2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

Adaptive Vision based Tracking Control of Robots with Uncertainty in Depth Information

C. C. Cheah, C. Liu and J. J. E. Slotine

Abstract— **In this paper, a vision based tracking controller with adaptation to uncertainty in depth information is presented. Depth uncertainty plays a special role in visual tracking as it appears nonlinearly in the overall Jacobian matrix and hence cannot be adapted together with other uncertain kinematic parameters. We propose a novel parameter update law to update the uncertain parameters of the depth. It is proved that system stability can be guaranteed for the visual tracking task in presence of uncertainties in depth information, robot kinematics and dynamics. Simulation results are presented to illustrate the performance of the proposed controller.**

I. INTRODUCTION

The kinematics and dynamics of robot manipulators are highly nonlinear. The robot manipulators are required to manipulate various tools and hence the overall kinematic and dynamic parameters of the robots vary during operations and are difficult to be predicted in advance. By exploring physical properties of the robot system, Takegaki and Arimoto [1] and Arimoto [2] showed using Lyapunov's method that PD plus gravity compensation and PID feedback are effective for setpoint control despite the nonlinearity and uncertainty of the robot dynamics. To deal with trajectory tracking control, several adaptive robot control laws have been proposed and much progress has been obtained in robot tracking control theories with uncertain dynamic parameters [3]-[11].

However, most research on robot control has assumed that the exact kinematics and Jacobian matrix of the robot manipulators from joint space to Cartesian space are known. Unfortunately, no physical parameters can be derived exactly. Moreover, when the robot handle tools of different lengths, unknown orientations and gripping points, the overall kinematics are changing and therefore difficult to derive exactly. To overcome the problem of uncertain kinematics, several Approximate Jacobian setpoint controllers [12]-[14] were proposed recently. The proposed controllers do not require the exact knowledge of kinematics and Jacobian matrix that is assumed in the literature of robot control. The results in [12]-[14] are focusing on setpoint control or point to point control of robot. In some applications, it is necessary to specify the motion in much more details than simply stating the desired final position. Thus, a desired trajectory should be specified.

C. C. Cheah is with School of Electrical and Electronic Engineering, Nanyang Technological University, Block S1, Nanyang Avenue, S(639798), Republic of Singapore, Email: ECCCheah@ntu.edu.sg; C. Liu is with Robotics Department, LIRMM, 161 Rue Ada, 34392 Montpellier, France, Email: liu@lirmm.fr; J. J. E. Slotine is with Nonlinear System Laboratory, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139 USA, Email: jjs@mit.edu.

Recently, an adaptive Jacobian controller is proposed for trajectory tracking control of robot manipulators with uncertainties in kinematics and dynamics [15]. The main idea is to introduce an adaptive sliding vector based on estimated task-space velocity, so that kinematic and dynamic adaptations can be performed concurrently. A novel dynamics regressor using the estimated kinematics parameters was proposed. The novelty of the adaptive regressor lies in that online updating information of the updated system parameters is utilized in the regressor instead of only using measurable system variables and parameters. It enables separate and simultaneous treatments of different kinds of uncertainties in robot system and presents a potential tool for studying more general nonlinear systems containing multiple uncertainties. It was shown that the end-effector's position converges to the desired position even when the kinematics and Jacobian matrix are uncertain.

To avoid singularities associated Euler angle representation, adaptive Jacobian tracking controller based on unit quaternion is proposed [16]. In [17], an adaptive Jacobian tracking controller is proposed for redundant robots with uncertain actuator torque transmission matrix. To improve the control performance especially in cases of high speed movement, an adaptive Jacobian control method is proposed for trajectory tracking control of rigid-link electrically-driven robot which can deal with the uncertainties in robot kinematics, dynamics and the actuator dynamics at the same time [18]. In this case, the actuator dynamics leads to a third order system and accelerations measurements are avoided in the control inputs by constructing observers to specify desired armature currents.

The above adaptive Jacobian tracking controllers consider the case where the unknown kinematic parameters of the Jacobian matrix from joint space to task space are linearly parameterizable. However, in visual tracking control problems, the image velocity is inversely proportional to the depth information and hence the depth information appears nonlinearly in the overall Jacobian matrix thus cannot be adapted together with other unknown kinematic parameters. Therefore these adaptive controllers are effective only in cases where the depth information is slowly time-varying. Though image-based visual servoing techniques are known to be robust to modeling and calibration errors in practice, only a few theoretical results been obtained for the stability analysis in presence of the uncertain camera parameters [19]-[22] and in most results the depth information is assumed to be known exactly. To deal with uncertainty in depth information, some vision based controllers [23], [24] are

1-4244-0602-1/07/$20.00 ©2007 IEEE.

proposed recently. However, these results are focusing on uncertainty in interaction matrix or image Jacobian matrix, and the effects of uncertain robot kinematics and dynamics are not considered. Hence, no rigorous theoretical result has been obtained for the stability analysis of visual tracking control with uncertainties in depth information, taking into consideration the uncertainties of the nonlinear robot kinematics and dynamics.

In this paper, we extends our recent results in [15] to visual tracking control with uncertainty in depth information, kinematics and dynamics. The main new point is the adaptation to depth uncertainty in addition to kinematics and dynamics uncertainty. Simulation results are presented to illustrate the performance of the proposed controller.

