# Towards application of a mechatronic platform for whole-body isometric force-torque measurements to functional assessment in neuro-rehabilitation

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Abstract—Great amounts of raw data acquired with the use of an innovative mechatronic platform during an extensive clinical trial in a neuro-rehabilitation setting needs an analysis and interpretation. The platform records data from eight 6 DOF force-torque sensors during an isometric functional assessment of post-stroke patients. The identification of preprocessing parameters and onset detection methods, developed thanks to the close collaboration between biomedical engineers and clinicians, is presented in the paper. The present work presents also the implementation and testing of the software for the data pre-processing.

#### I. INTRODUCTION

THE approach for assessing the recovery state of stroke patients presented in this paper relies on repeated measurements of motor efforts during movement initiations for specific tasks. As the emphasis in stroke rehabilitation is on the improvement of functional performance, an ideal measuring tool must use Activities of Daily Living (ADL) tasks [1]-[4] as a principle for its quantitative measurements.

The correctness of performing the tasks is in line with important functional milestones that stroke patients acquire during recovery. The basic assumption inspiring this research work, is that the initiation of a task has the same functional properties as performing the task [5]-[8].

The presented platform was designed in order to perform isometric measurements during ADL tasks; the motivation for the isometric approach is based on the neurophysiological assumptions that in the first days after stroke, the active range of motion is very limited and that an

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isometric analysis at the start of a functional directed movement can overcome this problem. The outcome of these measurements can contribute to verify the integrity of a post-stroke existing or altered "internal model" for a particular functional task.

The importance of isometric measurements as an assessment in rehabilitation has been widely demonstrated for specific parts of the human body [9]. The measurements consist of time trajectories of isometric forces and torques obtained by a diagnostic device (ADD, Alladin Diagnostic Device) (Figure 1) equipped with eight dedicated 6 DOF sensors.



Figure 1. A view of the proposed mechatronic platform (ALLADIN Diagnostic Device)

The complete description of the ADD, the tasks to be performed and the measurement method are explained in detail in two public project deliverables [10], [11] and in previous papers [12]-[14].

This paper deals with the interpretation of the data collected during the clinical trials. A number of research issues have been faced in collaboration with the clinicians. Among them:

- detection of the onset movement time and the identification of a time window of interest;
- data pre-processing with the aim of extracting useful information from the recorded force-torque signals;
- data mining algorithms.

The paper addresses the first two phases while papers dedicated to the third one have been already presented [15].

#### II. DEFINITIONS AND APPROACH

As a convention, the eight force-torque sensors (JR3 Inc., Woodland, USA) have been numbered as described in Table I.

Thumb	Index	Middle	Arm	Trunk	Seat	Foot	Big Toe
0	1	2	3	4	5	6	7
RS	RS	RS	TS	TS	RS	RS	RS

Table I. Correspondence among the force-torque sensors, the indices and the mounting side(RS=Robotic Side, TS=Tool Side)

The data are acquired at a sampling rate of 100 Hz through two PCI acquisition boards, which means that three force and three torque values are captured for each sensor every 10 ms.



Figure 2. The coordinate systems according to the installation manual for JR3 force-torque sensors

An important issue of a multi-sensor dynamometric platform like the ADD, is to define a coordinate system for each sensor. A general convention is crucial to make the user correctly understand the measured data. The orientation of the sensor coordinate system depends on the mounting of the sensor. The coordinate systems are shown in Figure 2.

The sensor has two sides, henceforth named as "robot side" and "tool side". The sensor measures positive force and torque when the tool side is fixed and the robot side is loaded and moved.

For structural reasons, the sensors were fixed to the frame either with the tool side (TS) or the robot side (RS), according to the Table I.

Once the sensors mounting sides are defined, the interpretation of forces and torques measured by any of the sensors can be homogenously defined for the whole ADD.

Other general definitions needed for extracting meaningful parameters from the measured signals are here presented. The block diagram for selection of the parameters that have to be extracted by a dedicated pre-processing software tool is illustrated in Figure 3, in terms of overall functional architecture. Data processing is based on the following basic assumptions:

1. Parameters should be calculated for both forces and torques measurements of all four attempts (baseline

and three repetitions of the specific task) for all the sensors and all the tasks in each session;



Figure 3. Overall architecture of the APT - Alladin Pre-processing Tool

- 2. The usefulness of torques in terms of added-value for an effective data mining should be evaluated.
- 3. Stroke patients typically demonstrate reduced ability in controlling generated force-torque, both in intensity and in spatial direction, therefore force-torque vector direction and amplitude should reflect the presence of impairments and can be visualized by comparing deviations between current force-torque signals and previous force-torque trace.
- 4. The amount of information is huge, which means that it should be processed by data mining algorithms. Relevant standard statistical parameters should be extracted from the distribution of deviation angles over time.
- 5. Stroke patients demonstrate abnormal time activation patterns due to limitation in forward model generation, motion planning and supervision, sensory-motor control. The sequence of activation of the different sensors and the relative time delays during the execution of the same task should be of clinical interest for estimating 'distance to normality'.

