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Robust Control using Recursive Design Method for Flexible Joint Robot Manipulator

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Abstract **– Flexible joint robot manipulators can be decomposed into two cascaded subsystems, a series connection of robot link dynamics and joint dynamics. For these flexible manipulators, we propose the robust controller using a recursive design method. The recursive design procedures are constructive and contain two steps. First, a fictitious robust controller for the robot link dynamics is designed as if the link dynamics had an independent control. As the fictitious control, a nonlinear H_∞ control using the energy dissipation is designed in the sense of L_2-gain attenuation from the disturbance caused by uncertainties to performance. Second, a real robust control is designed recursively by using a Lyapunov's second method. The designed robust control is applied to a 2 DOF robot manipulator with joint flexibilities.**

Index Terms - Flexible Joint, Robust Control, H_∞ Control, Backstepping Control, NLMI (Nonlinear Matrix Inequality)

I. Introduction

For a class of nonlinear system which is composed of series connection of finite number of nonlinear subsystems, a recursive design is applied for stabilizing control. Interesting progress in the recursive design has been achieved in adaptive control of feedback linearizable systems [1].

Since many systems inherently have uncertainties such as parameter variations, external disturbances, and unmodelled dynamics, the robust control can be considered. The Lyapunov's second method is widely used in designing robust controllers, as proceeded in existing results [2,3]. One of the difficulties in using Lyapunov's second method is that it is not easy to find an appropriate Lyapunov function for control design.

Another robust control method which has attracted attention of many researchers is H_∞ control. Although the nonlinear H_∞ control has been derived by the L_2-gain analysis based on the concept of the energy dissipation [4,5], its application is not easy due to the difficulty in solving the first-order partial differential inequality called as Hamilton Jacobi inequality (HJ inequality). The H_∞ control problem in nonlinear systems reduces to the existence of the solution to HJ inequality and many methods are proposed in recent papers [6,7,8,9].

In this paper, a robust controller is designed for cascaded nonlinear uncertain systems using a recursive design which is composed of two steps. In the first step, a fictitious robust controller for the first subsystem is designed as if the subsystem had independent control. As the fictitious control, the nonlinear H_∞ control is used to guarantee the robust stability of the system. The solution to HJ inequality can be obtained through a more tractable nonlinear matrix inequality (NLMI) method and the fact that the matrices forming the NLMI is bounded [9]. In second step, the actual robust control is designed recursively by Lyapunov's second method.

The designed control is applied to a 2 DOF robot manipulator with flexible joints. In the system, the two subsystems representing joint dynamics and link dynamics are connected in a series with stiffness terms. Simulations are performed for this system with uncertainties of inertia and stiffness.

This paper is organized as follows. In section Ⅱ, the recursive design procedures are presented for the system without uncertainties and the robust controls are designed for the uncertain system. In section Ⅲ, the robust design is designed for the robot manipulators with flexible joints using the suggested procedures in section Ⅱ. In section Ⅳ, the simulation is presented. In section Ⅴ, the conclusions are presented.

II. Robust Recursive Design

A. Recursive Design for Certain System

The system considered in this paper is described by

$$\dot{x}_1 = f(x_1) + F(x_1)x_2 \tag{1}$$

$$\ddot{x}_2 = g(x_1, x_2, \dot{x}_2) + G\tau \tag{2}$$

where x_1 and $x_2 \in \mathbb{R}^n$ are the states of systems and $\tau \in \mathbb{R}^n$ is a control input, and G is a constant matrix. Equation (2) of the second subsystem is a differential equation whose output is the input signal to the first subsystem. The recursive design exploits this structural property.

In recursive design, it is required that there exists a fictitious control, which stabilizes the first subsystem of (1).

Assumption 1 *(Global Stabilizability) When x_2 is assumed to be the fictitious control, there exists a C^2 fictitious* *control law* u_{f1} *such that the first system* $\dot{x}_1 = f(x_1) + F(x_1)u_{f1}$ *is globally stable. This is established with a* C^1 *positive definite function* $E(x_1)$ *such that*

$$\frac{\partial E}{\partial x_1}\left(f(x_1) + F(x_1)u_{f1}\right) \le 0 .$$

In the first subsystem, this control law is not implementable and its effect must be achieved through the second subsystem, that is, the real control τ must be determined so that $x_2 = u_{f1}$.