II. Robot Dynamics and Kinematics

The dynamics of the robot manipulator with n degree of degrees of freedom can be expressed as [2], [25]:

$$M(q)\ddot{q}+(\frac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q}+g(q)=u \tag{1}$$

where $q\in R^n$ is a vector of generalized joint coordinates, $M(q)\in R^{n\times n}$ is the inertia matrix, $u\in R^n$ is a vector of applied joint torques,

$$S(q,\dot{q})\dot{q} \;=\; \frac{1}{2}\dot{M}(q)\dot{q}-\frac{1}{2}\{\frac{\partial}{\partial q}\dot{q}^T M(q)\dot{q}\}^T$$

and $g(q)\in R^n$ is the gravitational force. Several important properties of the dynamic equation described by equation (1) are given as follows [2], [4], [25], [30]:

Property 1 The inertia matrix $M(q)$ is symmetric and uniformly positive definite for all $q\in R^n$. ◇

Property 2 The matrix $S(q,\dot{q})$ is skew-symmetric so that $v^T S(q,\dot{q})v=0$ for all $v\in R^n$. ◇

Property 3 The dynamic model as described by equation (1) is linear in a set of physical parameters $\theta_d=(\theta_{d1},\cdots,\ \theta_{dp})^T$ as

$$M(q)\ddot{q}+(\frac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q}+g(q)=Y_d(q,\dot{q},\dot{q},\ddot{q})\theta_d$$

where $Y_d(\cdot)\in R^{n\times p}$ is the dynamic regressor matrix. ◇

In most applications of robot manipulators, a desired path for the end-effector is specified in task space or operation space such as Cartesian space or camera image space [2], [12], [31]. In this work, a fixed pin-hole camera is utilized to monitor the motion of a feature point attached to the robot end-effector and hence the task space is defined as camera image space in pixels. Let $x\in R^2$ be the coordinates of the feature point's projection on the camera image plane and $r\in R^b$ denotes the coordinates of the feature point in robot base frame. r can be defined by [35]

$$r=h(q), \tag{2}$$

where $b<n$, $h(\cdot)\in R^b$ is a transformation describing the relation between the joint space and robot base frame. From the forward kinematics equation (2), it has

$$\dot{r}=J_m(q)\dot{q} \tag{3}$$

where $J_m(q)$ is the manipulator Jacobian matrix mapping from joint space to robot base frame.

The relationship between velocities in camera image space and robot base frame is represented by [23], [24]

$$\dot{x}=\frac{1}{z}L(x)\cdot\dot{r}, \tag{4}$$

where $z\in R$ represents the depth of the feature point with respect to the camera image frame, matrix $L(x)$ is a Jacobian matrix.

From the kinematics equations (3) and (4), the image-space velocity $\dot{x}$ can be related to joint-space velocity $\dot{q}$ as [23], [24], [26]:

$$\dot{x}=\frac{1}{z}L(x)\cdot\dot{r}=\frac{1}{z}L(x)J_m(q)\dot{q}=\frac{1}{z}J(q)\dot{q} \tag{5}$$

and $J(q)=L(x)J_m(q)$ is the lumped Jacobian matrix.

In the presence of uncertainties in robot kinematics and camera parameters, neither the Jacobian matrix $J(q)$ nor the depth z can be obtained exactly.

A property of the kinematic equation described by equation (5) is stated as follows [15]:

Property 4 Both z and $J(q)\dot{q}$ in equation (5) are linear in sets of kinematic parameters $\theta_k=(\theta_{k1},\cdots,\ \theta_{kq})^T$ and $\theta_z=(\theta_{z1},\cdots,\ \theta_{zj})^T$, such as robot link lengths, camera intrinsic and extrinsic parameters. Hence, z and $J(q)\dot{q}$ can be expressed as,

$$z=Y_z(q)\theta_z, \tag{6}$$

$$J(q)\dot{q}=Y_k(q,\dot{q})\theta_k, \tag{7}$$

where $Y_z(q)\in R^{1\times j}$ is called the depth regressor vector and $Y_k(q,\dot{q})\in R^{2\times q}$ is called the kinematic regressor matrix. ◇

Remark: Although both z and $J(q)\dot{q}$ are linear in kinematic parameters, the overall Jacobian matrix $\frac{1}{z}J(q)$ is not linearly parameterizable because it is is *inversely proportional* to z and hence the kinematic parameters in z and $J(q)$ cannot be extracted to form a lumped kinematic parameter vector.

III. Adaptive Visual Tracking Control

In this section, we present an Adaptive Jacobian Tracking Controller for robot with uncertain kinematics, dynamics and camera depth information.

In the presence of kinematics and camera parameter uncertainty, the camera depth information and the Jacobian matrix are uncertain and hence equation (5) can be expressed as

$$\hat{\dot{x}}=\frac{1}{\hat{z}(q,\hat{\theta}_z)}\hat{J}(q,\hat{\theta}_k)\dot{q} \tag{8}$$

where $\hat{\dot{x}}\in R^2$ denotes an estimated image-space velocity, $\hat{J}(q,\hat{\theta}_k)\in R^{2\times n}$ is the approximate Jacobian matrix and $\hat{\theta}_k\in R^q$ denotes a set of estimated kinematic parameters, $\hat{z}(q,\hat{\theta}_z)\in R$ is the estimated depth of the end-effector feature point and $\hat{\theta}_z\in R^j$ denotes the estimated parameters in $\hat{z}(q,\hat{\theta}_z)$. It's assumed that a predefined range $[\theta_{min},\theta_{max}]$ for θ_z is known such that $z(q)=Y_z(q)\theta_z$ is positive for all $\theta_z\in[\theta_{min},\theta_{max}]$.

Let us define a vector $\dot{x}_r\in R^2$ as

$$\dot{x}_r=\dot{x}_d-\alpha(x-x_d) \tag{9}$$

where $x_d \in R^2$ is a desired trajectory, $\dot{x}_d = \frac{dx_d}{dt} \in R^2$ is the desired velocity specified in camera image space and α is a positive constant. Differentiating equation (9) with respect to time, we have

$$\ddot{x}_r = \ddot{x}_d - \alpha(\dot{x} - \dot{x}_d) \quad (10)$$

where $\ddot{x}_d = \frac{d\dot{x}_d}{dt} \in R^2$ is the desired acceleration.

