since the derivative or velocity of a convolved double pulse is a two step velocity profile as used by Starr. The double step or double pulse technique in theory will completely eliminate the residual vibrations of a system with one known damped frequency. But if a system has multiple modes, or if the frequencies are not known exactly, the double pulse system degenerates and may actually increase the residual vibration. Singhose, Singer and Seering [11] took *Input Shaping* further by introducing a technique to reduce the sensitivity to frequency

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errors and to allow for multiple modes. Their improved technique implements three to five pulses to increase the sensitivity or width of the effective notch filter. The temporal spacing of the pulses in all the impulse-based *Input Shaping* methods by Starr and Singhose, Singer and Seering is equal to one half the period of the frequency to be eliminated.

II. Analysis of Existing Pulse-Based profiles

Profiles convolved with a series of pulses can create motion that will not result in residual vibration in a driven system (Figure 1). Actually, pulse-based profiles first introduce vibrations into a system, then cancel out the vibrations when the final series of pulses is applied. Figure 2 shows a series of frequency spectra produced by a three-hump extra-insensitive (EI) shaper. The figure shows the resulting spectra as successive pulses in an EI shaper are applied to a system. The first pulse creates a flat spectrum that excites all frequencies. As each additional pulse is added, the width of the notch increases and the magnitude of the spectrum decreases. If this pulse train is convolved with a step in position, a profile is generated that will cancel out all residual vibration over a given frequency range. This is the basic implementation of *Swing Free* motion (from a signal processing view) as first described by Starr (Figure 1).

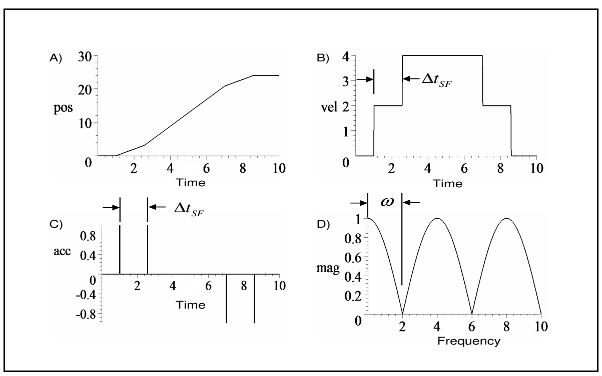

Fig. 1. *Swing Free* motion with $\Delta t_{SF} = \pi/\omega$. Step A) Position profile, B) Velocity profile C) Acceleration profile D) Frequency spectrum with null point at $\omega = 2$ rad/sec.

However, the position, velocity, and acceleration profiles that are created are not smooth. The pulses can be substituted with continuous shapes, but the profile will still have a series of large-amplitude short-duration surges. Increasing the width of the pulse decreases the required acceleration amplitudes, causing the overall length of the kernel to grow.

Input Shaping avoids this problem by convolving the pulse trains with an existing smooth base profile. The base profile acts like a filter which smooths out the pulses, thus avoiding the need to give the pulses width.

III. A New Signal Processing View of Profile Generation

The current approach of *Input Shaping* ignores the potential contributions the base profile and pulses shaping can

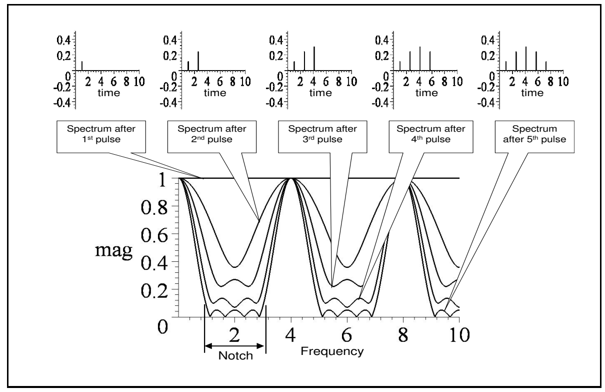

Fig. 2. *Input Shaping* excites a system with its first pulse. As each successive pulse is applied to the system, the frequency spectrum magnitude is reduced and the notch widens.

have on the profile. Traditional *Input Shaping* also ignores how pulse trains in the time domain create aliasing in the frequency domain. If the entire profile generation procedure is viewed as a signal processing problem including the base profile, simple pulse-based profiles can be generated that take advantage and build upon the concepts of *Swing Free*, *Input Shaping*, and aliasing.

