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Global Robust and Adaptive Output Feedback Dynamic Positioning of Surface Ships

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Abstract— **A constructive method is presented to design a global robust and adaptive output feedback controller for dynamic positioning of surface ships under environmental disturbances. Measurements of the ship's velocities are not required for feedback. The ship's parameters are not required to be known. An adaptive observer is first designed to estimate the ship's velocities and parameters. The control is then designed based on Lyapunov's direct method to force the ship's position and orientation to globally asymptotically converge to the desired values.**

I. Introduction

Offshore oilfield development has moved to a deeper, and more severe environment for new oil sources. Moreover the offshore oil-rigs have become small and light weight. In deep-water applications, floating production, storage and offloading units are very cost effective. However, the length of lines becomes excessive in a conventional chain and anchor mooring system, and maintaining the position of an offshore platform becomes difficult both technically and economically. Therefore, dynamic positioning systems using thrusters are often used in those applications. Dynamic positioning systems have been commercially available for marine vessels since the 1960s. Conventional dynamic positioning systems are designed based on linearization of the kinematic equations of motions about a set of predefined constant yaw angles so that linear control theory can be applied. The kinematic equations of motion are usually linearized about 36 different yaw angles. For each of these linearized models, optimal Kalman filters and feedback control gains are computed. These filters are used to provide estimates of the vessel velocities since only positions are usually measured in a dynamic positioning system, see for example [1], [2], [3].

Because of limitations of linear control techniques such as complexity in tuning control gains and no global stability results due to linearization, recently several researchers applied nonlinear control theory to design various control systems for dynamic positioning of surface vessels. In [4] and [5], Lyapunov methods [6] and backstepping technique [7] were used to design a passive nonlinear observer to estimate the vessel velocities. This observer is then incorporated into the control design, which is based on Lyapunov's direct method. The constant bias disturbances are also included in the dynamics for the observer design and control design. In addition, some interesting practical implementation results on a full-scale vessel were reported in these papers. In [8], the problem of weather optimal dynamic positioning was addressed based on the basic principle of pendulum. In this weather optimal dynamic positioning system, the control system automatically turns the vessel such that it heads to the direction of the constant environmental disturbances to minimize the load on the vessel. In [9], universal controllers were proposed for both trajectory tracking and stabilization for underactuated vessels. These types of controllers and observers designed in [10] and [11] can be used for dynamic positioning of underactuated ships as well. In [12] and [13], several control systems were proposed for a riser system where the goal is to maintain the top and bottom angles of the riser at desired values. In existing output feedback dynamic positioning systems, see for example [4] and [5], the system parameters such as mass of the vessel and hydrodynamic coefficients are required to be known for observer design. Any inaccuracy in these parameters directly affects the performance of the controlled systems. Furthermore, when there are uncertainties in the system parameters, no stability analysis results can be found in the existing output feedback dynamic positioning systems. These problems motivate the new output feedback dynamic positioning system proposed in this paper.

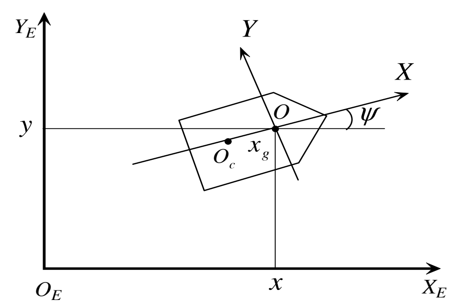

Fig. 1. Definition of the earth-fixed frame $O_E X_E Y_E$ and the body-fixed frame OXY.

In this paper, we propose a constructive method to design a global robust and adaptive output feedback controller for dynamic positioning of surface ships under environmental disturbances. The ship's parameters are not required to be known. A new adaptive observer is first designed to estimate the ship's velocities and parameters. The measurement noise of the ship positions is also filtered out through the adaptive observer. The output feedback controller is then proposed to force the ship's position and orientation to globally

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asymptotically converge to its desired values.

II. Problem statement

Assume that the ship has an xz-plane of symmetry; surge is decoupled from sway and yaw; heave, pitch and roll modes are neglected; the body-fixed frame coordinate origin is on the center-line of the ship (see Fig. 1). In this figure, $O_E X_E Y_E$ is the earth-fixed frame, OXY is the body-fixed frame, and O_c is the center of gravity of the vessel. The mathematical model of the ship used for dynamic positioning in a horizontal plane is described as [14]:

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}(\psi)\boldsymbol{v} \\ \boldsymbol{M}\dot{\boldsymbol{v}} &= -\boldsymbol{D}\boldsymbol{v} + \boldsymbol{\tau} + \boldsymbol{\tau_{dis}} \end{aligned} \tag{1}$$

where $\boldsymbol{\eta} = [x\ y\ \psi]^T$ denotes the position (x, y) and heading ψ of the ship coordinated in the earth-fixed frame, $\boldsymbol{v} = [u\ v\ r]^T$ denote the ship's surge, sway and yaw velocities coordinated in the body-fixed frame. The other terms in (1) are defined below:

The rotation matrix $\boldsymbol{J}(\psi)$, mass including added mass matrix $\boldsymbol{M}$, and damping matrix $\boldsymbol{D}$ are given by

$$\begin{aligned} \boldsymbol{J}(\psi) &= \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \boldsymbol{M} &= \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix}, \\ \boldsymbol{D} &= -\begin{bmatrix} X_u & 0 & 0 \\ 0 & Y_v & Y_r \\ 0 & N_v & N_r \end{bmatrix} \end{aligned} \tag{2}$$

where m is the vessel mass, I_z is the moment of inertia about the body-fixed z-axis, x_g is the distance from the origin O of the body-fixed frame to the center of gravity of the vessel. The other symbols in (2) are referred to as hydrodynamic derivatives, see [15].

