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A Global Asymptotic Stable Output Feedback PID Regulator for Robot Manipulators

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Abstract—**In this paper, we provide an answer to the long-standing question of designing global asymptotically stable proportional-integral-derivative (PID) regulators with only position feedback for uncertain robots. Our main contribution is to establish the global asymptotic stability of the controlled system by using Lyapunov direct method and LaSalle's invariance principle. We provide explicit conditions on the regulator gains to ensure global asymptotic stability. The proposed controller does not utilize the modeling information in the control formulation, and thus permits easy implementation. Simulations performed on a planar two degrees-of-freedom robot manipulator demonstrate the effectiveness of the proposed approach.**

I. INTRODUCTION

REGULATION of robot manipulators may be recognized as the simplest aim in robot control and at the same time finds its main application in the robotic field. Despite the success of modern control theory, it has been recognized that the majority of the controllers used in robotic manipulators are still the proportional-derivative (PD) or proportional-integral-derivative (PID)-type [1]-[3]. This is not only due to the simple structure which is conceptually easy to understand and explicit tuning procedures but also to the fact that the algorithm provides adequate performance in the vast majority of applications. Most of these controllers have been designed using linear models, or linearized ones, and some interesting new PID structures such as nonlinear and adaptive PID controllers have been proposed to overcome the limitations of traditional linear PID controllers for regulation tasks of the nonlinear dynamic systems [4]-[8].

It has been demonstrated by Arimoto that a local and independent PID servo-loop that replacing the linear position error term by a saturated position error one, which would give rise to global asymptotic stability of the setpoint position control for nonlinear mechanical systems [1]. Motivated by this work, Kelly [7] employed the tangent hyperbolic function to implement the position regulation of robot manipulator with velocity measurement. Alvarez-Ramirez *et al.* [8] proved the semiglobal stability of saturated linear PID control for robot manipulators with a standard saturated function. Zergeroglu *et al.* [9] formulated adaptive PID-type controller to solve the regulation of robot manipulators by using the tangent hyperbolic function, under the bounded inputs. Cheah *et al.* [10], [11] used adaptive saturated PD control to address the task-space regulation of uncertain robotic manipulators. Most recently, Su *et al.* [12], [13] incorporated a nonlinear saturated synchronized error into the available PD control law to implement the high-precision motion control of parallel manipulators.

A major drawback remains for these schemes, i.e. the requirement of measurements of both position and velocity. Velocity measurement increases cost and imposes constraints on the achievable bandwidth. To remove the requirement of the velocity measurements, several control techniques that stabilize arbitrary positions of robotic manipulators can be found in the literature. Berghuis and Nijmeijer [14] formulated a linear proportional plus the linear filtered position with the gravity compensation. Recently, Orlova *et al.* [15] extended this simple well-known controller to the friction manipulator cases, where a switched control action is added and if high frequency is affordable by the actuators, then the global stability is achievable. Kelly *et al.* [16] showed that a simple output PD plus desired gravity compensation preserve global asymptotic stability for position control of robot manipulators, by replacing the velocity by its dirty derivative. Loria *et al.* [17] showed that a class of Euler-Lagrange systems with bounded inputs that can be globally asymptotically stabilized, by incorporating some saturated function in the output controller. These strategies also require the knowledge of the gravity. To overcome the parametric uncertainties on the gravitational torque, an adaptive version of PD controller has been introduced in [18], [19], guaranteeing global asymptotic stability. The main draw of these approaches is that the regressor matrix has to be known.

On the other hand, most industrial robots are controlled by linear PID controllers which do not require any component of robot dynamics into its control law [8], [20]. In particular, Alvarez-Ramirez *et al.* [8] formulated a saturated linear PID controller by resorting to an additional saturated integral term to avoid the evaluation of the gravity term and replacing the velocity by its dirty derivative. Ortega *et al.* [20] presents a

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called PI^{2}D controller for position control of robots, by inclusion of two integral terms around the position error and the filtered position, respectively, into a commonly used PD controller. Unfortunately, the semi-global stability of the resulting closed system is proved only.

In this paper we introduce a new class of output global position controllers for robot manipulators which do not include their dynamics in the control laws. Motivated by the controllers reported in [16] and [20], we develop a new class of output regulators leading to a linear PID output feedback plus an integral action driven by a class of saturated functions of position error. We characterize the class of function and give simple explicit conditions on the controller parameters which guarantee global positioning.