Theorem 1 *If assumption 1 is satisfied with the fictitious control* u_{f1} *then the overall system has stable equilibrium point with the second fictitious control and the real control*

$$u_{f2} = \dot{u}_{f1} - (L_F E)^T - \alpha_1 e_1 \tag{3}$$
$$G\tau = \dot{u}_{f2} - g - e_1 - \alpha_2 e_2 \tag{4}$$

where $L_F E = \dfrac{\partial E}{\partial x_1} F$, $e_1 = x_2 - u_{f1}$, $e_2 = \dot{x}_2 - u_{f2}$ *and* α_1 *and* α_2 *are positive constants.*

proof : Using the fictitious control u_{f1}, we can rewrite (1) as

$$\dot{x}_1 = f(x_1) + F(x_1)u_{f1} + F(x_1)e_1 . \tag{5}$$

To show that the real control achieves stable equilibrium point, we use the positive definite function

$$V = E(x_1) + \frac{1}{2}e_1^T e_1 + \frac{1}{2}e_2^T e_2 . \tag{6}$$

Using (2) and (5), its time-derivative is

$$\begin{aligned}\dot{V} &= \frac{\partial E}{\partial x_1}\left(f(x_1) + F(x_1)u_{f1} + F(x_1)e_1\right) + e_1^T\left(\dot{x}_2 - \dot{u}_{f1}\right) + e_2^T\left(\ddot{x}_2 - \dot{u}_{f2}\right)\\ &= \frac{\partial E}{\partial x_1}\left(f(x_1) + F(x_1)u_{f1}\right) + e_1^T\left((L_F E)^T - \dot{u}_{f1} + u_{f2}\right)\\ &\quad + e_2^T\left(g(x_1, x_2, \dot{x}_2) + G\tau - \dot{u}_{f2} + e_1\right).\end{aligned}$$

By *assumption 1* and (3) and (4), stability is proved.

Q.E.D.

B. Robust Fictitious Control for Uncertain System

The recursive design in the previous section can be applied easily to a class of nonlinear systems without model uncertainties, but every system has modeling errors. For the reason model uncertainty must be considered in control design to obtain good performance. In this section the fictitious control is designed with the nonlinear H_∞ control to guarantee the robustness to model uncertainties.

To find the H_∞ control is to find a stabilizing state-feedback control input such that the closed-loop system has a L_2-gain equal to or less than γ in the input-to-output sense. In the nonlinear H_∞ control design, it is essential to find the solution to the associated Hamilton-Jacobi [HJ] inequality derived from the condition to satisfy L_2-gain property in the input-to-output sense [4]. If a solution exists, then it will guarantee the stability as well as the disturbance attenuation.

Before proceeding detailed design, we defined the fictitious control u_{f1} such as

$$u_{f1} = u_t + u_r$$

where u_t is the control input to transform (1) satisfying the following *assumption 2* and u_r is a robust control input to be designed with H_∞ theory.

Assumption 2 *There exist a* C^1 *control input* u_t *such that the first system (1) is transformed to*

$$\dot{s} = A(x_1, \dot{x}_1)s + B_1(x_1)w + B_2(x_1)u_r \tag{7}$$

where $s(x_1, x_{1d}) \in \mathbb{R}^n$ *is the new state,* $x_{1d} \in \mathbb{R}^n$ *is the desired trajectory,* $w \in \mathbb{R}^n$ *is the disturbances caused by model uncertainties and* $A(x_1, \dot{x}_1)$, $B_1(x_1)$ *and* $B_2(x_1)$ *are the matrix-valued function of suitable dimensions.*

By *assumption 2* and with the performance vector z, the first nonlinear subsystem (1) can be described as

$$\begin{aligned}&\dot{s} = A(x_1, \dot{x}_1)s + B_1(x_1)w + B_2(x_1)u_r\\ &z = Hs + Du,\ H^T D = 0,\ D^T D > 0\end{aligned} \tag{8}$$

where H and D are the constant matrices of suitable dimensions.