A. The Building Blocks of Motion Profiles

Using the convolution operator, complex motion profiles can be built up from simple components. The three basic components used in this paper will be (1) the rate-limit, (2) the Dirac pulse, and (3) the Boxcar function. The underlying foundation of all of the profiles will be a rate-limit. As the foundation of each profile, it will control the transient or slew velocity of the profile. Filtered pulse-based shaping will be added to the rate-limit.

B. The rate-limited Profile

The simplest profile generation algorithm is a rate-limit (Figure 3). If the desired end position, or required traverse distance (P_f), is passed through a rate-limit, a ramp position profile is generated with a corresponding ramped velocity profile (Figure 3). The actual velocity profile is generated by differentiating the position profile ($v = d/dt(P)$). The

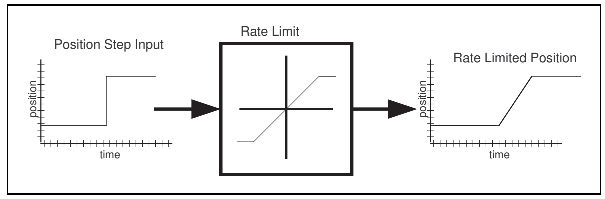

Fig. 3. Simple rate-limit profile.

magnitude of the velocity profile (V_{max}) is set by the rate-limit (V_{rl}). The slew velocity will then be held at the rate-limit value. This is not a very smooth profile (infinite acceleration and jerk). However, this simple profile generator has the advantage of requiring no initial computation, has no special cases and can be easily updated at any time with a new velocity or end position with no recomputing. Simply change the rate-limit (V_{rl}) or set a new end position. The rate-limit also facilitates recomputing the profile if a new end position is entered during motion. Again, simply pass the new end position into the rate-limit.

C. Smoothing the rate-limit with Boxcar Filters

The simple rate-limit of the previous section can create a versatile profile with many advantages over the classic method of blending polynomial segments. Two major concerns with a simple rate-limit are the discontinuous velocity and infinite acceleration. However, the velocity of the position profile can be smoothed by convolving the profile with simple Finite Impulse Response (FIR) filter kernels such as the Boxcar filter [21].

When passed through a boxcar filter, the velocity of the position profile is transformed from a rectangle to trapezoidal profile (Figure 4). Khalsa first developed this technique in 1982 and presented his findings in 1990 [4]. Fanuc Ltd. (patent 5,057,756) [28] and Samsung Electronics Co. Ltd. (patent 6,046,564) [30] used this same technique to smooth velocity profiles.

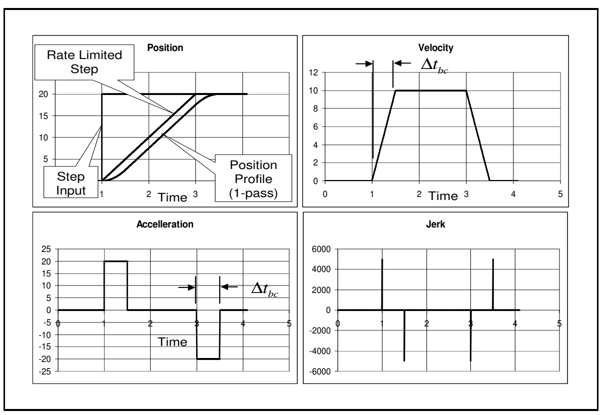

Fig. 4. Rate-limit profile with a boxcar filter ($\Delta t_{bc} = 0.5$ seconds).