Next, define an adaptive task-space sliding vector using equation (8) as,

$$\hat{s}_x = \hat{\dot{x}} - \dot{x}_r \quad (11)$$

Differentiating equation (11) with respect to time, we have,

$$\dot{\hat{s}}_x = \hat{\ddot{x}} - \ddot{x}_r = \frac{1}{\hat{z}(q,\hat{\theta}_z)}\hat{J}(q,\hat{\theta}_k)\ddot{q} + \frac{1}{\hat{z}(q,\hat{\theta}_z)}\dot{\hat{J}}(q,\hat{\theta}_k)\dot{q} - \frac{\dot{\hat{z}}(q,\hat{\theta}_z)}{\hat{z}^2(q,\hat{\theta}_z)}\hat{J}(q,\hat{\theta}_k)\dot{q} - \ddot{x}_r \quad (12)$$

where $\hat{\ddot{x}}$ denotes the derivative of $\hat{\dot{x}}$. In the redundant case, the null space of the approximate Jacobian matrix can be used to minimize a performance index [32]. Next, let

$$\dot{q}_r = \hat{z}(q,\hat{\theta}_z)\hat{J}^+(q,\hat{\theta}_k)\dot{x}_r + (I - \hat{J}^+(q,\hat{\theta}_k)\hat{J}(q,\hat{\theta}_k))\psi \quad (13)$$

where $\hat{J}^+(q,\hat{\theta}_k) = \hat{J}^T(q,\hat{\theta}_k)(\hat{J}(q,\hat{\theta}_k)\hat{J}^T(q,\hat{\theta}_k))^{-1}$ is the generalized inverse of the approximate Jacobian matrix $\hat{J}(q,\hat{\theta}_k)$, and $\psi \in R^n$ is minus the gradient of the convex function to be optimized, I represents the identity matrix. In this paper, we assume that the robot is operating in a finite work space such that the approximate Jacobian matrix is of full rank.

From equation (13), we have

$$\ddot{q}_r = \hat{z}(q,\hat{\theta}_z)\hat{J}^+(q,\hat{\theta}_k)\ddot{x}_r + \hat{z}(q,\hat{\theta}_z)\dot{\hat{J}}^+(q,\hat{\theta}_k)\dot{x}_r + \dot{\hat{z}}(q,\hat{\theta}_z)\hat{J}^+(q,\hat{\theta}_k)\dot{x}_r + (I - \hat{J}^+(q,\hat{\theta}_k)\hat{J}(q,\hat{\theta}_k))\dot{\psi} - \dot{\hat{J}}^+(q,\hat{\theta}_k)\hat{J}(q,\hat{\theta}_k)\psi - \hat{J}^+(q,\hat{\theta}_k)\dot{\hat{J}}(q,\hat{\theta}_k)\psi \quad (14)$$

Hence, we have an adaptive sliding vector in joint space as

$$s = \dot{q} - \dot{q}_r \quad (15)$$

and

$$\dot{s} = \ddot{q} - \ddot{q}_r. \quad (16)$$

From equations (13) and (15), we note that

$$\frac{1}{\hat{z}(q,\hat{\theta}_z)}\hat{J}(q,\hat{\theta}_k)s = \frac{1}{\hat{z}(q,\hat{\theta}_z)}\hat{J}(q,\hat{\theta}_k)\dot{q} - \dot{x}_r = \hat{\dot{x}} - \dot{x}_r = \hat{s}_x \quad (17)$$

Substituting equations (15) and (16) into equation (1), the equations of motion can be expressed as,

$$M(q)\dot{s} + (\tfrac{1}{2}\dot{M}(q) + S(q,\dot{q}))s + M(q)\ddot{q}_r + (\tfrac{1}{2}\dot{M}(q) + S(q,\dot{q}))\dot{q}_r + g(q) = u \quad (18)$$

From *Property 3*, the last five terms of equation (18) are linear in a set of dynamics parameters θ_d and hence can be expressed as,

$$M(q)\ddot{q}_r + (\frac{1}{2}\dot{M}(q) + S(q,\dot{q}))\dot{q}_r + g(q) = Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\theta_d$$

so the dynamic equation (18) can be written as,

$$M(q)\dot{s} + (\tfrac{1}{2}\dot{M}(q) + S(q,\dot{q}))s + Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\theta_d = u \quad (19)$$

The algorithm we shall now derive is composed of (i) a control law based on the approximate Jacobian matrix and approximate depth information as,

$$u = -\frac{1}{\hat{z}(q,\hat{\theta}_z)}\hat{J}^T(q,\hat{\theta}_k)(K_v\Delta\dot{x} + K_p\Delta x) + Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\theta}_d, \quad (20)$$

where $\Delta x = x - x_d$, $\Delta\dot{x} = \dot{x} - \dot{x}_d$, $K_v \in R^{2\times 2}$ and $K_p \in R^{2\times 2}$ are symmetric positive definite gain matrices, (ii) a dynamic adaptation law

$$\dot{\hat{\theta}}_d = -L_d Y_d^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)s \quad (21)$$

iii) a kinematic adaptation law

$$\dot{\hat{\theta}}_k = \frac{1}{\hat{z}(q,\hat{\theta}_z)}L_k Y_k^T(q,\dot{q})(K_v\Delta\dot{x} + K_p\Delta x) \quad (22)$$

