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Broadcast Feedback for Stochastic Cellular Actuator Systems Consisting of Nonuniform Actuator Units

Jun Ueda, Lael Odhner, and H. Harry Asada

Abstract— **In this paper, the concept of broadcast feedback for stochastic cellular control systems is expanded to a system with nonuniform cellular length and nonuniform transition probability. The cellular control architecture was originally inspired by skeletal muscles comprising a vast number of tiny functional units, called sarcomeres. The output of the actuator system is an aggregate effect of numerous cellular units, each taking a bistable ON-OFF state. A central controller broadcasts the error between the aggregate output and a reference input. Rather than dictating the individual units to take specific states, the central controller merely broadcasts the overall error signal to all the cellular units uniformly. In turn each cellular unit makes a stochastic decision with a state transition probability, which is modulated in relation to the broadcasted error. Stability conditions of the broadcast feedback system are obtained by using a stochastic Lyapunov function. It is demonstrated that, even in the presence of the distribution of the cell length and/or the distribution of the transition probability generated in each cell, the aggregate output of the cellular units can track a given trajectory stably and robustly.**

I. INTRODUCTION

The exact mechanism of skeletal muscle control is still unknown. However, from the reported muscle behavior in those references we can gain some insights as to how a vast number of sarcomeres can be controlled with much fewer sensors and motor neurons. It is known that the activation of sarcomeres is not governed by a deterministic control, but it contains a stochastic process due to the diffusion of calcium ions [1]. Other references argue that the actomyosin contraction process, the essential process of actuation, is a Brownian process [2]. It is also notable that a muscle can function properly although a significant fraction of the cellular units are not functional.

The authors have presented new control architecture inspired by the muscle behavior, which in turn has the potential to be a novel approach to the control of a vast number of cellular units [3] [4]. The proposed architecture, called "Broadcast Feedback", elucidates the stochastic nature of the cellular units as well as the relationship between many sarcomeres and few sensors and motor neurons. In the broadcast feedback architecture, a central control unit simply broadcasts the error between the reference input and the aggregate output of the cellular units. In turn individual cellular units make independent stochastic decisions based on the broadcasted signal of overall error. No addressing scheme is necessary for broadcast control, since information is sent to all the cells rather than to a specific cell. Hence the method is highly scalable to a vast number of cellular control systems.

This paper expands the concept of the broadcast feedback for stochastic cellular control systems to a system with nonuniform cellular length and nonuniform transition probability. We demonstrate that even in the presence of the distribution of the cell length and/or the distribution of the transition probability embedded in each cell, the aggregate output of the cellular units converge to a reference robustly by merely broadcasting the aggregate output error.

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II. INSPIRATION FROM BIOLOGICAL MUSCLE CONTROL

A skeletal muscle consists of five layers of hierarchical structure, starting with sarcomeres as the lowest functional units. At the molecular level, recent studies have reported that stochastic behavior is essential in explaining intracellular calcium transport [1] and actomyosin contraction itself [2]. At the macroscopic level, a skeletal muscle shows smooth motion although the muscle fibers are known to have either "ON" (producing tension) or "OFF" (relaxed) state [5], and they exhibit prominent hysteresis [6]. Today's artificial muscle actuators, although similar in some aspects, are significantly different in structure from a biological muscle. Assimilating the anatomical structure and motor control architecture of a skeletal muscle, we can gain some insights as to how an artificial muscle can be built and controlled. This leads to an alternative to the design of today's artificial muscle actuators, which is worth investigation for long-term research interests. The following are three major aspects inspired by the biological muscle.

