

Upper body posture estimation for standing function restoration

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Abstract—This paper addresses the problem of restoring standing in paraplegia via functional electrical stimulation (FES) and investigates the relationship between body posture and voluntary upper-body movements. A methodology is presented for upper-body posture estimation in the sagittal plane from force and torque measurements exerted on handles during human standing, in the hypothesis of quasi-static equilibrium. The method consists in setting up constraints related to the geometric equations and the hand-handle interaction. All measured quantities are subject to an uncertainty assumed unknown but bounded. The set membership estimation problem is solved via interval analysis. Guaranteed uncertainty bounds are computed for the estimated postures. The methodology is validated experimentally with spinal cord injured patients with lesions between T5 and T12. Possible applications of the developed methodology are lower limbs function rehabilitation within clinical centers, walk assistance and independent mobility for spinal cord injured patients.

I. INTRODUCTION

Paraplegia results from a severe spinal cord injury which causes the interruption of afferent and efferent signal paths from centralnervous system to lower limb muscles, and thus implies the inability to stand and walk. Functional movement restoration is possible for paraplegics by the use of Functional Electrical Stimulation (FES). Unfortunately, movement generation induced by FES remains mostly open looped and is tuned empirically. In order to design an efficient closed loop control, we need to understand how upper and lower limbs may cooperate. Indeed, generated artificial lower body movements should act in a cooperative way with upper voluntary actions. The so-obtained synergy between voluntary and controlled movements will reduce both patient's fatigue and electro-stimulation energy cost. A mean to solve this issue consists in characterizing upper body voluntary movements through posture estimation.

Research studies for posture and motion estimation using video or image data to extract parameters of a human body model are actively studied [1], [2], [3], [4], for instance in the area of motion analysis for sports and medical purposes [5], [6], [7], interactive applications, surveillance systems [8] and more. Since camera-based systems restrict the user to the constrained environment where the cameras are installed, other studies develop real-time posture tracking systems by using, instead of camera-based systems, miniature sensors such as accelerometers, goniometers or magnetic sensors [9], [10], [11]. However, these types of strategies require to attach devices onto the patient which is not desired in our study

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case. Indeed, we plan to take advantage of the available walker usually used by the patient.

The use of walkers is investigated by several research teams. In [12], [13], the walker is used for assisting gait, where the measurements of forces and torques applied to the walker's handles are used to infer navigational intent of the user. However, this method is intended for persons having functional ability but activity difficulties, and are not suitable for paraplegic persons. The measurements of arm forces applied by the patient on handles are also used to calculate stimulation patterns to apply on the lower limbs in order to maintain the body in a statically stable state [14].

The method introduced in this paper uses the forces exerted on a walker's handle for estimating the posture of a patient. It consists in setting up constraints related to the biomechanical system geometric equations and the hand-handle interaction. In order to insure a reliable estimation of posture, we shall use a guaranteed numerical method which takes in account both, uncertainties within measured forces and model parameters, as well as any modelling error. The theory presented here is evaluated with a 3 degrees of freedom model of the human body in the sagittal plane, but is expandable to more complex models.

The methodology is validated experimentally onto four paraplegic subjects.

II. A RELIABLE METHOD FOR POSTURE ESTIMATION

A. Modelling the Human Body

According to observations from human gait, most of joint movements during locomotion appear to take place in the sagittal plane. In our study, motion in the frontal plane during standing occurs at very low velocities. Moreover, stimulation on the different muscle groups of the lower limbs predominantly generate movement in the sagittal plane. For these reasons, the design of a two-dimensional model of the human body in the sagittal plane is sufficient for a preliminary study. During FES-standing, stimulation of the quadriceps and the hamstring locks the knee in extension, and therefore prevents knee movement. During stance, we considered that the distance between the thigh and the handle is constant, which allows us to assume that the ankle is immobilized. Hence, the lower limbs in this study is treated as a single rigid link. The human body is thus regarded as a four bar linkage with a three degrees of freedom dynamic structure defined in the sagittal plane, as shown in Fig. 1. All links are assumed to be rigid bodies.

