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PWM Control for a micro-robot moving on a discrete curvature trajectory set

Grigoris Lionis and Kostas J. Kyriakopoulos

Abstract— **In this paper we study the problem of controlling a micro robot which moves either on a line or on circle of predetermined radius. Locomotion constraints of this type are usual in micro robots, and therefore, it is important to have efficient and easy to implement ways of coping with these constraints. To solve the problem we present a multilevel motion controller that can be easily implemented on a robot with scarce computational resources as it does not rely on complex logical operations. It reduces the kinematics of the micro robot to the kinematics of a unicycle, screening in effect the micro behavior from the high level planner. Simulated results are provided to verify the proposed methodology.**

I. Introduction

For the past few years, a number of researches have been studying aspects of micro robotics behavior, i.e. robotics on micro-scales. Robotics in these scales is often not a simple use of well studied principles and ideas from traditional 'macro' robotics, but instead , new ideas and paradigms have to be used, in order to cope with the behavior of robots in these scales.

In this work we will focus on the control of a micro robot on which motion constraints are imposed, due to the nature of the locomotion system. In general, micro locomotion systems are not as versatile as locomotion systems found on larger robots, but instead are constrained by a number of reasons including

- Power Constraints: Simultaneous rotational and linear movement could be prohibited by the power available
- Computational Constraints: Calculations needed for a micro-hexapod to rotate and translate simultaneous could exceed the on board resources
- Locomotion Structure: The locomotion system may not allow complex motion patterns

In our case,the structure of the locomotion systems does not allow the micro-robot to move arbitrarily.More specifically, we are interested in controlling a micro-robot with piezo-actuated legs Fig. I that can move forwards and backwards but cannot turn arbitrarily, only with a specified turning radius Fig. 2. Motivation for studying such a micro-robot comes from the behavior of state of the art micro robots, having similar motion constraints [4]. Constraints similar to the constraints of the robot we are studying,are associated with a large class of micro-robots, and as a result of great practical significance.

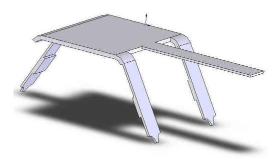

Fig. 1. Micro Robot

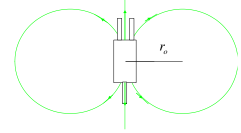

Fig. 2. Self Motion capabilities of the micro-robot

The system we are examining moves roughly as a tricycle in which the steering wheel is locked in two positions, corresponding to the state in which the system moves forwards or backwards and to the state where the system rotates around a constant center of rotation located at a distance ρ from the robot.

Unicycles are extremely well studied in the literature and a vast number of publications has appeared in the literature concerning planning, closed loop control, task execution, obstacle avoidance etc. To name a few,in [10] the authors study motion planning techniques for a non-holonomic unicycle in the presence of obstacles, while in [5] the authors present and compare a number of stabilizing controllers for unicycles.

On the other hand, the problem of bounded curvature vehicles has received, comparatively, less attention. In [2] the authors study the problem of stabilizing a kinematic unicycle on the plane, assuming that the robot moves with bounded curvature and that few sensory information are available to the robot. They propose a hybrid control technique that stabilizes the robot to a large class of trajectories. This model assumes a robotic vehicle that can only move forwards, and therefore does not utilize the full potential of a robot that can move both forwards and backwards. Moreover, this technique, involving a hybrid automaton, although very appealing in a large scale robot, could be problematic in a computationally scarce micro-robot.

In [8], the problem of calculating optimal paths for robots moving on bounded curvature trajectories,with the robot being able to move both forwards and backwards is studied.

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G. Lionis is a PhD Student at the School of Mechanical Engineering, National Technical University of Athens Athens,Greece glion@mail.ntua.gr

K. Kyriakopoulos is with Faculty of the School of Mechanical Engineering, National Technical University of Athens Athens,Greece kkyria@mail.ntua.gr

The authors give a procedure that, for any given endpoints on a plane free of obstacles, will generate a small number of trajectories, in which the optimal trajectory belongs. Thus, by comparing these trajectories the optimal one can be found. This result, solves completely the path planning problem, but is difficult to extend it in closed loop control and/or to multi agent systems.

