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Complex Packaging Line Modelling and Simulation

Andrea Grassi, Elisa Gebennini, Gabriele Goldoni, Cesare Fantuzzi, Rita Gamberini,
Robert Nevin and Bianca Rimini

***Abstract*— The paper presents advanced issues in modelling and simulation of complex packaging line. In particular, we developed a theoretical model of a line with two machines and a buffer, which is a simplified version of a real packaging line from Tetra Pak company. The paper reports also about simulation results that confirm theoretical supposals.**

I. Introduction

In recent years increasing competition in global markets has pushed industries to adopt innovative production systems, targeting improved production quality at reduced cost. In particular high volume production, demands a highly automation and organisation in *production lines*.

Production lines are an assembly of automated machines that work in a productive chain, each machine being tailored to perform specific operations on the raw material at the highest possible speed.

Since the machines in a production line are closely coupled to each others, the performance of the whole line depends strongly on both the performance of single machines and on how they are linked. Thus to optimize line performance, several aspects should be taken in account at the design stage.

The key issue concerning the line production efficiency is in the identification of key factors in the design of the single machine and in their assembly in the production line. An aspect which is generally acknowledged in the literature is that the machines interact with each other, and a machine blocking will produce a general failure of all of the other machines. To decrease this vulnerability, it is common to introduce a production decoupling buffer following most critical machines.

However, the correct choice of the buffer position and size depends on the production goal and on the machine characteristics. For example in a Tetra Pak production line, the filling machine has a critical behaviour as a stoppage because of a downstream line fault produces product waste and possibly a longer time to restoration with respect to nominal MTTR.

Continuous production lines design is a typical industrial problem which attracted, and currently attracts, the interest of scientists. When machines are connected each other in a continuous line, the performance of a generic machine is not only determined by its behavior, but only by the interactions with the other machines immediately located in the upstream and downstream. Hence, a *repaired* machine (that is, a machine able to perform operations on products) could be found in idle state as a consequence of failures occurred

– in the upstream, determining an interruption of the ingoing product flow, thus causing a *starvation*;
– in the downstream, determining a stop of the outgoing product flow, thus a *block* of the machine as a consequence of the impossibility to discharge products.

To mitigate the effects of such harmful interactions, buffers are allocated along the line to act as decoupling points to sustain the flow for a determined time span, when failures on machines occur.

Hence, the main concern in the design of continuous production lines is the determination of the right position and capacity of the buffers, these consistently affecting the overall throughput of the line itself, as can be seen in [2][3][4].

Scientists and technicians have addressed this problem in several ways, but the most attractive and promising approach is probably the mathematical modeling. By means of mathematical models, an analytic relation between the buffers structuring and the throughput of the line can be established, thus precise and useful insights can be obtained to address the optimal dimensioning (see [1] for a comprehensive representation).

One of the most important methodologies adopted to mathematically represent the behavior of the line is obtained considering that the line behaves as a continuous time Markov process. In this way, also inhomogeneous lines (i.e. those in which machines can have different productivities) can be addressed.

Nevertheless, deriving a model of a line as a whole is a very complex task. Hence, decomposition techniques have been developed in such a way as to allow the line to be fractionized in a sequence of bi-machine sub-problems, being these latter solvable in analytic terms. Once analytic models for each bi-machine sub-problem are obtained, the performance parameters of the whole line can be computed by means of iterative procedures.

Several works have been proposed in this direction, as can be noted by the wide literature produced [5][6][7][8][9][10].

As pointed up before, decomposition techniques fractionize the whole line in a sequence of sub-problems characterized by two-machines connected each other by means of a buffer, this latter modeling the accumulation capabilities

C. Fantuzzi, R. Gamberini, E. Gebennini, G. Goldoni, B. Rimini and A. Grassi are with *Dipartimento di Scienze e Metodi dell'Ingegneria, Università di Modena e Reggio Emilia, Via Amendola 2 – Pad. Morselli, 42100 Reggio Emilia, Italy.*
<name.surname>@unimore.it
R. Nevin is with *Tetra Pak Carton Ambient S.p.A., Via Delfini 1, 41100 Modena, Italy.*
robert.nevin@tetrapak.com

