2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

Robust Adaptive Motion Tracking Control of Piezoelectric Actuation Systems for Micro/Nano Manipulation

Hwee Choo Liaw, Bijan Shirinzadeh
Robotics and Mechatronics Research Laboratory
Department of Mechanical Engineering
Monash University, Clayton, VIC 3800, Australia
{hwee-choo.liaw, bijan.shirinzadeh}@eng.monash.edu.au

Julian Smith
Monash Medical Centre
Department of Surgery
Monash University, Clayton, VIC 3800, Australia
julian.smith@med.monash.edu.au

Abstract— **This paper presents a robust adaptive control methodology for piezoelectric actuation systems to track specified motion trajectories. This methodology is proposed to deal with the control problems of unknown or uncertain system parameters, non-linearities including the hysteresis effect, and external disturbances in the piezoelectric actuation systems, without any form of feed-forward compensation. In this paper, a special class of positive definite functions is employed to formulate the control methodology such that the closed-loop system stability can be guaranteed. The control formulation as well as the stability analysis is detailed. Furthermore, an experimental investigation is also conducted. Implementation of the control methodology is practical as only a knowledge of the estimated system parameters is required. In the experimental study, a promising tracking ability in following a specified motion trajectory is demonstrated. With the capability of motion tracking under the aforementioned conditions, the robust adaptive control methodology is very attractive in realising high performance control applications in the field of micro/nano manipulation.**

I. Introduction

Piezoelectric actuators have been recognised as the most popular device for accomplishing high-precision motion tasks in the field of micro/nano manipulation [1]. However, when these actuators are used, there exists a highly nonlinear relationship between the input (applied) voltage and the output displacement. This prevents the actuators from providing the desired high-precision motion resolution and accuracy. A considerable amount of research studies have therefore been performed to resolve this nonlinear behaviour in piezoelectric actuation systems. One area of research has been performed to model and compensate for the non-linearities, particularly for the hysteresis effect. Other areas of studies have been focused on the enhancement of positioning performance by proposing closed-loop control of the piezoelectric actuation systems.

A number of modelling techniques have been studied. These include a voltage-input electromechanical model [2], a charge steering model [3], and a model of physical hysteresis [4]. Other approaches to the modelling have been based on the established mathematical formulations to approximate the input/output behaviour due to the hysteresis. The examples include the Maxwell slip model [2], Duhem model [5], [6], and Preisach model [7], [8]. However, the hysteresis effect is very complex. It is difficult to obtain an accurate model and the model parameters are difficult to quantify in practice. Therefore, positioning accuracy cannot be guaranteed with hysteresis compensation in an open-loop system.

On the other hand, appropriate closed-loop control strategies have been formulated to achieve high-precision positioning of the piezoelectric actuation systems. Recent examples include a combination of a feed-forward model in a feedback control with an input shaper [9], an adaptive back-stepping approach [10], a tracking control of a piezo-ceramic actuator with hysteresis compensation [11], and a new mathematical model for improving the positioning accuracy of piezoelectric actuators [12]. In most of these studies, a complex hysteresis model has been adopted to compensate for the hysteresis effect.

In this paper, a number of ideas are gathered for the control of micro/nano manipulators, particularly for the piezoelectric actuation systems. First, a special class of positive definite functions is identified [13] for the purpose of control formulation. Second, a robust adaptive control methodology is established without any form of feed-forward compensation. Compared to our previous study [14], this approach does not require a prior information about the bounds of the unknown or uncertain system parameters. Third, a control objective of tracking a desired motion trajectory in position, velocity, and acceleration is proposed for the closed-loop control of the systems.

The proposed robust adaptive methodology is formulated to adjust the control signal for accommodating the unknown or uncertain system parameters, non-linearities including the hysteresis effect, and external disturbances in the piezoelectric actuation systems. The stability of the control systems is analysed. Both the position and velocity tracking errors are proved to be converging to zero in tracking of a desired motion trajectory. Implementation of the control methodology is appropriate as only the control gains and estimated system parameters are required. Furthermore, a promising tracking performance is demonstrated in the experimental study.

This paper is organised as follows. The model of a piezoelectric actuation system is introduced in Section II. Section III describes a special positive definite function for the purpose of control formulation. The proposed robust adaptive control methodology is formulated in Section IV followed by the stability analysis in Section V. The experimental study is detailed in Section VI and the results are presented and discussed in Section VII. Finally, conclusions are drawn in Section VIII.

II. Model of Piezoelectric Actuation System

An electromechanical model of a piezoelectric actuator has been identified based on recent studies [2], [3]. For control purposes, this piezoelectric actuator model [14] can be expressed as

$$m\,\ddot{x} + b\,\dot{x} + k\,x + v_h + f_e = v_{in}, \tag{1}$$

where m, b, and k are the effective mass, damping, and stiffness, respectively, x is the actuator output displacement, v_h is the voltage due to the hysteresis, f_e is related to the force imposed by the external mechanical load, and v_{in} is the applied (input) voltage. Furthermore, it is understood that the nonlinear hysteresis effect is bounded, i.e. $|\,v_h\,| \leq \delta v_h$, where δv_h is a constant number.