The control input vector $\boldsymbol{\tau} \in \mathbb{R}^3$ of forces and moment provided by the actuator system, and the disturbance vector $\boldsymbol{\tau_{dis}}$ of forces and moment induced by waves, wind and ocean currents are given by

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{G}\boldsymbol{u} \\ \boldsymbol{\tau_{dis}} &= \boldsymbol{J^T}(\psi)\boldsymbol{b} \end{aligned} \tag{3}$$

where the control inputs are denoted by $\boldsymbol{u} \in \mathbb{R}^n$ with $n \geq 3$ denoting the number of independent actuators, and $\boldsymbol{G} \in \mathbb{R}^{3\times n}$ is a constant matrix describing the actuator configuration. Unmodeled external forces and moment due to waves, wind, and ocean currents are lumped together into an earth-fixed constant vector $\boldsymbol{b} \in \mathbb{R}^3$. In this paper we impose the following assumption:

Assumption 2.1: a)The ship velocity vector $\boldsymbol{v} = [u\ v\ r]^T$ is not available for feedback.

b)The matrices $\boldsymbol{M}$ and $\boldsymbol{D}$ are positive definite.

c) Elements of the vector $\boldsymbol{b}$ and matrices $\boldsymbol{M}, \boldsymbol{D}, \boldsymbol{G}$ are unknown but constant and bounded, i.e. there exist positive constants $b_1, b_2, M_1, M_2, D_1, D_2, G_1, G_2$ such that

$$\begin{aligned} &b_1 \leq \|\boldsymbol{b}\| \leq b_2,\ M_1 \leq \|\boldsymbol{M}\| \leq M_2, \\ &D_1 \leq \|\boldsymbol{D}\| \leq D_2,\ G_1 \leq \|\boldsymbol{G^T G}\| \leq G_2. \end{aligned} \tag{4}$$

d) The matrix $(\boldsymbol{M^{-1}G})^{\boldsymbol{T}}(\boldsymbol{M^{-1}G})$ is invertible for all $\boldsymbol{M}$ and $\boldsymbol{G}$ that satisfy (4).

Remark 2.1: In most dynamic positioning systems, measurements of the ship velocities are not available. Moreover, if the ship velocities are measured by using sensors or are obtained by differentiating the ship position and orientation, they are significantly corrupted with noise. This in turn degrades performance of a control system. Therefore Item a) of Assumption 2.1 is particularly practical. Item b) of Assumption 2.1 is standard for low speed applications, see [14]. The upper and lower bounds of the ship parameters (those elements of the matrices $\boldsymbol{M}, \boldsymbol{D}, \boldsymbol{G}$) can be computed by some commercially available softwares such as VERES from Marintek. Moreover, the high frequency disturbances should not be compensated by a dynamic positioning system because they cause the propulsion system to quickly wear and extensively consume power. Hence, only earth-fixed low frequency or constant disturbances should be counteracted by the propulsion system. These observations imply that Item c) of Assumption 2.1 is reasonable. Item d) of Assumption 2.1 results from Item b) of Assumption 2.1 and the fact that $\boldsymbol{G} \in \mathbb{R}^n,\ n \geq 3$.

In this paper, we consider the following control objective.

Control objective. *Under Assumption 2.1, design the control input vector* $\boldsymbol{u}$ *to force the ship position* (x, y) *and orientation* ψ *to globally asymptotically converge to their desired constant values* $(x_d,\ y_d,\ \psi_d)$.

III. Observer design

In this section, we will design an adaptive observer to estimate all the ship velocities, ship parameters and the disturbances. We first convert (1) into a convenient parameterization form for the purpose of observer design. Substituting (3) into (1) results in

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}(\psi)\boldsymbol{v} \\ \boldsymbol{M}\dot{\boldsymbol{v}} &= -\boldsymbol{D}\boldsymbol{v} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{J^T}(\psi)\boldsymbol{b} \end{aligned} \tag{5}$$

which can be written in the following parameterization form

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}(\psi)\boldsymbol{v} \\ \dot{\boldsymbol{v}} &= \boldsymbol{\Phi_v}(\boldsymbol{v})\boldsymbol{\Theta_v} + \boldsymbol{\Phi_u}(\boldsymbol{u})\boldsymbol{\Theta_u} + \boldsymbol{\Phi_\psi}(\psi)\boldsymbol{\Theta_\psi} \end{aligned} \tag{6}$$

where $\boldsymbol{\Phi_v}(\boldsymbol{v}),\ \boldsymbol{\Phi_u}(\boldsymbol{u}),\ \boldsymbol{\Phi_\psi}(\psi)$ are matrices containing elements of $\boldsymbol{v},\ \boldsymbol{u}$ and ψ, respectively, and $\boldsymbol{\Theta_v},\ \boldsymbol{\Theta_u},\ \boldsymbol{\Theta_\psi}$ are vectors of elements of the matrices $\boldsymbol{M},\ \boldsymbol{D},\ \boldsymbol{G}$ and the vector $\boldsymbol{b}$. These matrices and vectors are such that

$$\begin{aligned} \boldsymbol{\Phi_v}(\boldsymbol{v})\boldsymbol{\Theta_v} &= -\boldsymbol{M^{-1}D}\boldsymbol{v}, \\ \boldsymbol{\Phi_u}(\boldsymbol{u})\boldsymbol{\Theta_u} &= \boldsymbol{M^{-1}G}\boldsymbol{u}, \\ \boldsymbol{\Phi_\psi}(\psi)\boldsymbol{\Theta_\psi} &= \boldsymbol{M^{-1}J^T}(\psi)\boldsymbol{b}. \end{aligned} \tag{7}$$

It is noted that the above parametrization is completely possible since $-\boldsymbol{M}^{-1}\boldsymbol{D}\boldsymbol{v}$ and $\boldsymbol{M}^{-1}\boldsymbol{G}\boldsymbol{u}$ are linear in $\boldsymbol{v}$ and $\boldsymbol{u}$, and $\boldsymbol{M}^{-1}\boldsymbol{J}^{\boldsymbol{T}}(\psi)\boldsymbol{b}$ is linear in $\cos(\psi)$ and $\sin(\psi)$. A simple calculation shows that

$$\begin{aligned}
\boldsymbol{\Phi_v}(\boldsymbol{v}) &= \begin{bmatrix} \boldsymbol{v}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} \\ \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{v}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} \\ \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{v}^{\boldsymbol{T}} \end{bmatrix} \in \mathbb{R}^{3\times 9}, \\
\boldsymbol{\Theta_v} &= \begin{bmatrix} \boldsymbol{\Theta_{v1}} & \boldsymbol{\Theta_{v2}} & \boldsymbol{\Theta_{v3}} \end{bmatrix} \in \mathbb{R}^{9\times 1} \\
\boldsymbol{\Phi_u}(\boldsymbol{u}) &= \begin{bmatrix} \boldsymbol{u}^{\boldsymbol{T}} & \boldsymbol{0}_{\boldsymbol{u}}^{\boldsymbol{T}} & \boldsymbol{0}_{\boldsymbol{u}}^{\boldsymbol{T}} \\ \boldsymbol{0}_{\boldsymbol{u}}^{\boldsymbol{T}} & \boldsymbol{u}^{\boldsymbol{T}} & \boldsymbol{0}_{\boldsymbol{u}}^{\boldsymbol{T}} \\ \boldsymbol{0}_{\boldsymbol{u}}^{\boldsymbol{T}} & \boldsymbol{0}_{\boldsymbol{u}}^{\boldsymbol{T}} & \boldsymbol{u}^{\boldsymbol{T}} \end{bmatrix} \in \mathbb{R}^{3\times 3n}, \\
\boldsymbol{\Theta_u} &= \begin{bmatrix} \boldsymbol{\Theta_{u1}} & \boldsymbol{\Theta_{u2}} & \boldsymbol{\Theta_{u3}} \end{bmatrix} \in \mathbb{R}^{3n\times 1} \\
\boldsymbol{\Phi_\psi}(\boldsymbol{\psi}) &= \begin{bmatrix} \boldsymbol{p}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} \\ \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{p}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} \\ \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{0}^{\boldsymbol{T}} & \boldsymbol{p}^{\boldsymbol{T}} \end{bmatrix} \in \mathbb{R}^{3\times 9}, \\
\boldsymbol{\Theta_\psi} &= \begin{bmatrix} \boldsymbol{\Theta_{\psi 1}} & \boldsymbol{\Theta_{\psi 2}} & \boldsymbol{\Theta_{\psi 3}} \end{bmatrix} \in \mathbb{R}^{9\times 1}
\end{aligned} \tag{8}$$