Throughout this paper, we use the notation $\lambda_m(A)$ and $\lambda_M(A)$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive-definite bounded matrix $A(x)$, for any $x \in \Re^n$. The norm of a vector x is defined as $\|x\| = \sqrt{x^T x}$ and that of a matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_M(A^T A)}$, and I denotes an identity matrix of the appropriate dimension.

II. Robot Manipulator Model and Properties

The dynamics of an n-degrees-of-freedom robot manipulator, with all actuated revolute joints described in joint coordinates, can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + G(q) = \tau \tag{1}$$

where $q,\dot{q},\ddot{q} \in \Re^n$ denotes the link position, velocity, and acceleration, respectively, $M(q) \in \Re^{n\times n}$ represents the symmetric inertia matrix, $C(q,\dot{q}) \in \Re^{n\times n}$ denotes the centrifugal-Coriolis matrix, $D \in \Re^{n\times n}$ stands for the diagonal positive definite matrix composed of damping friction coefficients for each joint, $G(q) = \partial U(q)/\partial q \in \Re^n$ is a gravitational force, $U(q)$ is the potential energy due to gravity, and $\tau \in \Re^n$ denotes the torque input vector.

The dynamic equation of (1) has the following properties that will be used in the stability analysis.

Property 1 [1], [11], [18], [21]-[23]: $M(q)$ and D are symmetric, positive definite matrices. Furthermore, $M(q)$ is bounded by

$$0 < \lambda_m(M) \le \|M(q)\| \le \lambda_M(M) \tag{2}$$

Property 2 [1], [21], [22]: The centrifugal-Coriolis matrix $C(q,\dot{q})$ is defined using Christoffel symbols, and $\frac{1}{2}\dot{M}(q) - C(q,\dot{q})$ is skew-symmetric, i.e.

$$\zeta^T\left(\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right)\zeta = 0, \ \forall \zeta \in \Re^n \tag{3}$$

where $\dot{M}(q)$ is the time derivative of the inertia matrix $M(q)$.

Property 3 [1], [21], [22]: The matrix $C(q,\dot{q})$ satisfies the following relationship:

$$C(q,\xi)\upsilon = C(q,\upsilon)\xi, \qquad \forall \xi,\, \upsilon \in \Re^n \tag{4a}$$

and is bounded by

$$0 < C_m\|\dot{q}\|^2 \le \|C(q,\dot{q})\dot{q}\| \le C_M\|\dot{q}\|^2, \quad \forall q,\dot{q} \in \Re^n \tag{4b}$$

Property 4 [1]: Since $G(q) = \partial U(q)/\partial q$ and $U(q)$ are trigonometric functions of q, there exists a positive-definite diagonal matrix A such that the following two inequalities, with a specified constant $a > 0$, are satisfied simultaneously for any fixed q_d and any q

$$U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{2}\Delta q^T A \Delta q \ge a\|\Delta q\|^2 \tag{5}$$

$$\Delta q^T[G(q) - G(q_d)] + \Delta q^T A \Delta q \ge a\|\Delta q\|^2 \tag{6}$$

where $\Delta q = q - q_d$ denotes the position error of the actuator, and q and q_d denote the actual and desired coordinates of actuators, respectively.

III. Main Results

A. Control Formulation

First, we define a class of scalar potential function as follows:

$$S(x) = \begin{cases} \dfrac{|x|^{\alpha+1}}{\alpha+1} + \dfrac{(\alpha-1)\delta^{\alpha+1}}{2(\alpha+1)}, & \delta < |x| < \beta \\ \dfrac{\delta^{\alpha-1}x^2}{2}, & |x| \le \delta \\ \beta^\alpha|x| + \dfrac{(\alpha-1)\delta^{\alpha+1} - 2\alpha\beta^{\alpha+1}}{2(\alpha+1)}, & |x| \ge \beta \end{cases} \tag{7}$$

where $\alpha, \delta \in (0\ \infty)$, and $\beta > \delta$ are design parameters. The first derivative of $S(x)$ with respect to x can be expressed as

$$s(x) = \begin{cases} |x|^\alpha \operatorname{sgn}(x), & \delta < |x| < \beta \\ \delta^{\alpha-1}x, & |x| \le \delta \\ \beta^\alpha \operatorname{sgn}(x), & |x| \ge \beta \end{cases} \tag{8}$$

where $\operatorname{sgn}(\cdot)$ being the standard signum function.