If this position profile is convolved with another Boxcar filter, the velocity profile is transformed into a trapezoidal with blended ends (Figure 5). Each time a Boxcar is convolved with the profile, the next derivative with respect to time becomes continuous and the time required to traverse the profile is increased by the length of the Boxcar (Δt_{bc}). The time spent in each of the trapezoidal sections is 2 Δt_{bc} and the time spent at the slew velocity (V_{max}) is reduced by 2 Δt_{bc}. Thus, the total time added to the profile is 2 Δt_{bc}. The general expression for the total time spent in a rate-limited profile convoluted with n_{bc} Boxcars is:

$$t_{pf} = \frac{P_f}{V_{rl}} + n_{bc}\Delta t_{bc} \tag{1}$$

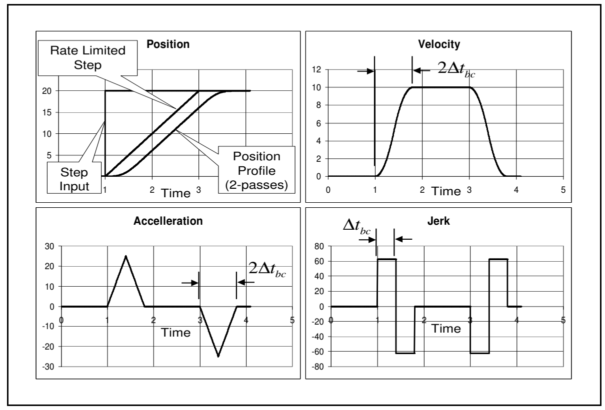

Fig. 5. Two-pass Boxcar filter profile ($\Delta t_{bc} = 0.4$ seconds).

D. The Convolution and Pulse-Based Profiles

In section III-C a smooth profile was created from a rate-limit and Boxcar filters. A pulse-based profile such as the three-hump EI can be convolved with this smooth profile to create a motion trajectory that will cancel out unwanted residual vibration. This is essentially the technique used in traditional *Input Shaping*. Using the associative property of the convolution, the order used to produce profiles with the convolution operator is not important. We can think of *Input Shaping* as a series of pulses convolved with a smoothing filter (Boxcar or any other FIR filter), the result of which is then convolved with a rate-limit foundation profile.

By changing the order in which the profile is generated, we can use the smoothing filter to improve the frequency spectrum of the final profile. Profiles can then be created in three steps. First, create a pulse-based profile that will eliminate residual vibrations at the desired frequencies. Second, add a smoothing filter that both smooths the profile and further improves the spectrum of the profile. Third, combine the pulses, the smoothing filter and the rate-limit to create a final profile.

E. Aliased Motion Profiles

By considering the discrete domain, greater understanding of the previous work by Starr, Singhose, Singer and Seering can be obtained. *Input Shaping* profiles are a series of pulses separated by a constant time T_S. For example, the three-hump EI shaper is a series of five pulses (Figure 6(A)). If the three-hump EI shaper is viewed as five samples of a continuous curve, and it is noted that sampling always causes aliasing in the frequency domain, the peaks in the frequency spectrum of Figure 6(B) can be interpreted as aliasing due to sampling. A quick calculation confirms that the peaks are at the aliasing frequency due to sampling.

$$\Omega_S = n\frac{2\pi}{T_S} \tag{2}$$

Where:

Ω_S = Aliasing frequencies due to sampling

n = Aliasing peaks ($n = 1, 2, 3, \ldots$)
T_S = Sampling period

When $T_S = 1$

$$\Omega_S = 2\pi, 4\pi, 6\pi, \cdots, 2n\pi.$$

The width of the frequency notch of three-hump EI *Input Shaping* can be doubled by simply halving the sampling period of the continuous curve. Since *Input Shaping* was not derived from sampling a continuous curve, an assumption on the shape of the continuous profile must be made. A first approximation of the curve can be made by using a linear interpolation between the original pulses of a three-hump EI *Input Shaping* profile ($T_{S\ new} = T_S/2$). The newly resampled three-hump EI shaper is shown in Figure 6(C) with its frequency spectrum in Figure 6(D). As can be seen, the aliasing peak moves as predicted by equation (2)($\Omega_S = 4\pi, \cdots, 4n\pi$).