In the form of (8), the derived HJ inequality is more tractable since it can be transformed to nonlinear matrix inequality (NLMI). The design of a nonlinear H_∞ controller for the nonlinear system in the form of (8) is summarized as the following theorem.

Theorem 2 *Given* $\gamma > 0$ *, suppose there exists a matrix* P *satisfying*

$$P^T A + A^T P + \frac{1}{\gamma^2} P^T B_1 B_1^T P + H^T H - P^T B_2 [D^T D]^{-1} B_2^T P \le 0 \tag{9}$$

and there exists a non-negative energy storage function

$E(s)=s^T Ps \geq 0$. *Then the control input satisfying* $L_2 - gain \leq \gamma$ *is*

$$u_r = -[D^T D]^{-1} B_2^T Ps \,. \tag{10}$$

proof : The derivative of energy storage function E along the trajectory of s is

$$\begin{aligned}\dot{E} &= \frac{\partial E}{\partial s}\dot{s} = 2s^T P^T (As + B_1 w + B_2 u_r) \\ &= s^T (P^T A + A^T P)s + 2s^T P^T (B_1 w + B_2 u_r).\end{aligned}$$

Introducing $\gamma^2 \|w\|^2 - \|z\|^2$ into above equation, we obtain

$$\begin{aligned}\dot{E} &= \gamma^2 \|w\|^2 - \|z\|^2 - \gamma^2 \left\|w - (1/\gamma^2) B_1^T Ps\right\|^2 + 2s^T H^T D u_r \\ &+ \left\|Du_r + D^{-T} B_2^T Ps\right\|^2 + s^T \{P^T A + A^T P \\ &+ (1/\gamma^2) P^T B_1 B_1^T P - P^T B_2 (D^T D)^{-1} B_2^T P + H^T H\} s.\end{aligned}$$

By using (9) and the control input u_r, the derivative of the storage function is arranged as

$$\dot{E} \leq \gamma^2 \|w\|^2 - \|z\|^2 .$$

Therefore it satisfies $L_2 - gain \leq \gamma$.

Q.E.D.

To obtain the solution of (9) easily, it is transformed to a nonlinear matrix inequality using the Schur complement. Firstly, premultiplying and postmultiplying the inequality (9) by the positive definite matrices P^{-T} and P^{-1} respectively, then by the Schur complement, the HJ inequality becomes

$$\begin{bmatrix} W & Q^T H \\ HQ & -I \end{bmatrix} \leq 0$$

where $W = AQ + Q^T A + \frac{1}{\gamma^2} B_1 B_1^T - B_2 [D^T D]^{-1} B_2$ and $Q = P^{-1}$.

Solving the above NLMI yields convex optimization problem. Unlike the linear case, this convex problem is not finite dimensional. However, if the matrices forming the NLMI are bounded, then we only need to solve a finite number of LMIs.

The overall stabilizing robust fictitious control becomes

$$u_{f1} = u_t - [D^T D]^{-1} B_2^T Q^{-1} s \,.$$

C. Robust Real Control for Uncertain System

If there exists the fictitious control u_t and u_r satisfying *assumption 2* and *theorem 2*, the choice of $x_2 = u_{f1}$, if permitted, would ensure robust stability. Since x_2 is not a real control input, we cannot let $x_2 = u_{f1}$ and its effect must be achieved through the second subsystem.

Using the fictitious control u_{f1}, we can rewrite (1) as

$$\dot{s} = As + B_1 w + B_2 u_r + B_2 e_1 \,.$$

To design the robust real control, which achieves L_2 -gain property, we use the function (6). Its time derivative is

$$\begin{aligned}\dot{V} &\leq \gamma^2 \|w\|^2 - (\|z\|^2 + \alpha_1 \|e_1\|^2 + \alpha_2 \|e_2\|^2) \\ &\quad + e_1^T (\Delta_1 + u_{f2}) + e_2^T (\Delta_2 + G\tau)\end{aligned} \tag{11}$$

where $\Delta_1 = \alpha_1 e_1 + 2B_2^T Ps - \dot{u}_{f1}$, $\Delta_2 = \alpha_2 e_2 + g + e_1 - \dot{u}_{f2}$ and α_1 and α_2 are positive constants.