Lemma 1: The functions $S(x)$ and $s(x)$ in (8) and (9) have the following properties:

(1) $S(x) > 0$ for $x \ne 0$ and $S(x) = 0$ for $x = 0$;

(2) $S(x)$ is continuously differentiable, and $s(x)$ is strictly increasing in x for $|x| < \beta$ and saturated for $|x| \ge \beta$;

(3) There is a constant $b > 0$ such that

$$S(x) \geq bs^2(x) > 0 \qquad \text{for} \quad x \neq 0 \tag{9}$$

(4) There are constant $\kappa > 0$ such that

$$x^2 \geq \kappa S(x) > 0 \qquad \text{for} \quad x \neq 0 \tag{10}$$

Property (1) is obvious. Proofs of properties (2) and (3) can be found in [13].

Example of the proposed quasi-natural potential function is shown in Fig. 1, with $\delta = 0.01$, $\beta = 1.0$, and $\alpha = 0.5$ (solid line) and $\alpha = 1.5$ (dash line), respectively.

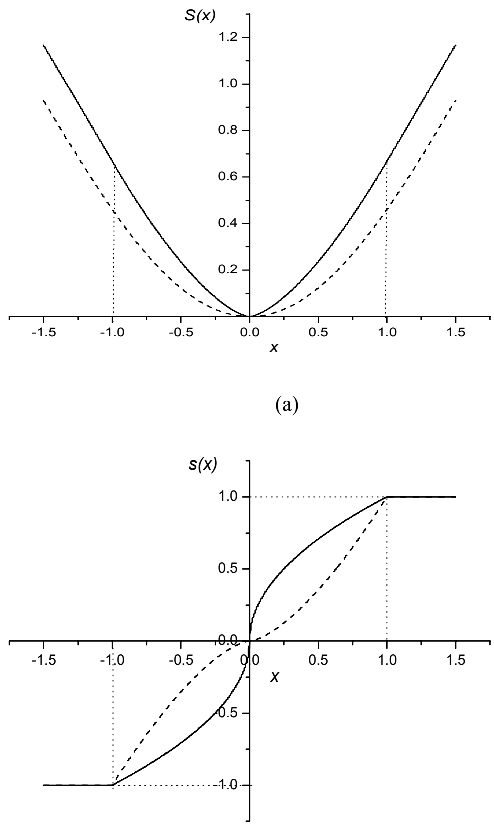

Fig. 1. (a) Quasi-natural potential $S(x)$. (b) Its derivative $s(x)$ ($\alpha = 0.5$ for the solid line; $\alpha = 1.5$ for the dash line).

Using the saturated function, the following linear output proportional-derivative (PD) plus nonlinear integral (I) control law can be formulated as

$$\tau = -(K_p + K_i)\Delta q - K_i \int_0^t [\lambda s(\Delta q(\sigma))] d\sigma - K_d \upsilon \tag{11}$$

$$\dot{q}_c = -Aq_c - ABq \tag{12}$$

$$\upsilon = q_c + Bq \tag{13}$$

where $K_p + K_i$, K_i and K_d are diagonal positive definite proportional, integral and derivative gain matrices, respectively, A and B are positive definite filter gains, λ is a small positive constant, and $s(\Delta q) \in \Re^n$ is defined as follows:

$$s(\Delta q) = [s(\Delta q_1), \ldots, s(\Delta q_n)]^T \tag{14}$$

Introducing the following vector:

$$z(t) = \Delta q + \int_0^t [\lambda s(\Delta q(\sigma))] d\sigma + K_i^{-1} G(q_d) \tag{15}$$

Substituting (11) and (15) into (1) yields the error equation for the closed-loop system as follows:

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D\dot{q} + G(q) - G(q_d) \\ + K_p \Delta q + K_i z + K_d \upsilon = 0 \end{aligned} \tag{16}$$

whose origin $\left[\Delta q^T \ \dot{q}^T \ \upsilon^T\right]^T = 0 \in \Re^{3n}$ is the unique equilibrium.

B. Stability Analysis

Given a target position q_d, we consider the global output regulation control problem that the designed controller does not involve any mode information, such that the robot manipulator approaches from any initial state $(q(0), \dot{q}(0))$ to the target state $(q_d, 0)$ asymptotically.