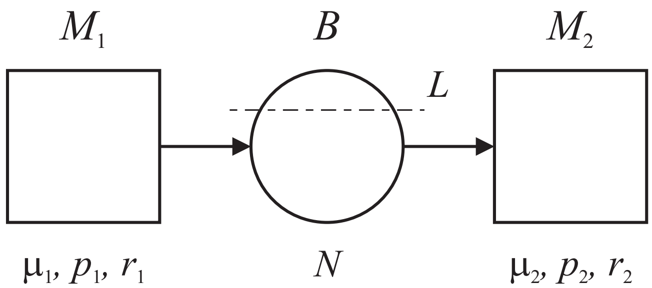

Fig. 1. Two machines system

of the transportation system (storages and/or accumulating conveyors).

Hence, the importance to have well performing mathematical models to correctly represent the behavior of each two-machines sub-system is emphasized.

The basic two-machines system, modeled as a continuous time Markov process, was developed in [11]. This model considers two machines, with equal or different speeds, continuously operating on a flow of products, while a storage, with a finite capacity N, is introduced to mitigate the harmful interactions that can be generated in case of machines failures. This model can be effectively adopted to represent the most cases findable in practice, such as automatic assembly lines.

Nevertheless, there are also manufacturing systems in which machines are subject to product wasting at each restart, as the filling machine of Tetra Pak. The filling machine is the first machine in a packaging line, and its goal is to fill the packages with the liquid product and to seal the package. Because process characteristics at each restart the filling machine has to waste some packages.

At the best of authors' knowledge, in the current literature there are none theoretical results that deals with this problems. Our approach is to define a buffer control algorithm that seek to prevent filling machine stoppage.

This system mathematical model is presented in Section II, while Section III describes the simulation results.

II. System model

The two machine finite buffer problem, with M_1's restart controlled by buffer level, is modeled as a continuous time, mixed state Markov process. The system is depicted in Figure 1.

The variables μ_i, p_i, and r_i represent the production rate, the failure rate, and the restoration rate, respectively, $\forall\, i = 1, 2$. As emphasized in Section I, in such a system $\mu_2 > \mu_1$ since the main issue is to prevent stops in M_1.

The system state is defined as

$$\mathcal{S} = \big(x, \beta, \alpha_1, \alpha_2, t\big)\,, \qquad (1)$$

being x the buffer level, $\beta = \{0, 1\}$ a binary parameter identifying the forced block state of the first machine, $\alpha_i = \{0, 1\}$ the repair state of the machine $i = \{1, 2\}$, and t the time variable.

In a generic time interval δt, the variation in the buffer level involved by the machines behaviour is $\big((1-\beta)\alpha_1\mu_1 - \alpha_2\mu_2\big)\delta t$, if x is far enough to its boundaries 0 and N.

When the buffer reaches the level N, the first machine can not discharge products and consequently goes blocked, that is, it could process units $(\alpha_1 = 1)$ but it has to stop production as a consequence of the impossibility to send products in the downstream. Moreover, as said in Section I, to reduce the number of stops of M_1, an immediate restart is prevented by putting M_1 in the forced block state $(\beta = 1)$ and maintaining it blocked until the buffer level decreases to a predefined value $L \in [0, N]$. As an additional consequence, while $\beta = 1$, M_1 can not go down since operational dependent failures are assumed. While $\beta = 0$, the probability of failure of M_1 at time $t + \delta t$, provided that $\alpha_1(t) = 1$, is $p_1\delta t$.

On the other side, M_2 can consume products at its nominal rate μ_2 only if the buffer is not empty, otherwise it is forced to slow down its speed to μ_1 (remeber the hypothesis $\mu_2 > \mu_1$). In this case the probability of failure of M_2 at time $t + \delta t$, provided that $\alpha_2(t) = 1$, is $p_2^b\delta t$, where

$$p_2^b = \frac{\mu_1}{\mu_2} p_2\,, \qquad (2)$$

since a failure rate proportional to machine operating speed is assumed. When the buffer is not empty, such a probability is $p_2\delta t$.

Finally, the probability to have a restoration at time $t + \delta t$ of a machine i failed in t $\big(\alpha_i(t) = 0\big)$ is $r_i\delta t$.

The model comprises a set of equations that represent the behavior of the system. It is advisable to distinguish two groups of equations, the one related to the boundary states (when the buffer is empty or full) and the other related to the intermediate buffer levels.