To design the second fictitious control with robustness, the bound of $\|\Delta_1\|$ must be obtained. Using (7) and (10), it can be obtained as

$$\|\Delta_1\| \leq \alpha_1 \|e_1\| + \beta_1 \|s\| + \|\dot{u}_{f1}\| \leq \alpha_1 \|e_1\| + \beta_1 \|s\| + \beta_2 \|\dot{s}\| + \beta_3 := \rho_1$$

where β_1, β_2 and β_3 are positive constants. Therefore the second fictitious control u_{f2} can be designed as

$$u_{f2} = -\frac{e_1}{\|e_1\| + \varepsilon_1} \rho_1 \,.$$

But to obtain the bound of $\|\Delta_2\|$, it is necessary to modify u_{f2} to be differentiable as following equation [13]

$$u_{f2} = -\frac{e_1 \|e_1\|^2}{\|e_1\|^3 + \varepsilon_1} \rho_1 \,. \tag{12}$$

Using (12), the bound of $\|\Delta_2\|$ can be obtained as

$$\begin{aligned}\|\Delta_2\| &\leq \alpha_2 \|e_2\| + \|g\| + \|e_1\| + \|\dot{u}_{f2}\| \\ &\leq \alpha_2 \|e_2\| + \|g\| + \|e_1\| + k_1 \|\rho_1\|^2 + k_2 := \rho_2 .\end{aligned}$$

If it is assumed that $G_{\min} \leq G \leq G_{\max}$ for known upper and lower bounding matrices, the real control input can be chosen as

$$\tau = -G_{\min}^{-1} \left(\frac{e_2}{\|e_2\| + \varepsilon_2} \rho_2 \right).$$

With the second fictitious control u_{f2} and the real control τ , the storage function satisfies

$$\dot{V} \le \gamma^2 \|w\|^2 - (\|z\|^2 + \alpha_1 \|e_1\|^2 + \alpha_2 \|e_2\|^2)$$

which achieve L_2-gain property.

III. Design for Flexible Robot Manipulators

A. Dynamics of flexible Robot Manipulators

Consider the dynamics of robot manipulators with joint flexibility. The dynamics is

$$M(x_1)\ddot{x}_1 + C(x_1,\dot{x}_1)\dot{x}_1 + G(x_1) = K(x_2 - x_1) \quad (13)$$

$$J\ddot{x}_2 + B\dot{x}_2 + K(x_2 - x_1) = \tau \quad (14)$$

where $x_1 \in \mathbb{R}^n$ is the link side angle, $x_2 \in \mathbb{R}^n$ is motor side angle, $M(x_1)$ is the positive definite symmetric inertia matrix, $C(x_1,\dot{x}_1)$ represents the centripetal and coriolis torque, $G(x_1)$ represents the gravitational torque, J denotes the diagonal inertia matrix of actuator about their principal axes of rotation multiplied by the square of the respective gear ratios, B is the viscous function matrix and K is the stiffness matrix [14]. Since model uncertainties exist in the above dynamics, the robust control is needed for the recursive design.

B. Robust Fictitious Control for Uncertain System

Transformation of Dynamics. To design the fictitious control be robust to model uncertainties, Equation (13) is rewritten as

$$\ddot{x}_1 = -M^{-1}C\dot{x}_1 - M^{-1}G - M^{-1}Kx_1 + M^{-1}Kx_2 . \quad (15)$$

Before proceeding with u_t in section II, the new state s, which is modified error for joint tracking, is defined as

$$s = \dot{x}_1 - \{\dot{x}_{1d} - \Lambda(x_1 - x_{1d})\} = \dot{x}_1 - \dot{x}_{1r}$$

where x_{1d} and $\dot{x}_{1d}$ are the desired position and velocity respectively. If the elements of s approach to zeros at $t \to \infty$, so do the tracking errors of joints.

A suitable control input satisfying *assumption 2* can be chosen as

$$u_t = \hat{K}^{-1}(\hat{M}\ddot{x}_{1r} + \hat{C}\dot{x}_{1r} + \hat{G}) + x_1$$

where $\hat{K}$, $\hat{M}$, $\hat{C}$ and $\hat{G}$ are the matrixes with nominal parameter values.