and iv) a depth adaptation law

$$\dot{\hat{\theta}}_z = -\frac{1}{\hat{z}(q,\hat{\theta}_z)}\Phi L_z Y_z^T(q)\dot{x}^T(K_v\Delta\dot{x} + K_p\Delta x) \quad (23)$$

where $L_k \in R^{q\times q}$, $L_d \in R^{p\times p}$, $L_z \in R^{j\times j}$ are symmetric positive definite matrices, Φ is a projection operator matrix defined as $\Phi = diag\{\phi_1,\cdots,\phi_j\}$ [33] and

$$\phi_i = \begin{cases} 0, & \text{if } \hat{\theta}_{zi} \geq \theta_{imax} \text{ and } \dot{\hat{\theta}}_{zi} \geq 0 \\ 0, & \text{if } \hat{\theta}_{zi} \leq \theta_{imin} \text{ and } \dot{\hat{\theta}}_{zi} \leq 0 \\ 1, & \text{otherwise.} \end{cases} \quad (24)$$

such that $\hat{\theta}_z \in [\theta_{min}, \theta_{max}]$.

The closed-loop dynamics is obtained by substituting (20) into equation (19) to give

$$M(q)\dot{s} + (\tfrac{1}{2}\dot{M}(q) + S(q,\dot{q}))s + Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Delta\theta_d + \frac{1}{\hat{z}(q,\hat{\theta}_z)}\hat{J}^T(q,\hat{\theta}_k)(K_v\Delta\dot{x} + K_p\Delta x) = 0 \quad (25)$$

where $\Delta\theta_d = \theta_d - \hat{\theta}_d$.

Let us define a Lyapunov-like function candidate as

$$V = \tfrac{1}{2}s^T M(q)s + \tfrac{1}{2}\Delta\theta_d^T L_d^{-1}\Delta\theta_d + \tfrac{1}{2}\Delta\theta_k^T L_k^{-1}\Delta\theta_k + \tfrac{1}{2}\Delta\theta_z^T L_z^{-1}\Delta\theta_z + \tfrac{1}{2}\Delta x^T(K_p + \alpha K_v)\Delta x \quad (26)$$

where $\Delta\theta_k = \theta_k - \hat{\theta}_k$. Differentiating with respect to time and using *Property 1*, we have

$$\dot{V} = s^T M(q)\dot{s} + \tfrac{1}{2}s^T\dot{M}(q)s - \Delta\theta_d^T L_d^{-1}\dot{\hat{\theta}}_d - \Delta\theta_k^T L_k^{-1}\dot{\hat{\theta}}_k - \Delta\theta_z^T L_z^{-1}\dot{\hat{\theta}}_z + \Delta x^T(K_p + \alpha K_v)\Delta\dot{x}$$

Substituting $M(q)\dot{s}$ from equation (25), $\dot{\hat{\theta}}_k$ from equation (22), $\dot{\hat{\theta}}_d$ from equation (21) and $\dot{\hat{\theta}}_z$ from equation (23) into the above equation, using *Property 2* and equation (17) yields,

$$\dot{V} = -\hat{s}_x^T(K_v\Delta\dot{x} + K_p\Delta x) - s^T Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Delta\theta_d - \Delta\theta_d^T L_d^{-1}\dot{\hat{\theta}}_d - \Delta\theta_k^T L_k^{-1}\dot{\hat{\theta}}_k - \Delta\theta_z^T L_z^{-1}\dot{\hat{\theta}}_z + \Delta x^T(\alpha K_v + K_p)\Delta\dot{x} \quad (27)$$

From equations (6), (7), (9) and (11), we have

$$\begin{aligned}\hat{s}_x &= \tfrac{1}{\hat{z}(q,\hat{\theta}_z)}\hat{J}(q,\hat{\theta}_k)\dot{q} - \dot{x}_d + \alpha\Delta x \\ &= \tfrac{1}{\hat{z}(q,\hat{\theta}_z)}[\hat{J}(q,\hat{\theta}_k)\dot{q} - J(q)\dot{q} + \dot{x}(z-\hat{z}(q,\hat{\theta}_z)) \\ &\quad + \hat{z}(q,\hat{\theta}_z)(\dot{x}-\dot{x}_d) + \alpha\hat{z}(q,\hat{\theta}_z)\Delta x] \\ &= \tfrac{1}{\hat{z}(q,\hat{\theta}_z)}[-Y_k(q,\dot{q})\Delta\theta_k + \dot{x}Y_z(q)\Delta\theta_z \\ &\quad + \hat{z}(q,\hat{\theta}_z)(\Delta\dot{x}+\alpha\Delta x)] \end{aligned} \tag{28}$$

where

$$Y_k(q,\dot{q})\Delta\theta_k = J(q)\dot{q} - \hat{J}(q,\hat{\theta}_k)\dot{q} = \dot{x} - \hat{\dot{x}} \tag{29}$$

Substituting parameter update laws (21), (22), (23) and equation (28) into equation (27) yields,

$$\begin{aligned}\dot{V} &= -\Delta\theta_z^T(I-\Phi)\tfrac{1}{\hat{z}(q,\hat{\theta}_z)}Y_z^T(q)\dot{x}^T(K_v\Delta\dot{x}+K_p\Delta x) \\ &\quad -\Delta\dot{x}^T K_v\Delta\dot{x} - \alpha\Delta x^T K_p\Delta x\end{aligned} \tag{30}$$

From the definition of (24), it can be easily verified that

$$\dot{V} \le -\Delta\dot{x}^T K_v\Delta\dot{x} - \alpha\Delta x^T K_p \Delta x \le 0 \tag{31}$$

We are now in a position to state the following Theorem:

Theorem 1 The adaptive Jacobian tracking control law (20) and the parameter update laws (21), (22)and (23) guarantee the stability and result in the convergence of position and velocity tracking errors of the adaptive control system. That is, $x - x_d \to 0$ and $\dot{x} - \dot{x}_d \to 0$, as $t \to \infty$.

