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Data Association in Bearing-Only SLAM using a Cost Function-based Approach

N. M. Kwok, Q. P. Ha and G. Fang

***Abstract*— When using an extended Kalman filter (EKF) in simultaneous localization and mapping (SLAM) for a mobile robot with bearing-only measurements, it is crucial to correctly assign correspondences between measurements and registered features in the map, otherwise the filter diverges or becomes inconsistent. Conventional methods based on the Mahalanobis distance metric may produce data association ambiguities. Its reliability may further be degraded in bearing-only SLAM due to the limited amount of information delivered from the sensor. The data association process is cast here as that of making a decision based on the sensor measurement as whether to update the EKF or not. For this, cost functions are applied taking into account the interferences from other features. The proposed approach enhances robustness of the data association and consequently assures the performance of bearing-only SLAM. Results from simulations and experiments are included to demonstrate the effectiveness of the method in a typical indoor scenario.**

I. Introduction

The simultaneous localization and map building (SLAM) problem is concerned with deploying a mobile robot in an unknown workplace and the robot is required to estimate its location as well as the feature locations within its operating environment [1] such that the assigned task can be accomplished, e.g., navigation, surveillance. The SLAM problem has been conventionally treated as a state estimation process where the Bayesian estimation approach is widely adopted. The extended Kalman filter (EKF), in particular, is the most popular candidate employed because of its efficiency and effectiveness. However, an implementation hurdle arises in practice where it is important to correctly obtain correspondences between sensor measurements and features. If the correspondence fails, the EKF can result in divergence or inconsistent updates. This difficulty then gives rise to the data association problem addressed in this paper.

The problem of data association was considered as decisions with uncertain observations in [2]. It was also named as probabilistic data association (PDA) [3], where estimator updates were conducted by an aggregation of all possible associations. The idea of multiple hypothesis for decision making, was adopted in [4] for signal detection and estimation. Other problems concerning multi-target tracking were reported in [5] and tackled with the multiple hypotheses tracking method. In the approach proposed therein, hypotheses are generated for each potential measurement-feature pairing, however, management strategies are needed to remove the inherent computation complexity due to employing multiple hypotheses. On the basis of PDA, data associations were considered by the use of the joint probability [6] and the joint compatibility [7] techniques. These methods employ screening and searching to obtain proper matches between measurements and features. Other attempts to solve the data association problem included simultaneous measurements [8] and adaptive time filter updates [9]. In the context of target tracking, a comparison of approaches was conducted in [10]. A tutorial on sensor managements in close connection with data association was presented in [11] and a review can also be found in [12].

The connections between data association and mobile robot localization and map building were established in [13] where the Mahalanobis distance (MD) gating method was used. In addition, the PDA approach was adopted in [14]. These works used the EKF as the state estimator, and the incorporation of MD gating have ever since become a convention. An alternative filtering approach, the particle filter, was used in [15] where data association was performed on a sample based domain with increased computation complexity. Other variations include the integer-programming optimization method adopted in [16] to search for matches between measurements and features after screened by a MD based validation gate. Moreover, a forgetting-factor was applied to remove features in the map in order to reduce the occurrence of possible mismatches [17].

The approaches reported in the literature often include a preliminary gating operation upon which sophisticated techniques are then built. Furthermore, the gating is mostly conducted with the MD or a χ^2-test against a pre-determined and fixed confidence threshold where there is no common guidelines to select its value. In addition, these tests do not consider the interferences from other features in the association process. That is, the MD gating concerns only the *power* [18] of a statistical test but the *significance* is not considered by the design of this association method.

In this paper, the data association problem in SLAM using bearing-only measurements is addressed with the application of cost function-based decisions. The contributions of the paper include i) the analysis on the difficulties of conventional MD gating to distinguish among association ambiguities, and ii) the proposal for a cost function-based data association approach which explicitly takes into account the effect of interference from other features.