Lemma 2: Under the subsequently conditions (21) and (22), the Lyapunov-like function V, defined below, is a positive definite function with respect to $\Delta q, \dot{q}, \upsilon$

$$\begin{aligned} V = &\frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\Delta q^T K_p \Delta q + \lambda s(\Delta q)^T M(q)\dot{q} \\ &+ \sum_{i=1}^n \lambda d_i S(\Delta q_i) + U(q) - U(q_d) - \Delta q^T G(q_d) \\ &+ \frac{1}{2} z^T K_i z + \frac{1}{2}\upsilon^T K_d B^{-1} \upsilon \end{aligned} \tag{17}$$

with d_i denotes the ith diagonal elements of matrix D.

Proof: First, we consider the following

$$\begin{aligned} &\frac{1}{4}\dot{q}^T M(q)\dot{q} + \frac{1}{4}\Delta q^T K_p \Delta q + \lambda s(\Delta q)^T M(q)\dot{q} \\ &= \frac{1}{4}(\dot{q} + 2\lambda s(\Delta q))^T M(q)(\dot{q} + 2\lambda s(\Delta q)) \\ &\quad - \lambda^2 s(\Delta q)^T M(q) s(\Delta q) + \frac{1}{4}\Delta q^T K_p \Delta q \\ &\geq \frac{1}{4}\sum_{i=1}^n b_i \kappa_i k_{pi} s_i^2(\Delta q_i) - \lambda^2 s(\Delta q)^T M(q) s(\Delta q) \\ &\geq \sum_{i=1}^n \{(1/4) b_i \kappa_i k_{pi} - \lambda^2 \lambda_M(M)\} s_i^2(\Delta q_i) \end{aligned} \tag{18}$$

where we used (2) of Property 1, and (9) and (10) in Lemma 1.

Substituting (18) into (17), we have

$$V \geq \frac{1}{4}\dot{q}^T M(q)\dot{q} + U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{4}\Delta q^T K_p \Delta q$$
$$+ \sum_{i=1}^{n} \{(1/4) b_i \kappa_i k_{pi} - \lambda^2 \lambda_M(M)\} s_i^2(\Delta q_i)$$
$$+ \sum_{i=1}^{n} \lambda d_i S(\Delta q_i) + \frac{1}{2} z^T K_i z + \frac{1}{2}\upsilon^T K_d B^{-1}\upsilon \tag{19}$$

Now we can choose the positive definite constant λ small enough to satisfy the following inequality

$$D \geq \lambda c_0 I \tag{20}$$

where c_0 is a positive constant subsequently defined in (29).

Then, we can choose the matrices A, B, and K_d appropriately to satisfy the following inequality:

$$\lambda_m(A)\lambda_m(B^{-1}) \geq \frac{1}{2}\lambda\lambda_M(K_d)\lambda_m^{-1}(K_d) \tag{21}$$

Once λ and K_d is chosen to satisfy (20) and (21), we can choose K_p so large as to satisfy the following inequalities for an appropriate specified positive constant a:

$$K_p \geq 4\lambda^2 \lambda_M(M) K^{-1} B_0^{-1} \tag{22}$$

$$U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{4}\Delta q^T K_p \Delta q \geq a\|s(\Delta q)\|^2 \tag{23}$$

$$s(\Delta q)^T [G(q) - G(q_d)] + s(\Delta q)^T K_p \Delta q \geq \{a + \frac{1}{2}\lambda_M(K_d)\}\|s(\Delta q)\|^2 \tag{24}$$

where $K = \mathrm{diag}(k_1, \ldots, k_n)$, and $B_0 = \mathrm{diag}(b_1, \ldots, b_n)$. Note that the inequalities (23) and (24) correspond to inequalities (6) and (7) of Property 4, respectively, and the existence of such a matrix K_p is confirmed by the same argument given in proposing (6) and (7), since each component $s(\Delta q_i)$ satisfies (8) is quadratic in the vicinity of $\Delta q = 0$.

From (22), (23) and (19), we have

$$V \geq \frac{1}{4}\dot{q}^T M(q)\dot{q} + a\|s(\Delta q)\|^2 + \sum_{i=1}^{n} \lambda d_i S(\Delta q_i)$$
$$+ \frac{1}{2} z^T K_i z + \frac{1}{2}\upsilon^T K_d B^{-1}\upsilon > 0 \tag{25}$$

for $\left[\Delta q^T \;\; \dot{q}^T \;\; \upsilon^T\right]^T \neq 0$.