Proof:

Since $M(q)$ is uniformally positive definite, V in equation (26) is positive definite in s, Δx, $\Delta\theta_k$, $\Delta\theta_d$ and $\Delta\theta_z$. Since $\dot{V} \le 0$, V is bounded, and therefore s, Δx, $\Delta\theta_k$, $\Delta\theta_d$ and $\Delta\theta_z$ are bounded vectors. This implies that $\hat{\theta}_k$, $\hat{\theta}_d$ are bounded and $\hat{\theta}_z$ is bounded by the projection algorithm, x is bounded if x_d is bounded, and $\hat{s}_x = \hat{J}(q,\hat{\theta}_k)s$ is also bounded. Since Δx is bounded, $\dot{x}_r$ in equation (9) is bounded if $\dot{x}_d$ is bounded. Therefore, $\dot{q}_r$ in equation (13) is also bounded if the approximate Jacobian matrix is of full rank. From equations (15), $\dot{q}$ is bounded and the boundedness of $\dot{q}$ means that $\dot{x}$ is bounded since the Jacobian matrix is bounded. Hence, $\Delta\dot{x}$ is bounded and $\ddot{x}_r$ in equation (10) is also bounded if $\ddot{x}_d$ is bounded. From equation (22), $\dot{\hat{\theta}}_k$ is therefore bounded since Δx, $\Delta\dot{x}$, $\dot{q}$ are bounded and $Y_k(\cdot)$ is a trigonometric function of q. Similarly, $\dot{\hat{\theta}}_z$ is bounded from equation (23) and $Y_z(q)$ is a trigonometric function of q. Therefore, $\ddot{q}_r$ in equation (14) is bounded. From the closed-loop equation (25), we can conclude that $\dot{s}$ is bounded. The boundedness of $\dot{s}$ implies the boundedness of $\ddot{q}$ as can be seen from equation (16). Then $\ddot{x} = \dot{J}(q)\dot{q} + J(q)\ddot{q}$ is bounded, which means that $\Delta\ddot{x} = \ddot{x} - \ddot{x}_d$ is also bounded if $\ddot{x}_d$ is bounded. From the boundedness of $\Delta\ddot{x}$ and $\Delta\dot{x}$, we can conclude that $\Delta\dot{x}$ and Δx are uniformly continuous. Since V is bounded and from inequality (31), it's clear that Δx and $\Delta\dot{x} \in L_2(0,\infty)$. Then according to [34], we have $\Delta x = x - x_d \to 0$ and $\Delta\dot{x} = \dot{x} - \dot{x}_d \to 0$ as $t \to \infty$. △△△

IV. Extension to Multiple Feature Points Tracking Control

In this section, we extend the result to the case where multiple feature points are used and the depth information of each feature point with respect to the camera frames are different.

In the presence of kinematics and camera parameter uncertainties, similar as equation (8) we have the estimated image space velocities vector

$$\hat{\dot{x}} = \hat{Z}^{-1}(q,\hat{\theta}_z)\hat{J}(q,\hat{\theta}_k)\dot{q} \tag{32}$$

where $\hat{\dot{x}} = [\hat{\dot{x}}_1^T, \hat{\dot{x}}_2^T, \cdots, \hat{\dot{x}}_m^T]^T \in R^{2m}$, $\hat{\dot{x}}_i$ denotes the approximate image velocity of the i^{th} feature point and m is the total number of feature points. $\hat{Z}(q,\hat{\theta}_z) \in R^{2m\times 2m}$ and $\hat{J}(q,\hat{\theta}_k) \in R^{2m\times n}$ are defined as

$$\hat{Z}^{-1}(q,\hat{\theta}_z) = \begin{bmatrix} \frac{1}{\hat{z}_1(q,\hat{\theta}_{z1})}I, & 0 & \cdots & 0 \\ 0 & \frac{1}{\hat{z}_2(q,\hat{\theta}_{z2})}I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\hat{z}_m(q,\hat{\theta}_{zm})}I \end{bmatrix} \tag{33}$$

$$\hat{J}(q,\hat{\theta}_k) = \begin{bmatrix} \hat{J}_1(q,\hat{\theta}_{k1}) & \hat{J}_2(q,\hat{\theta}_{k2}) & \cdots & \hat{J}_m(q,\hat{\theta}_{km}) \end{bmatrix}^T \tag{34}$$

and $\hat{J}_i(q,\hat{\theta}_{ki})$ is the approximate Jacobian matrix corresponding to the i^{th} feature point.

Similarly, define an adaptive task-space sliding vector as,

$$\hat{s}_x = \hat{\dot{x}} - \dot{x}_r \tag{35}$$

Next, let

$$\dot{q}_r = \hat{J}^+(q,\hat{\theta}_k)\hat{Z}(q,\hat{\theta}_z)\dot{x}_r + (I - \hat{J}^+(q,\hat{\theta}_k)\hat{J}(q,\hat{\theta}_k))\psi \tag{36}$$

Hence, we have an adaptive sliding vector in joint space as,

$$s = \dot{q} - \dot{q}_r \tag{37}$$

From equations (36) and (37), we note that

$$\begin{aligned}\hat{Z}^{-1}(q,\hat{\theta}_z)\hat{J}(q,\hat{\theta}_k)s &= \hat{Z}^{-1}(q,\hat{\theta}_z)\hat{J}(q,\hat{\theta}_k)\dot{q} - \dot{x}_r \\ &= \hat{\dot{x}} - \dot{x}_r = \hat{s}_x\end{aligned} \tag{38}$$