Binary Cellular Structure. Bistable ON-OFF control has salient features in coping with complex nonlinearities of actuator materials. Muscle fibers have prominent hysteresis as addressed by [6]. Most materials for artificial muscle actuators, too, have prominent hysteresis and state-dependent complex nonlinearities [7][8]. The control problem becomes much simpler for ON-OFF control, as demonstrated by [9]

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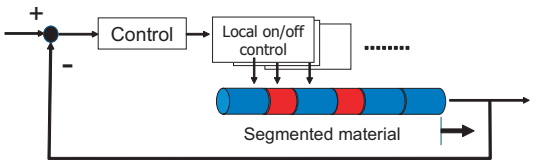


Fig. 1. Segmented Binary Control

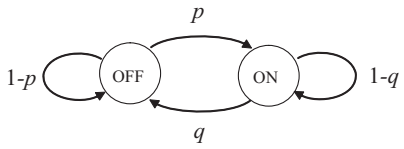


Fig. 2. Single Cell

for shape-memory alloy (SMA) and by [10] for dielectric elastomers.

Figure 1 shows an artificial muscle control system having a binary cellular structure. Instead of driving the whole actuator material as a bulk, the actuator material is divided into many small segments, each controlled as a bistable ON-OFF finite state machine [9].

Broadcast control. Increasing the number of cellular units and reducing the size of each cell can bring about improved resolution and faster response. However, as the number of cellular units increases, it is infeasible, or at least difficult, to control all the cellular units directly with a central controller. Each motor neuron transmits a control signal from the central nervous system to a target muscle fiber. The control signal is then disseminated through a network of T tubules to a number of sarcoplasmic reticula, which activate a bundle of sarcomeres. This anatomical fact implies that a signal from the central nervous system is broadcasted over a vast number of cellular units, rather than different information is delivered to individual units.

Distributed stochastic control. Stochastic behavior can be observed at various motor control processes, ranging from motor unit firing[11] to actomyosin motors[2]. Especially, molecular-level processes, such as calcium release, breakdown of ATP, etc., are influenced by thermal noise resulting in stochastic behavior. This implies that even though the control command, or nerve impulse, is sent uniformly to all units, the response of all the units may not be the same. Stochastic decision-making at local units regulates the aggregate output of the ensemble units without deterministic coordination.

Combining the above three aspects inspired by a skeletal muscle lead to the concept of stochastic cellular control system[3][4].

III. STOCHASTIC CELLULAR CONTROL SYSTEMS

A. Single Cells

A cell is defined to be the smallest functional unit having its own state and producing an output. Each individual cell takes bistable ON-OFF states as shown in Fig. 2. Each cell has a decision-making unit that changes the transition probability from one state to the other by receiving a broadcast signal. Let p^i be the transition probability from OFF to ON, and q^i be the transition probability from ON to OFF for cell i. We assume that the transition is performed in discrete time step, hence the behavior of the cell is modeled as a discrete-time, non-homogeneous Markov process. Each cell provides the following displacement:

$$y^i = \delta^i \eta^i = \begin{cases} \eta^i, & ON \\ 0, & OFF \end{cases}, \tag{1}$$

where y^i is the displacement of the ith cell. η^i is the displacement when the cell is ON. δ^i is given as

$$\delta^i = \begin{cases} 1, & ON \\ 0, & OFF \end{cases}. \tag{2}$$

Consider the case where cell i is OFF at time t, i.e., $\delta^i_t = 0$. If given a transition probability p^i_{t+1}, the expectation and the variance of y^i_{t+1} are given by

$$E[y^i_{t+1}|p^i_{t+1}] = \eta^i p^i_{t+1}. \tag{3}$$

and

$$\begin{aligned} Var[y^i_{t+1}|p^i_{t+1}] &= E[{y^i}^2_{t+1}|p^i_{t+1}] + E[y^i_{t+1}|p^i_{t+1}]^2 \\ &= {\eta^i}^2 p^i_{t+1}(1 - p^i_{t+1}). \end{aligned} \tag{4}$$

B. Cellular Control System

Consider a cellular control system in which N cells are connected in series. The output y of the system is given by the aggregate output of all the cells. From (1):

$$y = \sum_{i=1}^{N} \delta^i \eta^i, \tag{5}$$

In this system, the OFF cells do not contribute to the aggregate output. The gross stroke (the range of the output of the system) is then given by $L = \sum_{i=1}^{N} \eta^i$.