A. Boundary behavior

There are twelve boundary states: $(0, 0, \alpha_1, \alpha_2)$ where $\alpha_1 = 0$ or 1, $\alpha_2 = 0$ or 1 and $(N, \beta, \alpha_1, \alpha_2)$ where $\beta = 0$ or 1, $\alpha_1 = 0$ or 1 $\alpha_2 = 0$ or 1. Let us examine the equations to represent the probability of finding the system in a given boundary state.

Lower Boundary: x = 0*:* The equations that describe the behavior of the system at the lower boundary are very similar to those investigated in the previous literature. In this case the parameter β is included in the definition of the system state nevertheless its value is fixed to zero when the buffer is empty.

Hence the equations related to the lower boundary are only stated and not derived in the following (the reader is referred to [1] for more details).

- *Boundary-to-Boundary Equations*

$$\frac{d}{st}\mathbf{p}(0,0,0,0) = -(r_1 + r_2)\mathbf{p}(0,0,0,0)\,, \qquad (3)$$

$$\mathbf{p}(0,0,1,0) = 0\,. \qquad (4)$$

- *Interior-to-Boundary Equations*

$$\frac{d}{dt}\mathbf{p}(0,0,0,1) = r_2\mathbf{p}(0,0,0,0) - r_1\mathbf{p}(0,0,0,1) + \\ + p_1\mathbf{p}(0,0,1,1) + \mu_2 f(0,0,0,1)\,, \quad (5)$$

$$\frac{d}{dt}\mathbf{p}(0,0,1,1) = -(p_1 + p_2^b)\mathbf{p}(0,0,1,1) + \\ + r_1\mathbf{p}(0,0,0,1) + p_1\mathbf{p}(0,0,1,1) + \\ + (\mu_2 - \mu_1) f(0,0,1,1)\,. \quad (6)$$

- *Boundary-to-Interior Equations*

$$\mu_1 f(0,0,1,0) = r_1\mathbf{p}(0,0,0,0) + p_2^b\mathbf{p}(0,0,1,1)\,. \quad (7)$$

Upper Boundary: x = N: It is important to note that the variable β changes instantaneously from 0 to 1 when the buffer level reaches the value N. This implies that, when $x = N$, all the states with $\beta = 0$ are coincident with the corresponding states having $\beta = 1$.

$$(N,0,\alpha_1,\alpha_2) \equiv (N,1,\alpha_1,\alpha_2)\,,\ \forall\, \alpha_1,\alpha_2 = \{1,2\}\,. \quad (8)$$

- *Boundary-to-Boundary Equations*
  The probability of finding the system in some boundary states is equal to zero. In particular the system can not get to the states $(N,0,0,0)$ and $(N,1,0,0)$ because the buffer can not reach level N if M_1 is down.

$$\mathbf{p}(N,0,0,0) = \mathbf{p}(N,1,0,0) = 0\,. \quad (9)$$

  Moreover, as a consequence of the hypothesis $\mu_2 > \mu_1$, it follows that

$$\mathbf{p}(N,0,1,1) = \mathbf{p}(N,1,1,1) = 0\,, \quad (10)$$

  since, if the second machine is working, the buffer level can only decrease.
- *Interior-to-Boundary Equations*
  To be in state $(N,1,1,0)$ (or equally in state $(N,0,1,0)$) at time $t+\delta t$ the system could have been only in one of two sets of states at time t. It could have been in state $(N,1,1,0)$ (or $(N,0,1,0)$) with no repair of the second machine (the first could not have failed since it was blocked) or else in any interior state $(x,0,1,0)$, where $N - \mu_1\delta t \le x < N$, if repair of the second machine or failure of the first did not occur. Symbolically, ignoring the second order terms,

$$\mathbf{p}(N,\beta,1,0,t+\delta t) = (1 - r_2\delta t)\mathbf{p}(N,\beta,1,0,t) + \\ + \int_{N-\mu_1\delta t}^{N} f(x,0,1,0,t)dx\ ,\quad \forall\,\beta = 0,1\,.$$