The Gaussian curve produces the most compact frequency spectrum for a given width in the time domain [25]. Thus, it makes sense to fit the Gaussian curve to the three-hump EI *Input Shaper* to produce a profile with no aliasing. Figure 6(E) is a truncated Gaussian fitted to the five pulses of a three-hump EI *Input Shaper*. It can be seen that the frequency spectrum of the Gaussian matches that of the EI *Input Shaper*, but with no aliasing. Thus, the continuous curve increases the usable upper limit of the *Input Shaper* to infinity.

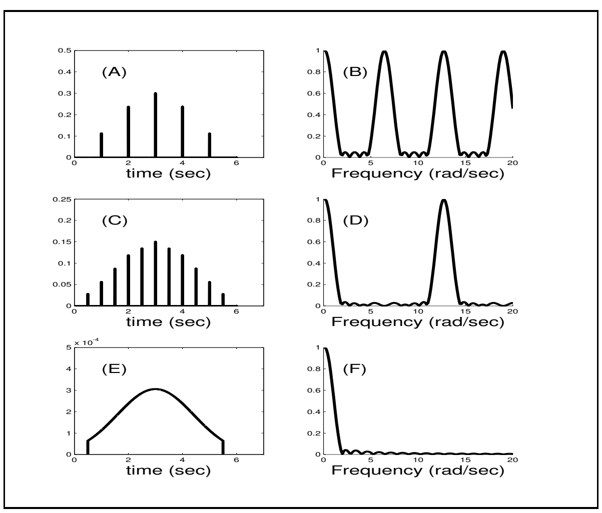

Fig. 6. Sampled view of three-hump EI shaper, (A) three-hump EI shaper, (B) Frequency spectrum of EI shaper, (C) three-hump EI shaper re-sampled with linear interpolation, (D) Frequency spectrum of re-sampled with linear interpolation, (E) Gaussian curve fitted to three-hump EI shaper points, (F) Frequency spectrum of fitted Gaussian curve.

F. Alias-Based Profiles

From Figure 6 it appears that continuous profiles will always outperform pulse-based profiles. Pulse profiles can be replaced with smooth continuous kernels with frequency spectra capable of eliminating residual vibration, or not inducing vibration initially. As stated above, one such kernel is the Gaussian curve.

A truncated continuous Gaussian curve as shown in Figure 6 gives superior results when a wide frequency spectrum notch is required. However when the implementation of a profile is taken into consideration, pulse-based profiles have an advantage. The profile in Figure 6(E) is 5 seconds long. If a digital system runs at 100 Hz then the FIR kernel will have 500 taps (500 terms in its kernel). This requires 500 multiplications and 500 additions to compute the convolution each frame. The genius of the three-hump EI shaper and any pulse-based *Input Shaping* is that even though it is 4 seconds long it can be implemented as only 5 taps requiring only 5 multiplications and 5 additions.

When *Input shaping* profiles are viewed as continuous profiles that have been sampled, the realm of pulse-based profiles can be expanded and generalized. The width of the frequency notch created from a sampled curve is defined by the original unsampled curve and the sampling period ΔT_S. The shape or the width of the original continuous curve defines the left side or low frequency side of the notch. Before a curve such as the Gaussian is sampled it can be thought of as a low-pass filter (Figure 7(A-B)). The sampling of the curve creates the alias right side or high frequency side of the notch (Figure 7(C-D)).