The dynamic equation can be similarly written as,

$$M(q)\dot{s} + (\tfrac{1}{2}\dot{M}(q) + S(q,\dot{q}))s + Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\theta_d = u \tag{39}$$

The algorithm we shall now derive is composed of (i) a control law based on the approximate Jacobian matrices and approximate depth information as,

$$\begin{aligned}u &= -\hat{J}^T(q,\hat{\theta}_k)\hat{Z}^{-1}(q,\hat{\theta}_z)(K_v\Delta\dot{x} + K_p\Delta x) \\ &\quad + Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\theta}_d,\end{aligned} \tag{40}$$

where $\Delta x = x - x_d \in R^{2m}$, $\Delta\dot{x} = \dot{x} - \dot{x}_d \in R^{2m}$, $K_v \in R^{2m\times 2m}$ and $K_p \in R^{2m\times 2m}$ are symmetric positive definite gain matrices, (ii) a dynamic adaptation law

$$\dot{\hat{\theta}}_d = -L_d Y_d^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)s \tag{41}$$

iii) a kinematic adaptation law

$$\dot{\hat{\theta}}_k = L_k Y_k^T(q,\dot{q})\hat{Z}^{-1}(q,\hat{\theta}_z)(K_v\Delta\dot{x} + K_p\Delta x) \tag{42}$$

and iv) a depth adaptation law

$$\dot{\hat{\theta}}_z = -\Phi L_z Y_z^T(q,\dot{x})\hat{Z}^{-1}(q,\hat{\theta}_z)(K_v\Delta\dot{x}+K_p\Delta x) \quad (43)$$

where L_k, L_d, L_z are symmetric positive definite matrices, Φ is defined in the same way as in (24) and regressor $Y_z(q,\dot{x})$ is defined as

$$\hat{Z}(q,\hat{\theta}_z)\dot{x} = Y_z(q,\dot{x})\hat{\theta}_z. \quad (44)$$

The closed-loop dynamics is obtained by substituting (40) into equation (39) to give

$$M(q)\dot{s}+(\tfrac{1}{2}\dot{M}(q)+S(q,\dot{q}))s+Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Delta\theta_d \\ +\hat{J}^T(q,\hat{\theta}_k)\hat{Z}^{-1}(q,\hat{\theta}_z)(K_v\Delta\dot{x}+K_p\Delta x)=0 \quad (45)$$

where $\Delta\theta_d = \theta_d - \hat{\theta}_d$. Let us define a Lyapunov-like function candidate as

$$V = \tfrac{1}{2}s^T M(q)s+\tfrac{1}{2}\Delta\theta_d^T L_d^{-1}\Delta\theta_d+\tfrac{1}{2}\Delta\theta_k^T L_k^{-1}\Delta\theta_k \\ +\tfrac{1}{2}\Delta\theta_z^T L_z^{-1}\Delta\theta_z+\tfrac{1}{2}\Delta x^T(K_p+\alpha K_v)\Delta x \quad (46)$$

where $\Delta\theta_k = \theta_k - \hat{\theta}_k$. Differentiating with respect to time, substituting equations (38), (41), (42), (43) and equation (45) yields,

$$\dot{V} = -\hat{s}_x^T(K_v\Delta\dot{x}+K_p\Delta x)-s^T Y_d(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Delta\theta_d \\ -\Delta\theta_d^T L_d^{-1}\dot{\hat{\theta}}_d-\Delta\theta_k^T L_k^{-1}\dot{\hat{\theta}}_k-\Delta\theta_z^T L_z^{-1}\dot{\hat{\theta}}_z \\ +\Delta x^T(\alpha K_v+K_p)\Delta\dot{x} \quad (47)$$

Similar as in (28), from equations (9), (32), (35) and (44), we have

$$\hat{s}_x = \hat{Z}^{-1}(q,\hat{\theta}_z)\hat{J}(q,\hat{\theta}_k)\dot{q}-\dot{x}_d+\alpha\Delta x \\ = \hat{Z}^{-1}(q,\hat{\theta}_z)[-Y_k(q,\dot{q})\Delta\theta_k+Y_z(q,\dot{x})\Delta\theta_z \\ +\hat{Z}(q,\hat{\theta}_z)(\Delta\dot{x}+\alpha\Delta x)] \quad (48)$$

Substituting parameter update laws (41), (42), (43) and equation (48) into equation (47) yields,

$$\dot{V} = -\Delta\theta_z^T(I-\Phi)Y_z^T(q,\dot{x})\hat{Z}^{-1}(q,\hat{\theta}_z)(K_v\Delta\dot{x}+K_p\Delta x) \\ -\Delta\dot{x}^T K_v\Delta\dot{x}-\alpha\Delta x^T K_p\Delta x \quad (49)$$

Similarly, from the definition of (24), we have

$$\dot{V} \leq -\Delta\dot{x}^T K_v\Delta\dot{x}-\alpha\Delta x^T K_p\Delta x \leq 0 \quad (50)$$

Theorem 2 The adaptive Jacobian tracking control law (40) and the parameter update laws (41), (42) and (43) guarantee the stability of the adaptive control system and the position and velocity tracking errors of the m feature points converge to 0 as $t \to \infty$.

The proof is similar as *Theorem* 1 and hence is omitted here.

V. Simulation Study

In this section, the simulation study for the control method proposed in this work is presented. The simulation is based on a 3 DOF anthropomorphic manipulator with a camera installed a distance away from the robot, as illustrated in Fig 1.

