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Optimal Admission Polices for a Retailer of Seasonal Products with Drop-Shipping

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Abstract— **This paper studies optimal inventory rationing policies for a retailer of perishable products who sells through its own stores and third party websites by an affiliate program. By posting on partners' webpages, an affiliate program allows the retailer to attract more customers who otherwise would be missed. However, the retailer needs to pay out a commission for each sale originated from the website operator that participates in the affiliate program. Thus, the net revenue of selling one unit of product to an online "referral" (online customer) is less profitable than that to a customer from a physical store. When the inventory at stores is running low, the retailer may further refer the online request to somebody else for fulfilling, which is equivalently to say that the retailer can reject online customer requests. Therefore, upon the arrival of any demand through the affiliate program, the retailer needs to decide whether or not to accept it; and if so, assign which of multiple outlets for the fulfillment. Based on a discrete-time dynamic programming model, the optimal admission policy of the retailer is analyzed in this paper, which is shown to be a two-dimensional threshold policy. The structural properties of the revenue function are analyzed, and numerical examples are given to show the revenue impact of optimal admission control.**

I. Introduction

Consider a retailer which sells through both physical stores and an online outlet, or with the so-called "clicks-and-bricks" business model (Gulati and Garino, 2000). The online outlet does not carry any inventory and operates under a drop-shipping arrangement with its physical stores, i.e., all online orders are fulfilled by the physical stores. In addition, the online outlet also uses third-party websites with some Affiliate Programs (APs) to attract more online customers. However, a certain amount of commission is incurred for each sale that comes from an AP. The extra cost that is associated with shoppers who first click through to the third-party websites makes them less attractive as customers than those who directly visit the etailer's online or physical store. Therefore, the fulfillment of in-store demand is always a priority relative to the AP's drop-shipping request. When the stock runs "low", the retailer has to decide whether to fulfill the AP's demand or further refer it to other sources (i.e., reject it) such as a distributor that supplies the retailer. Two interesting issues emerge from this scenario: given the initial inventory level, should the retailer accept the AP's order or reserve the item for its own future demand? If the drop-shipper decides to fulfill the online order, then which physical store should be assigned to execute the fulfillment?

This paper seeks to shed insight into the problems by considering a stylized drop-shipping model as follows. A retailer operates two physical stores which are located in different territories (say, territory 1 and 2) as well as an online store which keeps zero inventory. Thus, online orders to the online outlet will be fulfilled by drop-shipping of one of the physical stores, which depends on where an online customer originates – if she come from territory 1 (or 2), then her order will be met by the store in the same territory unless it runs out of stock. We do not differentiate two types of demands, i.e., selling either from the physical stores or from the online store earns the retailer the same revenue for each sale. Another stream of online orders is directed from other websites under an affiliate program, which is less profitable. We will call demand from this stream *online demand* for simplicity, and call all of the third party websites the *etailer*. In other words, the less profitable demand stream generates online demand to the etailer which then forwards it to the retailer for fulfillment. The product is seasonal and has a finite selling period during which it is un-replenishable. The retailer dynamically rations the demand from the etailer and assigns one of the stores for the order-fulfillment, based on the inventory levels of both stores and the information regarding where the online customer comes from, with the objective of maximizing its expected revenue.

We formulate the above setting as a discrete-time dynamic programming model which addresses the optimal admission policy of the retailer. It is shown that the optimal admission policy is of the two-dimensional threshold type. The structural properties of its revenue function are also characterized. In the rest of the paper, Section II briefly surveys the related literatures. Section III formulates the model and optimal admission policies. Numerical experiment is conducted in Section IV and finally, concluding remarks are given in Section V. All technical proofs are available from the authors.

II. Literature Review

The research presented in this paper relates closely to two areas: order fulfillment for online retailers, and revenue management and inventory rationing.

A. Order Fulfillment of On-line Retailing

It has been widely recognized that the potential integration of the on-line and physical operations has provided many traditional retailers with opportunities to both increase their market share and improve service. However, the academically-oriented research on such channels coordination and related issues focuses largely on the marketing aspect of Internet retailing, and is mostly qualitative in nature (for example, see Gulati and Garino, 2000; Ulaga, 2004; Biyalogorsky and Naik, 2003; and de Koster, 2003). Studies published in traditional operations management journals on the subject of electronic retailing are scant, and the literature which addresses the order fulfillment issues of online retailers through mathematical modeling is even more scant.