Hence, we can conclude that V is a positive definite Lyapunov function with respect to $\Delta q, \dot{q}, \upsilon$. ■

Based on Lemma 2, we have the following theorem.

Theorem 1: With the proposed output PID controller (11)-(13), the closed-loop system (16) is globally asymptotically stable, if λ is chosen small enough to satisfy (20); A, B, and K_d are chosen appropriate to satisfy (21); and K_p is chosen large enough to satisfy (22)-(24) simultaneously.

Proof: From Lemma 2, the function V defined in (17) can be selected as a Lyapunov candidate function. Differentiating V with respect to time, we have

$$\dot{V} = \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T M(q)\ddot{q} + \Delta\dot{q}^T K_p \Delta q + \lambda\dot{s}(\Delta q)^T M(q)\dot{q}$$
$$+ \lambda s(\Delta q)^T \dot{M}(q)\dot{q} + \lambda s(\Delta q)^T M(q)\ddot{q}$$
$$+ \lambda s(\Delta q)^T D\Delta\dot{q} + \dot{q}^T G(q) - \Delta\dot{q}^T G(q_d)$$
$$+ \dot{z}^T K_i z + \dot{\upsilon}^T K_d B^{-1}\upsilon \tag{26}$$

Substituting $M(q)\ddot{q}$ from (16) and $\dot{z}(t) = \Delta\dot{q} + \lambda s(\Delta q)$ from (15), into (26), and using (3) of Property 2, yields

$$\dot{V} = -\dot{q}^T D\dot{q} + \lambda\left\{s(\Delta q)^T C^T(q,\dot{q})\dot{q} + \dot{s}(\Delta q) M(q)\dot{q}\right\}$$
$$- \lambda\{s(\Delta q)^T [G(q) - G(q_d)] + s(\Delta q)^T K_p \Delta q\} \tag{27}$$
$$- \dot{q}^T K_d \upsilon - \lambda s(\Delta q)^T K_d \upsilon - \dot{q}^T K_i z + \dot{\upsilon}^T K_d B^{-1}\upsilon$$

Upon using $\dot{\upsilon} = -A\upsilon + B\dot{q}$ from (12) and (13), we have

$$\dot{V} = -\dot{q}^T D\dot{q} + \lambda\left\{s(\Delta q)^T C^T(q,\dot{q})\dot{q} + \dot{s}(\Delta q) M(q)\dot{q}\right\}$$
$$- \lambda\{s(\Delta q)^T [G(q) - G(q_d)] + s(\Delta q)^T K_p \Delta q\} \tag{28}$$
$$- \lambda s(\Delta q)^T K_d \upsilon - \upsilon^T K_d B^{-1} A\upsilon$$

By using (2) of Property 1 and (4) of Property 3 and the definition of $s(\Delta q)$ in (8), the second term of the right-hand side of (28) can be upper bounded by

$$\lambda\left\{s(\Delta q)^T C^T(q,\dot{q})\dot{q} + \dot{s}(\Delta q)^T M(q)\dot{q}\right\}$$
$$\leq \lambda(\sqrt{n}\beta^\alpha C_M + \lambda_M(\Lambda(\Delta q))M_M)\|\dot{q}\|^2 = \lambda c_0 \|\dot{q}\|^2 \tag{29}$$

where $\Lambda(\Delta q)$ being a diagonal matrix whose entries $\partial s(\Delta q_i)/\partial(\Delta q_i)$ are nonnegative, and can be determined by using the definition of $s(\Delta q)$ in (8). Note that the derivation of the first term of (29) utilizes $\|s(\Delta q)\| \leq \sqrt{n}\beta^\alpha$ according to (8).