In a practical environment, external disturbances have to be considered for the operation of the piezoelectric actuator. The piezoelectric actuator model (1) is therefore extended to include these additional effects,

$$m\,\ddot{x} + b\,\dot{x} + k\,x + v_h + f_e + v_{dc} + v_{dn} = v_{in}\,, \tag{2}$$

where v_{dc} and v_{dn} are the constant time-invariant and nonlinear time-varying external disturbances, respectively. Both terms v_{dc} and v_{dn} are assumed to be bounded.

The model (2) can be extended to describe a piezoelectric actuation system. With this established model, an advanced control methodology can be formulated to effectively control the piezoelectric actuation system.

III. Special Positive Definite Function

A class of positive definite functions plays an important role in formulating an advanced control strategy. In this class, a special function, which is twice continuously differentiable, gives rise to a saturation function and a strictly positive function in its first and second derivatives, respectively. With these characteristics, the saturation function can be used to derive the control strategy and the stability of the closed-loop system can be guaranteed. For example, in a control formulation, the special function, namely ρ, can be written as

$$\rho\,(e_p) = \sqrt{\varepsilon^2 + {e_p}^2} - |\,\varepsilon\,|\,, \tag{3}$$

where ρ is positive definite expressed as the function of a position tracking error e_p and an arbitrary constant ε. It must be noted that the value of the position tracking error e_p is bounded and the arbitrary constant ε is chosen in such a way that its absolute value $|\,\varepsilon\,| > 0$. The saturation function $s(e_p)$ is derived from the first derivative of the special function (3) and it is expressed as

$$s(e_p) = \frac{\mathrm{d}\rho(e_p)}{\mathrm{d}e_p} = \frac{e_p}{\sqrt{\varepsilon^2 + {e_p}^2}}\,. \tag{4}$$

The second derivative $\varrho(e_p)$ of (3) is obtained as

$$\varrho(e_p) = \frac{\mathrm{d}s(e_p)}{\mathrm{d}e_p} = \frac{\varepsilon^2}{\sqrt{(\varepsilon^2 + {e_p}^2)^3}}\,. \tag{5}$$

Furthermore, the time derivative of the saturation function $s(e_p)$ described by (4) can be written as

$$\dot{s}(e_p) = \varrho(e_p)\dot{e}_p = \frac{\varepsilon^2\,\dot{e}_p}{\sqrt{(\varepsilon^2 + {e_p}^2)^3}}\,, \tag{6}$$

where $\varrho(e_p)$ is given by (5).

The saturation function (4) will be employed in the following section to formulate the robust adaptive control methodology. The special positive function described by (3) and its properties given by (4) and (6) will be used in the stability analysis of the proposed control system.

IV. Robust Adaptive Control Methodology

For the piezoelectric actuation system described by (2), a robust adaptive control methodology can be formulated for the purpose of tracking a desired motion trajectory $x_d(t)$. Under the proposed control approach, the physical parameters of the system in (2) are assumed to be unknown or uncertain. Furthermore, there exist bounded nonlinear effects and external disturbances within the system. The $x_d(t)$ is assumed to be twice continuously differentiable and both $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ are bounded and uniformly continuous in $t \in [0,\,\infty)$. A combination of an adaptive scheme and a robust technique is used to realise the control methodology such that the closed-loop system will follow the required motion trajectory.

For the convenience of control formulation, the piezoelectric actuation system (2) is rewritten as

$$\boldsymbol{x}^T\boldsymbol{\varphi} + f_e + m_{eq}\,\ddot{x} + v_{hd} = v_{in}\,, \tag{7}$$

where

$$\begin{aligned} \boldsymbol{x} &= [\ddot{x},\,\dot{x},\,x,\,1]^T\,, & (8)\\ \boldsymbol{\varphi} &= [m^*,\,b,\,k,\,v_{dc}]^T\,, & (9)\\ m^* &= m - m_{eq}\,, & (10)\\ v_{hd} &= v_h + v_{dn}\,, & (11) \end{aligned}$$

and m_{eq} is a strictly positive scalar, i.e. $m_{eq} > 0$.