## III. ONSET DETECTION AND PRE-PROCESSING PARAMETERS

## A. Activation time of the sensors (onset detection)

There is latency between the start of the voluntary muscular contraction of different body segments of the subject and the start time of the force-torque recording. The determination of the movement onset time in the recorded signals represents a fundamental aspect of the preprocessing analysis.

Starting from the in-depth review of the state-of-the-art techniques and after an internal debate between engineers and clinical experts, candidate methodologies for automatic onset time estimation were identified by:

- 1. the point where the force-torque signal reaches 2% of its peak value;
- 2. using a 2nd order derivative of the force-torque signal (with low-pass filtering at 3 Hz or at 5 Hz);
- 3. using the Spectral Flatness Measure (SFM) of the force-torque signal, based on a maximal information redundancy criterion;

4. using a Probability Density Function (PDF) estimate of the force-torque signal through a kernel smoothing based method (ks-density).

- *The 2% rule.* Former neuro-rehabilitation research inspired the proposed technique [16]. The input to the threshold-based algorithm consists of the three components of the force  $F_x$ ,  $F_y$  and  $F_z$  (or torque) signals.

It computes the 2% of the peak value on the signal and finds the minimum time corresponding to that value for each component. This value is taken as onset time.

- The second derivative method. A previous study on the gait analysis inspired the present technique [17]. Three versions of the present algorithm (b, c, d) have been developed. The description of the single steps follows: *a*. it finds the threshold point on the 1st derivative of the input signal at the 15% of its maximum. *b*. it searches the nearest maximum peak of the second derivative of the 3 Hz filtered signal (2nd derivative-filtered 3 Hz). *c*. it searches the nearest maximum peak of the second derivative of the 5 Hz filtered signal (2nd derivative-filtered 5 Hz). *d*. it searches backward the zero crossing in the first derivative line (2nd derivative-zero crossing). This is similar to the 2% rule, except that it scans backward from a higher speed, so initial small velocity peaks are neglected.

- *The SFM method*. The SFM method is thoroughly described in [18].

- *The kernel smoothing based method (ks-density)*. The ks-density function computes a PDF estimate of the input vector. Typically stationary values (e.g. flat regions) of force-torque signal correspond to maxima of the PDF while values where the slope of the signal is high generally correspond to minima of the PDF.

The algorithm locates the minimum of the local minima (Minimum Density Point, MDP) in the ks-density function [19]. A first version of the PDF estimation algorithm outputs the MDP as the onset time. In a second version, the intersection of the line passing through the MDP with a slope equal to the mean value of the first derivatives of an arbitrary interval around the MDP is computed.

Table II presents the results of the comparative analysis among the performances of the different techniques with respect to the reference performance of three clinical experts.

Onset technique	Mean value (s)	Standard deviation (s)	Variance (s)	Median (s)	РОС
2% rule	0.5080	0.8524	0.7266	0.3887	0.57
SFM	0.0968	0.7870	0.6194	0.0990	0.69
ks-density	- 0.0252	0.7959	0.6335	- 0.1766	0.71
2nd derivative (zero crossing)	0.3361	0.6186	0.3827	0.0992	0.78
2nd derivative (filtered 3 Hz)	0.2136	0.6223	0.3873	- 0.0188	0.89
2nd derivative (filtered 5 Hz)	0.2044	0.6241	0.3894	- 0.0276	0.89

Table II. Results of the comparative analysis on the first reference dataset

First, the Mean Reference Vector (MRV) was derived by computing the mean of onset values provided by the experts. Then the Mean value, Standard Deviation, Variance and Median of the error vector related to each of the techniques were calculated (columns 2-5).

Finally, also a non-parametric statistical feature, defined as the Probability Of Correctness (POC), was computed. POC is calculated as the ratio Nc/N, where N is the total number of samples and Nc is the number of samples, which fall between the 5th-percentile and the 95th-percentile of the MRV.

The best technique for onset time estimation was selected by comparing the results obtained from the onset detection algorithms with the results given by the visual inspection of clinical experts (performed via a modified version of the Visualization Tool described in Section IV D). The comparison was performed on a reference data set by selecting 96 sample measurements. It was important that the chosen solution could guarantee a minimum loss of useful information needed as entry for the data mining module.