where $\boldsymbol{0} = [0\ 0\ 0]^T$; $\boldsymbol{0_u} = [\underbrace{0\ ...\ 0}_{n}]^T$; $\boldsymbol{p} = [\cos(\psi)\ \sin(\psi)\ 1]$; $\boldsymbol{\Theta_{vi}}, \boldsymbol{\Theta_{ui}}, \boldsymbol{\Theta_{\psi i}}$ with $i = 1, 2$ and 3 are the i^{th} row of $-\boldsymbol{M}^{-1}\boldsymbol{D}$, $\boldsymbol{M}^{-1}\boldsymbol{G}$, and $\boldsymbol{M}^{-1}\boldsymbol{B}$, respectlively, where $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{b_1} & \boldsymbol{b_2} & \boldsymbol{0} \\ \boldsymbol{b_2} & -\boldsymbol{b_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{b_3} \end{bmatrix}$, with $\boldsymbol{b} = [\boldsymbol{b_1}\ \boldsymbol{b_2}\ \boldsymbol{b_3}]$. We now observe that (7) is of the form that the unknown parameter vectors $\boldsymbol{\Theta_v}, \boldsymbol{\Theta_u}, \boldsymbol{\Theta_\psi}$ appear linearly. It is also of interest to point out that the second equation of (7) contains the product of the matrix $\boldsymbol{\Phi_v}(\boldsymbol{v})$, whose several elements are the unmeasured vector $\boldsymbol{v}$, and the unknown parameter vector $\boldsymbol{\Theta_v}$, whose elements are the unknown elements of $-\boldsymbol{M}^{-1}\boldsymbol{D}$. Due to this feature, the design of an observer is challenging. In the literature (see for example, [16], [7], [6]), it requires that the unknown parameters couple with functions of measured states to design an adaptive observer. As such, we propose the following adaptive observer

$$\begin{aligned}
\dot{\hat{\boldsymbol{\eta}}} &= \boldsymbol{J}(\psi)\hat{\boldsymbol{v}} + \boldsymbol{K_o}(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}) \\
\dot{\hat{\boldsymbol{v}}} &= \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}} + \boldsymbol{\Phi_u}(\boldsymbol{u})\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}} + \boldsymbol{\Phi_\psi}(\boldsymbol{\psi})\hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} + \\
&\quad \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{\psi})(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})
\end{aligned} \tag{9}$$

where $\boldsymbol{K_o}$ is a symmetric positive definite matrix; $\hat{\boldsymbol{\eta}} = [\hat{x}\ \hat{y}\ \hat{\psi}]^T$, $\hat{\boldsymbol{v}} = [u\ v\ r]^T$, $\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}$ are estimates of $\boldsymbol{\eta}, \boldsymbol{v}, \boldsymbol{\Theta_u}, \boldsymbol{\Theta_v}, \boldsymbol{\Theta_\psi}$, respectively; the matrix $\boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})$ is the matrices $\boldsymbol{\Phi_v}(\boldsymbol{v})$ with $\boldsymbol{v}$ replaced by $\hat{\boldsymbol{v}}$. Subtracting (9) from (7) results in the following observer error dynamics

$$\begin{aligned}
\dot{\tilde{\boldsymbol{\eta}}} &= \boldsymbol{J}(\psi)\hat{\boldsymbol{v}} + \boldsymbol{K_0}\tilde{\boldsymbol{\eta}} \\
\dot{\tilde{\boldsymbol{v}}} &= \boldsymbol{\Phi_v}(\boldsymbol{v})\boldsymbol{\Theta_v} - \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}} + \boldsymbol{\Phi_u}(\boldsymbol{u})\tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}} + \\
&\quad \boldsymbol{\Phi_\psi}(\boldsymbol{\psi})\tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} + \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{\psi})\tilde{\boldsymbol{\eta}}
\end{aligned} \tag{10}$$

where $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \hat{\boldsymbol{\eta}}, \tilde{\boldsymbol{v}} = \boldsymbol{v} - \hat{\boldsymbol{v}}$, $\tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}} = \boldsymbol{\Theta_u} - \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}$, $\tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} = \boldsymbol{\Theta_\psi} - \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}$. At this point, it is noted by construction that

$$\begin{aligned}
&\boldsymbol{\Phi_v}(\boldsymbol{v})\boldsymbol{\Theta_v} - \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}} \\
&= \boldsymbol{\Phi_v}(\boldsymbol{v})\boldsymbol{\Theta_v} - \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\boldsymbol{\Theta_v} + \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\boldsymbol{\Theta_v} - \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}} \\
&= \boldsymbol{\Phi_v}(\tilde{\boldsymbol{v}})\boldsymbol{\Theta_v} + \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}} \\
&= -\boldsymbol{M}^{-1}\boldsymbol{D}\tilde{\boldsymbol{v}} + \boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}}.
\end{aligned} \tag{11}$$