An adaptive scheme is established in dealing with the unknown system parameters. A set of estimated parameters $\hat{\boldsymbol{\varphi}}$ of $\boldsymbol{\varphi}$ is defined as

$$\hat{\boldsymbol{\varphi}} = [\hat{m}^*,\,\hat{b},\,\hat{k},\,\hat{v}_{dc}]^T\,. \tag{12}$$

An adaptive scheme [15] is employed to continuously update the control system through

$$\dot{\hat{\boldsymbol{\varphi}}} = -\boldsymbol{K}^{-1}\,\boldsymbol{x_d}\,\sigma\,, \tag{13}$$

where $\boldsymbol{K}$ is a 4×4 constant positive definite diagonal matrix, $\boldsymbol{x_d}$ is the desired motion expressed as $\boldsymbol{x_d} = [\ddot{x}_d,\,\dot{x}_d,\,x_d,\,1]^T$, and the error function σ is defined as

$$\sigma = \dot{e}_p + \alpha\,s(e_p)\,, \tag{14}$$

where $e_p(t) = x(t) - x_d(t)$, α is a strictly positive scalar, and $s(e_p)$ is the saturation function defined in (4). An adaptive signal $\hat{v}_{in}$ is therefore introduced as

$$\hat{v}_{in} = \boldsymbol{x_d}^T \hat{\boldsymbol{\varphi}} . \quad (15)$$

A robust control technique is utilised to accommodate the non-linearities in the system. It must be noted that the nonlinear effects v_{hd} given by (11) are bounded and there exists an upper bound δv_{hd} such that

$$|\, v_{hd} \,| \leq \delta v_{hd} . \quad (16)$$

In the robust control approach, a switching function is specified, which is the same as the error function σ given by (14). The time derivative of the switching function (14) is expressed as

$$\dot{\sigma} = \ddot{e}_p + \alpha\, \dot{s}(e_p) , \quad (17)$$

where the term $\dot{s}(e_p)$ is defined in (6).

For the piezoelectric actuation system described by (2), a robust adaptive control methodology is proposed, which is given as

$$\begin{aligned} v_{in} &= -k_p\, e_p - k_v\, \dot{e}_p + \hat{v}_{in} + f_e + \\ &\quad m_{eq}\, \ddot{x}_{eq} - k_s\, \sigma - d\, \frac{\sigma}{|\, \sigma \,|} , && (18) \\ \ddot{x}_{eq} &= \ddot{x}_d - \alpha\, \dot{s}(e_p) , && (19) \\ d &\geq \delta v_{hd} + \epsilon , && (20) \end{aligned}$$

where k_p and k_v are the proportional and derivative gains, respectively, and k_s and ϵ are any strictly positive scalars. Furthermore, the term $\frac{\sigma}{|\sigma|} \triangleq 0$ for $\sigma = 0$.

V. Stability Analysis

The closed-loop behaviour of the proposed control system must be carefully examined in the study of the system stability. In this investigation, the closed-loop dynamics will be derived for the stability analysis.

To find the closed-loop dynamics of the system, the control input (18) is substituted into the piezoelectric actuation system (7),

$$\begin{aligned} &\boldsymbol{x}^T \boldsymbol{\varphi} - \boldsymbol{x_d}^T \hat{\boldsymbol{\varphi}} + k_p\, e_p + k_v\, \dot{e}_p + \\ &m_{eq}(\ddot{x} - \ddot{x}_{eq}) + k_s\, \sigma + d\, \frac{\sigma}{|\, \sigma \,|} + v_{hd} = 0 , \quad (21) \end{aligned}$$

with the term $\hat{v}_{in}$ in (18) replaced by (15). Multiplying both sides of (21) by the error function σ defined in (14), the closed-loop dynamics is rewritten as

$$y + m_{eq}(\ddot{x} - \ddot{x}_{eq})\sigma + k_s\, \sigma^2 + d\, |\, \sigma \,| + v_{hd}\, \sigma = 0 , \quad (22)$$

where

$$y = (\boldsymbol{x}^T \boldsymbol{\varphi} - \boldsymbol{x_d}^T \hat{\boldsymbol{\varphi}} + k_p\, e_p + k_v\, \dot{e}_p)\, \sigma . \quad (23)$$

Due to the deviation of the estimated parameters from the actual values, an estimated adaptive error Δv can be obtained as

$$\Delta v = \boldsymbol{x_d}^T \Delta \boldsymbol{\varphi} , \quad (24)$$

where $\Delta \boldsymbol{\varphi} = \hat{\boldsymbol{\varphi}} - \boldsymbol{\varphi}$, and $\hat{\boldsymbol{\varphi}}$ and $\boldsymbol{\varphi}$ are given by (12) and (9), respectively. The estimated adaptive error (24) can also be written as

$$\boldsymbol{x}^T \boldsymbol{\varphi} - \boldsymbol{x_d}^T \hat{\boldsymbol{\varphi}} = \boldsymbol{x_e}^T \boldsymbol{\varphi} - \Delta v , \quad (25)$$

where $\boldsymbol{x_e} = \boldsymbol{x} - \boldsymbol{x_d} = [\ddot{e}_p,\, \dot{e}_p,\, e_p,\, 0]^T$. The term y described by (23) is then modified by (25) and (14),

$$y = (\boldsymbol{x_e}^T \boldsymbol{\varphi} + k_p\, e_p + k_v\, \dot{e}_p)[\dot{e}_p + \alpha\, s(e_p)] - \Delta v\, \sigma . \quad (26)$$