The segmental model is described in terms of Denavit-Hartenberg coordinate frames [15]. Frame (x_0, y_0, z_0) cor-

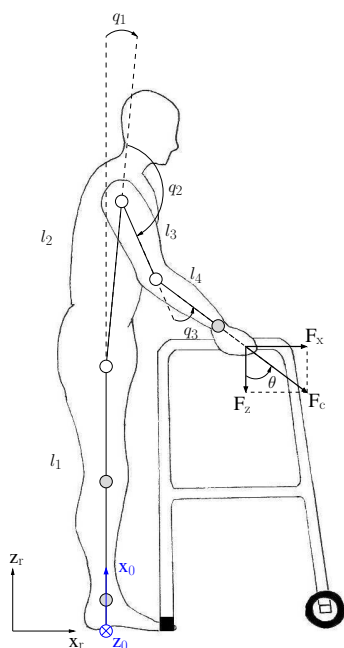


Fig. 1. The four bar linkage human model.

responds to the base frame defined at the distal end of the leg. The allowed movements were limited about the sagittal plane. The hip, shoulder and elbow extension and flexion have been modeled as a single degree of freedom hinge joint.

We define $\mathbf{q} = [q_1 \ q_2 \ q_3]^T$ as the joint angle vector, which is a function of time. It is expressed as a column vector with indices 1, 2 and 3 referring to the hip, the shoulder and the elbow joints respectively. The segments length are denoted by L_j . In Fig. 1, the variables q_1 and q_2 indicate positive angle directions while q_3 indicates a negative one, with respect to the zero position.

The coordinate transformation describing the position and orientation of the hand with respect to the reference frame is given by:

$$\mathbf{T}_n(\mathbf{q}) = \prod_{j=1}^n \mathbf{A}_j^{j-1}(\mathbf{q}) = \begin{bmatrix} \mathbf{R}_n(\mathbf{q}) & \mathbf{P}_n(\mathbf{q}) \\ \mathbf{0} & 1 \end{bmatrix} \quad (1)$$

where $\mathbf{R}_n(\mathbf{q})$ is a 3×3 matrix representing the orientation of the end-effector relative to the reference frame and $\mathbf{P}_n(\mathbf{q})$ is the 3×1 position vector of the origin of this frame with respect to the origin of the reference frame.

The Denavit-Hartenberg parameters of the table I are used to calculate each transformation matrix \mathbf{A}_j and determine the global transformation matrix ${}^w\mathbf{T}_h$ by using (1).

$${}^w\mathbf{T}_h = \begin{bmatrix} s_{123} & c_{123} & 0 & P_x^h \\ 0 & 0 & 1 & 0 \\ c_{123} & -s_{123} & 0 & P_z^h \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

with

$$P_x^h = l_2 s_1 + l_3 s_{12} + l_4 s_{123} \quad (3)$$

TABLE I
DH PARAMETERS FOR THE 3 DOF MODEL

Joint	α_j	d_j	θ_j	r_j
0 (Foot)	-90	0	-90	0
1 (Hip)	0	l_1	q_1	0
2 (Shoulder)	0	l_2	q_2	0
3 (Elbow)	0	l_3	q_3	0
h (Hand)	0	l_4	0	0

and

$$P_z^h = l_1 + l_2 c_1 + l_3 c_{12} + l_4 c_{123} \quad (4)$$

In each equations the following abbreviations were used:

$$\begin{aligned} c_1 &= \cos(q_1), & s_1 &= \sin(q_1), \\ c_{12} &= \cos(q_1 + q_2), & s_{12} &= \sin(q_1 + q_2), \\ c_{123} &= \cos(q_1 + q_2 + q_3), & s_{123} &= \sin(q_1 + q_2 + q_3). \end{aligned}$$

B. Modeling Arm Support

During FES-supported movements, paraplegic patients need their arms to maintain balance and sustain desired movement. Support is taken in charge by two handles, each equipped with a six axis force/torque sensor, mounted on a supporting frame. Measuring handle support forces relates of upper body manoeuvres. This is why it is convenient to display and interpret the handle reaction vector.

Contact between the human hand and the handle creates a closed chain kinematic linkage. This interaction is described by the components of the resultant force vector F_c measured in the the x and z directions. Under the assumption of working in the sagittal plane and considering that the orientation of the forearm is colinear to the resultant force f_c , which is true for $F_x \geq 0$ and $F_z < 0$, it is reasonable to write the following hypothesis :

$$\theta \approx q_1 + q_2 + q_3 - \pi \quad (5)$$

Parameter θ is a known quantity and can be obtained by f_x and f_z measurements :

$$\theta = \arctan\left(\frac{F_x}{F_z}\right) \quad (6)$$

At this point, determining the subject's posture, given by vector \mathbf{q} , could be solved analytically through state-of-the-art tools by using inverse kinematics. Although the measured quantities P_x , P_z and θ , as well as anthropometric parameters l_j , are subject to uncertainty, and thus the problem cannot be solved by classical techniques.