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N. M. Kwok* and Q. P. Ha are with the Centre of Excellence for Autonomous Systems, Faculty of Engineering, University of Technology, Sydney, Broadway, NSW, 2007, Australia. G. Fang is with the School of Engineering, University of Western Sydney, Penrith, NSW, 1797, Australia. *ngai.kwok@eng.uts.edu.au

The rest of the paper is organized as follows. In Section II, the bearing-only SLAM problem is briefly reviewed. Difficulties encountered with data association are discussed in Section III. In Section IV, the developments in applying cost functions in the data association problem are presented. Results from simulations and experiments are given in Section V and a conclusion is drawn in Section VI.

II. Bearing-Only SLAM

In the EKF stochastic mapping framework [1], bearing-only SLAM is considered as an estimation process on the system states under the constraint of noise corrupted bearing measurements. The estimation process, which contains the system and measurement models, is briefly described below.

A. System Model

The system state is consisted of the robot pose and feature or landmark locations

$$\hat{\mathbf{x}}_k = [\hat{\mathbf{x}}_{v,k}^T, \hat{\mathbf{x}}_{f_i,k}^T]^T, \tag{1}$$

where $\hat{\mathbf{x}}_{v,k} = [x_v, y_v, \phi_v]^T$ is an estimate of the robot pose (location x_v, y_v and orientation ϕ_v) with reference to a world coordinate, $\hat{\mathbf{x}}_{f_i,k} = [x_{f_i}, y_{f_i}]^T$ is the estimated static feature location. k is the time index, subscript i is the feature index and superscript T denotes vector or matrix transpose.

B. Bearing-only Measurement Model

The bearing-only SLAM is characterized by the fact that the measurement only contains the bearing from the robot to the i^{th} feature. The measurement model is

$$\mathbf{z}_i = \theta_i = \arctan\left(\frac{y_{f_i} - y_v}{x_{f_i} - x_v}\right) - \phi_v + n_\theta, \tag{2}$$

where the noise n_θ is assumed as a zero mean Gaussian given by $n_\theta \sim \mathbb{N}(0, \mathbf{R})$ with $\mathbf{R} = \sigma_\theta^2$ as the noise variance. Note here that, features may have similar bearing measurements aligning along a line from the robot to the features but they are spatially separated.

C. Estimation Process

Assume that features have been observed and incorporated into the filter through an initialization procedure, see the procedures described in [1], the EKF proceeds through the following steps recursively.

Prediction: The state is predicted using a process model $\mathbf{f}(\cdot)$ with motion control $\mathbf{u}_k$, giving the state estimate $\hat{\mathbf{x}}$ and error covariance $\mathbf{P}$ at time $k+1$ as

$$\begin{aligned} \hat{\mathbf{x}}_{k+1|k} &= \mathbf{f}(\hat{\mathbf{x}}_k, \mathbf{u}_k) \\ \mathbf{P}_{k+1|k} &= \nabla\mathbf{f}_x \mathbf{P}_{k|k} \nabla\mathbf{f}_x^T + \nabla\mathbf{f}_u \Sigma \mathbf{f}_u^T, \end{aligned} \tag{3}$$

where $\nabla\mathbf{f}_x$ and $\nabla\mathbf{f}_u$ are the Jacobians evaluated at the state estimates and control, Σ is the covariance matrix for control uncertainty which also includes the model uncertainty.

Measure: The sensor provides a measurement $\mathbf{z}_k$ (in bearing) from observing features and an innovation ν is calculated together with its covariance $\mathbf{S}$ and the Kalman gain $\mathbf{K}$, that is

$$\begin{aligned} \nu &= \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_k) \\ \mathbf{S} &= \nabla\mathbf{h}\mathbf{P}_{k+1|k}\nabla\mathbf{h}^T + \mathbf{R} \\ \mathbf{K} &= \mathbf{P}_{k+1|k}\nabla\mathbf{h}^T\mathbf{S}^{-1}, \end{aligned} \tag{4}$$

where $\mathbf{h}(\cdot)$ is the measurement model and $\nabla\mathbf{h}$ is the Jacobian, the sensor noise covariance is $\mathbf{R}$.