Through simulation Bendoly (2004) studies a wide range of substitute-inventory availability scenarios, where the multiple decentralized neighboring stores are sources for the on-line fulfillment of any given order, and the in-store demand is always a priority relative to on-line orders at any given store. In Ayanso et al. (2004), the online retailer uses both in-house and "drop-shipping" as order fulfillment options. Based on a simple mathematical model, Chen et al. (2005) use numerical experiments to assess two inventory strategies of online retailers: carrying own inventory and outsourcing inventory to another party. Netessine and Rudi (2003) evaluate three inventory options of the online retailer: outsourcing inventory, i.e., drop-shipping; carrying own inventory; and a combination of both, i.e., the retailer uses local inventory as a primary source and relies on drop-shipping as a backup. Netessine and Rudi (2004) analyze the interaction between a wholesaler and an online retailer for a drop-shipping supply chain, where the retailer is involved in the marketing and advertising activities and the wholesaler takes care of the fulfillment business.

B. Revenue Management/Inventory Rationing

The task of dynamically allocating inventory to different demand classes lies at the heart of many revenue management models. Examples of dynamic allocation models include Lee and Hersh (1993), Subramanian et al. (1999), Feng and Xiao (2000). Interested readers are referred to McGill and van Ryzin (1999) and Talluri and van Ryzin (2004) for an in-depth survey of revenue management problems with multi-fare classes.

Inventory rationing is also a revenue management methodology, where some inventory is reserved in anticipation of demand from higher margin customers. Veinott (1965) was one of the first to consider multiple demand classes in a multi-period model. Topkis (1968) extends Veinott's work by considering how inventory should be allocated between demand classes. With the initial quantity of inventory being given, Gerchak et al. (1985) study the dynamic optimal rationing policies within a fixed time horizon like ours in this paper. Frank et al. (2003) consider a periodic review inventory system with two priority demand classes, one deterministic and the other stochastic. The decision maker has the option to ration inventory to the stochastic source.

The research presented in this paper differs from the previous revenue management and inventory rationing literatures in that most of the previous models focus on the capacity control of a single resource. In contrast, we analyze a two-resource capacity control problem, in which the physical retailer has multiple stores that can be assigned to fulfill the etailer's drop-shipping requests. The retailer optimizes the global revenue by dynamically assigning one of the stores to fulfill the online order.

III. The Model

A. Description, Notation and Assumptions

The retailer operates two physical stores which are located at territories 1 and 2, respectively. The two stores have Q_1 and Q_2 units of identical products to sell within a finite time horizon $[0, T]$ without any replenishment opportunity. The physical transshipment of inventory between the two stores is not allowed (i.e., transhipment is rather expensive). The retail prices charged by different stores can be different, denoted as p_i, $i = 1, 2$, which are fixed at all times. For simplicity, assume demand at a physical store (including demand originated from the online outlet but better to be delivered by that physical store) follows a homogeneous Poisson process with arrival rate λ_i, $i = 1, 2$. However, the model is readily to be extended to consider time-varying demand rates.

The online orders through the etailer are fulfilled under the drop-shipping agreement with the retailer. The etailer's online demand rate is denoted as λ_e, which consists of customers coming from both territories 1 and 2. Denote by β_1 and β_2 the percentages of online customers who come from territory 1 and 2, respectively, where $\beta_1 + \beta_2 = 1$. Therefore, the arrival rate of online customers from territory i $(= 1, 2)$ is $\beta_i \lambda_e$.

For any product sold through the etailer, the retailer has to share some profit with the etailer. Generally, it is much more economical and preferable to supply the online customers from territory $i (= 1, 2)$ by store i, since it saves delivery costs and provides shorter response time. Let w_i and $c_i (i = 1, 2)$ denote the unit revenue deductions for each unit being sold by store i to territory-i and territory-$j (= 3 - i)$ online customers. That is, to deliver one unit of product to online customers located in territory 1 (or 2), if store 1 (or 2) is assigned, the unit revenue that can be collected will be $p_1 - w_1$ (or $p_2 - w_2$); otherwise if store 2 (or 1) is assigned, the unit revenue will be $p_2 - c_2$ (or $p_1 - c_1$). We assume the following assumptions:

Assumption 1: For each physical store i $(= 1, 2)$, the unit net revenue from selling to the online customers of the same territory is larger; i.e., $w_1 < c_1 < p_1$, and $w_2 < c_2 < p_2$.