C. A Set Membership Identification of Posture

Equations (3), (4) and (5) can be re-written as

$$\mathbf{g}(\mathbf{q}) = \mathbf{y} \quad (7)$$

where $\mathbf{y} = [P_x^h, P_z^h, \theta]^T$. The patient's posture is given by the \mathbf{q} vector, which can be obtained by solving (7). If

the measured quantities \mathbf{y} and anthropometric parameters L_j were known with no uncertainty, then the problem could be solved analytically through state-of-the-art tools by using inverse kinematics.

Solving (7) when \mathbf{y} is subject to uncertainty with classical techniques based on possibly weighted least squares optimisation for instance, derives reliable results only if the errors are stochastic and with known probability laws. In fact the measured data are subject to either stochastic or deterministic uncertainties and it is not easy to derive a reliable characterization of the probability distribution for these errors. Moreover, the model used may be based on some simplifying hypotheses for which a full probabilistic description might not be reliable. Consequently, it is more natural to assume all the uncertain quantities as unknown but bounded with known bounds and no further hypotheses about probability distributions. In such a bounded error context, the solution is no longer a point but is the set of all acceptable values of the \mathbf{q} vector, which makes the model output $\mathbf{g}(\mathbf{q})$ consistent with actual data \mathbf{y} and prior error bounds.

Denote \mathbb{E} a feasible domain for output error and $\mathbb{Y} = \mathbf{y} + \mathbb{E}$ the feasible domain for model output. The set \mathbb{S} to be estimated is the set of all feasible postures:

$$\mathbb{S} = \{\mathbf{q} \in \mathbb{Q} \mid \mathbf{g}(\mathbf{q}) \in \mathbb{Y}\} \quad (8)$$

where the set \mathbb{Q} is an initial search space for the \mathbf{q} vector. Characterizing the set \mathbb{S} is a set inversion problem since (8) can be rewritten as:

$$\mathbb{S} = \mathbf{g}^{-1}(\mathbb{Y}) \cap \mathbb{Q} \quad (9)$$

Equation (9) can be solved in a guaranteed way using a set inversion algorithm based on space partitionning, interval analysis [16], [17], [18] and constraint propagation techniques [19], [16], [17], [20], [21]. The algorithm SIVIA, *Set Inversion Via Interval Analysis* [19] explores all the search space without losing any solution. It makes it possible to derive a guaranteed enclosure of the solution set \mathbb{S} as follows:

$$\underline{\mathbb{S}} \subseteq \mathbb{S} \subseteq \overline{\mathbb{S}} \quad (10)$$

The solution set \mathbb{S} is enclosed between two approximation sets as shown in Fig. 2. The inner enclosure $\underline{\mathbb{S}}$ consists of the boxes that have been proved feasible. All elements of $\underline{\mathbb{S}}$ are solutions but there might be acceptable solutions that are not contained in $\underline{\mathbb{S}}$.

To prove that a box $[\mathbf{q}]$ is feasible it is sufficient to prove that $\mathbf{g}([\mathbf{q}]) \subseteq [\mathbb{Y}]$. If, on the other hand, it can be proved that $\mathbf{g}([\mathbf{q}]) \cap [\mathbb{Y}] = \emptyset$, then the box $[\mathbf{q}]$ is unfeasible. Otherwise, no conclusion can be reached and the box $[\mathbf{q}]$ is said undetermined. It is then bisected and tested again until its size reaches a threshold ε to be tuned by the user. Such a termination criterion ensures that SIVIA terminates after a finite number of iterations. The outer enclosure $\overline{\mathbb{S}}$ is defined by:

$$\overline{\mathbb{S}} = \underline{\mathbb{S}} \cup \Delta\mathbb{S} \quad (11)$$

where $\Delta\mathbb{S}$ is an uncertainty layer given by the union of all the undetermined boxes (with their widths not larger than ε).

