A Region Reaching Control Scheme for Underwater Vehicle-Manipulator Systems

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*Abstract***—In this paper, a new region control scheme is proposed for underwater vehicle-manipulator systems (UVMS). In the proposed control concept, the desired objective can be specified as a region instead of a point. The proposed region control concept is a generalization of setpoint control problem because when the desired region is specified arbitrarily small, the control objective reduces to a point. Lyapunov-like function is proposed for the stability analysis. Simulation studies are presented to demonstrate the effectiveness of the proposed controller.**

I. INTRODUCTION

Many research efforts have been devoted to the development of underwater robotics [1]-[8] as the need for exploring and preserving the oceanic environments has gained significant momentum. In the conventional setpoint control problems of underwater vehicles [1], the desired position is specified as a point. However, in some applications of underwater vehicles, the control objective is specified as a region instead of a point. For example, maintaining the underwater vehicle within a minimum and maximum depth in water; underwater vehicle traveling inside the pipeline for specific task; avoiding an obstacle located at a specified region.

Recently, a region reaching control scheme [9] is proposed for robot manipulator. In this new control concept, the desired objective can be specified as a region instead of a point. To deal with kinematic uncertainty, an approximate Jacobian region reaching control is proposed in [10]. However, the results in [9], [10] are limited to robot manipulator where a single desired region is specified for the end effector as illustrated in figure 1(a). Besides reaching tasks, region reaching concept is also useful for control of macro/mini structures, where a large region (secondary region) is specified for the macro system and a smaller region (primary region) is specified for the mini system. That is, a large region should be specified for the macro system instead of restricting its position to a point as shown in figure 1(b). This gives the macro system more freedoms to adjust its position while the mini system is performing various tasks.

This paper presents a region reaching control concept for an underwater vehicle (macro system) mounted with a manipulator (mini system), where the desired objectives can be specified by regions instead of points. For underwater vehicle-manipulator systems, two desired regions are being

specified, namely primary region and secondary region, as illustrated in Fig. 1(b). The proposed region control concept is also a generalization of setpoint control problem because when the desired region is specified arbitrarily small, the control objective reduces to a point.

Fig. 1. Illustration on region control of UVMS

II. DYNAMICS

In this section, the structure and properties of underwater vehicle-manipulator systems kinematics and dynamics are briefly reviewed. An underwater vehicle with an n -link manipulator attached on it, is considered. Two common vectors that being used in defining the underwater vehicle state vector are η and v. The vector η is defined as: $\eta = [\eta_1^T \ \eta_2^T]^T$, where $\eta_1 = [x \ y \ z]^T$ is the vehicle position vector in the earth fixed frame and $\eta_2 = [\phi \ \theta \ \psi]^T$ is the vehicle Euler angle in the earth fixed frame. The vector v is defined as: $v = [v_1^T v_2^T]^T$, where $v_1 = [u_v \ v_v \ w_v]^T$ is the body fixed linear velocity vector and $v_2 = [p_v q_v r_v]^T$ is the body fixed angular velocity vector.

The vehicle's motion path relative to the earth fixed frame coordinate system is given by the kinematics equation as follow [1]:

$$
\dot{\eta} = J_v(\eta)v = \begin{bmatrix} J_{v1}(\eta_2) & 0 \\ 0 & J_{v2}(\eta_2) \end{bmatrix} v, \quad (1)
$$

where $J_v(\eta)$ is a 6 x 6 kinematics transformation matrix or Jacobian matrix, with

$$
J_{v1}(\eta_2) = \begin{bmatrix} c_{\psi}c_{\theta} & s_{\psi}c_{\theta} & -s_{\theta} \\ -s_{\psi}c_{\phi} + c_{\psi}s_{\theta}s_{\phi} & c_{\psi}c_{\phi} + s_{\psi}s_{\theta}s_{\phi} & s_{\phi}c_{\theta} \\ s_{\psi}s_{\phi} + c_{\psi}s_{\theta}c_{\phi} & -c_{\psi}s_{\phi} + s_{\psi}s_{\theta}c_{\phi} & c_{\phi}c_{\theta} \end{bmatrix},
$$