Substituting u_{f1} for x_2, Equation (15) is transformed to

$$\dot{s} = A(x_1,\dot{x}_1)s + B_1(x_1)w + B_2(x_1)u_r \quad (16)$$

where $A = -M^{-1}C$, $B_1 = -M^{-1}$, $B_2 = M^{-1}K$ and $w = (M - K\hat{K}^{-1}\hat{M})\ddot{x}_{1r} + (C - K\hat{K}^{-1}\hat{C})\dot{x}_{1r} + (G - K\hat{K}^{-1}\hat{G})$ which is a disturbance vector caused by model uncertainties.

Robust Control. To derive the HJ inequality for the robust control input, each matrix term of (16) is substituted into (9), then

$$-(MP^{-T})^{-1}C + C^T(P^{-1}M^T)^{-1} + H^T H + \frac{1}{\gamma^2}(MP^{-T})^{-1}(P^{-1}M^T)^{-1}$$
$$-(MP^{-1})^{-1}K(D^T D)^{-1}K^T(P^{-1}M^T)^{-1} < 0.$$

Premultiplying and postmultiplying the inequality by the positive definite matrices MP^{-T} and $P^{-1}M^T$ respectively, then the HJ inequality becomes

$$-CQM^T - MQ^T C^T + CQ^T H^T HQM^T + \frac{1}{\gamma^2}I - K(D^T D)^{-1}K^T < 0 \quad (17)$$

where $Q = P^{-1}$. Using the Schur complement, Equation (17) can be described as a NLMI

$$\begin{bmatrix} W & MQ^T H^T \\ HQM^T & -I \end{bmatrix} \le 0 \quad (18)$$

where $W = -CQM^T - MQ^T C^T - K(D^T D)^{-1}K^T + \frac{1}{\gamma^2}I$. The matrices M and C is the nonlinear function of x_1 and $\dot{x}_1$ in (18). However, those matrices include trigonometric functions and can be bounded when each joint velocity range is bounded between two empirically determined external values. Using this fact, we consider that the matrices forming above NLMI vary in some bounded sets of the space of matrices, i.e.,

$$[M,C,K,H,D] \in Co\{[M_i,C_i,K_i,H,D]|_{i\in\{1,2,\cdots,L\}}\}$$

where Co represents the convex hull and L is the number of vertices of bounded space. Therefore, if there exists a solution Q to (19), then it is also a solution to (18) [9].

$$\begin{bmatrix} W & M_i Q^T H^T \\ HQM_i^T & -I \end{bmatrix} \le 0, \quad i \in \{1,2,\cdots,L\} \quad (19)$$

where $W = -C_i QM_i^T - M_i Q^T C_i^T - K_i(D^T D)^{-1}K_i^T + \frac{1}{\gamma^2}I$.

This approach provides a tractable method to get constant solution to NLMI, which can be used to design the robust control input. However, this approach generally leads to conservative results if the prescribed bound is large.

C. Robust Real Control for Uncertain System

To design the robust control, which achieves L_2-gain property, we use the positive definite function (6). Its time derivative is

$$\dot{V} \le \gamma^2 \|w\|^2 - \left(\|z\|^2 + \alpha_1 \|e_1\|^2 + \alpha_2 \|e_2\|^2\right) + e_1^T(\Delta_1 + u_{f2}) + e_2^T(\Delta_2 + J^{-1}\tau) \tag{20}$$

where $\Delta_1 = \alpha_1 e_1 + 2K^T M^{-T} Ps - \dot{u}_{f1}$, $\Delta_2 = \alpha_2 e_2 + e_1 - J^{-1}K(x_1 - x_2) - J^{-1}B\dot{x}_2 - \dot{u}_{f2}$, α_1, and α_2 are positive constants. This inequality contains the motor side dynamics and the real control input τ.