Expanding the right-hand side of (26) gives

$$\begin{aligned} y &= m^*\, \ddot{e}_p\, \dot{e}_p + (b + k_v)\, \dot{e}_p{}^2 + (k + k_p)\, e_p\, \dot{e}_p + \\ &\quad [m^*\, \ddot{e}_p + (b + k_v)\, \dot{e}_p + (k + k_p)\, e_p]\, \alpha\, s(e_p) - \Delta v\, \sigma , \\ &= \dot{u}_1 + w - \Delta v\, \sigma , && (27) \end{aligned}$$

and

$$\begin{aligned} u_1 &= \frac{1}{2}\, m^*\, \dot{e}_p{}^2 + \alpha\, (b + k_v)\, \rho(e_p) + \frac{1}{2}\, (k + k_p)\, e_p{}^2 + \\ &\quad \alpha\, m^*\, s(e_p)\, \dot{e}_p , && (28) \\ w &= (b + k_v)\, \dot{e}_p{}^2 + \alpha\, (k + k_p)\, e_p\, s(e_p) - \\ &\quad \alpha\, m^*\, \dot{s}(e_p)\, \dot{e}_p , && (29) \end{aligned}$$

where $\rho(e_p)$ is given by (3) and its time derivative is expressed as $\dot{\rho}(e_p) = s(e_p)\, \dot{e}_p$. The term u_1 in (28) can be rewritten as

$$\begin{aligned} u_1 &= \frac{1}{4}\, [\dot{e}_p + 2\alpha\, s(e_p)]\, m^*\, [\dot{e}_p + 2\alpha\, s(e_p)] + \frac{1}{4}\, m^*\, \dot{e}_p{}^2 - \\ &\quad \alpha^2\, m^*\, s^2(e_p) + \alpha\, (b + k_v)\, \rho(e_p) + \\ &\quad \frac{1}{2}\, (k + k_p)\, e_p{}^2 . && (30) \end{aligned}$$

The term $s^2(e_p)$ in (30) can be replaced by using (4),

$$\begin{aligned} u_1 &= \frac{1}{4}\, m^*\, [\dot{e}_p + 2\alpha\, s(e_p)]^2 + \frac{1}{4}\, m^*\, \dot{e}_p{}^2 + \\ &\quad [\, \frac{k + k_p}{2} - \frac{\alpha^2\, m^*}{\varepsilon^2 + e_p{}^2}\,]\, e_p{}^2 + \\ &\quad \alpha\, (b + k_v)\, \rho(e_p) . && (31) \end{aligned}$$

As $1/\varepsilon^2 \geq 1/(\varepsilon^2 + e_p{}^2)$, u_1 will be positive definite if the control gains k_p and k_v in (31) are chosen as

$$\begin{aligned} k_p &> \frac{2\, \alpha^2\, m^*}{\varepsilon^2} - k , \\ k_v &> -b . && (32) \end{aligned}$$

The terms $s(e_p)$ and $\dot{s}(e_p)$ in (29) can be replaced by using (4) and (6), respectively,

$$w = \frac{\alpha\, (k + k_p)}{\sqrt{\varepsilon^2 + e_p{}^2}}\, e_p{}^2 + [\, b + k_v - \frac{\alpha\, m^*\, \varepsilon^2}{\sqrt{(\varepsilon^2 + e_p{}^2)^3}}\,]\, \dot{e}_p{}^2 . \quad (33)$$

To ensure a positive definite w, the control gains in (33) must be selected in such a way that

$$\begin{aligned} k_p &> -k , \\ k_v &> \frac{\alpha\, m^*}{|\, \varepsilon \,|} - b . && (34) \end{aligned}$$

It is possible that the term $\Delta v\,\sigma$ in (27) is related to a positive definite function u_2 given as

$$u_2 = \frac{1}{2}\,\Delta\boldsymbol{\varphi}^T\,\boldsymbol{K}\,\Delta\boldsymbol{\varphi}\,. \tag{35}$$

Differentiating u_2 given by (35) with respect to time yields

$$\dot{u}_2 = \Delta\boldsymbol{\varphi}^T\,\boldsymbol{K}\,\Delta\dot{\boldsymbol{\varphi}}\,. \tag{36}$$

Due to the fact that the parameters $\boldsymbol{\varphi}$ given by (9) are time-invariant, i.e. $\dot{\boldsymbol{\varphi}} = 0$,

$$\Delta\dot{\boldsymbol{\varphi}} = \dot{\hat{\boldsymbol{\varphi}}} - \dot{\boldsymbol{\varphi}} = \dot{\hat{\boldsymbol{\varphi}}}\,, \tag{37}$$

where the term $\dot{\hat{\boldsymbol{\varphi}}}$ is given by (13). The term $\Delta\dot{\boldsymbol{\varphi}}$ in (36) is therefore replaced by (13),

$$\begin{aligned} \dot{u}_2 &= \Delta\boldsymbol{\varphi}^T\,\boldsymbol{K}\,[-\boldsymbol{K}^{-1}\,\boldsymbol{x_d}\,\sigma]\,, \\ &= -[\boldsymbol{x_d}^T\,\Delta\boldsymbol{\varphi}]^T\,\sigma\,, \\ &= -\Delta v\,\sigma\,, \end{aligned} \tag{38}$$

where the scalar Δv is defined in (24).