$$
J_{\nu 2}(\eta_2) = \begin{bmatrix} 1 & s_{\phi} t_{\theta} & c_{\phi} t_{\theta} \\ 0 & c_{\phi} & -s_{\phi} \\ 0 & s_{\phi}/c_{\theta} & c_{\phi}/c_{\theta} \end{bmatrix}.
$$
 (2)

where c_{α} , s_{α} and t_{α} are short notations for $cos(\alpha)$, $sin(\alpha)$ and $tan(\alpha)$, respectively. Notice that $J_{\nu2}(\eta_2)$ is undefined for 90 degrees in pitch angle, θ .

For the n -link manipulator, the common vectors that being used in defining the manipulator position variables are q , p and p_e . The joint position vector is defined by $q =$ $[q_1, \dots, q_n]$. The vector p is defined as: $p = [p_1^T \ p_2^T]^T$, where $p_1 = [x_l \ y_l \ z_l]^T$ and $p_2 = [\phi_l \ \theta_l \ \psi_l]^T$ is the position and orientation of the end effector in the vehicle body fixed frame, respectively. The position and orientation of the end effector in the earth fixed frame is defined by $p_e =$ $[p_{e1}^T \ p_{e2}^T]^T$, where $p_{e1} = [x_e \ y_e \ z_e]^T$ and $p_{e2} = [\phi_e \ \theta_e \ \psi_e]^T$.

The relation between the end effector velocity and the joint velocity is expressed by usual manipulator Jacobian matrix as follows:

$$
\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} {}^{B}R_O J_{m1}(q) \\ {}^{B}R_O J_{m2}(q) \end{bmatrix} \dot{q} \Longleftrightarrow \dot{p} = J'_{m}(q)\dot{q}, \quad (3)
$$

where $J_{m1}(q)$ and $J_{m2}(q)$ represent the position and orientation Jacobian matrix from manipulator base frame to end effector. ${}^{B}R_{O}$ denotes the transformation matrix from the vehicle body fixed frame to the manipulator base frame. If these two coordinate frames are coincidence, then ${}^{B}R_{O} = I$ and $J'_m(q)$ is the manipulator Jacobian matrix itself. By introducing the vector $\zeta = [v^T \dot{q}^T]^T$, it is possible to rewrite the relations between velocities in a compact form as [1], [2], [5]:

$$
\left[\begin{array}{c}\n\dot{\eta} \\
\dot{p}_e\n\end{array}\right] = J(\eta, q) \left[\begin{array}{c}v \\
\dot{q}\n\end{array}\right],\n\tag{4}
$$

where

$$
J(\eta, q) = \begin{bmatrix} J_v(\eta) & 0 \\ 0 & J_v(\eta) \end{bmatrix} \begin{bmatrix} I & 0 \\ J_1(\eta, q) & J'_m(q) \end{bmatrix},
$$

\n
$$
J_1(\eta, q) = \begin{bmatrix} I & -J_{v1}^T S (J_{v1} p_1) J_{v2} \\ 0 & I \end{bmatrix},
$$

\n
$$
S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.
$$
 (5)

The equation of motion for the underwater vehiclemanipulator systems is given as [5], [6]:

$$
M(q)\dot{\zeta} + C(q,\zeta)\zeta + D(q,\zeta)\zeta + g(\eta,q) = \tau \tag{6}
$$

where $M(q) \in R^{(6+n)\times(6+n)}$ is the inertia matrix including added mass terms, $C(q, \zeta)\zeta \in R^{6+n}$ is the vector of Coriolis and centripetal terms, $D(q, \zeta)\zeta \in R^{6+n}$ is the vector of dissipative effects, $g(\eta, q) \in R^{6+n}$ is the vector of gravity and buoyancy effects, $\tau \in R^{6+n}$ is the vector of force or moment acting on the vehicle as well as joint torques. Several important properties of the UVMS dynamics equation described in (6) are [5]:

Property 1: The inertia matrix, $M(q)$ including the added mass is symmetric and positive definite.

Property 2: The matrix, $M(q) - 2C(q, \zeta)$ is skew-symmetric.