In this simulation study, the true values of the robot link lengths are set as $l_1 = l_2 = l_3 = 0.5$ m, the distance from the camera frame to the robot base frame is $d = 2$ m and the camera scaling factors are $\alpha_x = \alpha_y = 1000$. To examine the performance of the proposed control method with the presence of uncertainties in robot kinematics and camera parameters, the approximate link lengths are set as $\hat{l}_1 = \hat{l}_2 = \hat{l}_3 = 0.4$ m, the distance from camera frame to robot base frame is estimated as $\hat{d} = 1.5$ m and the scaling factors of the camera are estimated as $\hat{\alpha}_x = \hat{\alpha}_y = 800$. The robot is then requested to follow a desired path defined in camera image space (pixel) as:

$$x_d = \begin{pmatrix} 250+30sin(t+\frac{\pi}{2}) \\ 250+30cos(t+\frac{\pi}{2}) \end{pmatrix}$$

With the control gains set as $K_p = 0.3I$, $K_v = 0.03I$, $\alpha = 0.1$ and the parameter adaptation gains $L_d = 10I$, $L_k = 10I$, $L_z = 10I$, the robot end-effector trajectory and the tracking errors are illustrated in Fig 2 and Fig 3 respectively. The simulation results show that the proposed control method can guarantee the convergence of the camera image space tracking errors even with the presence of kinematics and dynamics uncertainties.

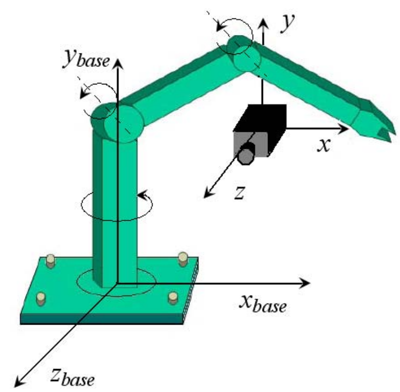

Fig. 1. Visual Servoing of 3 DOF Robot

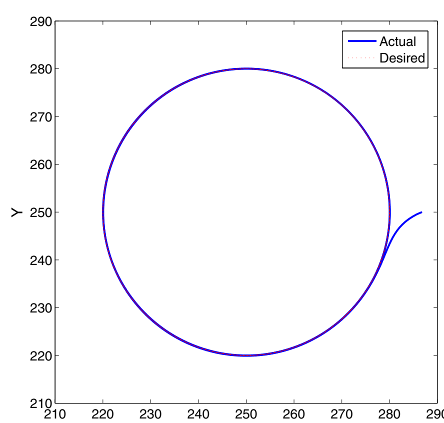

Fig. 2. Desired v.s. Actual Path

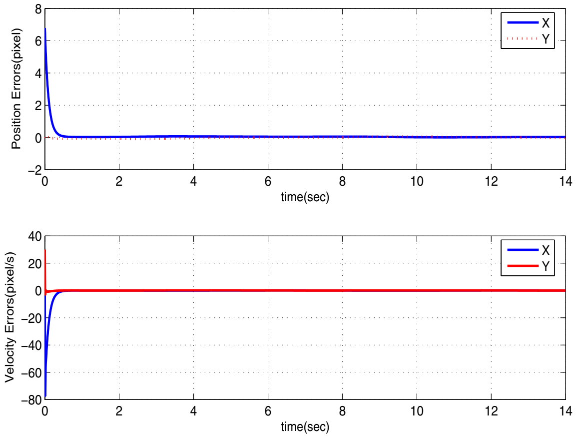

Fig. 3. Position and Velocity Errors

VI. Conclusion

We have proposed an adaptive vision based controller for tracking control of robot with uncertain depth information, kinematics and dynamics. Novel parameter update law for the depth information is proposed to update the uncertain parameters of the depth. We have shown that the robot is able to track a desired trajectory with the uncertain parameters being updated online by the proposed parameter update laws. Simulation results illustrate the performance of the proposed controller.

References

[1] M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators," *ASME J. of Dynamic Systems, Measurement and Control*, vol. 102, pp. 119–125, 1981.

[2] S. Arimoto, *Control Theory of Nonlinear Mechanical Systems - A Passivity-Based and Circuit-Theoretic Approach*. Oxford: Clarendon Press, 1996.

[3] J. J. Craig, P. Hsu, and S. S. Sastry, "Adaptive control of mechanical manipulators," *Int. J. Robotics Research*, vol. 6, no. 2, pp. 16–28, 1987.

[4] J. J. E. Slotine and W. Li, "On the adaptive control of robot manipulators," *Int. J. Robotics Research*, no. 6, pp. 49–59, 1987.

[5] R. H. Middleton and G. C. Goodwin, "Adaptive computed torque control for rigid link manipulators," *Systems and Control Letter*, vol. 10, pp. 9–16, 1988.
R. Kelly and R. Carelli, and R. Ortega, "Adaptive motion control design of robot manipulators. An input-output approach," *International Journal of Control*, vol. 50 no. 6, pp 2563-2581 1989.

[6] R. Ortega and M. W. Spong, "Adaptive motion control of rigid robots: a tutorial," *Automatica*, vol. 25, no. 6, pp. 877–888, 1989.

[7] N. Sadegh and R. Horowitz, "Stability and robustness analysis of a class of adaptive controllers for robotic manipulators," *International Journal of Robotics Research*, vol. 9, no. 3, pp. 74–92, 1990.

[8] H. Berghuis, R. Ortega and H. Nijmeijer, "A robust adaptive robot controller", *IEEE Transactions on Robotics and Automation*, vol. 9, no. 6, pp 825 - 830, 1993.