The analysis on the dataset has enlightened the following results:

i) all the four proposed onset detection techniques work properly for detecting the onset in terms of Signal to Noise Ratio;

ii) the proposed techniques can be used to remove those parts of the signal which are useless;

iii) the first phase of the data mining stage should be dedicated to the identification and recognition of typical pattern, which then could allow a narrower time windowing.

A second and larger reference dataset was identified, prepared and delivered to the clinical experts in order to perform a second onset estimation. The additional comparative analysis between the automatic techniques and an extended set of manually detected onset times demonstrated that the results were not significantly different from those previously obtained.

## B. Time window of interest

As the length of the recorded force-torque signal increases, also the computational burden increases. In order to keep it as low as possible, a novel approach for the development of pre-processing techniques was proposed: the basic idea is that only the portion of signal with sound content will be used for further processing, instead of the whole raw signal.

The selection of a suitable time window must handle the trade-off between keeping any useful information and reducing computational burden.

The measurement recording time during different ADL tasks ranges from a minimum of 2.4 s to a maximum of 5.4 s, depending on the specific ADL task (Table III).

From a clinical point of view, the data of interest to be extracted from the ADD measurements are conveyed by the very initial part of each recording, before the patient starts to adapt to the isometric constraint.

Therefore, in order to extract meaningful parameters, the complete force and torque signals at a given sensor were

considered only within a finite-length analysis frame. Time window starts from the estimate of the onset time and lasts a few hundreds of milliseconds, corresponding to a finite number N of samples.

Task	Baseline (0)	Video (1)	1 <sup>st</sup> rep (2)	2 <sup>nd</sup> rep (3)	3 <sup>rd</sup> rep (4)
Glass	3.0	5.4	5.4	5.4	5.4
Key	3.0	3.7	3.7	3.7	3.7
Spoon	3.0	3.4	3.4	3.4	3.4
Bag	3.0	2.4	2.4	2.4	2.4
Reaching	3.0	4.0	4.0	4.0	4.0
Moving	3.0	6.0	6.0	6.0	6.0

Table III. Duration times (s) of the different recordings during a typical ADD session

At the moment there is no definite frame length and the duration of the time window can be easily adapted in order to cope with possible uncertainties in the onset time estimation as illustrated in the following sub-section.

After a time window was identified and applied to the original signals of the input data set, a set of parameters was extracted from these data.

The choice of these parameters is the result of an internal debate between clinical partners, bioengineers and data mining experts. The ultimate aim was to identify parameters with a clinical soundness and appropriate to be processed by the data mining algorithms in order to estimate 'distance from normality' of the different patients along rehabilitation period.

#### C. Parameters definitions

A recording was defined as the set of force and torque measurements at a given measurement site, for a given patient, during a given session and for a given task. Hence, every recording is uniquely identified by the site identifier, the patient identifier, the session number, the task and attempt number.

The recordings for all these combinations represent a large amount of raw data to be processed in order to capture relevant characteristic features with respect to stroke patient recovery.



Figure 4. Example of deviation from the mean direction vector (red) for a sample measurement from a normal control

The iterative identification process of suitable parameters was done in collaboration with the clinicians and yielded to the following four main categories:

- Angular deviations from the mean direction. The underlying hypothesis relies on the consideration that trajectories in pathological subjects could show larger deviations from the mean direction than in normal controls. Figure 4 shows an example of such deviations from the mean trajectory in a normal control.

- Angular deviations between successive effort samples. The smoothness of the effort can be evaluated by computing the angle between successive force and torque samples.

- *Cumulative sum of effort series*. The integrals of the effort signals are expected to convey some information on the velocity of the imaginary movements and on the stroke patient ability to perform some movement velocity patterns thereof.

- *Cross-sensor time delay estimation*. For each of the proposed ADL task, a correct synchronization among the different parts of the body is needed for an optimal performance. The synchronization among the forces and torques during the recording of the isometric task can be computed by means the theoretical statistical dependency, known as mutual information [20].

## IV. IMPLEMENTATION AND TESTING OF DATA PRE-PROCESSING SOFTWARE

The data pre-processing software is composed by different modules (Figure 3), which have been implemented using the Matlab environment v6.5 (The Mathworks, Inc. Natick, USA) whose functional description follows.

## A. Alladin Pre-processing Tool

The Alladin Pre-processing Tool (APT) is a software tool that automatically derives specific parameters from the ADD recordings; store the output data into a structure using a format for subsequent data mining analysis that has to lead to the extraction of clinical markers and milestones, relevant for functional assessment of patients.