Property 3: The hydrodynamic damping matrix, $D(q, \zeta)$ is strictly positive such that $D(q, \zeta) > 0$.

III. REGION CONTROL LAW FOR UVMS

For underwater vehicle-manipulator systems, two desired regions are being specified, namely primary region and secondary region. As illustrated in Fig. 1(b), a primary region is specified for the manipulator end effector and a secondary region is specified for an underwater vehicle. Usually, a bigger region is specified for the secondary region to give the underwater vehicle more freedoms while the manipulator is performing various tasks.

The desired region for underwater vehicle, which is the secondary region, can be specified as:

$$
f_V(\delta \eta_o) = \begin{bmatrix} f_{V_1}(\delta \eta_{o_1}) \\ f_{V_2}(\delta \eta_{o_2}) \\ \vdots \\ f_{V_{N_2}}(\delta \eta_{o_{N_2}}) \end{bmatrix} \leq 0, \tag{7}
$$

where $\delta \eta_{o_j} = \eta - \eta_{o_j}, \eta_{o_j}$ is the reference point of the j th desired region, $j = 1, 2, \dots, N_2, N_2$ is the total number of secondary objective functions, $f_{V_j}(\delta \eta_{o_j}) \in R$ are scalar functions with continuous partial derivatives.

In addition, the primary region, which is the desired region for the manipulator end effector, can be specified as:

$$
f_M(\delta p_{e_o}) = \begin{bmatrix} f_{M_1}(\delta p_{e_{o_1}}) \\ f_{M_2}(\delta p_{e_{o_2}}) \\ \vdots \\ f_{M_{N_1}}(\delta p_{e_{o_{N_1}}}) \end{bmatrix} \leq 0, \tag{8}
$$

where $\delta p_{e_{o_i}} = p_e - p_{e_{o_i}}$, $p_{e_{o_i}}$ is the reference point of the i th desired region, $i = 1, 2, \dots, N_1, N_1$ is the total number of primary objective functions, $f_{M_i}(\delta p_{e o_i}) \in R$ are scalar functions with continuous partial derivatives.

For example, the desired regions can be specified as spheres using the following objective function:

$$
f_1(\delta \eta_{o_1}) =
$$

$$
(x - x_{o_1})^2 + (y - y_{o_1})^2 + (z - z_{o_1})^2 - r_1^2 \le 0,
$$
 (9)

They can also be specified as an intersection of two spherical regions, by adding an additional objective function as follows:

$$
f_2(\delta \eta_{o_2}) =
$$

$$
(x - x_{o_2})^2 + (y - y_{o_2})^2 + (z - z_{o_2})^2 - r_2^2 \le 0,
$$
 (10)

An graphical illustration of the desired region is shown in Fig. 2.

Another example of the desired region is shown in Fig. 3. The inequality functions that describe this region are specified as:

$$
f_1(\delta \eta_{o_1}) = (x - x_o)^2 - \epsilon_x^2 \le 0,
$$

\n
$$
f_2(\delta \eta_{o_2}) = (y - y_o)^2 - \epsilon_y^2 \le 0,
$$

\n
$$
f_3(\delta \eta_{o_3}) = (z - z_o)^2 - \epsilon_z^2 \le 0,
$$
\n(11)

where ϵ_x , ϵ_y and ϵ_z are the individual regional bounds for each axis.

Fig. 2. Desired region as the intersection of two spherical regions.

Fig. 3. Desired region specified as a rectangular block.

Besides the desired regions, repulsive regions can also be specified for obstacle avoidance purpose. For example, an objective function for repulsive region can be added as follows,

$$
f_4(\delta \eta_{o_4}) =
$$

$$
r_o^2 - (x - x_{o_4})^2 - (y - y_{o_4})^2 - (z - z_{o_4})^2 < 0, \quad (12)
$$

Figure 4 shows the graphical illustration of desired region and repulsive region.