To determine update laws for $\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}$, we consider the following Lyapunov function candidate

$$\begin{aligned}
V_o =& \frac{1}{2}\big(\|\tilde{\boldsymbol{\eta}}\|^2 + \|\tilde{\boldsymbol{v}}\|^2 + \\
& \tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{v}}^{-1}\tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}} + \tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{u}}^{-1}\tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}} + \tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{\psi}}^{-1}\tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}\big)
\end{aligned} \tag{12}$$

where $\boldsymbol{\Gamma_v}, \boldsymbol{\Gamma_u}, \boldsymbol{\Gamma_\psi}$ are symmetric positive definite matrices. Differentiating both sides of (12) along the solutions of (10) results in

$$\begin{aligned}
\dot{V}_o =& -\tilde{\boldsymbol{\eta}}^{\boldsymbol{T}}\boldsymbol{K_o}\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{v}}\boldsymbol{M}^{-1}\boldsymbol{D}\tilde{\boldsymbol{v}} + \tilde{\boldsymbol{v}}^{\boldsymbol{T}}\boldsymbol{\Phi_v}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}} + \\
& \tilde{\boldsymbol{v}}^{\boldsymbol{T}}\boldsymbol{\Phi_u}(\boldsymbol{u})\tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}} + \tilde{\boldsymbol{v}}^{\boldsymbol{T}}\boldsymbol{\Phi_\psi}(\boldsymbol{\psi})\tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} - \\
& \tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{v}}^{-1}\dot{\hat{\boldsymbol{\Theta}}}_{\boldsymbol{v}} - \tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{u}}^{-1}\dot{\hat{\boldsymbol{\Theta}}}_{\boldsymbol{u}} - \tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{\psi}}^{-1}\dot{\hat{\boldsymbol{\Theta}}}_{\boldsymbol{\psi}}
\end{aligned} \tag{13}$$

where we have used (11), which suggests that we choose the update laws for $\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}$ as follows

$$\begin{aligned}
\dot{\hat{\boldsymbol{\Theta}}}_{\boldsymbol{v}} &= \boldsymbol{\Gamma_v}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{v}}^{\boldsymbol{T}}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}\Big), \\
\dot{\hat{\boldsymbol{\Theta}}}_{\boldsymbol{u}} &= \boldsymbol{\Gamma_u}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{u}}^{\boldsymbol{T}}(\boldsymbol{u})\tilde{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}\Big), \\
\dot{\hat{\boldsymbol{\Theta}}}_{\boldsymbol{\psi}} &= \boldsymbol{\Gamma_\psi}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{\psi}}^{\boldsymbol{T}}(\boldsymbol{\psi})\tilde{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}\Big)
\end{aligned} \tag{14}$$

where the operator proj represents the Lipschitz projection algorithm [17] as

$$\begin{aligned}
&\text{proj}(\varpi, \hat{\omega}) = \varpi \ \text{ if } \ \Xi(\hat{\omega}) \le 0, \\
&\text{proj}(\varpi, \hat{\omega}) = \varpi \ \text{ if } \ \Xi(\hat{\omega}) > 0 \text{ and } \Xi(\hat{\omega})_{\hat{\omega}}\varpi \le 0, \\
&\text{proj}(\varpi, \hat{\omega}) = (1 - \Xi(\hat{\omega})\varpi \text{ if } \Xi(\hat{\omega}) > 0 \text{ and } \Xi(\hat{\omega})_{\hat{\omega}}\varpi > 0
\end{aligned} \tag{15}$$

where $\Xi(\hat{\omega}) = \frac{\hat{\omega}^2 - \omega_M^2}{(\xi^2 + 2\xi\omega_M)}$, $\Xi(\hat{\omega})_{\hat{\omega}} = \frac{\partial \Xi(\hat{\omega})}{\partial \hat{\omega}}$, ξ is an arbitrarily small positive constant, and $\|\omega\| \le \omega_M$. The projection algorithm is such that if $\dot{\hat{\omega}} = \Gamma\,\text{proj}(\varpi, \hat{\omega})$ with Γ a symmetric positive definite matrix, and $\|\hat{\omega}(t_0)\| \le \omega_M$ then

$$\begin{aligned}
&a)\ \hat{\omega}(t) \le \omega_M + \xi, \forall\ 0 \le t \le \infty, \\
&b)\ \text{proj}(\varpi, \hat{\omega}) \text{ is Lipschitz continuous}, \\
&c)\ \|\text{proj}(\varpi, \hat{\omega})\| \le \|\varpi\|, \\
&d)\ \tilde{\omega}^T\text{proj}(\varpi, \hat{\omega}) \ge \tilde{\omega}^T\varpi, \text{ with } \tilde{\omega} = \omega - \hat{\omega}.
\end{aligned} \tag{16}$$

Now substituting (14) into (13) and use Property d) of the proj algorithm in (16) results in

$$\dot{V}_o \le -\tilde{\boldsymbol{\eta}}^{\boldsymbol{T}}\boldsymbol{K_o}\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{v}}\boldsymbol{M}^{-1}\boldsymbol{D}\tilde{\boldsymbol{v}}. \tag{17}$$

We now state our first main result in the following theorem.

Theorem 3.1: Assume that the control vector $\boldsymbol{u}$ is designed in such a way that it only depends on $\hat{\boldsymbol{\eta}}, \boldsymbol{\psi}, \hat{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}$ and $\boldsymbol{\eta_d} = [x_d\ y_d\ \psi_d]^T$, and that it guarantees $\boldsymbol{\eta}(t)$ and $\boldsymbol{v}(t)$ bounded for all $0 \le t < \infty$.

Then the adaptive observer (9) with the update laws (14) ensures that the observer errors $\tilde{\boldsymbol{\eta}}(t)$ and $\tilde{\boldsymbol{v}}(t)$ globally asymptotically converge to zero. The updates $\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}(t)$, $\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}(t)$ and $\hat{\boldsymbol{\Theta}}_{\psi}(t)$ are bounded for all $0 \leq t < \infty$. Moreover the update laws (14) can be calculated based on only measured and known signals.

Remark 3.1: The assumption on boundedness of $\boldsymbol{\eta}(t)$ and $\boldsymbol{v}(t)$, and dependence of the control vector $\boldsymbol{u}$ on known signals in Theorem 3.1 will be completely relaxed in the next section where the control $\boldsymbol{u}$ is designed.

Proof of Theorem 3.1. Since the matrices $\boldsymbol{K_o}, \boldsymbol{M}, \boldsymbol{D}$ are positive definite, from (17) and definition of V_o, see (12), by integrating both sides of (17) then using Barbalat's lemma found in [6] it is not hard to show that $\lim_{t\to\infty}(\|\tilde{\boldsymbol{\eta}}(t)\| = 0)$ and $\lim_{t\to\infty}(\|\tilde{\boldsymbol{v}}(t)\| = 0)$. This means that the observer errors $\tilde{\boldsymbol{\eta}}(t)$ and $\tilde{\boldsymbol{v}}(t)$ globally asymptotically converge to zero. At the first glance, the update laws for $\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\psi}$ given in (14) depend on the unmeasured signal $\boldsymbol{v}$ since $\tilde{\boldsymbol{v}} = \boldsymbol{v} - \hat{\boldsymbol{v}}$. We will however show that these update laws can be calculated without using the unmeasured signal $\boldsymbol{v}$. To do so, integrating both sides of (14) from t_0 to t results in