C. Broadcast Feedback Control

In order to control the cellular system, the concept of broadcast feedback control has been proposed [3][4]. The output is measured at discrete time $t = 0, 1, 2, \cdots$. The central controller generates a *universal* control command based on the aggregate output error, i.e.,the difference between the reference input and the current aggregate output: $e_t = r - y_t$, and broadcasts it to all the cells. The simplest control is to set the universal control command as $u_t = e_t$ [3][4]. Each cell

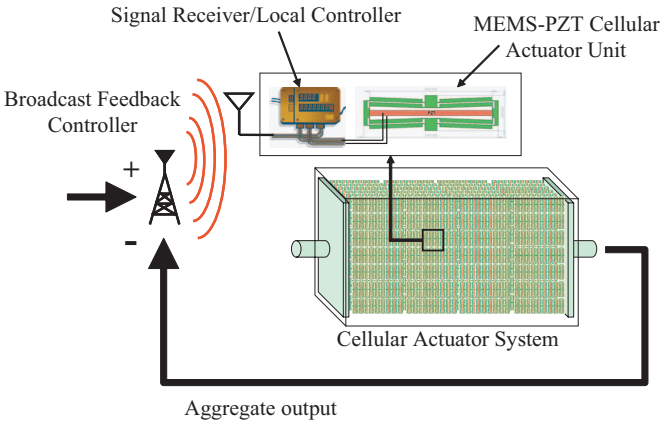


Fig. 3. Implementation of Cellular Actuator System with Broadcast Feedback

generates the transition probabilities, p^i and q^i, by receiving u: $e_t \rightarrow p^i_{t+1}, q^i_{t+1}$.

In order to simplify the problem, the transition from ON to OFF is prohibited when $e_t > 0$, and the transition from OFF to ON is prohibited when $e_t < 0$ at each cell. We call this control law a unilateral transition control:

$$p^i(e) = \begin{cases} 0 & (e \leq 0) \\ p^i(e) & (e > 0) \end{cases} \quad (6)$$

$$q^i(e) = \begin{cases} q^i(e) & (e < 0) \\ 0 & (e \geq 0) \end{cases} . \quad (7)$$

D. Implementation of Cellular Actuator System

Figure 3 shows an implementation example of the cellular actuator system. MEM-PZT cellular actuators [3] are connected to each other directly or through mechanical impedance in series and parallel, composing in totality a single actuator. In stead of wiring many control lines to each individual cells, each cellular actuator has a local control unit that receives the broadcasted signal from the central control unit, and turn its state in a stochastic manner. The local control unit controls the cellular actuator unit by ON-OFF manner, which overcomes the hysteresis of the material and simplifies the amplifier.

IV. ROBUSTNESS AGAINST UNIFORMITY OF THE CELLS

Since the cellular actuator system consists of a vast number of small cellular units, several problems are considered unavoidable due to the limitation of micro manufacturing. One of these problems is the difficulty in maintaining the uniformity of the cells, such as response to the signal, displacement, and life cycle. The cellular actuator system is expected to *sustain* a sufficient response capability even in the presence of these nonuniformities. We address several major problems as follows.

Failure of the Cells It is difficult to maintain all the cells functional; some of the cells may die or do not respond to the inputs. This problem may be due to the creep of the material, disconnection of the power lines, or break in the receiver circuit. In addition, the number of the *dead cells* may vary during the operation. It is notable that a muscle can function robustly and stably although a significant fraction of the cellular units are not functional.

Nonuniformity of Cell Length Even with the recent rapid progress in micro manufacturing technologies, it is difficult to produce many micro actuator units that have uniform displacement or force. A certain degree of variations is unavoidable. In contrast, the length of sarcomere in biological muscle system is not strictly uniform, varying from 2 to 3 μm, and it is considered difficult for the central nervous system to know the whole distribution of the length. However, the muscle control system seems to be working without major problems regarding this point.