Fig. 4. Illustration of desired region and repulsive region

The potential energy functions for the desired regions described in inequalities (7) and (8) can be specified respectively as:

$$
P_V(\delta \eta_o) = \sum_{j=1}^{N_2} P_{V_j}(\delta \eta_{o_j}),
$$
 (13)

$$
P_M(\delta p_{e_o}) = \sum_{i=1}^{N_1} P_{M_i}(\delta p_{e_{o_i}}),
$$
\n(14)

where

$$
P_{V_j}(\delta \eta_{o_j}) = \frac{k_{p_{V_j}}}{2} [max(0, f_{V_j}(\delta \eta_{o_j}))]^2.
$$
 (15)

$$
P_{M_i}(\delta p_{e_{o_i}}) = \frac{k_{p_{M_i}}}{2} [max(0, f_{M_i}(\delta p_{e_{o_i}}))]^2.
$$
 (16)

That is,

$$
P_{V_j}(\delta \eta_{o_j}) = \begin{cases} 0, & f_{V_j}(\delta \eta_{o_j}) \leq 0, \\ \frac{k_{p_{V_j}}}{2} f_{V_j}^2(\delta \eta_{o_j}), & f_{V_j}(\delta \eta_{o_j}) > 0, \end{cases}
$$
 (17)

$$
P_{M_i}(\delta p_{e_{o_i}}) = \begin{cases} 0, & f_{M_i}(\delta p_{e_{o_i}}) \le 0, \\ \frac{k_{p_{M_i}}}{2} f_{M_i}^2(\delta p_{e_{o_i}}), & f_{M_i}(\delta p_{e_{o_i}}) > 0, \end{cases}
$$
 (18)

where $k_{p_{V_j}}, k_{p_{M_i}}$ are positive constants. Note that $P_V(\delta \eta_o) = 0$, $P_M(\delta p_{e_o}) = 0$ only if all the inequality functions (13), (14) are satisfied, respectively.

Partial differentiating the potential energy function described by (17) with respect to η and the potential energy function (18) with respect to p_e , which can be written as:

$$
\left(\frac{\partial P_{V_j}(\delta \eta_{o_j})}{\partial \eta}\right)^T =
$$
\n
$$
k_{p_{V_j}} max(0, f_{V_j}(\delta \eta_{o_j})) \left(\frac{\partial f_{V_j}(\delta \eta_{o_j})}{\partial \eta}\right)^T, \qquad (19)
$$
\n
$$
\left(\frac{\partial P_{M_i}(\delta p_{e_{o_i}})}{\partial p_e}\right)^T =
$$
\n
$$
k_{p_{M_i}} max(0, f_{M_i}(\delta p_{e_{o_i}})) \left(\frac{\partial f_{M_i}(\delta p_{e_{o_i}})}{\partial p_e}\right)^T, \qquad (20)
$$

Hence, from (13) and (14) , partial differentiating the potential energy functions give,

$$
\left(\frac{\partial P_V(\delta \eta_o)}{\partial \eta}\right)^T = \sum_{j=1}^{N_2} \frac{\partial P_{V_j}(\delta \eta_{o_j})^T}{\partial \eta} = \sum_{j=1}^{N_2} k_{p_{V_j}} max(0, f_{V_j}(\delta \eta_{o_j})) \left(\frac{\partial f_{V_j}(\delta \eta_{o_j})}{\partial \eta}\right)^T, \quad (21)
$$

$$
\left(\frac{\partial P_M(\delta p_{e_o})}{\partial p_e}\right)^T = \sum_{i=1}^{N_1} \frac{\partial P_{M_i}(\delta p_{e_{o_i}})}{\partial p_e}^T = \sum_{i=1}^{N_1} k_{p_{M_i}} max(0, f_{M_i}(\delta p_{e_{o_i}})) \left(\frac{\partial f_{M_i}(\delta p_{e_{o_i}})}{\partial p_e}\right)^T, (22)
$$

From (21) and (22), the region controller with gravity and buoyancy force compensation is proposed as:

$$
\tau = -J^T(\eta, q) \left[\left(\frac{\partial P_V(\delta \eta_o)}{\partial \eta} \right)^T \right] - K_v \zeta + g(\eta, q), \quad (23)
$$

where $J(\eta, q)$ is the kinematics transformation matrix defined in (5), and $K_v \in R^{(m+n)\times(m+n)}$ is a positive definite diagonal matrix gain.