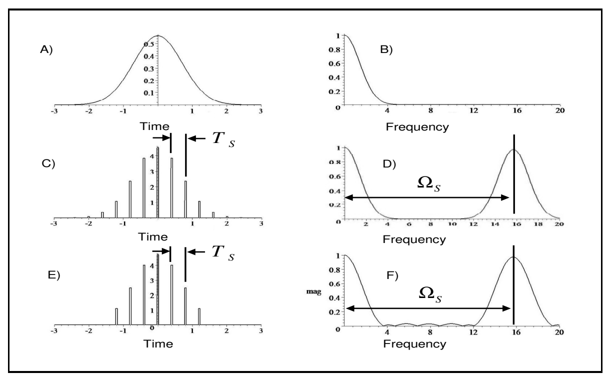

Fig. 7. Effects of sampling of a continuous curve, (A,B) Continuous Gaussian curve and frequency spectrum (C,D) Continuous Gaussian curve with sampling period of $T_S = 0.4$ from $t = -2.8$ to 2.8 with aliasing at $\omega = 2\pi/T_S = 5\pi$ (E,F) Continuous Gaussian curve with sampling period of $T_S = 0.4$ from $t = -1.2$ to 1.2 with aliasing at $\omega = 2\pi/T_S = 5\pi$.

Creating an alias-based profile is done in two steps. First, design a low-pass filter to form the left side of the notch ω_L (Figure 8). The design of the low-pass filter can be done with any technique. But since only pure DC is required to be passed, the use of a windowing filter is all that is required. Second, determine the right side or width of the notch by the placement of the aliasing peak (Ω_S). The sampling period T_S as defined by equation (2) (with $n = 1$) controls the location of the peak in the frequency domain. The sampling period is a simple function of ω_H and ω_L defining the notch frequencies (Figure 8). The duration of the low-pass filter can have any length, but it is best to have the duration of low-pass filter be an integer multiple of T_S.

$$T_S = \frac{2\pi}{\omega_H + \omega_L} \quad (3)$$

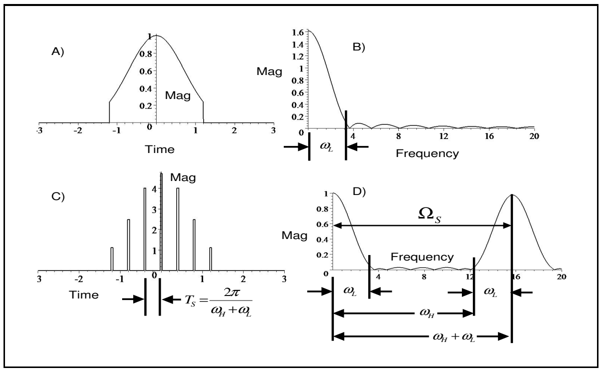

Fig. 8. Designing a pulse-based profile in the frequency domain. First, design a low-pass filter to form the left side of the notch ω_L. Second, the right side or width of the notch is controlled by the sample size $T_S = 2\pi/(\omega_H - \omega_L)$.

As stated previously, an advantage of the pulse-based profile is the reduction in the number of multiplies and additions required to convolve a kernel in real time. As discussed in section III-F, a profile that requires hundreds of taps in a FIR filter can be reduced by orders of magnitude with aliased pulse profiles. But the existence of pulses needs to be addressed. In hardware, pulses can only be represented as Boxcars. Even though a Boxcar FIR has many taps, it can be programed in software as one tap requiring only a single multiplication, an addition, and a subtraction. This reduction in mathematical operations occurs because the magnitudes of all the taps in a Boxcar FIR are the same. Thus, a Boxcar kernel can be considered a single tap FIR filter.

G. *Pulse-Based Profiles Converted to IFIR Filters*

Aliased pulse-based profiles can be taken one step further by taking advantage of converting the pulses into Boxcars. If the width of the Boxcar pulses Δt_{bc} are made equal to the alias sampling period T_S, the first null point of the Boxcar will align with the first alias peak of the pulse-base profile (Figure 9). This eliminates almost all of the alias peaks caused by sampling.