Theorem 1: For the piezoelectric actuation system described by (2), the robust adaptive control law (18) assures the convergence of the motion trajectory tracking with $e_p(t) \to 0$ and $\dot{e}_p(t) \to 0$ as $t \to \infty$ under the conditions of (32) and (34).

Proof: It must be noted that for the system described by (2) with the proposed control law (18), the functions, u_1, u_2, and w, from (31), (35), and (33), respectively, are positive definite in all possible values of $e_p(t)$ and $\dot{e}_p(t)$ under the conditions of (32) and (34).

A Lyapunov function u_3 is proposed for the closed-loop system,

$$u_3 = \frac{1}{2}\,m_{eq}\,\sigma^2\,, \tag{39}$$

which is continuous and non-negative. Differentiating u_3 with respect to time yields

$$\dot{u}_3 = m_{eq}\,\sigma\,\dot{\sigma}\,. \tag{40}$$

The term $\dot{\sigma}$ in (40) can be derived from (17) and (19),

$$\dot{\sigma} = \ddot{x} - \ddot{x}_{eq}\,, \tag{41}$$

and (40) is rewritten as

$$\dot{u}_3 = m_{eq}\,(\ddot{x} - \ddot{x}_{eq})\,\sigma\,. \tag{42}$$

With the closed-loop dynamics (22), the time derivative of the Lyapunov function (42) becomes

$$\dot{u}_3 = -y - k_s\,\sigma^2 - d\,|\,\sigma\,| - v_{hd}\,\sigma\,. \tag{43}$$

Replacing the term y in (43) by using (27) and (38), and considering (16) and (20), yields

$$\begin{aligned} \dot{u}_3 &= -\dot{u}_1 - w - \dot{u}_2 - k_s\,\sigma^2 - d\,|\,\sigma\,| - v_{hd}\,\sigma\,, \\ \dot{u} &= -w - k_s\,\sigma^2 - d\,|\,\sigma\,| - v_{hd}\,\sigma\,, \\ &\leq -w - k_s\,\sigma^2 - d\,|\,\sigma\,| + \delta v_{hd}\,|\,\sigma\,|\,, \\ &\leq -w - k_s\,\sigma^2 - \epsilon\,|\,\sigma\,|\,, \end{aligned} \tag{44}$$

where $u = u_1 + u_2 + u_3$ is a Lyapunov function. This shows that $u \to 0$ and implies $e_p(t) \to 0$ and $\dot{e}_p(t) \to 0$ as $t \to \infty$. Both the system stability and tracking convergence are guaranteed by the control law (18) driving the system (2) closely tracking the desired motion trajectory. ■

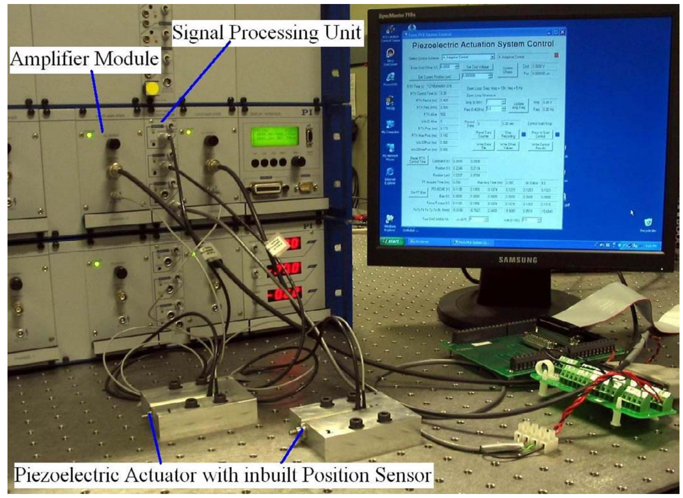

Fig. 1. Piezoelectric actuation experimental research facility

Remark 1: In the implementation of the control law (18), the discontinuous function $\frac{\sigma}{|\,\sigma\,|}$ will give rise to control chattering due to imperfect switching in the computer control. This is undesirable, as un-modelled high frequency dynamics might be excited. To eliminate this effect, the concept of boundary layer technique [16] is applied to smooth the control signal. In a small neighbourhood of the sliding surface ($\sigma = 0$), the discontinuous function is replaced by a boundary saturation function which is defined as