Update: The predicted state and the error covariance are updated by

$$\begin{aligned} \hat{\mathbf{x}}_{k+1|k+1} &= \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}\nu \\ \mathbf{P}_{k+1|k+1} &= \mathbf{P}_{k+1|k} - \mathbf{K}\mathbf{S}\mathbf{K}^T. \end{aligned} \tag{5}$$

The EKF procedure then repeats from the prediction stage until the termination of the estimation process. However, it is observed that in order to proceed with the recursive estimation, one needs to calculate the innovation in (4) which requires a proper data association between the measurement and the feature. On the other hand, an erroneous association will cause the EKF to diverge or become inconsistent. Before presenting the proposed method, the difficulties encountered in the conventional data association approach are discussed in the next section.

III. Conventional Data Association

The basic idea of the conventional data association in SLAM is to define a metric, the squared innovation, as a test statistic to decide on an association between a measurement and a feature. In the following, the MD gating approach is presented, its limitations in the bearing-only SLAM context revealed and then the motivation for this work is stated.

A. Mahalanobis Distance Gating

Notice that the innovation is a realization of a random variable, assuming the sensor and estimation uncertainties are characterized by Gaussian distributions, the squared innovation metric then follows a χ^2-distribution. In the conventional data association, one usually uses the MD with the innovation calculated from the physical (real) measurement and an expected measurement derived from the estimated locations of the robot and feature. A pre-determined confidence gating threshold γ is defined. A match between a measurement and a feature is declared if the measurement falls within the gate. The normalized-squared innovation and the threshold are related by

$$d^2 = \nu^T \mathbf{S}^{-1} \nu < \gamma, \tag{6}$$

where the threshold can be determined from statistical tables with the corresponding confidence and a degree-of-freedom (dof) equal to the dimension of the measurement (dof=1 for bearing-only measurement, hence the threshold is obtained from $\chi^2_{0.05,1}$ for a $\alpha = 0.95$ confidence level).

By referring to the decision making theory [18], the declaration for an association by the MD gating would incur a *missed-detection* error, $1 - \alpha$, about the decision provided that the association is true. On the other hand, a decision will be subjected to a *false-alarm* error of β if the association is false but declared as true. Evidently, the false-alarm could be caused by other measurements and features and the MD gating is not capable of considering this source of error. Furthermore, there are other problems found in bearing-only data associations discussed next.

B. Feature Uncertainties in Bearing-only SLAM

Let there be two features registered and suppose the measurement is originated from feature 1, see Fig. 1(a), then the Jacobian and predicted error covariance are given by

$$\nabla \mathbf{h} = [\mathbf{h}_v, \mathbf{h}_{f_1}, \mathbf{0}]$$
$$\bar{\mathbf{P}} = \begin{bmatrix} \mathbf{P}_{vv} & \mathbf{P}_{vf_1} & \mathbf{P}_{vf_2} \\ \mathbf{P}_{vf_1}^T & \mathbf{P}_{f_1f_1} & \mathbf{P}_{f_1f_2} \\ \mathbf{P}_{vf_2}^T & \mathbf{P}_{f_1f_2}^T & \mathbf{P}_{f_2f_2} \end{bmatrix}. \tag{7}$$

The innovation covariance in (4) becomes

$$S = \mathbf{h}_v \mathbf{P}_{vv} \mathbf{h}_v^T + 2\mathbf{h}_{f_1} \mathbf{P}_{vf_1}^T \mathbf{h}_v^T + \mathbf{h}_{f_1} \mathbf{P}_{f_1f_1} \mathbf{h}_{f_1}^T + R, \tag{8}$$

which becomes a scalar in the case of a single bearing measurement. The first term $\mathbf{h}_v \mathbf{P}_{vv} \mathbf{h}_v^T$ is a function of the robot uncertainty only, the second term $2\mathbf{h}_{f_1} \mathbf{P}_{vf1}^T \mathbf{h}_v^T$ is related to the robot-feature correlation. The third term $\mathbf{h}_{f_1} \mathbf{P}_{f_1f_1} \mathbf{h}_{f_1}^T$ is determined by the feature uncertainty and critically affects the innovation variance S. The last term is the sensor uncertainty R which is independent of the features.