Substituting (29) into (28) and upon using (23), we have

$$\dot{V} \leq -\dot{q}^T (D - \lambda c_0 I)\dot{q} - \lambda\{a + \frac{1}{2}\lambda_M(K_d)\}\|s(\Delta q)\|^2$$
$$- \lambda s(\Delta q)^T K_d \upsilon - \upsilon^T K_d B^{-1} A\upsilon$$
$$\leq -\dot{q}^T (D - \lambda c_0 I)\dot{q} - \lambda\{a + \frac{1}{2}\lambda_M(K_d)\}\|s(\Delta q)\|^2$$
$$+ \frac{1}{2}\lambda\lambda_M(K_d)(\|s(\Delta q)\|^2 + \|\upsilon\|^2) - \upsilon^T K_d B^{-1} A\upsilon$$
$$= -\dot{q}^T (D - \lambda c_0 I)\dot{q} - \lambda a\|s(\Delta q)\|^2$$
$$- \{\lambda_m(K_d)\lambda_m(A)\lambda_m(B^{-1}) - \frac{1}{2}\lambda\lambda_M(K_d)\}\|\upsilon\|^2 \tag{30}$$

From (20) and (21) and λ and a are positive constants, we conclude that $\dot{V} \leq 0$. In fact, $\dot{V} = 0$ means $s(\Delta q) = 0$, $\dot{q} = 0$, and $\upsilon = 0$. By definition of $s(\Delta q)$ in (8), we have

$\Delta q = 0$. Therefore, by LaSalle's invariance theorem [24], we have $\Delta q(t) \to 0$, $\dot{q}(t) \to 0$, and $\upsilon \to 0$ as $t \to \infty$ for any initial state $(q(0), \dot{q}(0))$. This completes the proof. ■

IV. Illustration Example

Simulations on a two-DOF planar robot manipulator were conducted to illustrate the effectiveness of the proposed simple output PID controller. The entries to model the robot manipulator are, respectively [25]

$$M = \begin{bmatrix} \theta_1 + 2\theta_2 \cos(q_2) & \theta_3 + \theta_2 \cos(q_2) \\ \theta_3 + \theta_2 \cos(q_2) & \theta_3 \end{bmatrix}$$
$$C = \begin{bmatrix} -2\theta_2 \sin(q_2)\dot{q}_2 & -\theta_2 \sin(q_2)\dot{q}_2 \\ \theta_2 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \quad (31)$$
$$G = \begin{bmatrix} \theta_4 \sin(q_1) + \theta_5 \sin(q_1 + q_2) \\ \theta_5 \sin(q_1 + q_2) \end{bmatrix}$$

Furthermore, a Coulomb friction is also considered in the simulations. To keep the notation used for model (1), it is defined $D = \mathrm{diag}(\theta_6, \theta_7)$, and

$$f_c(\dot{q}) = \begin{bmatrix} \theta_8 \,\mathrm{sgn}(\dot{q}_1) \\ \theta_9 \,\mathrm{sgn}(\dot{q}_2) \end{bmatrix} \quad (32)$$

where the parameters in the simulation are summarized in Table I.

Table I
PARAMETERS OF THE ROBOT MANIPULATOR

Notation	Value	Units
θ_1	2.351	$\mathrm{kg/m^2}$
θ_2	0.084	$\mathrm{kg/m^2}$
θ_3	0.102	$\mathrm{kg/m^2}$
θ_4	38.465	$\mathrm{N \cdot m}$
θ_5	1.825	$\mathrm{N \cdot m}$
θ_6	2.288	$\mathrm{N \cdot m \cdot s}$
θ_7	0.175	$\mathrm{N \cdot m \cdot s}$
θ_8	7.170 if $\dot{q}_1 > 0$ and 8.049 if $\dot{q}_1 < 0$	$\mathrm{N \cdot m}$
θ_9	1.724	$\mathrm{N \cdot m}$

The final desired positions were $q_d = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]^T$ (rad). The sampling period was determined as $T = 1\,\mathrm{ms}$. All the initial parameters are set as zero. The parameters of the used saturated function were $\alpha = 0.7$, $\beta = 1.0$ and $\delta = 0.01$. The gains for the proposed output PID controller were chosen in accordance with stability conditions (20)-(24) as $\lambda = 1.0$, $K_p + K_i = \mathrm{diag}(155,155)$, $K_i = \mathrm{diag}(150,15)$, $K_d = \mathrm{diag}(20,15)$, $A = \mathrm{diag}(50,80)$, and $B = \mathrm{diag}(50,60)$. First the regulation of the robot targeted at the desired positions without noise was conducted out, and the position errors and the required input torques are shown in Figs 1 and 2. After that a white noise with an amplitude of 0.01 rad is added to the position signals to imitate the measurement noise, and the obtained results are illustrated in Figs. 3 and 4. It can be seen that both of the cases the robot targeted at the final desired position correctly, and after a transient due to errors in initial condition, the position errors tend asymptotically to zero.