Substituting (23) into (6), the closed-loop equation is obtained as:

$$
M(q)\dot{\zeta} + C(q,\zeta)\zeta + D(q,\zeta)\zeta + K_v\zeta
$$

$$
+J^T(\eta,q)\left[\left(\frac{\partial P_V(\delta\eta_o)}{\partial \eta}\right)^T\right] = 0.
$$
(24)

A Lyapunov-like function is proposed as:

$$
V = \frac{1}{2}\zeta^T M(q)\zeta + P_V(\delta \eta_o) + P_M(\delta p_{e_o}).\tag{25}
$$

Differentiating (25) with respect to time and using **Property 1**, yields:

$$
\dot{V} = \zeta^T M(q)\dot{\zeta} + \frac{1}{2}\zeta^T \dot{M}(q)\zeta + \dot{P}_V(\delta \eta_o) + \dot{P}_M(\delta p_{e_o}). \tag{26}
$$

where

$$
\dot{P}_V(\delta \eta_o) = \sum_{j=1}^{N_2} \frac{d}{dt} P_{V_j}(\delta \eta_{o_j}) =
$$
\n
$$
\left(\sum_{j=1}^{N_2} k_{p_{V_j}} max(0, f_{V_j}(\delta \eta_{o_j})) (\frac{\partial f_{V_j}(\delta \eta_{o_j})}{\partial \eta}) \right) \dot{\eta}, \qquad (27)
$$
\n
$$
\dot{P}_M(\delta p_{e_o}) = \sum_{i=1}^{N_1} \frac{d}{dt} P_{M_i}(\delta p_{e_{o_i}}) =
$$
\n
$$
\left(\sum_{i=1}^{N_1} k_{p_{M_i}} max(0, f_{M_i}(\delta p_{e_{o_i}})) (\frac{\partial f_{M_i}(\delta p_{e_{o_i}})}{\partial p_e}) \right) \dot{p}_e, (28)
$$

Substituting (21), (22), (27), (28) and the closed-loop equation (24) into (26), yields,

$$
\dot{V} = -\zeta^T K_v \zeta - \zeta^T D(q, \eta) \zeta + \frac{1}{2} \zeta^T (\dot{M}(q) - 2C(q, \eta)) \zeta \n+ \dot{\eta}^T \sum_{j=1}^{N_2} k_{p_{V_j}} max(0, f_{V_j}(\delta \eta_{o_j})) \left(\frac{\partial f_{V_j}(\delta \eta_{o_j})}{\partial \eta} \right)^T \n+ \dot{p}_e^T \sum_{i=1}^{N_1} k_{p_{M_i}} max(0, f_{M_i}(\delta p_{e_{o_i}})) \left(\frac{\partial f_{M_i}(\delta p_{e_{o_i}})}{\partial p_e} \right)^T \n- \zeta^T J^T(\eta, q) \left[\left(\frac{\partial P_V(\delta \eta_{o})}{\partial \eta_e} \right)^T \right],
$$
\n(29)

since $\left[\dot{\eta}^T \dot{p}_e^T\right]^T = J(q,\eta)\zeta$ as in (4). Simplifying (29) and applying **Property** 2 and **Property** 3, \dot{V} reduces to:

$$
\dot{V} = -\zeta^T K_v \zeta - \zeta^T D(q, \eta) \zeta \le 0. \tag{30}
$$

A compact set Ω is considered in the state space as:

$$
\Omega = \{ (\zeta, \eta, q) : V \le \gamma \}
$$
\n(31)

where $P_V(\delta \eta_o)$ and $P_M(\delta p_{e_o})$ have isolated minimums at the respective desired regions with a positive γ . If $\zeta(0), \eta(0),$ $q(0) \in \Omega$, then $(\zeta, \eta, q) \in \Omega$ since $V \leq 0$.