The third term can be further expanded as

$$\mathbf{h}_{f_1} \mathbf{P}_{f_1f_1} \mathbf{h}_{f_1}^T \propto \sin^2 \varphi_1 P_{x_1x_1} + \cos^2 \varphi_1 P_{y_1y_1}, \tag{9}$$

where $P_{x_1x_1}$ and $P_{y_1y_1}$ are the diagonal elements of the $\mathbf{P}_{f_1f_1}$ matrix and φ_1 is the angle from the robot to the feature. This angle determines the magnitude of the feature uncertainty when it is projected towards the robot.

Consider a measurement returns from the sensor such that it gives equivalent innovations for two features, $\nu_1^2 \simeq \nu_2^2$ but $S_2 > S_1$ gives $d_2^2 < d_1^2$. According to the MD gating, feature 2 will be associated to the measurement although the estimated feature may be spatially separated from its true location. From another point of view, since $S_2 > S_1$, then large innovations may lead to associations of equal confidences with features of higher uncertainties. However, this kind of association is not desirable due to the uncertain *a priori* caused by the relative orientation of the feature uncertainty to the robot.

C. Motivation

A drawback of the above conventional MD based data association approach is that the test is a one-to-one matching process without taking the effect of other features into consideration. It should be emphasized here that the squared error metric used in the MD test is one of the rationales in quantifying the magnitude of the innovation, any other metric may also serve the purpose of validating the match of a measurement to an estimated feature, e.g., cost functions. In this research, it is attempted to derive an enhanced validation process such that the association ambiguities can be reduced. In the next section, a procedure based on a cost function approach will be proposed.

IV. Cost Function-Based Association

Without loss of generality in the following development, assume again that there are two features already registered in the estimator. Based on the proposed cost function, example association cases are considered. They include measurements falling on the two sides of the features and cases for equal innovations and MDs.

A. Proposed Approach

Recall that in the stochastic mapping framework, the EKF maintains a state estimate $\hat{\mathbf{x}}$ and the corresponding error covariance $\mathbf{P}$ (the time index k is dropped here). Each feature in the Cartesian coordinate is projected onto the bearing measurement space, giving

$$\hat{\theta}_{f_i} = \arctan\left(\frac{\hat{y}_{f_i} - \hat{y}_v}{\hat{x}_{f_i} - \hat{x}_v}\right) - \hat{\phi}_v. \tag{10}$$

The i^{th} feature maintains an uncertainty attributed from the robot and its own, the variance is given by

$$S_i = \nabla \mathbf{h}_{f_i} \bar{\mathbf{P}} \nabla \mathbf{h}_{f_i}^T + \sigma_\theta^2, \tag{11}$$

where $\nabla \mathbf{h}_{f_i}$ is the Jacobian for feature i, $\bar{\mathbf{P}}$ is the prediction uncertainty. Note that for a bearing-only measurement $z = \theta$, the uncertainty in the measurement domain is given by

$$p_i(\theta) = \frac{1}{\sqrt{2\pi S_i}} \exp(\frac{-(\theta - \hat{\theta}_{f_i})^2}{2S_i^2}). \tag{12}$$

Furthermore, the cost in associating feature i to a measurement can be defined as

$$\bar{\mathcal{C}} = \sum_{i=1}^{2} \mathcal{C}_i = \sum_{i=1}^{2} \int_{\theta}^{\hat{\theta}_{f_i}} p_i(\zeta) d\zeta, \tag{13}$$

where ζ is an integration variable. A table of size $\mathcal{I} \times \mathcal{J}$, where $i \in [1, \mathcal{I}]$ and $j \in [1, \mathcal{J}]$ ($\mathcal{I}$ is the number of features, $\mathcal{J}$ is the number of measurements), is then setup with the cost functions as its entries of all possible matches between features and measurements. Finally, an association is declared for the minimal among costs $\mathcal{C}_{ij}$.