$$\begin{aligned}
\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}(t) &= \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}(t_0) + \\
&\boldsymbol{\Gamma_v}\int_{t_0}^{t}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{v}}^{\boldsymbol{T}}(\hat{\boldsymbol{v}}(\boldsymbol{\tau}))(\boldsymbol{v}(\boldsymbol{\tau}) - \hat{\boldsymbol{v}}(\boldsymbol{\tau})), \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}\Big)d\boldsymbol{\tau}, \\
\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}(t) &= \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}(t_0) + \\
&\boldsymbol{\Gamma_u}\int_{t_0}^{t}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{u}}^{\boldsymbol{T}}(\boldsymbol{u}(\boldsymbol{\tau}))(\boldsymbol{v}(\boldsymbol{\tau}) - \hat{\boldsymbol{v}}(\boldsymbol{\tau})), \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}\Big)d\boldsymbol{\tau}, \\
\hat{\boldsymbol{\Theta}}_{\psi}(t) &= \hat{\boldsymbol{\Theta}}_{\psi}(t_0) + \\
&\boldsymbol{\Gamma}_{\psi}\int_{t_0}^{t}\text{proj}\Big(\boldsymbol{\Phi}_{\psi}^{\boldsymbol{T}}(\psi(\boldsymbol{\tau}))(\boldsymbol{v}(\boldsymbol{\tau}) - \hat{\boldsymbol{v}}(\boldsymbol{\tau})), \hat{\boldsymbol{\Theta}}_{\psi}\Big)d\boldsymbol{\tau}
\end{aligned} \tag{18}$$

where we have used $\tilde{\boldsymbol{v}} = \boldsymbol{v} - \hat{\boldsymbol{v}}$.
On the other hand, from the first equation of (7), we have $\boldsymbol{v} = \boldsymbol{J}^{-1}(\psi)\frac{d\boldsymbol{\eta}}{dt}$, which is substituted into (18) to yield

$$\begin{aligned}
\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}(t) &= \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}(t_0) - \boldsymbol{\Gamma_v}\int_{t_0}^{t}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{v}}^{\boldsymbol{T}}(\hat{\boldsymbol{v}}(\boldsymbol{\tau}))\hat{\boldsymbol{v}}(\boldsymbol{\tau})d\boldsymbol{\tau}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}\Big) + \\
&\boldsymbol{\Gamma_v}\int_{\eta(t_0)}^{\eta(t)}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{v}}^{\boldsymbol{T}}(\hat{\boldsymbol{v}}(\boldsymbol{\sigma}))\boldsymbol{J}^{-1}(\psi(\boldsymbol{\sigma}))d\boldsymbol{\sigma}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}\Big), \\
\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}(t) &= \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}(t_0) - \boldsymbol{\Gamma_u}\int_{t_0}^{t}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{u}}^{\boldsymbol{T}}(\boldsymbol{u}(\boldsymbol{\tau}))\hat{\boldsymbol{v}}(\boldsymbol{\tau})d\boldsymbol{\tau}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}\Big) + \\
&\boldsymbol{\Gamma_u}\int_{\eta(t_0)}^{\eta(t)}\text{proj}\Big(\boldsymbol{\Phi}_{\boldsymbol{u}}^{\boldsymbol{T}}(\boldsymbol{u}(\boldsymbol{\sigma}))\boldsymbol{J}^{-1}(\psi(\boldsymbol{\sigma}))d\boldsymbol{\sigma}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}\Big), \\
\hat{\boldsymbol{\Theta}}_{\psi}(t) &= \hat{\boldsymbol{\Theta}}_{\psi}(t_0) - \boldsymbol{\Gamma}_{\psi}\int_{t_0}^{t}\text{proj}\Big(\boldsymbol{\Phi}_{\psi}^{\boldsymbol{T}}(\psi(\boldsymbol{\tau}))\hat{\boldsymbol{v}}(\boldsymbol{\tau})d\boldsymbol{\tau}, \hat{\boldsymbol{\Theta}}_{\psi}\Big) + \\
&\boldsymbol{\Gamma}_{\psi}\int_{\eta(t_0)}^{\eta(t)}\text{proj}\Big(\boldsymbol{\Phi}_{\psi}^{\boldsymbol{T}}(\psi(\boldsymbol{\sigma}))\boldsymbol{J}^{-1}(\psi(\boldsymbol{\sigma}))d\boldsymbol{\sigma}, \hat{\boldsymbol{\Theta}}_{\psi}\Big).
\end{aligned} \tag{19}$$

It is now seen that the right hand side of (19) contains only the known terms. This means that the estimates $\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\psi}$ should be updated based on (19) instead of (14). In general, the integrals in the right hand side of (19) do not have an analytical solution. A simple way to get around this problem is to use a discrete-time approximation of the update laws (19). We here present this approximation for $\boldsymbol{\Omega_v}$. An approximation for $\boldsymbol{\Omega_u}$ and $\boldsymbol{\Omega}_{\psi}$ can be carried out similarly. Assuming that the sampling interval Δ is sufficiently small, then the first equation of (19) (together with proj operator) can be approximated as follows:

$$\begin{aligned}
&\text{If } \boldsymbol{\Xi}(\boldsymbol{i}-\boldsymbol{1}) \leq 0 \\
&\quad \hat{\boldsymbol{\Theta}}_{v}(\boldsymbol{i}) = \hat{\boldsymbol{\Theta}}_{v}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{\Gamma}_{v}\boldsymbol{\Phi}_{v}^{\boldsymbol{T}}(\boldsymbol{i}-\boldsymbol{1}) \times \\
&\qquad \Big(\boldsymbol{\Delta}\hat{\boldsymbol{v}}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{J}^{-1}(\boldsymbol{i}-\boldsymbol{1})(\boldsymbol{\eta}(\boldsymbol{i}) - \boldsymbol{\eta}(\boldsymbol{i}-\boldsymbol{1}))\Big), \\
&\text{If } \boldsymbol{\Xi}(\boldsymbol{i}-\boldsymbol{1}) > 0 \text{ and } \boldsymbol{\Xi}_{\hat{\boldsymbol{\Theta}}_{v}}(\boldsymbol{i}-\boldsymbol{1})\boldsymbol{\Phi}_{v}^{\boldsymbol{T}}(\boldsymbol{i}-\boldsymbol{1}) \times \\
&\qquad \Big(\boldsymbol{\Delta}\hat{\boldsymbol{v}}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{J}^{-1}(\boldsymbol{i}-\boldsymbol{1})(\boldsymbol{\eta}(\boldsymbol{i}) - \boldsymbol{\eta}(\boldsymbol{i}-\boldsymbol{1}))\Big) \leq 0 \\
&\quad \hat{\boldsymbol{\Theta}}_{v}(\boldsymbol{i}) = \hat{\boldsymbol{\Theta}}_{v}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{\Gamma}_{v}\boldsymbol{\Phi}_{v}^{\boldsymbol{T}}(\boldsymbol{i}-\boldsymbol{1}) \times \\
&\qquad \Big(\boldsymbol{\Delta}\hat{\boldsymbol{v}}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{J}^{-1}(\boldsymbol{i}-\boldsymbol{1})(\boldsymbol{\eta}(\boldsymbol{i}) - \boldsymbol{\eta}(\boldsymbol{i}-\boldsymbol{1}))\Big), \\
&\text{If } \boldsymbol{\Xi}(\boldsymbol{i}-\boldsymbol{1}) > 0 \text{ and } \boldsymbol{\Xi}_{\hat{\boldsymbol{\Theta}}_{v}}(\boldsymbol{i}-\boldsymbol{1})\boldsymbol{\Phi}_{v}^{\boldsymbol{T}}(\boldsymbol{i}-\boldsymbol{1}) \times \\
&\qquad \Big(\boldsymbol{\Delta}\hat{\boldsymbol{v}}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{J}^{-1}(\boldsymbol{i}-\boldsymbol{1})(\boldsymbol{\eta}(\boldsymbol{i}) - \boldsymbol{\eta}(\boldsymbol{i}-\boldsymbol{1}))\Big) > 0, \\
&\quad \hat{\boldsymbol{\Theta}}_{v}(\boldsymbol{i}) = \hat{\boldsymbol{\Theta}}_{v}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{\Gamma}_{v}\boldsymbol{\Phi}_{v}^{\boldsymbol{T}}(\boldsymbol{i}-\boldsymbol{1})(\boldsymbol{1} - \boldsymbol{\Xi}(\boldsymbol{i}-\boldsymbol{1})) \times \\
&\qquad \Big(\boldsymbol{\Delta}\hat{\boldsymbol{v}}(\boldsymbol{i}-\boldsymbol{1}) - \boldsymbol{J}^{-1}(\boldsymbol{i}-\boldsymbol{1})(\boldsymbol{\eta}(\boldsymbol{i}) - \boldsymbol{\eta}(\boldsymbol{i}-\boldsymbol{1}))\Big)
\end{aligned} \tag{20}$$