The stability of the region control with gravity and buoyancy force compensation is specified by the following theorem:

Theorem: *The closed-loop system described by (24) gives rise to the convergence of* η *to the desired region* $f_V(\delta \eta_o) = [f_{V_1}(\delta \eta_{o_1}) \quad f_{V_2}(\delta \eta_{o_2}) \quad \cdots \quad f_{V_{N_2}}(\delta \eta_{o_{N_2}})]^T \leq$ 0 , p_e to the desired region $f_M(\delta p_{e_o})$ = $[f_{M_1}(\delta p_{e_{o_1}})$ $f_{M_2}(\delta p_{e_{o_2}})$ \cdots $f_{M_{N_1}}(\delta p_{e_{o_{N_1}}})]^T \leq 0$, v *to* 0 *and* \dot{q} *to* 0 *as* $t \rightarrow \infty$ *.* **Proof:**

Since $\dot{V} = 0$ implies $v = 0$ and $\dot{q} = 0$, the maximum invariant set satisfy $J^T(\eta, q) \begin{bmatrix} \frac{\partial P_v(\delta \eta_o)}{\partial \eta} & \frac{\partial P_m(\delta p_{e_o})}{\partial p_e} \end{bmatrix}$ $\left[\frac{1}{\partial p_{e}}\right]^{T}$ = 0. Hence $\left(\frac{\partial P_V(\delta \eta_o)}{\partial \eta}\right)^T \rightarrow 0$, $\left(\frac{\partial P_M(\delta p_{e_o})}{\partial p_e}\right)^T$ $\frac{d\left(\delta p_{e_o}\right)}{\partial p_e}\bigg)^T \;\;\rightarrow\;\; \overset{\circ}{0},$ $v \rightarrow 0$ and $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$ if $\dot{J}^T(q,\eta)$ is nonsingular. This implies finally that $f_v(\delta \eta_o)$ = $\left[f_{V_1}(\delta\eta_{o_1})\ f_{V_2}(\delta\eta_{o_2})\ \cdots\ \ f_{V_{N_2}}(\delta\eta_{o_{N_2}})\right]^T \leq 0$ and $f_M(\delta p_{e_o}) = \int f_{M_1}(\delta p_{e_{o_1}}) f_{M_2}(\delta p_{e_{o_2}}) \cdots$ $f_{M_{N_1}}(\delta p_{e_{o_{N_1}}})\bigg]^T \leq 0$ in Ω as $t \to \infty$.

Remark: Note that, in the region control where η and p_e converge to the desired regions specified by $f_V(\delta\eta_o) = [f_{V_1}(\delta\eta_{o_1}) \ f_{V_2}(\delta\eta_{o_2}) \ \cdots \ f_{V_{N_2}}(\delta\eta_{o_{N_2}})]^T \leq 0$ and $f_M(\delta p_{e_o})$ = $\left[f_{M_1}(\delta p_{e_{o_1}})$ $f_{M_2}(\delta p_{e_{o_2}})\right]$ $\cdots f_{M_{N_1}}(\delta p_{e_{o_{N_1}}})\bigg]^T \leq 0$, respectively, is a generalization of setpoint control where the position error $\delta \eta = \eta - \eta_d$ and $\delta p_e = p_e - p_{e_d}$, converge to zero or a bound specified as $\|\delta\eta\| \leq \epsilon_V$ and $\|\delta p_e\| \leq \epsilon_M$ as $t \to \infty$. Using multiple objective functions, desired regions with arbitrary shapes can be specified as intersections of several regions.

IV. SIMULATION STUDIES

In this section, simulation studies are presented to illustrate the effectiveness of the proposed region controllers. The simulation studies are performed on the Omni Directional Intelligent Navigator (ODIN) [11] equipped with a 2-link manipulator with revolute joints. The parameters of the dynamic model of ODIN can be found in [11]. For simplicity and effective presentation, we consider only planar motion for both vehicle and manipulator in the simulations.

In the simulations, the vehicle is required to move from an initial position $[0.5, 0.5]$ to a desired secondary region while the end-effector reaching the desired primary region. The masses and length of the manipulator link 1, 2 and the load are set as $m_{L_1} = 0.50kg$, $m_{L_2} = 0.45kg$, $m_{load} = 0.05kg$, $L_1 = 0.35m$, $L_2 = 0.30m$ and $L_{load} = 0.05m$, respectively.