Replacing pulses with Boxcars (convolving the pulse profile with a Boxcar) effectively turns a pulse profile into an IFIR (Interpolated FIR) filter. The Boxcar filter acts like an interpolation function between the pulses; attempting to recreate the original unsampled profile. Any interpolation function can be used. But the use of simple functions will keep the computations required to a minimum. If two Boxcar functions (of equal duration) are cascaded together a triangle filter is formed. If the width of the triangle filter is made twice the width of the sampling period, the triangle filter will perform linear interpolation between the pulses of a pulse-based profile. The resulting profile and frequency spectrum starts to resemble the original curve. Figure 10

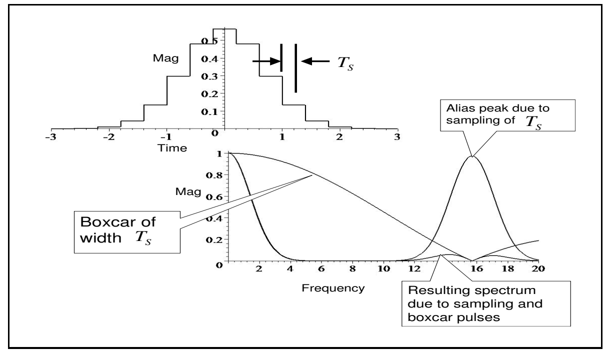

Fig. 9. Effect of aligning null point of Boxcar function with aliasing peak.

shows a truncated Gaussian reconstructed with a triangle interpolation filter. The interpolating function acts like a low-pass filter that removes aliasing (anti-aliasing filter). As with replacing pulses with Boxcars, the overall length of the resulting kernel increases when triangles are used. A Boxcar interpolator with a width equal to the sampling period T_S increases the length of the overall kernel by T_s. The use of a triangle interpolator increases the length of the kernel by $2T_S$

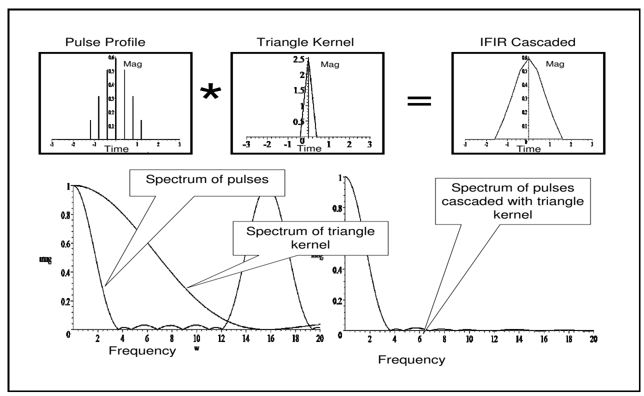

Fig. 10. Turning pulse-based profiles into a linear segment continuous profile using an IFIR interpolating triangle filter.

H. *Putting it All Together to Create a Rate-Limited Boxcar Aliased IFIR (RBAI) Profile*

In section III-F an expression for the sampling time to create an aliasing low-pass filter was shown (3). It was shown in section III-G that for a Boxcar generated interpolation filter to remove the aliasing peaks its width must be equal to the sampling period T_S. In section III-C an expression was shown for the length of a rate-limited smoothed profile. When combined, we get a expression for the total time required to traverse a pulse generated profile from the starting position to the end position.

$$T_{IFIR} = \frac{P_f}{V_{rl}} + n_{pulse}\left(\frac{2\pi}{\omega_H + \omega_L}\right) + n_{bc}\left(\frac{2\pi}{\omega_H + \omega_L}\right) \quad (4)$$

The first term in equation 4 is the duration of a rate-limited profile. The second term is the duration of the aliasing pulses. The third term is the duration of the smoothing interpolating filter. The first term will usually be a constant for a given trajectory with a required slew velocity. Adjusting the number of pulses (n_{pulse}) and high frequency cutoff (ω_H) controls the number and spacing of the side lobes (Figure 7). The number of Boxcar filters (n_{bc}) controls the smoothness of the final curve. The equivalent number of filter taps will be the sum of $n_{pulse} + n_{bc}$.