B. Examples

Example scenarios and illustrations of the association process are depicted in Fig. 1 through Fig. 4 for the following cases with the costs shown in shaded areas. In these examples, the robot is estimated to locate at $x_v = 0, y_v = 0, \phi_v = 0.2rad$ as shown in Fig. 1(a). The features are estimated at $x_{f_1} = 3m,\ y_{f_1} = 2m$ and $x_{f_2} = 3m,\ y_{f_2} = 3m$. The expected bearings are $\theta_1 = 0.3880rad,\ \theta_2 = 0.5854rad$ while the innovation variances are $S_1 = 0.0349^2,\ S_2 = 0.1653^2$, respectively, depending on the orientations towards the robot. Note that $\chi^2_{0.05,1} = 3.8415$ is the threshold used in the conventional MD test.

1) Measurement on Left of Features: Figure 1(a) shows the example setting. When it is attempted to associate the measurement to f_1, the cost is 0.0094 (Fig. 1(b)) while the cost for associating f_2 is 0.0012 (Fig. 1(c)). Hence, the measurement is associated to feature 2 and is confirmed by the MD of $0.1263 < \chi^2_{0.05,1}$.

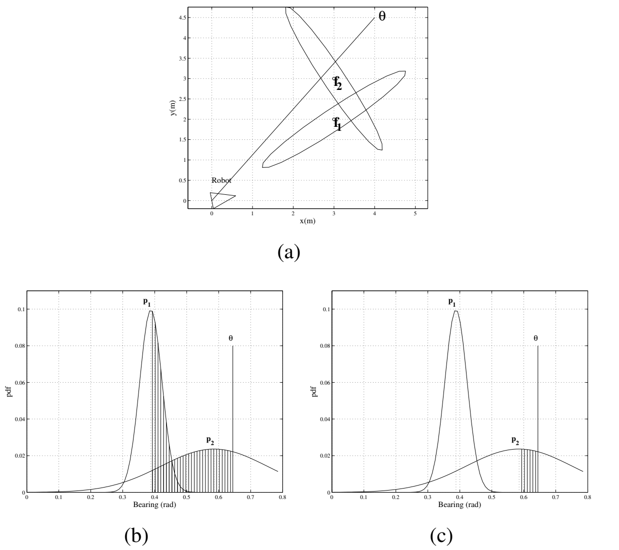


Fig. 1. Measurement on left hand side of features; (a) robot-feature locations; (b) association to feature f_1; (c) association to f_2.

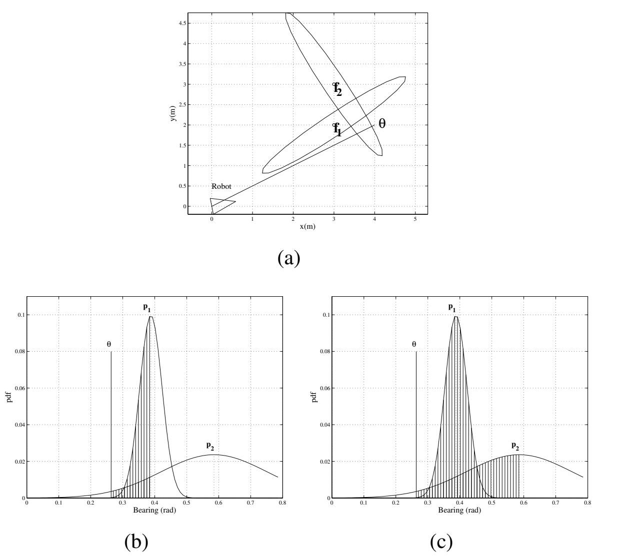


Fig. 2. Measurement on right hand side of features; (a) robot-feature locations; (b) association to feature f_1; (c) association to f_2.