Assumption 2: For each online demand coming from territory i $(= 1, 2)$, the unit net revenue is larger if it is fulfilled by the store that is located within the same territory; i.e., $p_1 - w_1 \geq p_2 - c_2$, and $p_2 - w_2 \geq p_1 - c_1$.

Each customer demand from either channel is of unit size. For tractability, we assume that the three customer flows, λ_1, λ_2, and λ_e are independent of each other. We consider a discrete-time setting, i.e., the time horizon $[0,T]$ is divided into small intervals of equal length Δt, each of which will be called a period. Δt is sufficiently small such that not more than one demand arrives within each period. Without loss of any generality, we let $\Delta t = 1$. (This can be done by re-scaling time and adjusting inter-arrival times of customer arrivals). A slight modification will convert the discrete time model to a continuous time model (see Gerchak et al., 1985).

Suppose at the beginning of period t, the inventory levels at each store are n_1 and n_2, respectively, then the state of the retailer can be characterized by a three-dimensional vector (t, n_1, n_2), where $t \in \{1, 2, \cdots, T\}$, $0 \leq n_i \leq Q_i (i = 1, 2)$. For notational convenience but without loss of generality, the salvage value of any items that are left unsold at the end of the sales horizon is assumed to be zero. The retailer is risk-neutral, and seeks to maximize its own expected revenue of the two stores, by dynamically rationing the finite inventory to its own demand and the etailer's online demand.

B. Optimal Admission Policy

Let $R_t(n_1, n_2)$ be the maximal total expected revenue of both stores that can be achieved in interval $[t, T]$ when the state is (t, n_1, n_2). In the last period, the retailer should accept any request as long as there are some inventories left (or else, the inventories will become worthless). Because the net revenues of assigning different stores to supply online customers from different territories may not be the same, the retailer should arrange the store with the higher margin to supply the online customer. Conditioning on all the possible events that may happen in period T, we have

$$R_T(n_1,n_2) = \begin{cases} 0, (n_1 = 0, n_2 = 0); \\ \lambda_1 p_1 + \lambda_e \beta_1 (p_1 - w_1) + \lambda_e \beta_2 (p_1 - c_1), (n_1 > 0, n_2 = 0); \\ \lambda_2 p_2 + \lambda_e \beta_1 (p_2 - c_2) + \lambda_e \beta_2 (p_2 - w_2), (n_1 = 0, n_2 > 0); \\ \lambda_1 p_1 + \lambda_2 p_2 + \lambda_e \beta_1 (p_1 - w_1) + \lambda_e \beta_2 (p_2 - w_2), \\ \qquad (n_1 > 0, n_2 > 0). \end{cases}$$

Next we proceed to analyze the revenue functions for $1 \leq t \leq T-1$. The recursive formulation depends on the inventory levels of both stores. For $n_1 = n_2 = 0$, i.e., both stores have sold out the product, it is apparent that

$$\forall t, 1 \leq t \leq T-1,\ R_t(0,0) = 0. \tag{1}$$

When one of the stores (say, store 1) runs out of its inventory, i.e., $n_1 = 0$, upon the arrival of any order-fulfillment requests from the etailer and after observing where the online customer comes from, the retailer decides whether or not to supply the online customer with store 2. For notational convenience, we denote

$$R_t^1(n_1) := R_t(n_1, 0),\ R_t^2(n_2) := R_t(0, n_2).$$

Consider the case when $n_i > 0, n_j = 0 (i = 1, 2, j = 3 - i)$. The problem faced by the retailer is similar to the inventory rationing with 3 demand classes: the in-store demands, the online demands from territory i, and the online demands from territory j. Following the literature on revenue management, we have for $\forall n_i, n_i > 0, n_j = 0$:

$$\begin{aligned} R_t^i(n_i) = R_{t+1}^i(n_i) &+ \lambda_i (p_i + R_{t+1}^i(n_i - 1) - R_{t+1}^i(n_i)) \\ &+ \lambda_e \beta_i (p_i - w_i + R_{t+1}^i(n_i - 1) - R_{t+1}^i(n_i))^+ \\ &+ \lambda_e \beta_j (p_i - c_i + R_{t+1}^i(n_i - 1) - R_{t+1}^i(n_i))^+, \end{aligned} \tag{2}$$

where $a^+ := \max(0, a)$. The following optimal decision rule and structural properties regarding the revenue functions are well known and easy to verify.