IV. Experimental Validation

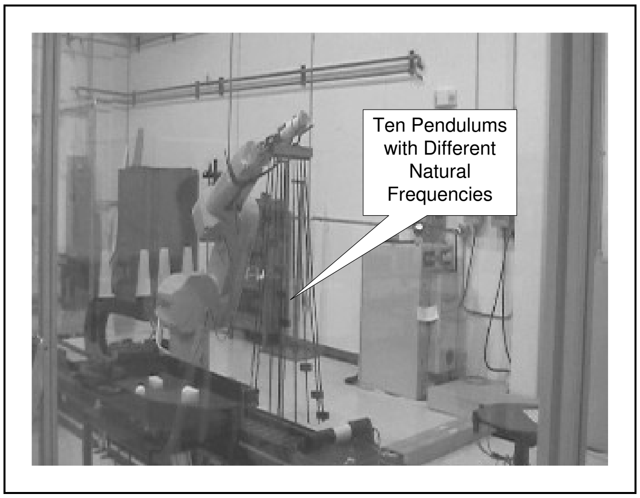

Fig. 11. Multiple pendulums of different lengths (the comb) being driven at the University of New Mexico by the RTU in the MTTC robotics lab.

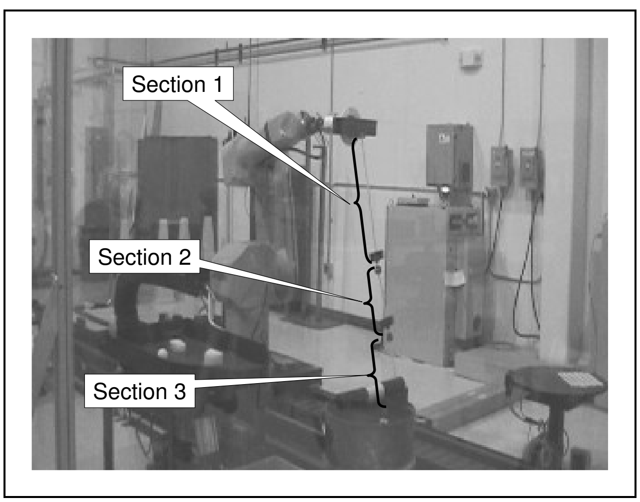

Fig. 12. Triple pendulum being driven at the University of New Mexico by the RTU in the MTTC robotics lab.

RBAI profiles were tested on several multiple-mode systems: a multiple pendulum device (Figure 11) and a triple pendulum (Figure 12). The profiles used to drive the systems were designed to have a low frequency cutoff below the lowest mode in the driven system. A Gaussian window was used for the low frequency aliasing filter with 15 pulses. The IFIR interpolation filter was designed as three passes of a Boxcar function to have an infinite upper frequency limit creating a low-pass filter profile.

Computer models of the pendula were first written to test the feasibility of RBAI filters in MATLAB and C++. Figures 13 and 14 show the resulting numerical simulation of the pendula when driven with the profiles. It can be seen that even though only an equivalent of 18 taps were used to create the kernels their shape is nearly identical to a Gaussian. The only noticeable difference is the RBAI generated profiles have blended ends. Because three Boxcars where used to create the kernels, they have continuous position, velocity and acceleration at the truncation points. Once convolved with the base rate-limit profiles, the final profiles have continuous position, velocity, acceleration, jerk, and jerk derivative.