On the other hand, a number of works regarding control of switching systems using averaging have been reported in the literature.In [9], the authors propose a framework for controlling hybrid systems, by establishing that when the hybrid system is periodic, the dynamics of the averaged system lie on the convex hull of the individual continuous system, and the control of the system is accomplished by controlling via classical control techniques the averaged continuous system. Our method, on the other hand, lies on controlling the form of the averaged kinematics, by constructing a special switching controller that gives to the averaged system a specific behavior, so that its control is easy.

The wealth of publications in unconstrained non-holonomic vehicles, motivates us to try to utilize these techniques by masking the bound constraints, instead of trying to extend the necessary results to curvature bounded vehicles. We propose to solve this problem,using a multilevel control scheme, that utilizes a pulse width modulated control in the lower level, accepts in the upper level unconstrained non-holonomic velocity inputs. Our goal is to capitalize on the vast number of motion planning and closed loop controllers for unicycles, both for the single and the multi agent case,and to use our control strategy as a low level control scheme, under more capable controllers handling obstacle avoidance and task execution. The multilevel control scheme, allows the robots to follow -in any desired accuracy- the motion of a unicycle.

II. Problem Definition

The state of the vehicle is given by

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{1}$$

where (x, y) are the Cartesian coordinates of the center of the robot w.r.t a global coordinate frame and θ is the orientation of the robot w.r.t. to the sam coordinate frame. The kinematics of the vehicle are given by (2)

$$\dot{q} = V \cdot \begin{bmatrix} \cos\theta \\ \sin\theta \\ \rho \end{bmatrix} \tag{2}$$

$$V \in [-V_{max}, -V_{min}] \cup\ 0 \cup [V_{min}, V_{max}], \rho \in \{-\rho_0, 0, \rho_0\}$$

with ρ being the curvature (i.e. the inverse of the turning radius) So the vehicle can either move forwards or backwards (w.r.t. its local position) or can rotate with a fixed turning radius ($\frac{1}{\rho_0}$). The problem is to control this robot, i.e. to find a control input so that the robot executes a given task. The tasks include stabilization to a point, tracking a trajectory, swarming, etc.

III. Controllability

It is known from the literature [8] that this system is globally controllable and in their work, the authors present an optimal global controllability scheme. A more challenging and interesting problem is how to control the system locally. Our solution to this question -although it is a well known fact that this car is small time locally controllable - is to show that this system 2 can follow any trajectory followed by a unicycle, and hence is locally controllable.

A. Discretization of the Control Inputs

In the subsequent analysis, we will assume that the linear velocity of the robot belongs in $V \in \{Vmax, -Vmax, 0\}$. By doing so we get results that are obviously compatible with system 2, since we merely commit the control input to a discrete subset of original control input.

Moreover, in this case the control problem becomes a a purely discrete one. We have a number of vector fields on which the micro-robot can move on. In particular, in this case we can decompose the kinematics of the system into 4 vector fields, that cannot be simultaneously activated, and that characterize all the possible forward motion directions the robot, can achieve from a point in the state space. These vector fields, labeled $g_1, ..., g_4$ are the following

$$g_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad g_2 = V \cdot \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}$$

$$g_3 = V \cdot \begin{bmatrix} \cos\theta \\ \sin\theta \\ \rho_0 \end{bmatrix} \qquad g_4 = V \cdot \begin{bmatrix} \cos\theta \\ \sin\theta \\ -\rho_0 \end{bmatrix}$$

Furthermore, the robot can move along v.f. $-g_2, -g_3, -g_4$, with the set of all these vector fields completely characterizing the system's available motion directions.