$$\mathrm{sat}(\frac{\sigma}{\Delta}) = \begin{cases} -1 & : \quad \sigma < -\Delta, \\ \sigma/\Delta & : \quad -\Delta \leq \sigma \leq \Delta, \\ +1 & : \quad \sigma > \Delta, \end{cases} \tag{45}$$

where Δ is the boundary layer thickness, and the robust adaptive control law (18) becomes

$$\begin{aligned} v_{in} &= -k_p\,e_p - k_v\,\dot{e}_p + \hat{v}_{in} + f_e + \\ &\quad m_{eq}\,\ddot{x}_{eq} - k_s\,\sigma - d\,\mathrm{sat}(\frac{\sigma}{\Delta})\,. \end{aligned} \tag{46}$$

VI. Experimental Study

In order to investigate the proposed robust adaptive control methodology, an experimental research facility has been established as shown in Fig. 1. The architecture of the experimental set-up is detailed in the block diagram as shown in Fig. 2. It consists of a piezoelectric actuator together with an inbuilt position sensor, an amplifier module, a signal processing unit, and a control PC comprising a digital-to-analogue (D/A) board and an analog-to-digital (A/D) board.

The piezoelectric actuator employed is a PI (Physik Instrumente) multi-layer PZT stacked ceramic translator capable of displacement of up to 45 μm corresponding to a range of operating voltage up to 100 V. The piezoelectric actuator is preloaded 300 N by an internal spring and is incorporated with a high-resolution strain gauge sensor for

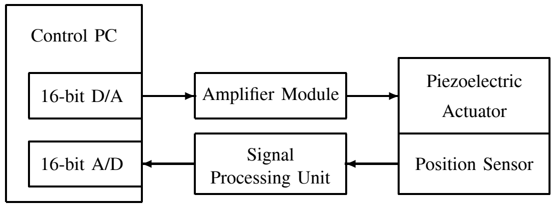

Fig. 2. Block diagram of the experimental architecture

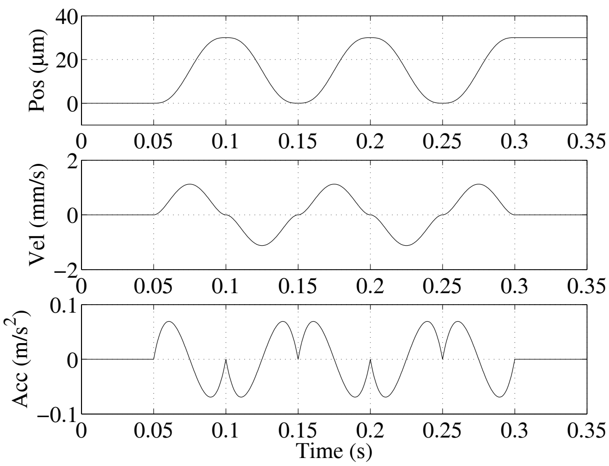

Fig. 3. Desired motion trajectory

position feedback. The PI amplifier module has a fixed output gain of 10 providing voltage ranges from -20 to +120 V. The signal processing unit is used to interface with the actuator position sensor and is connected between the position sensor and control PC. A standard desktop computer is used as the control PC. It is equipped with a Pentium 4 3.2 GHz processor running on an operating system capable of hard real-time control. The D/A and A/D boards within the control PC are of 16-bit resolution, and they are used to generate the control signal and to acquire the actuator position, respectively. In the experiments, the sampling frequency of the control loop is set at 2.5 kHz.

The control experiments serve not only to validate the theoretical formulation of the control algorithms but also to examine the effectiveness of the proposed control methodology in a physical system. In the experimental study, the closed-loop system is required to follow a desired motion trajectory, which is shown in Fig. 3 for position, velocity, and acceleration. The desired motion trajectory is formed by segments of quintic polynomials [17] for the implementation and analysis of the motion tracking and steady-state performances of the control system.

For the piezoelectric actuation system described by (2), the robust adaptive control law (46) is implemented in the control PC as shown in Fig. 2. With the desired motion trajectory, the tracking ability of the control system can be closely evaluated experimentally in the presence of parametric uncertainties, non-linearities, and external disturbances.

In the experimentation of the proposed control methodology, the initial estimate $\hat{\varphi}(0)$ for (13) is chosen to be zero. The control gains, k_p and k_v, of (46) are tuned to the values $5000\,(V/m)$ and $200\,(Vs/m)$, respectively. The diagonal constant matrix $\boldsymbol{K}$ in (13) is selected as $\boldsymbol{K}^{-1} = 6.5 \times 10^5\, diag\{1, 1, 1, 1\}$, where the units are (Vs^4/m^3), (Vs^2/m^3), (V/m^3), and (V/m), respectively. The saturation function $s(e_p)$ in (14) is implemented as given by (4). The arbitrary constant ε in (4) and positive scalar α in (14) are selected as $1 \times 10^{-6}\,(m)$ and $1 \times 10^{-2}\,(m/s)$, respectively. Furthermore, the term m_{eq} and the boundary layer thickness Δ of (46) are chosen as $0.1\,(Vs^2/m)$ and $0.16\,(m/s)$, respectively. The bound δv_{hd} and positive scalar ϵ in (20) are specified as $30\,(V)$ and $1\,(V)$, respectively. Lastly, the term k_s in the control law (46) is set to $150\,(Vs/m)$. It is assumed that no external force is applied to the control system and the term f_e in (46) is ignored in the control experiments.