To verify the effectiveness of the controller proposed in section III, the simulation is carried out by specifying the desired primary and secondary regions using the objective functions. The objective functions for the secondary region are specified as follows:

$$
f_{V_1} = (x - x_{o_V})^2 - \epsilon_{x_V}^2 \le 0,
$$

\n
$$
f_{V_2} = (z - z_{o_V})^2 - \epsilon_{z_V}^2 \le 0,
$$

where $[x_{o_V}, z_{o_V}] = [5.5, 5.3]$ is the reference point within the desired region, and the tolerance for each axis are specified as $\epsilon_{xy} = 0.1$ and $\epsilon_{zy} = 0.1$. Similarly, the objective functions for the primary region are specified as follows:

$$
f_{M_1} = (x - x_{o_M})^2 - \epsilon_{x_M}^2 \le 0,
$$

$$
f_{M_2} = (z - z_{o_M})^2 - \epsilon_{z_M}^2 \le 0,
$$

where $[x_{o_M}, z_{o_M}] = [6.0, 6.0]$ is the reference point within the desired region, and the tolerance for each axis are specified as $\epsilon_{x_M} = 0.1$ and $\epsilon_{z_M} = 0.1$. Both the vehicle and manipulator position converged to the desired square region, with $K_{p_V} = 2.5I, K_{v_V} = 35I, K_{p_M} = 0.6I$ and $K_{v_M} = 0.1I$. Figure 5 shows both of the vehicle and manipulator position converged to the desired square regions. Paths of the vehicle and manipulator's end-effector are shown in Fig. 6.

In the above simulation, the desired regions are specified as square region. Various desired regions could be specified based on different applications. Figure 7 and 8 show both of the vehicle and manipulator position converged to the desired circular regions, with $K_{p_V} = 2I$, $K_{v_V} = 45I$, $K_{p_M} = 0.6I$ and $K_{v_M} = 0.1I$.

In the practical implementation, region control can also be used for obstacle avoidance purposes. For example, when there is an obstacle within the operating area, repulsive regions can be specified for the region occupied by the obstacle, therefore, the system can be prevented from collide with the obstacle. By referring to position plot in Fig. 8, circular repulsive regions are purposely introduced near to the system path. The circular repulsive regions are specified by the objective functions as follows:

$$
f_{r_1} = r_{r_1}^2 - (x - x_r)^2 - (z - z_r)^2 \le 0,
$$

\n
$$
f_{r_2} = r_{r_2}^2 - (x - x_r)^2 - (z - z_r)^2 \le 0,
$$

where $[x_r, z_r] = [3.0, 2.5]$ is the reference point of the repulsive region, $r_{r_1} = 0.1$ and $r_{r_2} = 0.9$ are the radius of the circular repulsive regions. The larger repulsive region specified by f_{r_2} served as the active region for the system to react on the obstacle avoidance, and f_{r_1} represents the region where the obstacle is located. Therefore, once the system entered the larger repulsive region specified by f_{r_2} , the controller will pull the system out from the repulsive region, while moving towards the final desired region, as shown in Fig. 9. The controller gains for the simulation in Fig. 9 are chosen as $K_{pV} = 0.7I$, $K_{vV} = 28I$, $K_{pM} = 0.007I$, $K_{v_M} = 0.001I$, and the control gain for repulsive region is chosen as $K_{p_r} = 500$.

V. CONCLUSION

In this paper, a region control scheme is proposed for UVMS. Two desired regions, namely primary and secondary region are specified for manipulator and vehicle position, respectively. Lyapunov-like functions have been presented for the stability analysis of the region controller. Simulation results illustrated the performance of the proposed controllers. As a by product of the result, the proposed region control concept is also a generalization of setpoint control problem because when the desired region is specified arbitrarily small, the control objective is reduced to setpoint control.

Fig. 5. Stable response for the region controller using square regions.

Fig. 6. Region control using square regions.

Fig. 7. Stable response for the region controller using circular regions.

Fig. 8. Region control using circular regions.

Fig. 9. Region control using repulsive regions.

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