The APT also includes a Visualization Module which allows visual inspection of data during the pre-processing operations.

## B. Alladin Download Module

The Alladin Download Module, that was developed in Visual Basic, provided a user-friendly access to the Alladin Global Database. The implementation of ADM is foreseen for future upgrade of the APT in view of its final integration.

## C. Alladin Filtering Module

A two-channel parallel low-pass filtering, one featuring a cut-off frequency at 40 Hz and another with a cut-off frequency at 2 Hz was proposed and implemented in order to provide two separate data sets for subsequent processing. The two cut-off frequencies were selected taking into account that, on one hand, human muscles can generate mechanical signals up to a maximum frequency of 40 Hz

(muscle sound) [21], while, on the other hand, human voluntary movement typically generates signals within the frequency range 0-2 Hz [22].

The 40 Hz-channel is the main channel used for feature extraction, while the 2 Hz-channel is used for visualization and onset time estimation operations.

#### D. Alladin Visualization Module

The ALLADIN Visualization Module (AVM) was developed in order to visualize the ALLADIN measurements (Figure 5). Through the controls positioned on the main window, the patient ID, session, task and measurement number can be selected. Data filtering, calculations (minimum, maximum, mean) and coordinate transformations can be applied to the measurements, and plotted for inspection.

Also a slightly different version of AVM was implemented with the aim of simplifying the clinical experts' task. The module allows manual selection of the onset time directly on the plot, by simply clicking on the window by using the PC mouse.



Figure 5. Main AVM window

#### E. Alladin Feature Extraction Module (AFEM)

The Alladin Feature Extraction Module (AFEM) receives the filtered data from the 40 Hz-filtered channel of the AFM and generates the output data containing statistical and temporal features calculated for all the ADD measurements of the input data set.

The AFEM computes the complete list of parameters based on the assumptions given in Section II and the definitions described in Section III C. The APT generates, through the AFEM module, an output data structure variable (F).

The extracted parameters for every recording were stored in the above hierarchical structure of strings, arrays and cell arrays containing the identification information as well. Every stored parameter presented a description and a value.

An example follows.

$$= [0.077 \ 0.012 \ 0.021 \ 1.578 \ 4.287 \ 2.799 \ 0.139]$$

pF is the vector containing the parameters calculated on the angular deviations force vector, where MaxdF is the maximum values, MeandF is the mean value, StddF is the standard deviation, SkewdF is the value for the skewness, KurtdF is the value for the kurtosis [23], colatdF and azimdF corresponds to the colatitude angle and azimuth angle, respectively [24]. All parameters are computed on the vector of angular deviations from the mean force vector.

## F. AFEM testing

Test signals were created by Matlab scripts: for each Cartesian reference axis (x, y, z), force and torque trigonometric signals having a length of of 400 ms were generated. The choice of such signals (sine and cosine with different amplitudes) and the relative time interval ( $6\pi$ ) is adequate for reproducing a signal with 3 peaks, ideally corresponding to the 3 repetitions of the typical recording during a measurement session (Figure 6).



Figure 6. Testing signals for the force (a) and the torque (b) components

Each test signal was passed as input vector to the AFEM and the output vector was compared with the explicit calculation of the statistical parameters. As expected, a null vector was been obtained as the difference between the previous two vectors.

The test was performed for both force and torque signals. The results obtained from the tests performed on the AFEM module, after its validation, suggests that the AFEM module properly calculates all the parameters defined so far.

#### V. CONCLUSIONS AND FUTURE WORKS

This paper describes the technical issues on the use of a mechatronic platform for whole-body isometric force-torque measurements for functional assessment in neuro-rehabilitation. In particular, thanks to the close collaboration between medical doctors (physiotherapists, neurologists, etc.) and biomedical engineers, after several clinical tests, a multidisciplinary approach could be proposed to simplify the problem of handling the great amount of acquired raw data. In the proposed approach the relevant part of the raw signal (i.e., the part in which the force-torque exerted by the patient is clearly visible) was selected through the use of a series of movement onset detection algorithms. Then a first set of parameters were extracted as possible feature candidates in a preprocessing stage.

These pre-elaborated data input to data mining, will strongly decrease the computational workload. The thorough analysis performed during this work will be used to further investigate if specific body segments are involved during particular tasks and/or if the addition of one or more sensors to the platform could provide further useful dynamic information. All this information will lead to a possible redesign of the platform, with the aim to improve the present version of the device for functional assessment and for basic research in the Neuroscience domain as well.

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