  It is not necessary to consider transitions directly from states like $(x,0,1,1)$, since, if the second machine is working in t, the buffer level cannot reach N in $t+\delta t$. As $\delta t \to 0$, the equation becomes

$$\frac{d}{dt}\mathbf{p}(N,\beta,1,0) = -r_2\mathbf{p}(N,\beta,1,0) + \mu_1 f(N,\beta,1,0)\,, \\ \forall\,\beta = 0,1\,. \quad (11)$$

- *Boundary-to-Interior Equations*
  The only possible internal states reachable from the upper boundary are those with $\beta = 1$ and $\alpha_1 = 1$ because the first machine is forced to be blocked and can not fail. In addition, it is possible to leave the upper boundary $x = N$ only by repairing the second machine, then it results $\alpha_2 = 1$ and the buffer level decreases according to the productivity of the second machine (μ_2). To be in the state $(x,1,1,1)$ at time $t+\delta t$ the system can have been at the boundary state $(N,1,1,0)$ some time during the time interval $(t, t+\delta t)$, then

$$\int_{N}^{N-\mu_2\delta t} f(x,1,1,1,t+\delta t)dx = \\ \int_{t}^{t+\delta t} r_2\mathbf{p}(N,1,1,0,s)ds\,.$$

  Letting $\delta t \to 0$, the equation becomes

$$\mu_2 f(N,1,1,1) = r_2\mathbf{p}(N,1,1,0)\,. \quad (12)$$

B. Intermediate buffer level

The transition equations represent the behavior of the system at intermediate storage levels, that is, when the buffer is neither empty nor full. The set of equations reported below characterizes the system when the forced block state in M_1 is not reached. This is the case in which the parameter β equals 0, thus those equations are the same as the ones reported in [1].

$$\frac{\partial f}{\partial t}(x,0,1,1) = -(p_1+p_2)f(x,0,1,1) + \\ + (\mu_2-\mu_1)\frac{\partial f}{\partial x}(x,0,1,1) + \\ + r_1 f(x,0,0,1) + r_2 f(x,0,1,0), \quad (13)$$

$$\frac{\partial f}{\partial t}(x,0,0,0) = -(r_1+r_2)f(x,0,0,0) + \\ + p_1 f(x,0,1,0) + p_2 f(x,0,0,1), \quad (14)$$

$$\frac{\partial f}{\partial t}(x,0,0,1) = \mu_2\frac{\partial f}{\partial x}(x,0,0,1) + \\ - (r_1+p_2)f(x,0,0,1) + \\ + p_1 f(x,0,1,1) + r_2 f(x,0,0,0), \quad (15)$$

$$\frac{\partial f}{\partial t}(x,0,1,0) = -\mu_1\frac{\partial f}{\partial x}(x,0,1,0) + \\ - (p_1+r_2)f(x,0,1,0) + \\ + p_2 f(x,0,1,1) + r_1 f(x,0,0,0). \quad (16)$$

In the estimate case other equations are needed to model the entire behavior of the system. In fact, when the buffer level is between L and N, M_1 could be or not in the forced block state, hence there is another set of transient equation in which $\beta = 1$. Such equations are defined for $x \in \big[L, N\big]$.

When $\beta = 1$, M_1 is operational but in the forced block state, thus failures can not occur. Hence, the only transient states available in such a situation are $(x,1,1,1)$ and $(x,1,1,0)$.

Let us consider the first state $(x,1,1,1)$ representing the situation in which the machine M_2 is operational. The probability of finding such a state with a storage level between x

and $x+\delta x$ at time $t+\delta t$ is given by $f(x,1,1,1,t+\delta t)\delta x$, where:

$$f(x,1,1,1,t+\delta t) = (1-p_2\delta t)f(x+\mu_2\delta t,1,1,1,t) + \\ + r_2\delta t f(x,1,1,0,t) + \circ(\delta t)\,.$$

This derives from the following considerations:

1) If M_2 is operational at time t and the buffer level is $x+\mu_2\delta t$ (with $\delta x = \mu_2\delta t$), then, at time $t+\delta t$, the storage level will be x if failures do not occur in M_2 during δt, thus involving probability $\big(1-p_2\delta t\big)$.
2) If M_2 is down at time t, it can be up at time $t+\delta t$ if it will be repaired in δt, thus implying probability $r_2\delta t$. Moreover, there is not variation in the buffer level since M_1 is in the forced block state.
3) States characterized by $\alpha_1 = 0$ (M_1 is down) are not possible since, being M_1 in the forced block state, it can not fail.