where $i = 1, 2, ...$, $\boldsymbol{\Xi}(\boldsymbol{i}-\boldsymbol{1}) = \boldsymbol{\Xi}(\hat{\boldsymbol{\Theta}}(\boldsymbol{i}-\boldsymbol{1}))$, $\boldsymbol{\Xi}_{\hat{\boldsymbol{\Theta}}_{v}}(\boldsymbol{i}-\boldsymbol{1}) = \boldsymbol{\Xi}_{\hat{\boldsymbol{\Theta}}_{v}}(\hat{\boldsymbol{\Theta}}(\boldsymbol{i}-\boldsymbol{1}))$, $\boldsymbol{\Phi}_{v}^{\boldsymbol{T}}(\boldsymbol{i}-\boldsymbol{1}) = \boldsymbol{\Phi}_{v}^{\boldsymbol{T}}(\hat{\boldsymbol{\Theta}}(\boldsymbol{i}-\boldsymbol{1}))$ and $\boldsymbol{J}^{-1}(\boldsymbol{i}-\boldsymbol{1}) = \boldsymbol{J}^{-1}(\psi(\boldsymbol{i}-\boldsymbol{1}))$. Finally, Property a) of the projection algorithm shows that $\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}$(t), $\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}$(t) and $\hat{\boldsymbol{\Theta}}_{\psi}$(t) are bounded for all $t_0 \leq t < \infty$. This completes the proof of Theorem 3.1.
To prepare for the control design presented in the next section, we rewrite (9) as follows

$$\begin{aligned}
\dot{\hat{\boldsymbol{\eta}}} &= \boldsymbol{J}(\psi)\hat{\boldsymbol{v}} + \boldsymbol{K_o}\tilde{\boldsymbol{\eta}} \\
\dot{\hat{\boldsymbol{v}}} &= -\hat{\boldsymbol{A}}\hat{\boldsymbol{v}} + \hat{\boldsymbol{B}}\boldsymbol{u} + \boldsymbol{\Phi}_{\psi}(\psi)\hat{\boldsymbol{\Theta}}_{\psi} + \boldsymbol{J}^{\boldsymbol{T}}(\psi)\tilde{\boldsymbol{\eta}}
\end{aligned} \tag{21}$$

where $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ are estimates of $\boldsymbol{M}^{-1}\boldsymbol{D}$ and $\boldsymbol{M}^{-1}\boldsymbol{G}$, which are constructed from $\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}$ and $\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}$, respectively.

Remark 3.2: We will use $\hat{\boldsymbol{\eta}}$ and $\hat{\boldsymbol{v}}$ generated by (21) for control design instead of $\boldsymbol{\eta}$ and $\hat{\boldsymbol{v}}$ since (21) acts as a filter to filter out high frequency noise of position measurements. Moreover, thanks to Property a) of the projection algorithm given in (16) and assumption on the ship parameters given in (4), we have $\hat{\boldsymbol{B}}^{\boldsymbol{T}}\hat{\boldsymbol{B}}$ is invertible.

IV. Control Design

In this section, we will design the control $\boldsymbol{u}$ as a function of $\hat{\boldsymbol{\eta}}, \boldsymbol{\psi}, \hat{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\psi}$ and $\boldsymbol{\eta_d}$ to guarantee that $\boldsymbol{\eta}(t)$ and $\boldsymbol{v}(t)$ are bounded for all $0 \leq t < \infty$, and that $\lim_{t\to\infty}\|\boldsymbol{\eta}(t) - \boldsymbol{\eta_d}\| = \boldsymbol{0}$. Since (21) is of a strict feedback form [7], we will use the backstepping technique found in [7] to design the control $\boldsymbol{u}$. The control design consists of two steps. At the first step, we consider the first equation of (21) with $\hat{\boldsymbol{v}}$ being treated as a control. At the second step, the second equation of (21) is considered where the actual control $\boldsymbol{u}$ is designed.