If the uncertain model matrices satisfy

$$M_{\min} \le M \le M_{\max}, \qquad J_{\min} \le J \le J_{\max}$$
$$K_{\min} \le K \le K_{\max}, \qquad B_{\min} \le B \le B_{\max}$$

then the bound of Δ_1 and Δ_2 can be obtained as

$$\|\Delta_1\| \le \alpha_1 \|e_1\| + \delta_1 \|s\| + \delta_2 := \rho_1$$
$$\|\Delta_2\| \le \alpha_2 \|e_2\| + k_1 \|q_2 - q_1\| + k_2 \|\rho_1\|^2 + k_3 := \rho_2 .$$

The second fictitious control can be designed as

$$u_{f2} = \frac{e_1 \|e_1\|^2}{\|e_1\|^3 + \varepsilon_1} \rho_1 .$$

And the real control input can be chosen as

$$\tau = -J_{\max}\left(\frac{e_2}{\|e_2\| + \varepsilon_2} \rho_2\right).$$

By using u_{f2} and τ, the storage function is arranged as

$$\dot{V} \le \gamma^2 \|w\|^2 - \left(\|z\|^2 + \alpha_1 \|e_1\|^2 + \alpha_2 \|e_2\|^2\right).$$

Therefore it satisfies L_2-gain property.

IV. SIMULATION

In this section, the robust performance of the proposed controller for the 2 DOF robot manipulators against inertia and stiffness uncertainties is verified through simulation. The performance of the proposed robust controller is compared with that of a model based dynamic controller. The model based dynamic controller is composed of a PID controller and a feedforward dynamic controller considering joint flexibility. Table 4.1 shows the nominal values of the physical parameters of the 2-DOF robot manipulator.

TABLE I PHYSICAL PARAMETERS

m_1	6 kg	l_{c2}	0.15 m
m_2	4 kg	k_1	1500 Nm/rad
l_1	0.3 m	k_2	1200 Nm/rad
l_2	0.3 m	b_1	0.05 Nms/rad
l_{c1}	0.15 m	b_2	0.05 Nms/rad

At the initial pose, the two links are paralleled to the surface of land straightly against the gravity. The first joint θ_1 moves 180 degree in counterclockwise, and the second joint θ_2 moves 90 degree in counterclockwise. And there is disturbance torque (100 Nm) at 2.3 sec.

The angles and angular velocities of the proposed robust controller and a model based dynamic controller are shown in the Fig. 1 and Fig. 2. The results have similar behaviors, but there is no oscillation in case of the proposed robust control, moreover a convergence is very quick after disturbance torque. Also simulation is performed for perturbed parameter cases when the mass of links varies by 20% and the stiffness of joints varies by 20%. The results of parameter uncertainties, as shown in the Fig. 3 and Fig. 4, show the proposed controller has robustness to model uncertainties.

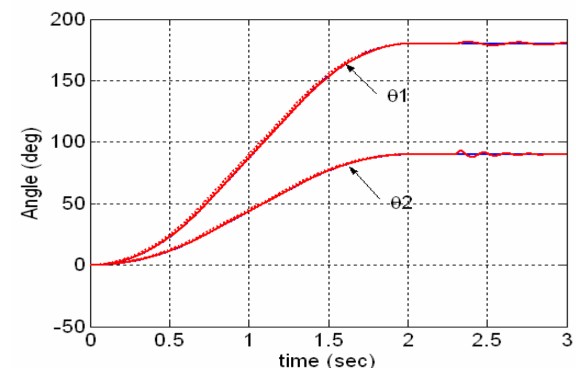


Fig. 1 Link angle.