VII. Results and Discussion

Given the desired motion trajectory as shown in Fig. 3, the piezoelectric actuator was commanded to travel in a range of $30\,\mu m$ with a maximum velocity and an acceleration reaching $1.1\,mm/s$ and $0.07\,m/s^2$, respectively. The resulting piezoelectric actuator positions and estimated velocities are shown in Fig. 4. Despite parametric uncertainties, nonlinear effects, and external disturbances in the system, the proposed control law (46) showed a promising tracking ability. The error or switching function σ, as shown in Fig. 5, indicates that the system operated well within the specified boundary layer thickness Δ, i.e. the system tracked the desired motion trajectory closely with the error or switching function kept to a minimum. The position tracking errors are also shown in Fig. 5. The resulting position tracking errors indicate that the control law had successfully accommodated the non-linearities including the hysteresis effect in the control system. Furthermore, it was observed that the position tracking errors, as presented in Fig. 5, were confined within $0.5\,\mu m$ during motion and less than $0.03\,\mu m$ at steady-state. The control input is shown in Fig. 6. In addition, the effectiveness of the proposed control methodology is also shown in Fig. 6 in the plot of actual against desired position.

In summary, the proposed robust adaptive control methodology for the piezoelectric actuation system is shown to be stable, robust, and capable of following the desired motion trajectory under unknown or uncertain system parameters, non-linearities including the hysteresis effect, and external disturbances. Implementation of the control methodology is appropriate and practical as only the control gains and estimated values of the system parameters are required.

VIII. Conclusions

A robust adaptive control methodology has been proposed and investigated for piezoelectric actuation systems to track specified motion trajectories. Without any form of feed-forward compensation, this methodology is formulated to accommodate parametric uncertainties, nonlinear effects, and external disturbances.

Using the saturation function derived from a special positive definite function in formulating the robust adaptive