A. Step 1

Define

$$\boldsymbol{\eta_e} = \hat{\boldsymbol{\eta}} - \boldsymbol{\eta_d},\ \boldsymbol{v_e} = \hat{\boldsymbol{v}} - \boldsymbol{\alpha_{\hat{v}}} \tag{22}$$

where $\boldsymbol{\alpha_{\hat{v}}}$ is a virtual control of $\hat{\boldsymbol{v}}$. Substituting (22) into the first equation of (21) results in

$$\dot{\boldsymbol{\eta}}_{\boldsymbol{e}} = \boldsymbol{J}(\psi)(\boldsymbol{\alpha_{\hat{v}}} + \boldsymbol{v_e}) + \boldsymbol{K_o}\tilde{\boldsymbol{\eta}}. \tag{23}$$

To design the virtual control $\boldsymbol{\alpha_{\hat{v}}}$, we consider the following Lyapunov function

$$V_1 = 0.5\|\boldsymbol{\eta_e}\|^2 \tag{24}$$

whose derivative along the solutions of (23) satisfies

$$\dot{V}_1 = \boldsymbol{\eta_e^T J}(\psi)(\boldsymbol{\alpha_{\hat{v}}} + \boldsymbol{v_e}) + \boldsymbol{\eta_e^T K_o}\tilde{\boldsymbol{\eta}} \tag{25}$$

which suggests that we choose the virtual control $\boldsymbol{\alpha_{\hat{v}}}$ as

$$\boldsymbol{\alpha_{\hat{v}}} = -\boldsymbol{J^{-1}}(\psi)\boldsymbol{K_1\eta_e} \tag{26}$$

where $\boldsymbol{K_1}$ is a symmetric positive definite matrix. It is of interest to note from (26) that the virtual control $\boldsymbol{\alpha_{\hat{v}}}$ is a smooth function of $\hat{\boldsymbol{\eta}}, \boldsymbol{\eta_d}$ and ψ. Substituting (26) into (25) results in

$$\dot{V}_1 = -\boldsymbol{\eta_e^T K_1 \eta_e} + \boldsymbol{\eta_e^T J}(\psi)\boldsymbol{v_e} + \boldsymbol{\eta_e^T K_o}\tilde{\boldsymbol{\eta}}. \tag{27}$$

B. Step 2

Differentiating both sides of the second equation of (22) along the solutions of (26) and the second equation of (21) gives

$$\begin{aligned}\dot{\boldsymbol{v}}_{\boldsymbol{e}} = &-\hat{\boldsymbol{A}}\hat{\boldsymbol{v}} + \hat{\boldsymbol{B}}\boldsymbol{u} + \boldsymbol{\Phi_\psi}(\psi)\hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} + \boldsymbol{J^T}(\psi)\tilde{\boldsymbol{\eta}} - \\ &\tfrac{\partial \boldsymbol{\alpha_{\hat{v}}}}{\partial \hat{\boldsymbol{\eta}}}(\boldsymbol{J}(\psi)\hat{\boldsymbol{v}} + \boldsymbol{K_o}\tilde{\boldsymbol{\eta}}) - \tfrac{\partial \boldsymbol{\alpha_{\hat{v}}}}{\partial \boldsymbol{\psi}}(\hat{r} + \tilde{r} + \boldsymbol{K_{o3}}\tilde{\boldsymbol{\eta}})\end{aligned} \tag{28}$$

where $\tilde{r}$ is the third element of $\tilde{\boldsymbol{v}}$, and $\boldsymbol{K_{o3}}$ is the third row of $\boldsymbol{K_o}$. To design the actual control $\boldsymbol{u}$, we consider the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\|\boldsymbol{v_e}\|^2. \tag{29}$$

Differentiating both sides of (29) along the solutions of (28) and (27) gives

$$\begin{aligned}\dot{V}_2 = &-\boldsymbol{\eta_e^T K_1 \eta_e} + \boldsymbol{\eta_e^T K_o}\tilde{\boldsymbol{\eta}} + \\ &\boldsymbol{v_e^T}\Big(\boldsymbol{J^T}(\psi)\boldsymbol{\eta_e} - \hat{\boldsymbol{A}}\hat{\boldsymbol{v}} + \hat{\boldsymbol{B}}\boldsymbol{u} + \boldsymbol{\Phi_\psi}(\psi)\hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} + \boldsymbol{J^T}(\psi)\tilde{\boldsymbol{\eta}} - \\ &\tfrac{\partial \boldsymbol{\alpha_{\hat{v}}}}{\partial \hat{\boldsymbol{\eta}}}(\boldsymbol{J}(\psi)\hat{\boldsymbol{v}} + \boldsymbol{K_o}\tilde{\boldsymbol{\eta}}) - \tfrac{\partial \boldsymbol{\alpha_{\hat{v}}}}{\partial \boldsymbol{\psi}}(\hat{r} + \tilde{r} + \boldsymbol{K_{o3}}\tilde{\boldsymbol{\eta}})\Big)\end{aligned} \tag{30}$$

which suggests that we choose the actual control $\boldsymbol{u}$ as follows

$$\begin{aligned}\boldsymbol{u} = &(\hat{\boldsymbol{B}}^{\boldsymbol{T}}\hat{\boldsymbol{B}})^{-1}\hat{\boldsymbol{B}}^{\boldsymbol{T}}\Big[-\boldsymbol{K_2 v_e} - \boldsymbol{J^T}(\psi)\boldsymbol{\eta_e} + \\ &\hat{\boldsymbol{A}}\hat{\boldsymbol{v}} - \boldsymbol{\Phi_\psi}(\psi)\hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} + \tfrac{\partial \boldsymbol{\alpha_{\hat{v}}}}{\partial \hat{\boldsymbol{\eta}}}\boldsymbol{J}(\psi)\hat{\boldsymbol{v}} + \tfrac{\partial \boldsymbol{\alpha_{\hat{v}}}}{\partial \boldsymbol{\psi}}\hat{r} - \\ &\epsilon\Big(\Big\|\tfrac{\partial \boldsymbol{\alpha_{\hat{v}}}}{\partial \hat{\boldsymbol{\eta}}}\boldsymbol{K_o}\Big\|^2 + \Big\|\tfrac{\partial \boldsymbol{\alpha_\psi}}{\partial \hat{\boldsymbol{\eta}}}\boldsymbol{K_o}\Big\|^2 + \Big\|\tfrac{\partial \boldsymbol{\alpha_\psi}}{\partial \hat{\boldsymbol{\eta}}}\boldsymbol{K_{o3}}\Big\|^2\Big)\boldsymbol{v_e}\Big]\end{aligned} \tag{31}$$

where $\boldsymbol{K_2}$ is a symmetric positive definite matrix, ϵ is a positive constant. The terms inside the bracket multiplied by ϵ are damping terms, which are included to overcome effects of the observer errors. Substituting (31) into (30), after a simple calculation, yields

$$\begin{aligned}\dot{V}_2 \leq &-(\lambda_{min}(\boldsymbol{K_1}) - \epsilon)\|\boldsymbol{\eta_e}\|^2 - (\lambda_{min}(\boldsymbol{K_2}) - \epsilon)\|\boldsymbol{v_e}\|^2 + \\ &\frac{1}{4\epsilon}\Big((\lambda_{max}^2(\boldsymbol{K_o}) + 2)\|\tilde{\boldsymbol{\eta}}\|^2 + \|\tilde{\boldsymbol{v}}\|^2\Big)\end{aligned} \tag{32}$$

where $\lambda_{min}(\bullet)$ and $\lambda_{max}(\bullet)$ are the minimum and maximum eigenvalues of $\bullet$. We now state the main result of the paper in the following theorem.