Decision Rule 1: When store j's inventory level decreases to zero, store i holds a positive inventory $(j = 1, 2, i = 3 - j)$, and an online order from territory k $(= 1, 2)$ arrives in period t, the retailer should assign store i to fill the online customer if $k = i$ and the following relation holds:

$$p_i - w_i + R_{t+1}^i(n_i - 1) \geq R_{t+1}^i(n_i);$$

the retailer should assign store i to fill the online customer if $k = j$ and the following relation holds:

$$p_i - c_i + R_{t+1}^i(n_i - 1) \geq R_{t+1}^i(n_i);$$

otherwise, the retailer should reject the etailer's drop-shipping request.

Theorem 1: The value function $R_t^i(n_i)$ exhibits the following structural properties, i=1,2:

(a) $R_t^i(n_i)$ is increasing in n_i and decreasing in t;
(b) $R_t^i(n_i) - R_t^i(n_i + 1)$ is increasing in n_i and t.

Here and below, "increasing" and "decreasing" are used in the non-strict sense, meaning non-decreasing and non-increasing, respectively. Theorem 1 means that the revenue function $R_t^i(n_i)$ is increasing concave in the inventory level n_i, and decreasing concave in the time period t.

Next, we proceed to investigate the cases when both stores hold inventories (i.e., $n_1 > 0, n_2 > 0$) at the beginning of period t. When an online demand occurs, the retailer needs to decide not only whether or not to accept it, but also which store to fulfill the order (if it decided to accept). Therefore, the retailer faces three options: arrange store 1 to fulfill the order, arrange store 2 to fulfill the order, or just reject the drop-shipping request. The optimal decision depends on the relative outcomes of the three options. Conditioning on the possible events in period t, we have the following recursive formulation:

$$\begin{aligned} R_t(n_1, n_2) = &\lambda_1 [p_1 + R_{t+1}(n_1 - 1, n_2)] \\ &+ \lambda_2 [p_2 + R_{t+1}(n_1, n_2 - 1)] \\ &+ \lambda_e \beta_1 \max \begin{cases} p_1 - w_1 + R_{t+1}(n_1 - 1, n_2) \\ p_2 - c_2 + R_{t+1}(n_1, n_2 - 1) \\ R_{t+1}(n_1, n_2) \end{cases} \\ &+ \lambda_e \beta_2 \max \begin{cases} p_1 - c_1 + R_{t+1}(n_1 - 1, n_2) \\ p_2 - w_2 + R_{t+1}(n_1, n_2 - 1) \\ R_{t+1}(n_1, n_2) \end{cases} \\ &+ (1 - \lambda_1 - \lambda_2 - \lambda_e) R_{t+1}(n_1, n_2), \end{aligned} \tag{3}$$

where on the right-hand side (RHS), the first and second blocks represent the expected revenues from the physical channels. Within the maximum operator of the third and fourth blocks, a comparison is made between assigning store 1 to fulfill the online order, assigning store 2 to fulfill the online order, and rejecting the drop-shipping request. The last block represents the revenue corresponding the event that no demand arrives. All of possibilities have been taken into consideration in the right-hand side. Accordingly, we state the optimal admission policy as follows.

Decision Rule 2: When both stores hold inventories on-hand at the beginning of period t, and an online order placed by an online customer from territory i $(=1,2)$ arrives in this period, the retailer should assign store i to fulfill the order if the following relation holds:

$$\begin{aligned} & p_i - w_i + R^i_{t+1}(n_i - 1, n_j) \\ \geq \quad & \max\{p_j - c_j + R^j_{t+1}(n_j - 1, n_i), R_{t+1}(n_1, n_2)\}, \end{aligned}$$

where $j = 3 - i$, $R^i_{t+1}(n_i - 1, n_j)$ equals $R_{t+1}(n_1 - 1, n_2)$ if $i = 1$ and $R_{t+1}(n_1, n_2 - 1)$ otherwise; the retailer should assign store j to fulfill the order if:

$$\begin{aligned} & p_j - c_j + R^j_{t+1}(n_j - 1, n_i) \\ > \quad & \max\{p_i - w_i + R^i_{t+1}(n_i - 1, n_j), R_{t+1}(n_1, n_2)\}; \end{aligned}$$

otherwise, the retailer should reject the etailer's drop-shipping request.