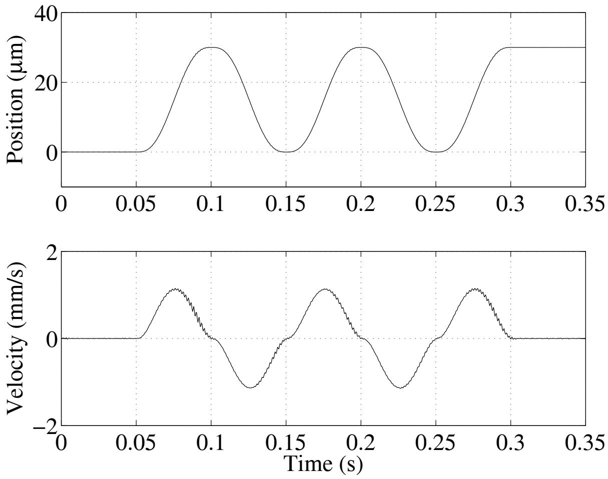

Fig. 4. Actual positions and estimated velocities

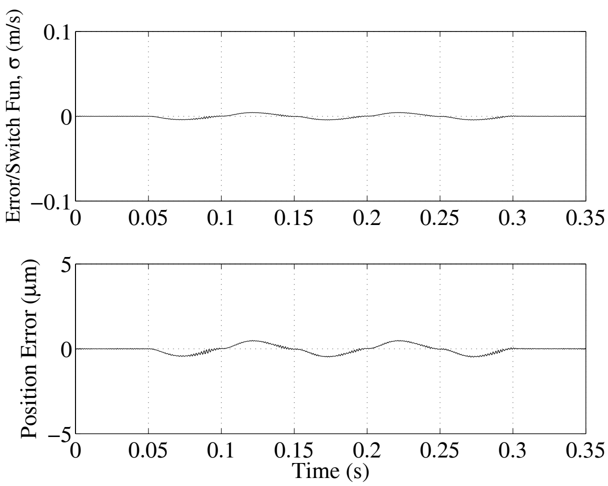

Fig. 5. Error or switching function and position tracking errors

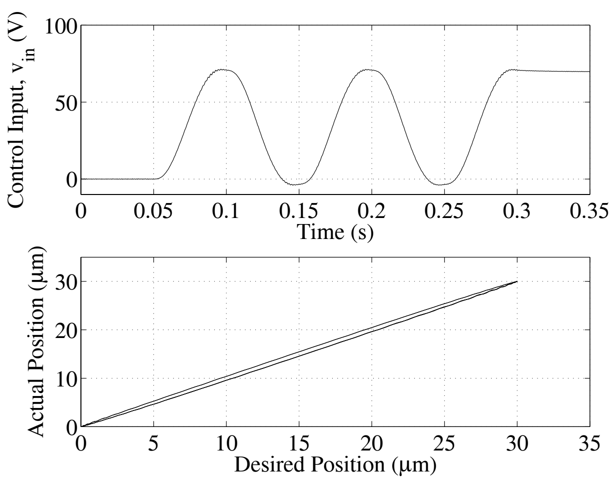

Fig. 6. Control input and actual against desired position

control methodology, the stability of the closed-loop system is guaranteed. Furthermore, a promising tracking ability has been demonstrated in the experimental study.

ACKNOWLEDGMENT

This work is supported by an Australian Research Council (ARC) Linkage Infrastructure, Equipment and Facilities (LIEF) grant and an ARC Discovery grant.

REFERENCES

[1] K. Spanner and S. Vorndran, "Advances in piezo-nanopositioning technology," in *Proceedings of 2003 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, vol. 2, Kobe, Japan, 20–24 July 2003, pp. 1338–1343.

[2] M. Goldfarb and N. Celanovic, "Modeling piezoelectric stack actuators for control of micromanipulation," *IEEE Control Systems Magazine*, vol. 17, no. 3, pp. 69–79, June 1997.

[3] H. J. M. T. A. Adriaens, W. L. de Koning, and R. Banning, "Modeling piezoelectric actuators," *IEEE/ASME Transactions on Mechatronics*, vol. 5, no. 4, pp. 331–341, December 2000.

[4] Y. I. Somov, "Modelling physical hysteresis and control of a fine piezo-drive," in *Proceedings of 2003 International Conference Physics and Control*, vol. 4, St. Petersburg, Russia, 20–22 August 2003, pp. 1189–1194.

[5] R. Banning, W. L. de Koning, H. J. M. T. A. Adriaens, and R. K. Koops, "State-space analysis and identification for a class of hysteretic systems," *Automatica*, vol. 37, no. 12, pp. 1883–1892, December 2001.

[6] J. Oh and D. S. Bernstein, "Semilinear Duhem model for rate-independent and rate-dependent hysteresis," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 631–645, May 2005.

[7] P. Ge and M. Jouaneh, "Modeling hysteresis in piezoceramic actuators," *Precision Engineering*, vol. 17, no. 3, pp. 211–221, July 1995.

[8] P. Ge and M. Jouaneh, "Generalized Preisach model for hysteresis nonlinearity of piezoceramic actuators," *Precision Engineering*, vol. 20, no. 2, pp. 99–111, March 1997.

[9] T. Chang and X. Sun, "Analysis and control of monolithic piezoelectric nano-actuator," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 1, pp. 69–75, January 2001.

[10] H.-J. Shieh, F.-J. Lin, P.-K. Huang, and L.-T. Teng, "Adaptive tracking control solely using displacement feedback for a piezo-positioning mechanism," *IEE Proceedings of Control Theory and Applications*, vol. 151, no. 5, pp. 653–660, 23 September 2004.

[11] G. Song, J. Zhao, X. Zhou, and J. A. D. Abreu-García, "Tracking control of a piezoceramic actuator with hysteresis compensation using inverse Preisach model," *IEEE/ASME Transactions on Mechatronics*, vol. 10, no. 2, pp. 198–209, April 2005.

[12] C. Ru and L. Sun, "Improving positioning accuracy of piezoelectric actuators by feedforward hysteresis compensation based on a new mathematical model," *Review of Scientific Instruments*, vol. 76, no. 9, pp. 095 111–1–8, September 2005.

[13] H. C. Liaw, D. Oetomo, G. Alici, and B. Shirinzadeh, "Special class of positive definite functions for formulating adaptive micro/nano manipulator control," in *Proceedings of 9th IEEE International Workshop on Advanced Motion Control*, Istanbul, Turkey, 27–29 March 2006, pp. 517–522.

[14] H. C. Liaw, D. Oetomo, B. Shirinzadeh, and G. Alici, "Robust motion tracking control of piezoelectric actuation systems," in *Proceedings of 2006 IEEE International Conference on Robotics and Automation*, Orlando, Florida, 15–19 May 2006, pp. 1414–1419.

[15] J.-J. E. Slotine and W. Li, "Adaptive manipulator control: A case study," *IEEE Transactions on Automatic Control*, vol. 33, no. 11, pp. 995–1003, November 1988.

[16] J.-J. E. Slotine and W. Li, *Applied nonlinear control*. Englewood Cliffs, New Jersey: Prentice-Hall, 1991.

[17] J. J. Craig, *Introduction to robotics: mechanics and control*. Reading, MA: Addison-Wesley, 1989.