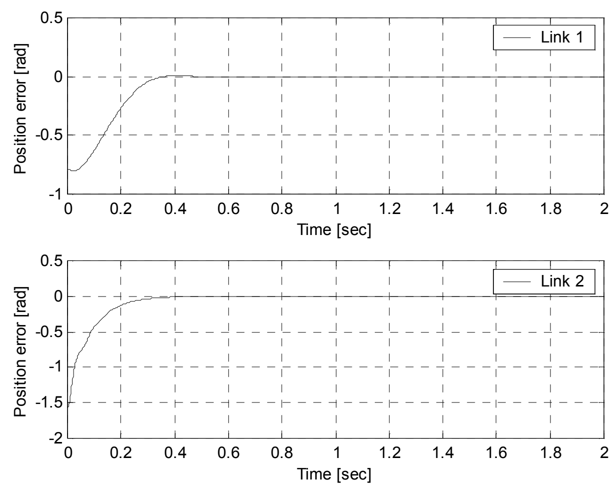

Fig.1. Position errors without noise.

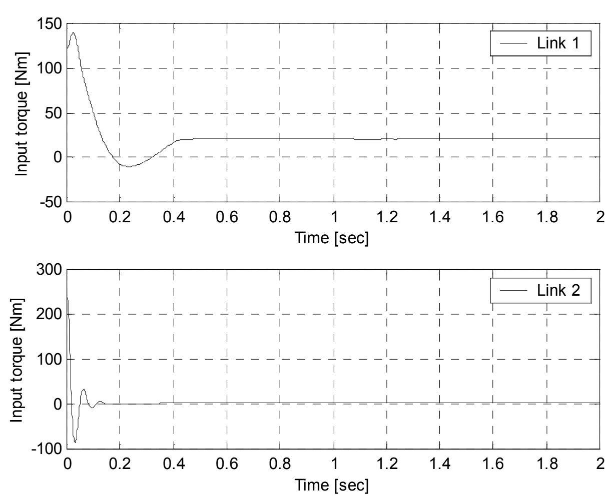

Fig. 2. Input torques without noise.

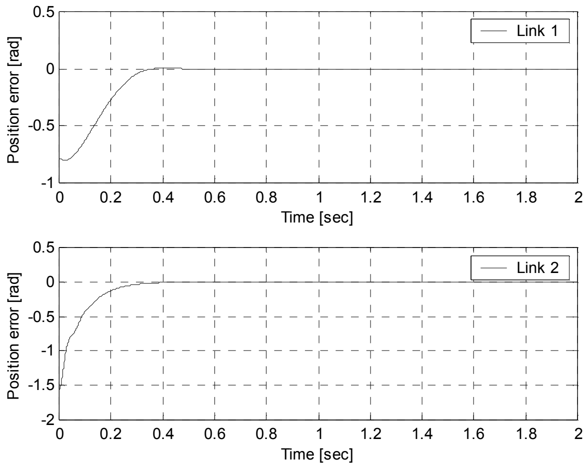

Fig.3. Position errors with noise.

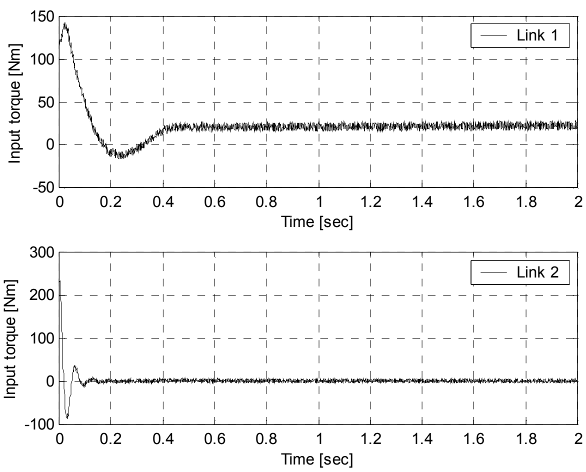

Fig. 4. Input torques with noise.

V. CONCLUSION

We have proven the global asymptotically output regulation of robot manipulators with a simple PID control with Lyapunov direct method and LaSalle's invariance principle. The proposed controller does not use the modeling parameters in the controller formulation and the gains of the controller can be explicitly determined in terms of a few bounds extracted from the robot dynamics and the developed saturated function, and thus permits easy implementation. Simulations preformed on a two-DOF robot demonstrate the effectiveness of the proposed approach.

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