Nonuniformity of Embedded Transition Probability Similarly, it is difficult to let all the cells have uniform local controllers that generate uniform transition probability from the broadcasted signal; the generated probabilities at local controllers may have some fluctuations due to noise, offset, or signal attenuation. Needless to say, these problems can be observed in biological systems, which are affected by thermal noise.

In the previous papers [3] [4], we analyzed the system by assuming that the cell length is uniform and the transition probability at each cell is the same. In this paper, we focus on the problems regarding the cell nonuniformities. We demonstrate that even in the presence of these nonuniformities, the aggregate output of the cellular units tracks a reference robustly. If these nonuniformities are allowed, the requirement for developing cellular actuators could be drastically relaxed.

V. STABILITY ANALYSIS

Due to the limitation of space, we give a detailed analysis only for the nonuniformity of the transition probability with the failure of the cells. Other cases will be reported in our future publications.

A. Assumptions

In this section, the stochastic stability of a cellular control system consisting of N cells with the broadcast feedback is analyzed. The control system is distributed with no inter-cellular communications, i.e., (1) the central controller does not know the number of active or dead cells. It does not know neither the distribution of cell length nor the transition probability. It broadcasts only the error between the aggregate output and reference. (2) the local control at each cell generates its own transition probability by broadcasted information and changes its state based on the probability if the transition is possible.

In general, stability and performance can be improved by observing internal states of the system. However, the controller of individual cell becomes more complex, which is not acceptable for a large-scale cellular control system.

We assume that the sampling rate of the broadcast feedback is sufficiently slow compared to the cell dynamics, so that each cell completes transition within the sampling period. This design concept is achieved without difficulty, when micro piezoelectric actuators are used [3][8].

B. Stochastic Lyapunov Analysis

We apply the stability theory using a stochastic Lyapunov function by Kushner [12][13] to this problem (See Appendix). Assume that all cells are functional. The dynamics of the error is represented by

$$E[e_{t+1}|e_t, r] = e_t - \sum_{i=1}^{N} p_{t+1}^i (1-\delta_t^i)\eta^i \quad (e_t > 0) \tag{8}$$

$$E[e_{t+1}|e_t, r] = e_t + \sum_{i=1}^{N} q_{t+1}^i \delta_t^i \eta^i \quad (e_t < 0) \tag{9}$$

Note that the stationary condition of Markov chain will be preserved once $e_t = 0$ holds since (6) and (7) provide

$$E[e_{t+1}|e_t = 0] = 0. \tag{10}$$

Assume that the reference r is constant. Change of r merely means the change of the coordinate origin, i.e., $x_t = e_t = r - y_t$. Let us consider $V^S = e_t^2$ for a candidate of the stochastic Lyapunov function. The change to the Lyapunov function candidate, ΔV_t^S, is calculated as follows:

$$\begin{aligned} \Delta V_t^S &= E[V_{t+1}^S|e_t] - V_t^S \\ &= Var[e_{t+1}|e_t] + E[e_{t+1}|e_t]^2 - e_t^2 \\ &= -k(e_t) \le 0, \end{aligned} \tag{11}$$

where $E[e_{t+1}^2] = Var[e_{t+1}] + E[e_{t+1}]^2$ has been applied. Note that the variance appears in (11), indicating the effect of the variance on the stability condition. If the process is deterministic and, thereby, the variance is zero, the stability condition has no difference from that of a deterministic Lyapunov function. Due to the stochastic nature of the process, the left hand side of the above inequality condition is larger with the added variance term. Therefore, more strict (conservative) stability condition must be met for the stochastic process. It is obvious that $Var[e_{t+1}] \to 0$ and $e_{t+1} \to E[e_{t+1}]$ if $N \to \infty$, resulting in deterministic analysis shown in [3].

When the inequality condition, (11), is satisfied, the process is called a nonnegative supermartingale, for which the Lyapunov function is guaranteed to converge to a nonnegative limit with probability one.