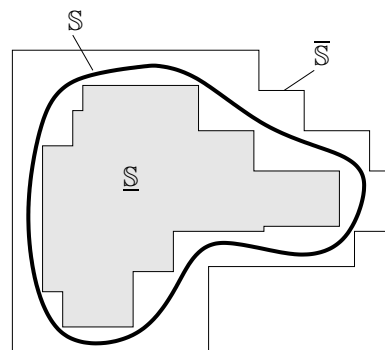


Fig. 2. Inner and outer enclosures of solution set \mathbb{S} .

The outer enclosure $\overline{\mathbb{S}}$ contains all solutions, if they exist, without losing any of them. It contains also some elements that are not solution.

III. THE EXPERIMENTAL PROCEDURE

A. Participants

Four spinal cord injured male subjects, with complete (or nearly complete) spinal lesions between T6 and T12, participated in the standing study program. The subjects physical characteristics are listed in Table II. The main selection criteria were the following: (1) participants show high motivation to the study, (2) post-injury standing experience, (3) appropriate contractions of the leg muscles in response to electrical stimulation, (4) sufficient upper body arm support strength to lift oneself up and maintain standing, (5) no cardiac or respiratory illness, (6) no previous stress fractures of upper and lower extremities, (7) no excessive body weight, (8) acceptable amount of spasticity and contracture in legs, (9) no psychological pathology.

The study was approved from the Consultative Committee for Protection of People Participating in Biomedical Research (CCPPRB Nîmes, France) and each patient who volunteered to participate in the experiments provided a written informed consent acknowledging the nature of the experiments and the risks involved.

B. Materials and Instrumentation

For leg muscle stimulation during standing, an eight channel stimulator was used. The self-adhesive surface electrodes were placed over the motor points area of the quadriceps, the gluteus maximus, the tibialis anterior and the biceps femoris

TABLE II
PHYSICAL CHARACTERISTICS OF PARTICIPANTS

	Subject 1	Subject 2	Subject 3	Subject 4
Gender	M	M	M	M
Age (yr)	30	50	18	29
Height (cm)	184	185	188	180
Weight (kg)	78	82	100	80
Level of injury	T8	T7	T12	T5
Post-injury (yrs)	2	13	1	1/2

(hamstrings) muscles of each leg (see Fig. 4). The stimulation device (Prostim, CE marked opto coupling) was driven directly in real time through a serial link by a PC. During active standing, patients were stimulated to predetermined FES constant currents, set up for each channel, in order to ensure safe standing.

A VICON 370 motion analysis system (Oxford Metrics, Oxford, UK), which included four infrared cameras, was used to acquire kinematic data at a sampling rate of 50 Hz. Sixteen reflective markers were captured and placed bilaterally on the following positions (Fig. 3): lateral malleolus, lateral femoral condyle, greater trochanter, lateral projection of the 12th thoracic/1st lumbar vertebrae, glenohumeral joint center (3 cm under the acromion), lateral humeral epicondyle, ulnar styloid process and on the auditory channel. The coordinates of the markers placed on each body segment defined the coordinate frame for each limb segment.

The handle reaction measuring system, comprising two six-axis transducers (Nano25, ATI Industrial Automation, Inc.), were attached to handles on a adjustable supporting parallel bars. The six components of the handle reactions are measured and displayed throughout a real time implemented force sensor interface software. The handles height and separation were set to comfort for each patient.

To collect plantar pressure distribution and determine ground reaction force (GRF), two flexible pressure insoles (pedar-system, Novel GmbH, Germany), each containing 99 sensors in a matrix design, operating at 40 Hz, were used. The insoles were fitted into the shoes of the subject. The pedar-system emitted an additional signal switching from 0 to 5 V was used to synchronize the VICON, the force sensors and insoles measuring systems.

A video recording of the experiments was made. The experimental arrangement is shown in Fig. 5.

C. Description of the Protocol

In a first session, the subjects have been exposed to daily FES exercises, for up to 1 hour per day during 5 days, in order to strengthen their quadriceps, gluteal maximus/medius, biceps femoris and tibialis anterior muscles. In a second session, following a thorough explanation of the study procedure, the patients, under FES, were instructed to stand up from a chair, assisted by parallel bars, and stay in standing position and sit back down. The stand phase was as long as one minute. This training phase has been repeated several times in order for the participants to become familiar with the testing equipment.