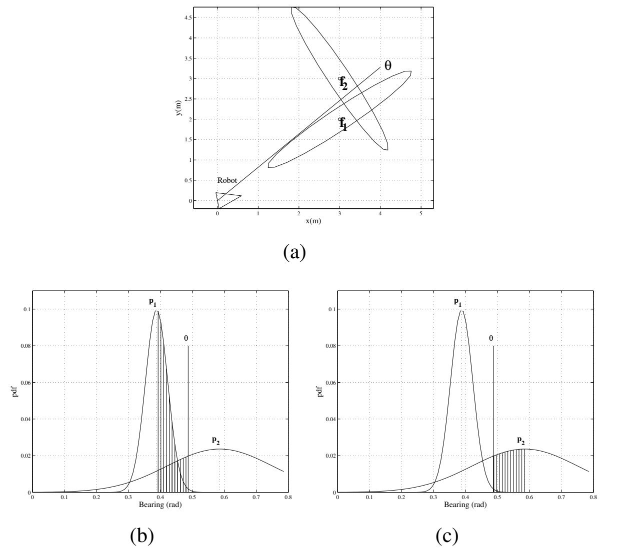


Fig. 3. Measurement giving equal innovations; (a) robot-feature locations; (b) association to feature f_1; (c) association to f_2.

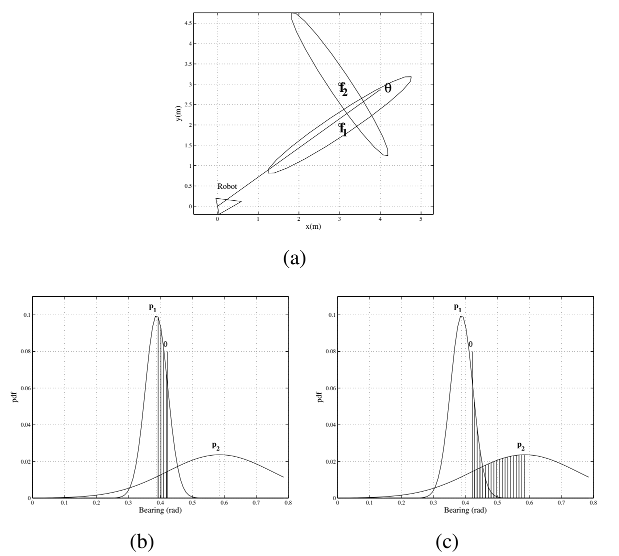


Fig. 4. Measurement giving equal Mahalanobis distance; (a) robot-feature locations; (b) association to feature f_1; (c) association to f_2.

2) Measurement on Right of Features: The scenario is shown in Fig. 2. The costs are 0.0053 and 0.0134 for associations to f_1 and f_2 respectively. However, in this case, the MDs are 12.6915 and $3.7871 < \chi^2_{0.051}$ with f_2 associated by the conventional approach. However, this contradicts with the cost functions and institutively the measurement is away from f_2. Therefore, the measurement is associated to f_1, instead of f_2, by the cost function approach and alleviates the error caused by the MD test.

3) Measurement giving Equal Innovations: The results are depicted in Fig. 3. The innovations are equal, i.e., $v_1 = v_2 = 0.0987$ where the costs are 0.0058 and 0.0024 suggesting an association to f_2. Observe that the cost contributed from f_2 when attempting to associate f_1 is higher than associating f_2. Therefore, f_2 is associated with a lower cost. This is confirmed by the MDs of $0.3561 < \chi^2_{0.05,1}$.

4) Measurement giving Equal Mahalanobis Distances: In this case, Fig. 4, the MDs are equal to 0.0718. Due to the equality, conventional MD test is not able to make any decision and the measurement will be discarded in practice. Here, the cost functions are 0.0034 and 0.0047 for features 1 and 2 respectively. According to the proposed approach, f_1 is associated and is endorsed by the small innovation (closeness) of the measurement to f_1.

C. Discussion

It has been illustrated that the cost function-based data association is able to mitigate ambiguities caused by the MD gating in case 2 where the measurement is located away from the un-associated feature. In addition, the proposed method is also capable of distinguishing association ambiguities among equal MDs, as illustrated in case 4, instead of discarding the measurement which is commonly found in practices using the conventional MD gating. With the enhanced robustness in data association, the quality of bearing-only SLAM is anticipated to be ensured. Results are included in the next section to verify this method.