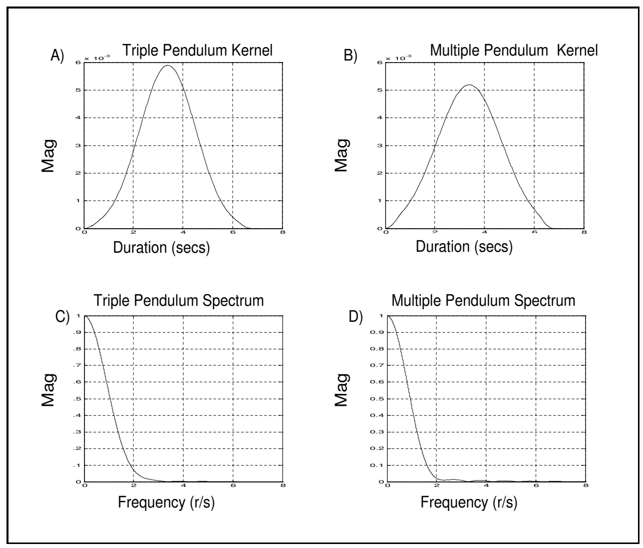

Fig. 13. (A-B) RBAI generated kernel ($n_{pulses} = 15$, $n_{bc} = 3$, $\Delta T_s = 0.4$) (C-D) Corresponding frequency spectrum of profiles.

Next the actual test articles were driven by the same profiles to validate the profiles and the models. When driven with the RBAI profiles, the systems showed no transient and no noticeable residual vibration. The only noticeable motion in the systems was a slight start up lag in the pendula. The lag quickly disappeared with no oscillation as the systems reached the slew velocity and when the systems came to rest.

These experiments show that using pulse-based profiles with an interpolation IFIR filter can create profiles that mimic Gaussian profiles and result in wider vibration suppression bandwidth than previous methods.

V. Conclusion

This paper demonstrates a simpler generalized view of the creation of wide-band pulse-based profiles based on digital signal processing windowing techniques. The method turns

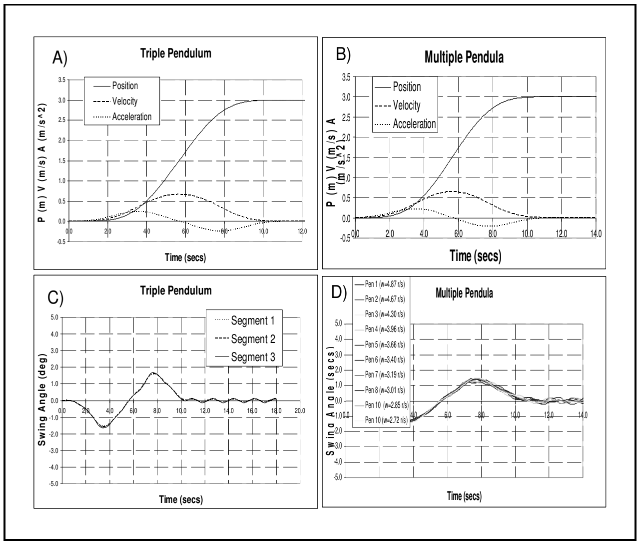

Fig. 14. RBAI generated profiles ($\Delta P = 3$m, $n_{pulses} = 15$, $n_{bc} = 3$, $\Delta T_s = 0.4$) for the triple and multiple pendulum experimental validation (A-B) Resulting position, velocity, and acceleration profiles (C-D) Numerical simulation of pendula.

the windowing functions into an IFIR filter using Boxcar functions for the interpolation. The IFIR filter smooths the pulse-based profiles, allowing the resultant filter to be applied directly over a simple rate-limited base profile. These Rate-limited Boxcar Aliased IFIR (RBAI) filters create profiles that have the same computational efficiency of *Input Shaping* profiles such as the three-hump EI Shaper but with capability to create profiles that act as low-pass filters. The use of RBAI profiles allow for the width of the notch and the magnitude of the residual vibration in the frequency spectrum to be adjusted separately with no limitations on the number of pulses. Because the frequency spectrum and residual vibration can be controlled separately, the RBAI method produce profiles that are superior to the current *Input Shaping* techniques resulting in wider vibration suppression bandwidth than previous methods.

As implemented RBAI profiles allow for real-time update of end position and velocity. This is achieved by creating profiles solely from combinations of simple FIR filters.

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