C. Nonuniform Transition Probability

Assume that the length of the cells is uniform, i.e, $\eta^i = \bar{\eta}$ $(i = 1, \cdots, N)$. Consider the case in which $p^i(e)$ and $q^i(e)$ are affected by noise:

$$p^i(e) = f_p(e) + w_p^i \tag{12}$$

$$q^i(e) = f_q(e) + w_q^i \tag{13}$$

where $f_p(e)$ and $f_q(e)$ are deterministic functions of e uniformly given to all cells. w_p^i and w_q^i are white noise that cause the nonuniformity. Let $\bar{p} = 1/N \cdot \sum_{i=1}^N p^i$ and $\bar{q} = 1/N \cdot \sum_{i=1}^N q^i$ be the mean of generated transition probabilities.

If $e_t > 0$, the expected aggregate output is given by

$$\begin{aligned} E[y_{t+1}|y_t] &= E\left[E\left[\sum_{i=1}^{N} p_{t+1}^i (1-\delta_t^i)\bar{\eta} \mid p_{t+1}^i\right]\right] \\ &= y_t + (N-n)\cdot \bar{p}_{t+1} \cdot \bar{\eta}. \end{aligned} \tag{14}$$

Therefore,

$$E[e_{t+1}|e_t, r] = e_t + \bar{p}_{t+1}(r - L - e_t). \tag{15}$$

These are the same results as obtained in the previous section.

Suppose $\delta_t^i = 0$. By applying the law of total variance, the variance of a single cell is given by

$$\begin{aligned} Var[y_{t+1}^i|y_t^i] &= E[Var[y_{t+1}^i|p_{t+1}^i]|y_t^i] + Var[E[y_{t+1}^i|p_{t+1}^i]|y_t^i] \\ &= \bar{\eta}^2(\bar{p} - \bar{p}^2). \end{aligned} \tag{16}$$

Therefore, the variance of the error is given by

$$\begin{aligned} Var[e_{t+1}|e_t, r] &= Var[y_{t+1}|y_t, r] \\ &= \sum_{i=1}^{N} Var[y_{t+1}^i|y_t^i] \\ &= \bar{\eta}^2 \cdot (N-n)\cdot \bar{p}(1-\bar{p}) \end{aligned} \tag{17}$$

Similarly, the expectation and variance for $e_t < 0$ are obtained.

The variance of p^i or q^i does not show in (15) or (17) explicitly. Furthermore, these results are similar to the case where the transition probabilities are uniform, i.e., $p^i = \bar{p}$, $q^i = \bar{q}$ $(i = 1, \cdots, N)$. This implies that the closed loop stability is determined only by $\bar{p}$ and $\bar{q}$, which will be given in the following section.

D. Condition of Stochastic Stability for the Nonuniform Transition Probability

Suppose that a broadcast feedback controller performs proportional control of the output y, i.e., only the error e_t is broadcasted. If each cell has the following set of transition probabilities *as a function of the broadcasted error*, the error asymptotically converges to P_m, which is a small region

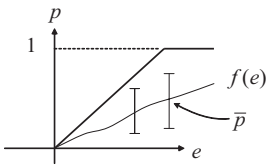


Fig. 4. Nonuniform Transition Probabilities due to Noise

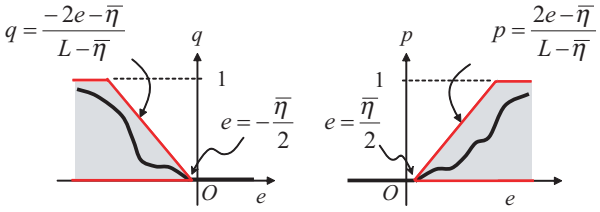


Fig. 5. Stable Transition Probabilities

including $e = 0$, with probability one. Figure 5 shows the obtained transition probabilities.

$$\bar{p}(e) = \begin{cases} 0 & e \le \eta/2 \\ 0 < \bar{p}(e) < min(\frac{2e-\bar{\eta}}{L-\bar{\eta}}, 1) & e > \bar{\eta}/2 \end{cases} \quad (18)$$

$$\bar{q}(e) = \begin{cases} 0 < \bar{q}(e) < min(\frac{-2e-\bar{\eta}}{L-\bar{\eta}}, 1) & e < -\bar{\eta}/2 \\ 0 & e \ge -\eta/2 \end{cases} \quad (19)$$