At session three, measurements were performed. The protocol here consisted in six trials, each composed of three phases (stand up, standing and sitting). Between each trial 5 minutes of rest were imposed. During the first three trials, subjects were asked to maintain an upright posture without any disturbances and as still as possible. In the last three trials, we proposed them to carry out a task, mobilizing their valid upper limbs. This task consisted in moving back and forth a small ball placed on a gutter beside the patient. The total duration of the experimentation for each subjects

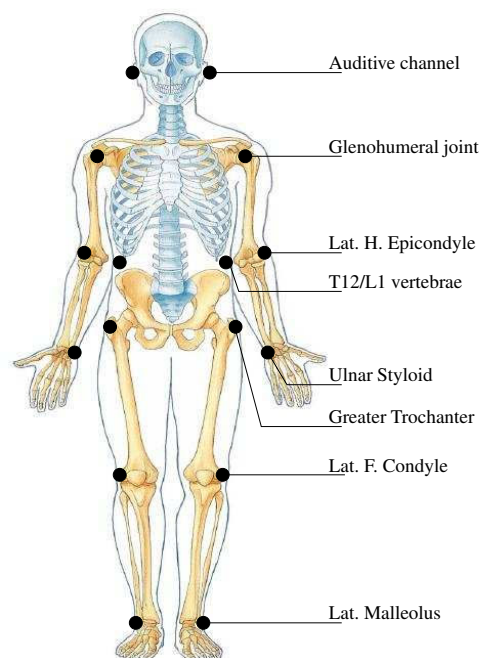


Fig. 3. Anatomical location of reflective markers.



Fig. 4. Electrode placement.

was of one hour and a half. In order to prevent falling, two experimenters stood on each side of the patient.

IV. RESULTS: POSTURE ESTIMATION

Recordings from the left and right side handles and from the shoe insoles were found to be very informative as for outlining the subjects' behavior during the experiments. The video and VICON recordings of each session were also very helpful when the results were later analysed. A typical measurement is shown in Fig.6 for a complete standing and sitting down procedure. Horizontal and vertical handle reaction components (F_x , F_z) were used in this case study in combination with the shoe insoles sensory signals.

Posture estimation was done during the standing phase at the beginning of the experiment in a one second interval, between 15 and 16 seconds, on Fig.6. The subject's actual



Fig. 5. Experimental set up.

posture during that time interval is shown in Fig.7 and were measured as :

$$q_1^r \approx 0^\circ, q_2^r \approx 192^\circ \text{ and } q_3^r \approx 10^\circ,$$

representing respectively the hip, shoulder and elbow joint angles. The body segment lengths were directly measured on the patient and are given by :

$$l_1 \approx 0.954 \text{ m}, l_2 \approx 0.518 \text{ m},$$

$$l_3 \approx 0.334 \text{ m}, l_4 \approx 0.262 \text{ m}$$

The feasible domain for model output are taken as:

$$\begin{aligned} P_x^h &\in [-0.02, 0.02] \text{ m} \\ P_z^h &\in [0.895, 0.995] \text{ m} \\ \theta &\in [-18.63, -15.63] \text{ degrees} \end{aligned} \quad (12)$$

The prior search space \mathbb{Q} , corresponding to the joints articular motion limit, is taken as:

$$\begin{aligned} q_1 &\in [-11, 90] \text{ degrees} \\ q_2 &\in [90, 210] \text{ degrees} \\ q_3 &\in [-103, 0] \text{ degrees} \end{aligned} \quad (13)$$

The SIVIA algorithm is implemented with the PROFIL/BIAS¹ interval library. The projections of the computed inner and outer solution sets, $\underline{\mathbb{S}}$ and $\overline{\mathbb{S}}$, onto the $q_i \times q_j$ planes are given in Fig.8 (with $\varepsilon = 0.01$).

These figures clearly show that the solution sets contain the actual posture (see also table III). The calculated subsets are consistent with the modelling (7) and prior domains (12) chosen for model output. The solutions obtained reflect the fact that for a fixed position of the forearm, defined by parameter θ , calculated by force measurements in the sagittal plane only, the hip, shoulder and elbow joints still have the possibility to reach other positions, while still consistent with the defined geometrical constraints. Contrary to any

optimization based techniques, there are no optimal solution, therefore any posture taken within the solution set is an acceptable one.