We have characterized the structural properties of the revenue function, $R_t(n_1, n_2)$, and the major results are summarized in the following Theorem.

Theorem 2: For $\forall t (1 \leq t \leq T)$, the retailer's revenue function $R_t(n_1, n_2)$ holds the following properties:

(a) $R_t(n_1, n_2) - R_t(n_1 + 1, n_2)$ is increasing in n_1 and n_2;
(b) $R_t(n_1, n_2) - R_t(n_1, n_2 + 1)$ is increasing in n_1 and n_2;
(c) $R_t(n_1 + 1, n_2) - R_t(n_1, n_2 + 1)$ is decreasing in n_1, and increasing in n_2.
(d) $R_t(n_1, n_2) - R_{t+1}(n_1, n_2)$ is decreasing in $n_i (i = 1, 2)$, i.e., $R_t(n_1, n_2)$ is sub-modular in (t, n_i);
(e) $R_t(n_1, n_2) - R_{t+1}(n_1, n_2)$ is increasing in t, i.e., $R_t(n_1, n_2)$ is concave in t.

Decision Rule 2 implies that the optimal admission policy is essentially characterized by the marginal expected revenues with respect to the inventory levels at both stores. Based on the above properties (a) – (d) of the revenue function $R_t(n_1, n_2)$, we have the following properties regarding the optimal admission policy of the retailer.

Proposition 1: The retailer's optimal admission policy exhibits the following structures. When the state is (t, n_1, n_2) and an online customer from territory i $(=1,2)$ arrives:

- If the retailer's optimal decision is to assign store i for the fulfillment, then the retailer should also assign store i to fill the online customer from territory i for all the states $(t, \tilde{n}_i, n_j)$ where $\tilde{n}_i > n_i$.
- If the retailer's optimal decision is to assign store $j (= 3 - i)$ for the fulfillment, then the retailer should also assign retailer j to fill the online customer from territory i for all the states $(t, n_i, \tilde{n}_j)$ where $\tilde{n}_j > n_j$.
- If the retailer's optimal decision is to reject the drop-shipping request of the etailer, then the retailer should also reject to fulfill the online customer from territory i for all the "lower" states $(\tilde{t}, \tilde{n}_1, \tilde{n}_2) \leq (t, n_1, n_2)$.

To further characterize the optimal admission policy upon the arrival of an online customer from territory i (say, $i = 1$), we define the following three curves:

$$\begin{aligned} L^1_t(n_2) := \min\{n_1 : & R_{t+1}(n_1 - 1, n_2) - R_{t+1}(n_1, n_2) \\ & \geq w_1 - p_1\}; \\ L^2_t(n_1) := \min\{n_2 : & R_{t+1}(n_1, n_2 - 1) - R_{t+1}(n_1, n_2) \\ & \geq c_2 - p_2\}; \\ L^3_t(n_2) := \min\{n_1 : & R_{t+1}(n_1 - 1, n_2) - R_{t+1}(n_1, n_2 - 1) \\ & \geq w_1 - p_1 - c_2 + p_2\}. \end{aligned}$$

Theorem 3: The running trends of the above three curves go as follows:

(a) $L^1_t(n_2)$ is decreasing in n_2;
(b) $L^2_t(n_1)$ is decreasing in n_1; and
(c) $L^3_t(n_2)$ is increasing in n_2.

Now we consider the situation when an online customer from territory 1 arrives in period t:

- When the retailer's inventory levels, (n_1, n_2) is above $L^1_t(n_2)$, i.e., $n_1 \geq L^1_t(n_2)$, the retailer will be better off to assign store 1 to fulfill the online customer than to reject the etailer's request;
- Similarly, when (n_1, n_2) lies above $L^2_t(n_1)$, i.e., $n_2 \geq L^2_t(n_1)$, assigning store 2 to fulfill the online customer is better than denying the etailer;
- The third curve, $L^3_t(n_2)$ splits the areas that assigning store 1 dominates assigning store 2 for the fulfillment. That is, when (n_1, n_2) lies above $L^3_t(n_2)$, it's better to fulfill the online customer that comes from territory 1 by store 1 than by store 2; or else, it's better to fulfill by retailer 2.