Using a scaling transformation of the form $\hat{x} = s \cdot x$,$\hat{y} = s \cdot y$ we can, by an appropriate choice of s, to re-write the kinematic equations into the equations of a robot moving with unitary velocity ($s = V^{-1}$), or into the equations of a robot that when rotates, rotates on the unitary circle $s = (V \cdot \rho)^{-1}$) . It is thus obvious, that we may utilize all these representations of the motion of the robot interchangeably.

IV. Pulse Width Modulated Control

We propose to control this micro robot using the concept of pulse width modulated control. It is intuitively obvious that if the robot swiftly alternates between two of these vector fields, its overall motion will lie somewhere between these two vector fields. So, by exploiting this fact, we can make the robot -by alternating continuously between 2 (or more) vector field, moves as a uncicycle. In Fig. 3 the concept of controlling the micro robot with alternating between vector fields (v.f.) is depicted. The robot starts moving from Point 1 , along the red curves (with curvature 1) up to point 2. From Point 2 to Point 3 the robot simply move backwards, and

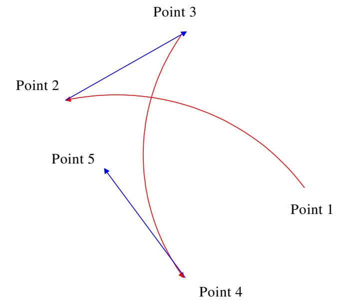

Fig. 3. Vector Field Alternation

then from point 3 to point 4 again with curvature 1. Finally from Point 4 to Point 5 the robot moves backwards.

To move between points 1 and 5 without alternating, the robot would have to turn much tighter, or equivalently, the robot follows a curve of different -larger- curvature than its self motion curves. By rapidly interchanging between vector fields, the micro-robot could move as closely to a unicycle as desired.

To make this a precise concept, we need a little notation. We define a switching function

$$\sigma(t,a,T) = \begin{cases} 1, 0 \leq t < T \\ 0, T \leq t < (a+1)T \\ \sigma(t-(a+1)\cdot T, a, T), T > (a+1)T \end{cases}$$

We define as $C^a_{g_i,g_j}(q_0)$ the solution of the differential equation

$$\dot{q} = \sigma(t,a,T)g_i + (1-\sigma(t,a,T))g_j, T \to 0 \tag{3}$$

The right hand of this differential equation, for any $T > 0$, is piecewise analytic, and therefore, for any $T > 0$, the solution of the differential equation is well defined.

The motion of the robot, when the control is pulsating between v.f. i,j with time constant a is therefore $C^a_{g_i,g_j}$ Our control strategy lies in controlling the motion of the microrobot by constructing a suitable $C^a_{g_i,g_j}$, i.e. by choosing appropriately i,j and a . Since we are interested in moving the micro-robot as a unicycle, we want to construct motion modes that move the system with arbitrary curvature.

We can prove the following : when we alternate between vector fields g_2 and g_3 , on the limit of $T \to 0$, the robot will move along vector field

$$g_{23a} = V \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \frac{a}{a+1} \end{bmatrix} \tag{4}$$

in the sense that the solution of 3 and the solution of 4 for the same initial condition q_0 will be the same $\forall t$

Formally, we can show that

$$C^a_{g_2,g_4} \sim V \cdot [\cos\theta \;\; \sin\theta \;\; -\frac{a}{a+1}]^T \tag{5}$$

$$C^a_{-g_2,g_4} \sim V^{\perp} \cdot [\cos\theta \;\; \sin\theta \;\; -\frac{a}{a-1}]^T \tag{6}$$

with $\sim$ meaning that the produce the same trajectory, starting from the same initial conditions, $V^{\perp}$ states the fact that the linear velocity of the overall motion will not be equal with the original linear velocity, but will be less. In the previous equation it is obvious that when $a = 1$, the v.f. pair $\{-2, 4\}$, will result in the agent rotating in place, i.e. will have infinite curvature.