For large N, we have given a stable transition control for the stochastic cellular control system [3] which is given by (24) and (25) in Appendix. The transition control given by (18) and (19) is an extension to the general case. Although this transition control is slightly more conservative than (24) and (25) since a small *dead band* is required, it takes the stochastic variance into account and guarantees the converge for any N. The system is also stable for any number of dead cells for any feasible r if the system is stable when all the cells are functional. Further, as described previously, the system allows a variation of (18) and (19) due to noise. *Sketch of derivation and proof.* Substitute (18) and (19) for (8) and (9), and check (11) . Use $L \ge L + e_t - r > 0$ to simplify the expression. Use $E[e_{t+1}|e_t, r] = e_t - \bar{p}_{t+1}(L - r + e_t - \bar{\eta}N_{dead}^{OFF})$ instead of (15) where N_{dead}^{OFF} is the number of dead cells staying in the OFF state.

VI. SIMULATION

A. Simulation Model

We demonstrate that the new transition control given by (18) and (19) provides higher robustness against the cell nonuniformities than the previous transition control given by (24) and (25). A position control of a series of MEMS-PZT cellular actuators with 7% strain [3] is examined for $N = 25$ and $N = 1000$. A step-like desired displacement is given and the response is examined in terms of stability. The new transition control laws are implemented as follows.

$$\bar{p}(e) = \begin{cases} 0 & e \le \eta/2 \\ min(\frac{1.5e-\bar{\eta}}{L-\bar{\eta}}, 1) & e > \bar{\eta}/2 \end{cases} \quad (20)$$

$$\bar{q}(e) = \begin{cases} min(\frac{-1.5e-\bar{\eta}}{L-\bar{\eta}}, 1) & e < -\bar{\eta}/2 \\ 0 & e \ge -\eta/2 \end{cases} \quad (21)$$

Similarly, $g_p = g_q = 1.5$ are used for (24) and (25).

The following three cases are examined:

1) $N = 25$. Nonuniform transition probability. Uniform cell length.
2) $N = 25$. Uniform transition probability. Nonniform cell length.
3) $N = 1000$. Nonuniform transition probability. Nonniform cell length. 200 units (20%) are not functional, staying in OFF state.

Figure 6 shows the distribution of cell length where the mean of the length is 280[μm]. Also, noise with mean 0 and variance 4.0×10^{-4} is applied to (12) and (13) for the uniformity of the transition probability.

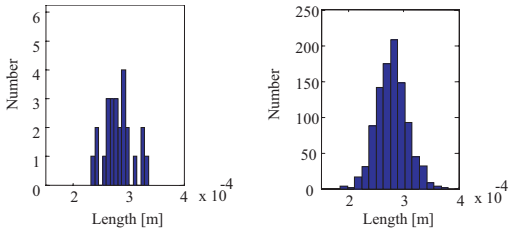
The broadcast signal e_t is updated in every 0.005[sec] so that state transition in each individual cell is performed in sync with this update. Sampling delay $T = 0.005$[sec] is added to the observation of e_t.

B. Simulation Results and Discussion

As shown in Fig. 7(a), the output stably tracks the given trajectory even the embedded transition probability is affected by noise. However, as shown in Fig. 7(b), the output becomes unstable if the previous transition control is applied. It should be noted again that the previous transition control does not take into account the stochastic variance, which may lead to oscillatory response. Although the robustness against the nonuniformity of cell length has not been discussed in this paper, the new transition control shows a potential to cope with this nonuniformity in Fig. data4. Figure 9 shows another potential of the cellular control system. That is, if the number of cells is large enough, the effect of the variance by nonuniformities as well as by stochastic transition control becomes negligible, resulting in high robustness.