Therefore, the three curves, $L^k_t(n_2)$, $k = 1, 2, 3$, split the two-dimensional inventory space into three areas, and the optimal admission decisions regarding whether or not to fulfill the online customer from territory 1, and which store is assigned for the fulfillment depend on the location of the inventory levels, which are depicted in Figure 1.

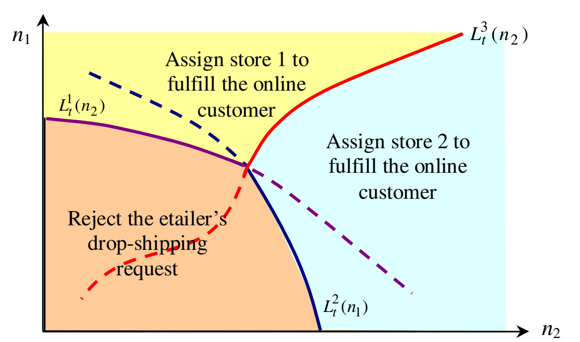

Fig. 1. Optimal Decision upon the Arrival of an Online Customer

The optimal admission decision upon the arrival of a online customer from territory 2 can be characterized in a similar way.

In all, the optimal admission control of the retailer follows a threshold policy which is two-dimensional, in contrast with the one-dimensional threshold polices of the single-resource revenue management models.

Finally, we investigate the relationship between the optimal admission policy and the time period t. The major result is summarized in the following theorem.

Theorem 4: As the time period approaches toward the end of the selling horizon (i.e., t increases), the "rejection area", i.e., the area that is below $L_t^1(n_2)$ and $L_t^2(n_1)$ shrinks (see Figure 2).

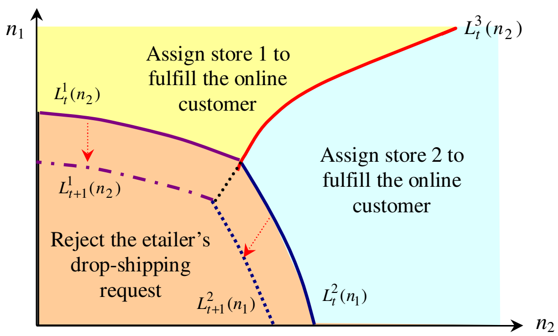

Fig. 2. The Movement of the "Rejection Area" *w.r.t.* Time Period t

Theorem 4 implies that, for the same inventory level (n_1, n_2), as the time left-to-go decreases, the retailer becomes more inclined to accept the etailer's drop-shipping request. This is consistent with our intuition.

IV. Numerical Experiments and Analysis

Revenue aside, compared with the optimal dynamic admission strategy, a static strategy (we call it the "heuristic control") that always accepts the etailer's drop-shipping requests and assigns the store that is close to the online customer as much as possible is more desirable because it is more convenient. There are possible additional costs that are associated with dynamic admission control. Therefore, a natural question is that of when a dynamic control strategy should be used and what the magnitude of the revenue gain will be. In this section, we report the results of several sets of experiments that are designed to develop some intuition with regard to these questions.

Consider the following example. A retailer sells Christmas Gifts through two separate stores as well as through a third-party etailer. The gifts are quite seasonal and should be sold out in a period of one months. The selling horizon is divided into $T = 5000$ small intervals with equal lengths. The initial inventory levels are $(Q_1, Q_2) = (100, 100)$; the selling prices are $p_1 = 5$ and $p_2 = 6$, respectively; other parameters are $\lambda_1 = 80/T$, $\lambda_2 = 50/T$, $\lambda_e = 120/T$, $\beta_1 = 0.7$, $\beta_2 = 0.3$, $w_1 = w_2 = 1$, and $c_1 = 1.5$, $c_2 = 2.5$. The physical retailer dynamically rations its inventories, with the objective to maximize the total revenue.

In the numerical experiments, we alter the value(s) of only one parameter at a time. First, the expected revenue of optimal admission control is compared with that of the heuristic control with different demand rates, λ_2's and λ_e's. The results are reported in Table I. It seems that when λ_2 is very small (say $\lambda_2 = 0$) or is very large (say $\lambda_2 = 100/T$), implementing the heuristic control results in a substantial revenue loss. This may due to the fact that by heuristic controlling more inventory is sold to the online customers that should have been reserved for sell to the retailer's in-store demands. Table I also shows that as the online demand accounts for a larger proportion of the total demand, the performance of the heuristic control strictly decreases. This, again, due to the fact that the retailer over-sells to the online customers. Note that, when λ_e is very large (say $\lambda_e = 200/T$), the performance of the optimal admission control over that of the heuristic control is significant.