With few steps the derivative form can be obtained.

$$f(x,1,1,1,t{+}\delta t) - f(x,1,1,1,t) = \\ (1-p_2)f(x+\mu_2\delta t,1,1,1,t) + \\ - f(x,1,1,1,t) + r_2\delta t f(x,1,1,0,t)\,,$$

$$\lim_{\delta t\to 0}\frac{f(x,1,1,1,t+\delta t)-f(x,1,1,1,t)}{\delta t} = \\ \lim_{\delta t\to 0}\big(-p_2 f(x+\mu_2\delta t,1,1,1,t)\big) + \\ + \lim_{\delta t\to 0}\big(r_2 f(x,1,1,0,t)\big) + \\ + \lim_{\delta t\to 0}\left(\frac{f(x+\mu_2\delta t,1,1,1,t)-f(x,1,1,1,t)}{\delta t}\right),$$

$$\frac{\partial f}{\partial t}(x,1,1,1,t) = -p_2 f(x,1,1,1,t) + r_2 f(x,1,1,0,t) + \\ + \mu_2 \lim_{\delta x\to 0}\left(\frac{f(x+\delta x,1,1,1,t)-f(x,1,1,1,t)}{\delta x}\right).$$

The final equation is here reported, where the t argument is suppressed.

$$\frac{\partial f}{\partial t}(x,1,1,1) = -\,p_2 f(x,1,1,1) + r_2 f(x,1,1,0) + \\ + \mu_2\frac{\partial f}{\partial x}(x,1,1,1)\,. \tag{17}$$

The same reasoning is adopted to obtain the other transient equation, reported in the following.

$$\frac{\partial f}{\partial t}(x,1,1,0) = -r_2 f(x,1,1,0) + p_2 f(x,1,1,1)\,. \tag{18}$$

C. Normalization and flow conservation

The normalization equation must be satisfied to assure that the sum of the probabilities of all possible states (transient and boundary) is 1.

$$\sum_{\alpha_1=0}^{1}\sum_{\alpha_2=0}^{1}\left[\int_0^N f(x,0,\alpha_1,\alpha_2)dx + \mathbf{p}(0,0,\alpha_1,\alpha_2)\right] + \\ + \sum_{\alpha_2=0}^{1}\left[\int_L^N f(x,1,1,\alpha_2)dx\right] + \mathbf{p}(N,1,1,0) = 1\,. \tag{19}$$

Moreover, the flow conservation must be established. The flow conservation is expressed by the following equality

$$P_1 = P_2\,, \tag{20}$$

where P_i is the throughput of the machine $i, \forall\, i = 1,2$.

Material leaves the second machine at rate μ_2 only if the buffer level is different from zero, otherwise the rate is equal to μ_1. Consequently,

$$P_2 = \mu_2\left[\int_0^N \big(f(x,0,0,1) + f(x,0,1,1)\big)dx + \\ + \int_L^N f(x,1,1,1)dx + \mathbf{p}(N,\beta,1,1)\right] + \\ + \mu_1\mathbf{p}(0,0,1,1)\,.$$

According to (10), $\mathbf{p}(N,\beta,1,1)$ goes to zero, then

$$P_2 = \mu_2\left[\int_0^N \big(f(x,0,0,1) + f(x,0,1,1)\big)dx + \\ + \int_L^N f(x,1,1,1)dx\right] + \mu_1\mathbf{p}(0,0,1,1)\,. \tag{21}$$

For what concerns the expression of P_1, it is necessary to consider also that material can not enter the first machine if the machine is forced to be blocked. Thus,

$$P_1 = \mu_1\left[\int_0^N \big(f(x,0,1,0) + f(x,0,1,1)\big)dx + \\ + \mathbf{p}(0,0,1,1)\right]. \tag{22}$$

Subtracting (22) from (21) yields

$$P_2 - P_1 = \int_0^N \big[(\mu_2-\mu_1)f(x,0,1,1) + \mu_2 f(x,0,0,1) + \\ - \mu_1 f(x,0,1,0)\big]dx + \int_L^N \mu_2 f(x,1,1,1)dx\,,$$

or

$$P_2 - P_1 = \int_0^N \big[(\mu_2-\mu_1)f(x,0,1,1) + \mu_2 f(x,0,0,1) + \\ - \mu_1 f(x,0,1,0) + \\ + \Phi(x)\mu_2 f(x,1,1,1)\big]dx\,, \tag{23}$$

where $\Phi(x)$ is defined as

$$\Phi(x) = \begin{cases} 0 & \text{if } 0 \le x < L\,, \\ 1 & \text{otherwise}\,. \end{cases} \tag{24}$$