Theorem 4.1: Under Assumption (2.1), the robust and adaptive output feedback control system consists of the control law (31), the observer (9), and the update laws (19) solves the control objective stated in Section II, i.e. $\lim_{t\to\infty}(\|\boldsymbol{\eta}(t) - \boldsymbol{\eta_d}\|) = 0$. In particular, the control vector $\boldsymbol{u}$ depends only on measured and available signals: $\hat{\boldsymbol{\eta}}, \boldsymbol{\psi}, \hat{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}$ and $\boldsymbol{\eta_d}$. Moreover, $\boldsymbol{\eta}(t)$ and $\boldsymbol{v}(t)$ are bounded for all $0 \leq t < \infty$.

Proof of Theorem 4.1. To prove this theorem, we consider the Lyapunov function

$$V_{\sum} = V_2 + \varrho V_o \tag{33}$$

where ϱ is a large positive constant to be picked later. Differentiating both sides of (33) along the solutions of (32) and (17) gives

$$\begin{aligned}\dot{V}_{\sum} \leq &-(\lambda_{min}(\boldsymbol{K_1}) - \epsilon)\|\boldsymbol{\eta_e}\|^2 - (\lambda_{min}(\boldsymbol{K_2}) - \epsilon)\|\boldsymbol{v_e}\|^2 + \\ &\frac{1}{4\epsilon}\Big((\lambda_{max}^2(\boldsymbol{K_o}) + 2)\|\tilde{\boldsymbol{\eta}}\|^2 + \|\tilde{\boldsymbol{v}}\|^2\Big) - \\ &\varrho\Big(\lambda_{min}(\boldsymbol{K_o})\|\tilde{\boldsymbol{\eta}}\|^2 + \lambda_{min}(\boldsymbol{M^{-1}D})\|\tilde{\boldsymbol{v}}\|^2\Big).\end{aligned} \tag{34}$$

Now we pick ϵ and ϱ such that

$$\begin{aligned}&\lambda_{min}(\boldsymbol{K_1}) - \epsilon \geq \kappa_1, \\ &\lambda_{min}(\boldsymbol{K_2}) - \epsilon \geq \kappa_2, \\ &\varrho\lambda_{min}(\boldsymbol{K_o}) - \frac{1}{4\epsilon}(\lambda_{max}^2(\boldsymbol{K_o}) + 2) \geq \kappa_3, \\ &\varrho\lambda_{min}(\boldsymbol{M^{-1}D}) - \frac{1}{4\epsilon} \geq \kappa_4\end{aligned} \tag{35}$$

where $\kappa_i, i = 1, ..., 4$ are strictly positive constants. Note that we can always pick sufficiently small ϵ and sufficiently large ϱ such that (35) holds. Moreover, the constants ϵ and ϱ are only used in the proof of Theorem 4.1, and are not used in the control implementation. Now using (35), we can write (34) as

$$\begin{aligned}\dot{V}_{\sum} &\leq -\kappa_1\|\boldsymbol{\eta_e}\|^2 - \kappa_2\|\boldsymbol{v_e}\|^2 - \kappa_3\|\tilde{\boldsymbol{\eta}}\|^2 - \kappa_4\|\tilde{\boldsymbol{v}}\|^2 \\ &\leq 0\end{aligned} \tag{36}$$

From $\dot{V}_{\sum} \leq 0$ and definition of $V_{\sum}$, see (33) with its components given in (12), (29) and (24), we have

$$W_{\sum}(t) \leq W_{\sum}(t_0), \qquad \forall\ 0 \leq t_0 \leq t < \infty \tag{37}$$

where $W_{\sum} = \|\tilde{\boldsymbol{\eta}}\|^2 + \|\tilde{\boldsymbol{v}}\|^2 + \tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{v}}^{-1}\tilde{\boldsymbol{\Theta}}_{\boldsymbol{v}} + \tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{u}}^{-1}\tilde{\boldsymbol{\Theta}}_{\boldsymbol{u}} + \tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{\psi}}^{-1}\tilde{\boldsymbol{\Theta}}_{\boldsymbol{\psi}} + \|\boldsymbol{\eta_e}\|^2 + \|\boldsymbol{v_e}\|^2$. The inequality (37) readily implies that all signals $\hat{\boldsymbol{\eta}}(t)$, $\hat{\boldsymbol{v}}(t)$, $\hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}(t)$, $\hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}(t)$, $\hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}(t)$, $\boldsymbol{\eta}(t)$ and $\boldsymbol{v}(t)$ are bounded. This means that the solutions of the controlled system consisting of the ship dynamics (1), the observer dynamics (9), the update laws (19) and the control (31) exist and forward complete. Integrating both sides of the first inequality of (36) results in

$$\int_{t_0}^{\infty}\Big(\kappa_1\|\boldsymbol{\eta_e}(t)\|^2+\kappa_2\|\boldsymbol{v_e}(t)\|^2+\kappa_3\|\tilde{\boldsymbol{\eta}}(t)\|^2+ \kappa_4\|\tilde{\boldsymbol{v}}(t)\|^2\Big)dt \le W_{\sum}(t_0)-W_{\sum}(\infty)\le W_{\sum}(t_0). \quad (38)$$

On the other hand it is noted that $\boldsymbol{\eta_e}(t)$, $\boldsymbol{v_e}(t)$, $\tilde{\boldsymbol{\eta}}(t)$, and $\tilde{\boldsymbol{v}}(t)$ are continuous. Therefore, the last inequality of (38) implies from Barbalat's lemma found in [6] that $\lim_{t\to\infty}(\|\boldsymbol{\eta_e}(t)\|) = \lim_{t\to\infty}(\|\boldsymbol{\eta}(t)-\boldsymbol{\eta_d}\|) = 0$. Finally, from (31) it is clear that the control u depends only on measured and available signals: $\hat{\boldsymbol{\eta}}, \boldsymbol{\psi}, \hat{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{v}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{u}}, \hat{\boldsymbol{\Theta}}_{\boldsymbol{\psi}}$ and $\boldsymbol{\eta_d}$. Proof of Theorem (4.1) is completed.

V. CONCLUSIONS

We have developed a constructive method to design a global robust and adaptive output feedback controller for dynamic positioning of surface ships under environmental disturbances. Many assumptions such as full state measurements and completely known system parameters, which are often made in the literature are relaxed. An attractive feature of the proposed controller is that the novel adaptive observer can act as both an estimator and a filter to estimate unavailable states and filter out high frequency noise. Although dynamic positioning vessels are usually fully actuated, it is of interest to investigate a combination the proposed technique in this paper and our techniques for trajectory tracking and path following of underactuated surface vessels in [9], [10] and [11] to design a robust and adaptive output feedback for dynamic positioning of underactuated vessels.

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