TABLE I

Numerical Results with Different λ_2's and λ_e's

λ_e	λ_2	Optimal	Heursitc	Revenue Loss
120	0	862.2	743.6	13.75%
120	20	927.1	860.1	7.23%
120	40	977.4	938.1	4.01%
120	60	1024.5	970.0	5.32%
120	80	1053.7	990.8	5.97%
120	100	1071.7	1007.1	6.03%
0	50	699.9	699.9	0.00%
40	50	864.4	836.7	3.21%
80	50	977.6	927.3	5.14%
120	50	1001.8	956.9	4.48%
160	50	1017.9	940.2	7.63%
200	50	1025.4	927.1	9.58%

The percentage parameters, β_1 and β_2, determines the "split" of online demands over the two territories. They have a great impact on the retailer's performance, since under the reasonable assumption that fulfilling the online customer by a nearby store is generally more economical (Assumption 2), an appropriate β_1 (and β_2) can achieve a better match between each store's inventory supply and demand. Table II shows that when β_1 (or β_2) is low, implementing the heuristic control results in a larger revenue loss. This is intuitive.

TABLE II

Numerical Results with Different β_1's

β_1	Optimal	Heuristic	Revenue Loss
0.00	1016.0	981.6	3.38%
0.20	1024.1	993.6	2.99%
0.40	1025.3	997.6	2.71%
0.60	1016.3	972.9	4.26%
0.80	985.0	939.7	4.60%
1.00	950.5	902.3	5.07%

The revenue deduction from fulfilling the online customers is the underlying reason that makes the retailer less willing to accept the etailer's drop-shipping requests. Finally we change the value of c_2 and the numerical comparisons are reported in Table III. When c_2 is large, the revenue loss by implementing the heuristic control strictly increases. This again, is consistent with our intuition.

TABLE III
NUMERICAL RESULTS WITH DIFFERENT c_2'S

c_2	Optimal	Heuristic	Revenue Loss
2.00	1009.4	968.6	4.05%
2.50	1001.8	956.9	4.48%
3.00	994.4	945.2	4.95%
3.50	987.3	933.5	5.45%
4.00	980.4	921.8	5.97%
4.50	973.7	910.1	6.53%

V. CONCLUDING REMARKS

We have studied a dynamic admission control model in which a physical retailer sells seasonal products through two separate stores and a third-party etailer. For each order collected from the website, the etailer directs to the retailer for fulfillment, intending to share some profit. The retailer needs to decide when to accept such drop-shipping requests, and if so, assign which store for the order fulfillment. The decisions are made based on the on-hand inventory levels, the remaining time, and the information regarding where the online customer comes from. We have established several structural properties for the optimal admission policy, which is shown to be a two-dimensional threshold policy. Numerical experiments show that implementing a simple heuristic control of always accepting the etailer's drop-shipping request and assigning the store that is more close to the online customers for the fulfillment as much as possible may result in substantial revenue losses.

The products studied in this paper are seasonal items, and we have assumed that the selling prices remain unchanged during the entire selling season. In reality, their prices are usually decreasing over time, instead of remaining constant. Our model can be easily extended to incorporate a time-varying price pattern. If the pricing trajectory over the season can be estimated at the beginning of the season (e.g., both physical store will offer significant discounts during the period of "Black Friday" and the weeks following Christmas), most of the results obtained in the previous sections (e.g., properties in Theorem 2) will remain unchanged with p being replaced by $p(t)$. Also, the demand rates, λ_1, λ_2, and λ_e can depend on time.

There are two directions for extending the current model. First, as our model is based on the assumption that each customer orders only one unit of item, an natural problem for future research is to consider batch demands. Second, we have assumed that the etailer carries zero inventory. However, it is important to note that some etailers adopt a hybrid inventory strategy: carrying a certain amount of inventory and employing physical retailers as backups. In such situations, the etailer also needs to ration his own inventory; that is, should he fullfill the online customer by his own inventory, or by requesting the physical retailer? This involves a competitive game between the etailer and the physical retailer, which is an interesting topic we are currently investigating.

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