V. Results

Simulations and experiments are conducted to verify the effectiveness of the proposed approach. For the bearing-only SLAM problem considered, a Gaussian sum filter consists of a bank of EKFs is employed here as the estimator [19]. In the experiment, a black-and-white camera is used as the bearing-only sensor. A laser scanner is also used in a range-bearing SLAM as a performance reference.

A. Simulations

In the simulations, the test scenario is an emulated indoor environment consisting of a square region scattered with eight stationary features. Features are place intentionally at locations such that groups of two features (located at the north, west and south regions) will provide measurements in close proximity and cause ambiguities in conventional MD tests. Performances are assessed on the basis of SLAM quality on estimation errors.

1) Test 1: Conventional Data Association: The trajectory of the estimated robot is plotted in Fig. 5(a) with the true trajectory superimposed (dotted line). The feature location estimates are also illustrated with the 3σ uncertainty ellipse. Fig. 5(b) shows the time history of the robot location and orientation estimation error together with the 3σ error bound. The RMS errors obtained for the robot states are $0.07m$, $0.05m$, $1.28°$. The box plot for the feature location estimates are drawn in Fig. 5(c) and will be compared to the results in the test for using the proposed data-association method.

2) Test 2: Cost Function-based Data Association: The same robot trajectory is used in this test, Fig. 6(a). The resulting robot RMS error in the x-coordinate, y-coordinates and orientation are $0.06m$, $0.04m$ and $0.91°$ respectively. The robot orientation error shown in Fig. 6(b) also indicates a reduction in the error from a maximum of $5°$ to less than $3°$. A reduction in 3σ error bound is also noticeable, around $50 \sim 55$sec, where the robot orientation estimation error is reduced from $10°$ to $6°$. The box plot of the feature-location estimation errors is given in Fig. 6(c). Reductions in landmark location uncertainties are noticeable from the 3^{rd} and 6^{th} feature.

3) Discussion: The success of the approach is mainly attributed to the increased number of robust associations and updates performed by the estimator. Evidently, growth of robot pose uncertainties during the early estimation periods have been effectively attenuated as reported in Test 2. However, with converged estimations in later stages, the reduction in uncertainties are not so noticeable for landmarks away from the robot.

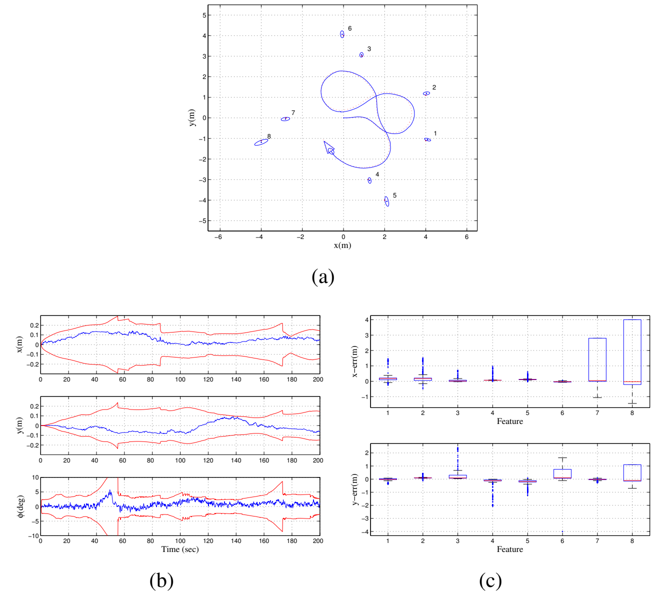


Fig. 5. Results from conventional data association method; (a) estimation result; (b) robot pose error; (c) feature position error.