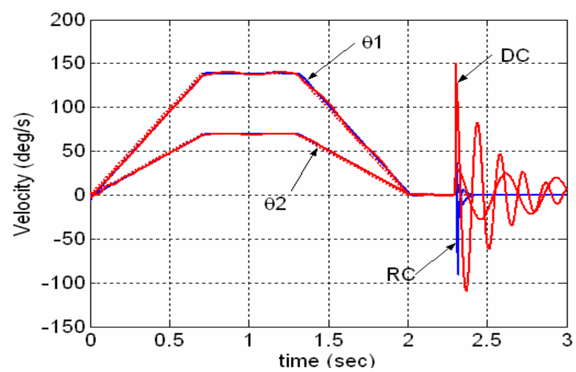
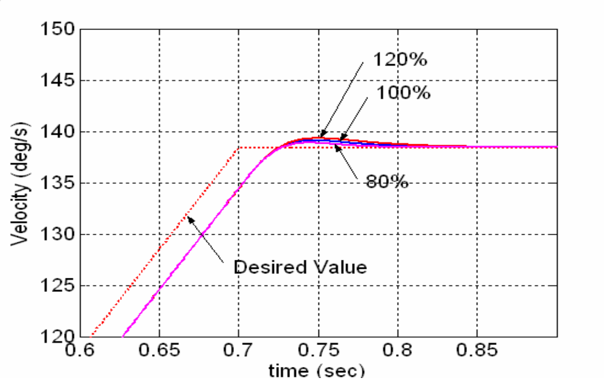
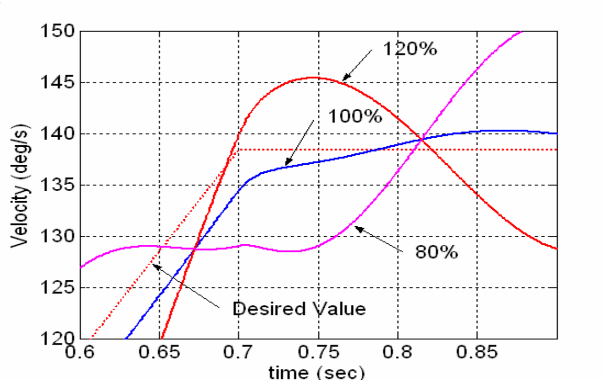


Fig. 2 Link velocity.
(RC : Robust control, DC : Model based dynamic control)

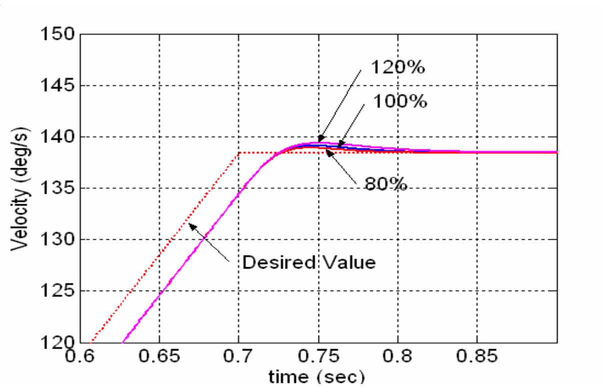


(a) Proposed robust control

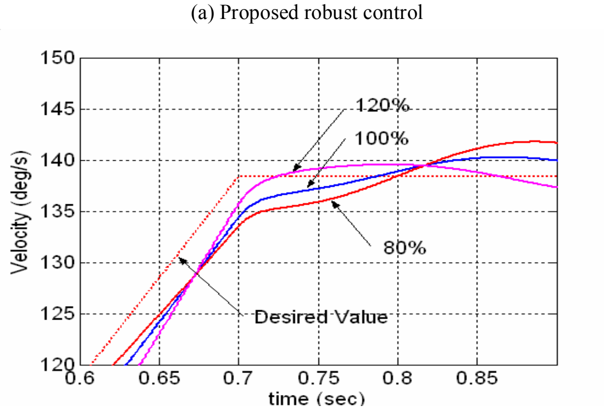


(b) Model based dynamic control

Fig. 3 Angular velocity under inertia uncertainty.



(a) Proposed robust control



(b) Model based dynamic control

Fig. 4 Angular velocity under stiffness uncertainty.

V. CONCLUSION

Using recursive design, a robust control was designed for the robot manipulators with flexible joint, which can be decomposed into two cascaded subsystem, i.e. the link dynamics and the joint dynamics. First, the fictitious robust control for the link dynamics was designed using nonlinear H_∞ control. The associated HJ inequality was transformed to NLMI and its solution was obtained from the fact that the terms in matrices can be bounded. The application of proposed method was simple since the gain matrix can be determined easily by an efficient convex optimization algorithm. Second, the real robust control for the joint dynamics was designed recursively using a Lyapunov's second method. In the results, the proposed robust controller had robustness to model uncertainties.

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