The integrand of the (23) can also be obtained by adding the steady state versions $\left(\frac{\partial f}{\partial t} = 0\right)$ of the internal differential equations (13)–(18). It follows that, when the steady state is reached,

$$(\mu_2-\mu_1)f(x,0,1,1) + \mu_2 f(x,0,0,1) + \\ -\mu_1 f(x,0,1,0) + \Phi(x)\,\mu_2 f(x,1,1,1) = 0\,, \\ 0 \le x \le N\,. \tag{25}$$

Equation (25) implies that the integral in the (23) vanishes and therefore the flow conservation equation (20) is proved.

III. Simulation Results

In this section a simulation model for the non-homogeneous-two-machine line is presented. Such a model is used to asses the expected improvements that should be achieved by introducing a restart level for the filler machine. According to results shown in Section II, three scenarios have been evaluated by changing the ratio μ_1/μ_2 as shown in Table I:

Scenario	μ_1	μ_2	μ_1/μ_2
1	20.000	22.000	0.91
2	20.000	21.500	0.93
3	20.000	21.000	0.95

TABLE I
Simulation scenarios

The inputs of the model are shown in table II :

Parameters	***Value***
$MTBF_2$	20 min
$MTTR_2$	4 min
$waste$	100 items
$fillerRestartPoint$	4.000 items

TABLE II
Input of the model

Note that the reliability of the first machine is set equal to 1, thus it can be in the down state only if it is blocked. It should be outlined that at each restart one hunderd of packages should be withdraw because machine process characteristics. The operational model has been implemented on Flexsim©simulation platform.

The simulations were performed changing the value of L in the interval $[N, 0]$ for each scenario. Figure 2 shows the result of the simulation campaign.

In particular throughput performance over different value of the ratio L/N for each scenario is depicted. The three throughput curves have a similar shape: they present a maximum value somewhere in the $0-1$ range. This result underlines the advantage of introducing a level policy on the buffer that delays the restart of the filler machine in case the buffer has reach the maximum value N. In particular it is clear that it can be identified a level for ratio L/N which maximize the productivity level for each scenario, that fulfill the target of this study.

IV. Conclusions

The paper presented some issues in the modelling and simulation of complex packaging line. In particular a theoretical model of a line with two machines and a buffer has been presented, starting from a real case stay from Tetra Pak company. Simulation results proved effectiveness of the proposed solution.

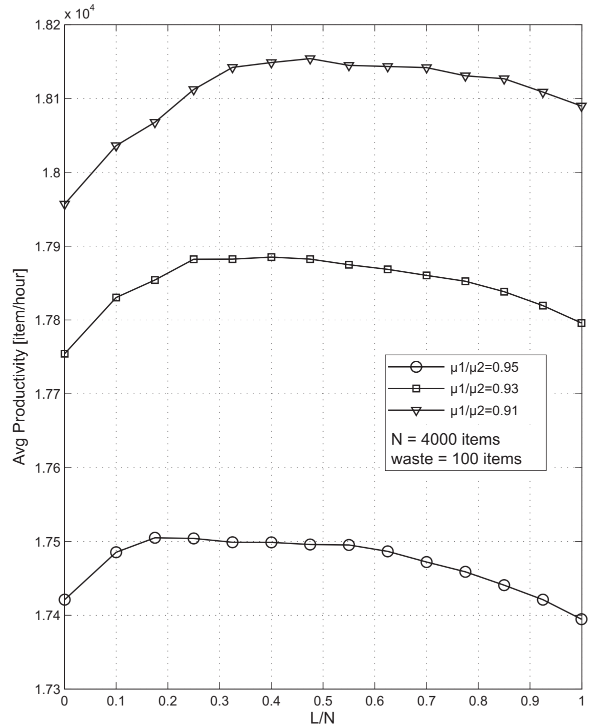

Fig. 2. Throughput performances

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