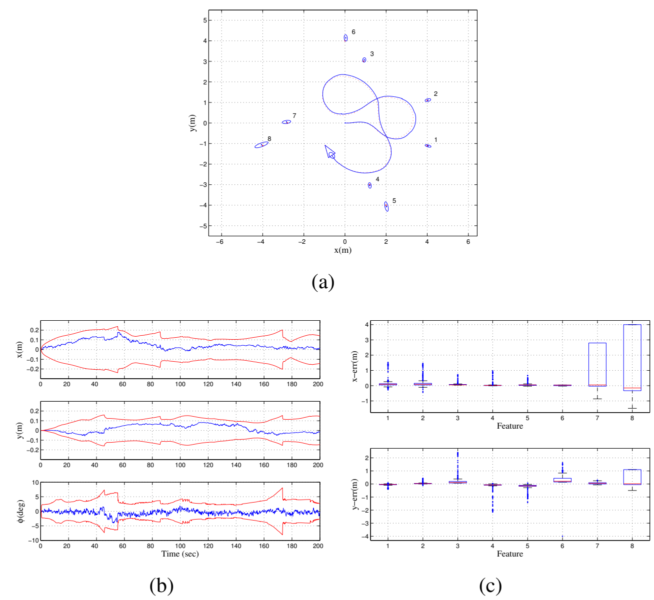


Fig. 6. Results from cost function-based data association method; (a) estimation result; (b) robot pose error; (c) feature position error.

B. Experiment

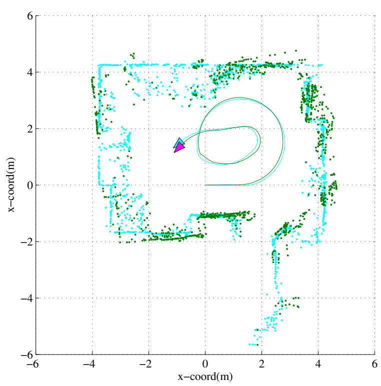
The proposed method is also tested on a real-lift experiment using a camera as a sensor. In this case, a Pioneer DX2 robot mounted with the camera and a laser scanner is driven in the laboratory environment. Moreover, the laser scan is also imposed on the results as an aid for visual indication of the test environment. Typical images captured by the robot are included in Fig. 7 showing the laboratory environment containing chairs and cabinets. Edges of the furniture are chosen and detected by an image processing algorithm as features where their angles relative to the robot are treated as a bearing-only measurement.

The trajectory traced by the robot together with the laser scan for the laboratory environment is shown in Fig. 8(a). It is indicated that the robot has firstly moved in a circle with decreasing radius and finally in a near-straight line. It could be conclude that the robot is able to keep track of its position as the imposed laser scans, indicated as dots, are consistent. Depicted in Fig. 8(b) is the estimation errors of the robot states and the corresponding 3σ uncertainty bounds. The errors have been obtained from comparing to the results obtained from a laser scanner based range-bearing SLAM as a reference. RMS errors are $0.11m$, $0.04m$ and $3.28°$ for the xy-coordinate and orientation. These errors are within practical satisfactory margins and are within the 3σ uncertainty bounds.

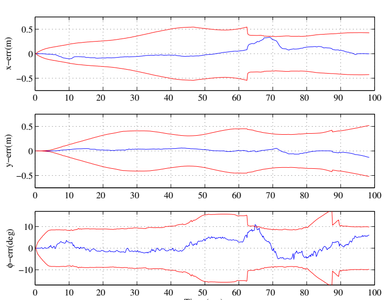
(a) Captured image 1

(b) Captured image 2

Fig. 7. Typical images captured in the experiment.



(a) Estimated robot trajectory



(b) Estimation errors

Fig. 8. Experimental results.

VI. Conclusion

This paper has presented a data association approach for bearing-only SLAM. The orientation of the feature uncertainty is found critical in associations using the conventional Mahalanobis distance gating. The proposed approach, invoking the decision-theoretic philosophy, using cost functions and taking into account the interferences from other features, is developed to resolve ambiguities arising from measurements or features in close angular proximity. Owing to enhanced robustness in the data association and hence an increased number of estimator updates, the SLAM performance is improved. The effectiveness of the proposed method is verified with simulations and experiments. Results obtained are satisfactory in the context of robot and feature location estimation errors.

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