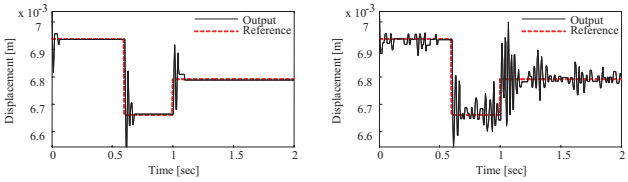
VII. CONCLUSION

In this paper, a broadcast feedback approach has been proposed for a large-scale stochastic cellular control system with nonuniformity. It has been demonstrated that, the aggregate output of the cellular units can track a given trajectory stably and robustly even in the presence of the distribution of the cell length and/or the distribution of the transition probability embedded in each cell. Detailed analyses including the case for nonuniform cell length will be presented in our future publications.



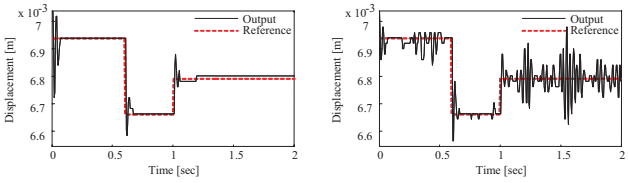
(a) N=25 (b) N=1000

Fig. 6. Distribution of the Cell Length



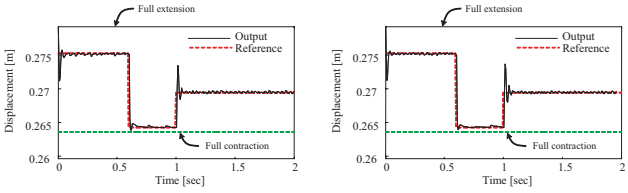
(a) new transition control (b) previous transition control

Fig. 7. Simulation Result for $N = 25$ with Nonuniform transition probability



(a) new transition control (b) previous transition control

Fig. 8. Simulation Result for $N = 25$ with Nonuniform cell length



(a) new transition control (b) previous transition control

Fig. 9. Simulation Result for $N = 1000$ with Nonuniform cell length, Nonuniform transition probability, and 20% of dead cells

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APPENDIX

A. Asymptotic Stability of Discrete Stochastic Systems

Let $V^S(x)$ be a scalar-valued, non-negative, continuous function, satisfying $V^S(0) = 0$, $V^S(x) > 0$, $x \neq 0$. $V^S(x)$ has continuous first derivatives in the bounded set $Q_m = \{x : V^S(x) < m\}$, $m < \infty$. Let $x_0, x_1 \cdots$ be a scalar-valued discrete parameter Markov process, where x_0 is the initial condition in Q_m. If a non-negative, real, scalar function $k(x_t)$ exists, such that the difference between $V^S(x)$ at time t and the conditional mean $E[V^S(x_{t+1})|x_t]$ at time $t+1$ is bounded as

$$E[V^S(x_{t+1})|x_t] - V^S(x_t) = -k(x_t) \leq 0 \qquad (22)$$

in Q_m, then x_t converges to

$$x_t \to P_m = Q_m \cap \{x : k(x) = 0\} \qquad (23)$$

with a probability no less than $1 - V^S(x_0)/m$. $V^S(x)$ is called a stochastic Lyapunov function.

B. Transition Control for Uniform Cellular Actuators with Large N

Assume that N is large enough, and all the cellular units are uniform in term s of displacenemt and transition probability, i.e., $p^i = \bar{p}$, $q^i = \bar{q}$ and $\eta^i = \bar{\eta}$ $(i = 1, \cdots, N)$. The stable transition probabilities are given by [3]:

$$\bar{p}(e) = \begin{cases} 0 & (e \leq 0) \\ min(g_p e/L, 1) & (e > 0) \end{cases} \qquad (24)$$

$$\bar{q}(e) = \begin{cases} min(-g_q e/L, 1) & (e < 0) \\ 0 & (e \geq 0) \end{cases} \qquad (25)$$

where g_p and g_q are control gains. The closed loop stability is guaranteed by $0 < g_p, g_q < 2$.