[9] L. L. Whitcomb, A. Rizzi, and D.E. Koditschek, "Comparative Experiments with a New Adaptive Controller for Robot Arms", *IEEE Transactions on Robotics and Automation*, vol. 9, no. 1, pp 59-70, 1993.

[10] K.W. Lee and H. Khalil, "Adaptive output feedback control of robot manipulators using high gain observer", *Int. J. Control*, vol. 67, no. 6, 1997

[11] P. Tomei, "Robust adaptive friction compensation for tracking control of robot manipulators," *IEEE Transactions on Automatic Control*, vol. 45, no. 11, pp. 2164 – 2169, 2000.

[12] C. C. Cheah, S. Kawamura, and S. Arimoto, "Feedback control for robotic manipulators with an uncertain jacobian matrix," *Journal of Robotic System*, vol. 12, no. 2, pp. 119–134, 1999.

[13] C. C. Cheah, M. Hirano, S. Kawamura, and S. Arimoto, "Approximate jacobian control for robots with uncertain kinematics and dynamics," *IEEE Trans. on Robotics and Automation*, vol. 19, no. 4, pp. 692–702, 2003.

[14] W. E. Dixon, "Adaptive regulation of amplitude limited robot manipulators with uncertain kinematics and dynamics," in *Proc. of American Control Conference*, (Boston, USA), pp. 3939–3844, 2004.

[15] C. C. Cheah, C. Liu and J.J.E. Slotine, "Adaptive Tracking Control for Robots with Unknown Kinematic and Dynamic Properties", *International Journal of Robotic Research*, Vol. 25, No. 3, pp 283-296, 2006.

[16] D. Braganza, W. E. Dixon, D. M. Dawson, and B. Xian, Tracking Control for Robot Manipulators with Kinematic and Dynamic Uncertainty, *Proc. of IEEE Conference on Decision and Control*, (Seville, Spain), pp 5293-5297, 2005.

[17] C. C. Cheah, C. Liu and J. J. E. Slotine, "Task-space Adaptive Setpoint Control for Robots with Uncertain Kinematics and Actuator Model," *IEEE Trans. on Automatic Control*, vol. 51, no. 6, pp. 1024 – 1029, Jun. 2006.

[18] C. Liu, C. C. Cheah and J. J. E. Slotine, "Adaptive Jacobian Tracking Control of Rigid-Link Electrically Driven Robots based on Visual Task-Space Information," *Automatica*, vol. 42, issue. 9, pp. 1491 – 1501, Sep. 2006.

[19] E. Malis and F. Chaumette, "Theoretical improvements in the stability analysis of a new class of model-free visual servoing methods", *IEEE Transaction on Robotics and Automation*, Vol. 18, No. 2, pp 176-186, April 2002.

[20] B. Espiau, "Effect of camera calibration errors on visual servoing in robotics," in *Proc. of International Symposium on Experimental Robotics*, (Kyoto, Japan),r 1993.

[21] L. Deng, F. Janabi-Sharifi, and W. J. Wilson, "Stability and robustness of visual servoing methods," in *Proc. of IEEE International Conference on Robotics and Automation*, pp. 1605-1609, (Washington D.C.), 2002.

[22] E. Malis, "Visual servoing invariant to changes in camera intrinsic parameters", *IEEE Transaction on Robotics and Automation*, Vol. 20, No. 1, pp 72-81, 2004.

[23] E. Malis and P. Rives, "Robustness of Image-Based Visual Servoing with Respect to Depth Distribution Errors", *Proc. of IEEE International Conference on Robotics and Automation*, pp 1056 - 1061, 2003.

[24] H. Wang and Y.H. Liu, "Uncalibrated Visual Tracking Control without Visual Velocity", *Proc. of IEEE International Conference on Robotics and Automation*, pp 2738 - 2743, 2006.

[25] F. L. Lewis, C. T. Abdallah, and D. M. Dawson, *Control of Robot Manipulators*. New York: Macmillan Publishing Company, 1993.

[26] S. Hutchinson, G. Hager and P. Corke, "A tutorial on visual servo control," *IEEE Trans. on Robotics and Automation*, vol. 12, no. 5, pp. 651 – 670, 1996.

[27] L. E. Weiss, A. C. Sanderson and C. P. Neuman, "Dynamic sensor-based control of robots with visual feedback," *IEEE Trans. on Robotics and Automation*, vol. RA-3, no. 5, pp. 404 – 417, 1987.

[28] B. Espiau, F. Chaumette and P. Rives, "A new approach to visual servoing in robotics", *IEEE Trans. on Robotics and Automation*, Vol. 8 , No. 3, pp 313 - 326, 1992.

[29] N.P. Papanikolopoulos, P.K. Khosla, and T. Kanade, "Visual tracking of a moving target by a camera mounted on a robot: a combination of control and vision", *IEEE Trans. on Robotics and Automation*, Vol. 9 , No. 1, pp 14 - 35, 1993.

[30] J. J. E. Slotine and W. Li, *Applied nonlinear control*. Englewood Cliffs, New Jersey: Prentice Hall, 1991.

[31] O. Khatib, "A unified approach for motion and force control of robot manipulators: the operation space formulation," *IEEE J. Robotics and Automation*, vol. 3, no. 1, pp. 43–53, 1987.

[32] Y. Nakamura, *Advanced Robotics*. Reading MA: Addison-Wesleyn, 1985.

[33] P. Ioannou and J. Sun, *Robust Adaptive Control*. Prentice Hall, 1996.

[34] C. Desoer and M. Vidyasagar, *Feedback systems: input-output properties*. Academic Press, New York, 1975.

[35] S. Arimoto, *Control Theory of Non-Linear Mechanical Systems*. Oxford University Press, 1996.