TABLE III
PROJECTION OF SOLUTION POSTURE ($\varepsilon = 0.01$)

Joints	Projection of inner enclosure	Projection of outer enclosure
q_1	[-1.35 , 25.52]	[-4.14 , 28.79]
q_2	[192.5 , 213.66]	[190.34 , 215.32]
q_3	[-74.10 , -31.05]	[-77.81 , -28.28]

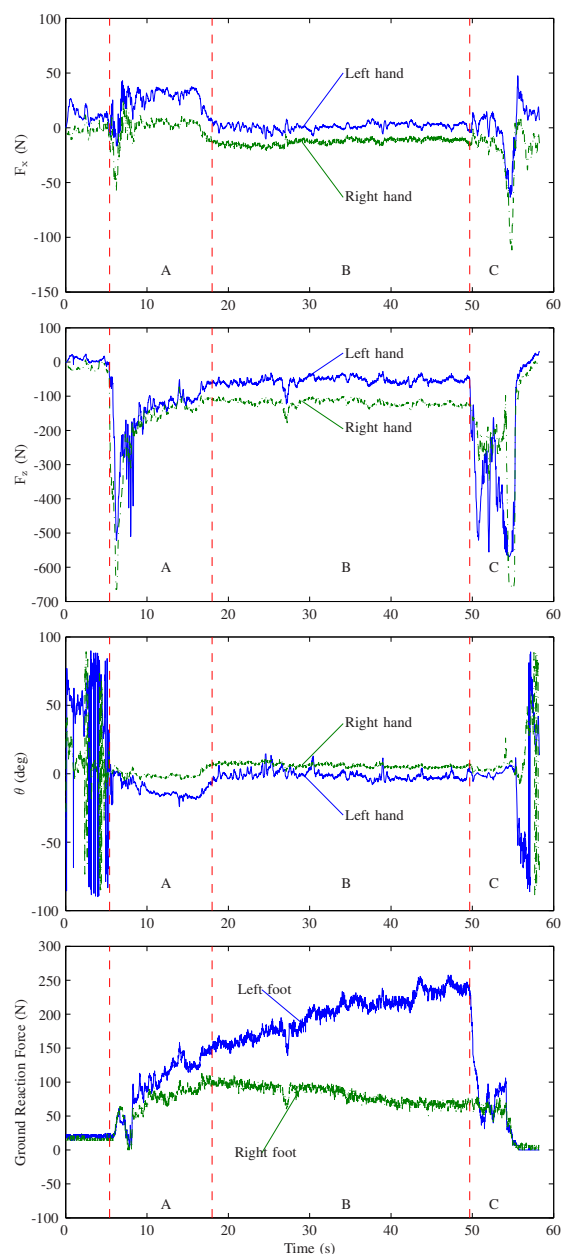
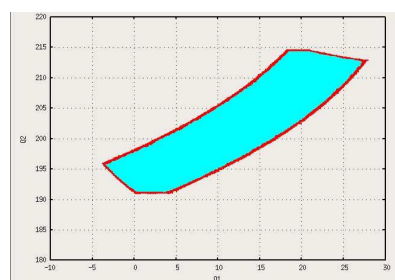


Fig. 6. Force recordings for subject S4 during standing. A : Standing up, B: Standing, C : Sitting down

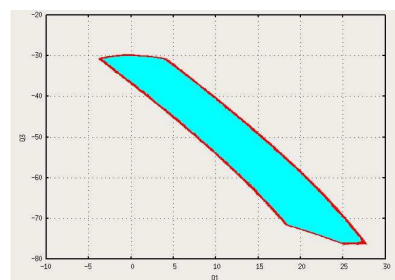
¹<http://www.ti3.tu-harburg.de/>



Fig. 7. Patient during a standing trial.



(a)



(b)

Fig. 8. Projection of the solution set onto (a) $q_1 \times q_2$ and (b) $q_1 \times q_3$ planes.

V. CONCLUSION

A method for reliable upper body posture estimation, based on measuring forces exerted on handles, has been introduced. The problem is solved in a set membership context and is tackled by interval analysis tools. An experimental study was carried out onto paraplegic subjects in order to validate the proposed methodology. Satisfactory results were obtained using a 2D model of the human body since we were able to guarantee that the real posture, i.e. joint positions, was included in the estimated domains. We were also capable of computing guaranteed bounds on the estimated postures which take into account all source of uncertainty.

The solution sets can be further reduced by introducing new constraints. This could be achieved by considering a 3D-based model of the human body and taking into account the three dimensionnal resultant force vector of each handle as well as the interactions between the feet and the ground by measuring ground reaction forces.

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