V.F. Comb.	Robots Behavior
g_1	Robot is Still
g_2	Robot Moves Straight
g_3	Robot Turns Left $\rho = 1$
g_4	Robot Turns Right $\rho = 1$
$C^a_{g_2,g_4}$	Robot Turns Forward Right $\rho = \frac{a}{a+1}$
$C^a_{-g_2,g_4}$	Robot Turns Forward Right $\rho = -\frac{a-1}{a}$

TABLE I
SUMMARY OF VECTOR FIELD AND ASSOCIATED MOTION

We can summarize some of the motion results when alternating between different v.f.[1] in table I.

V. PROOF

In this section we will show that when the switching frequency tends to infinity, the behavior of the system is as described in the previous section. We will only show this for the combinations of v.f. $g_2, -g_2$ with g_3 as the other cases are symmetrical to this one. This proofs show a well known fact in PWM control, and it is included here only for completeness.

We begin by examining the switching between v.f. g_2 and g_3 . We examine the motion of the agent when the applied control input is $C^a_{g_2,g_3}$, with $T \to 0$. We define as a step the time interval $T + aT$ in which the robot undergoes motion on g_2 for time T being followed by motion on v.f g_3 for time aT . We begin by examining the motion of the robot during a single step. Assuming the robot stars from (x_i, y_i, θ_i) we can calculate the motion of the robot during the $(i+1)-th$ time step as

$$\begin{aligned} \Delta x_{i+1} &= \cos(\theta_i)T + \int_0^{aT} \cos(\theta_i + \omega\tau)d\tau \\ &= \cos(\theta_i)T + 1/\omega[\sin(\theta_i + aT\omega) - \sin(\theta_i)] \\ \Delta y_{i+1} &= \sin(\theta_i)T + \int_0^{aT} \sin(\theta_i + \omega\tau)d\tau \\ &= \sin(\theta_i)T - 1/\omega[\cos(\theta_i + aT\omega) - \cos(\theta_i)] \\ \Delta\theta_{i+1} &= aT\omega \end{aligned}$$

By keeping in mind that

$$x_i = x_0 + \sum_{j=1}^{i} \delta x_j$$

$$y_i = y_0 + \sum_{j=1}^{i} \delta y_j$$

$$\theta_i = \theta_0 + \sum_{j=1}^{i} \delta\theta_j$$

[1]We do not present all the combinations, as all others are symmetrical with the ones on the table.

we get that

$$x_n = x_0 + T\sum_{j=0}^{n-1}\cos(\theta_0 + ja\omega T)+ \\ +1/\omega[\sin(\theta_0 + naT) - \sin\theta_0]$$
$$y_n = y_0 + T\sum_{j=0}^{n-1}\sin(\theta_0 + ja\omega T)- \\ -1/\omega[\cos(\theta_0 + naT) - \cos\theta_0]$$
$$\theta_n = \theta_0 + an\omega T$$

Let $\mathcal{T}$ be the time interval at the end of which we want to compare the evolution of the micro robot, under the actual interchanged vector fields, and vector field $[\cos\theta\ \sin\theta\ a/(1+a)]^T$

If the robot was moving along $[\cos\theta\ \sin\theta\ a/(1-a)]^T$, after the same time $\mathcal{T} = n(a+1)T$ its position would be

$$\hat{x}_n = x_0 + \int_0^{n(a+1)T}\cos(\theta_0 + a/(1+a)\omega\tau)d\tau =$$
$$= x_0 + \frac{(a+1)}{a\omega}[\sin(\theta_0 + an\omega T) - \sin(\theta_0)]$$
$$\hat{y}_n = y_0 + \int_0^{n(a+1)T}\sin(\theta_0 + a/(1+a)\omega\tau)d\tau =$$
$$= y_0 - \frac{(a+1)}{a\omega}[\cos(\theta_0 + an\omega T) - \cos(\theta_0)]$$
$$\hat{\theta}_n = \theta_0 + n(a+1)T\frac{a}{a+1}\omega = \theta_0 + na\omega T$$

Therefore we have that

$$x_n - \hat{x}_n =$$
$$= T\sum_{j=0}^{n-1}\cos(\theta_0 + ja\omega T) - \frac{1}{a\omega}[\sin(\theta_0 + an\omega T) - \sin(\theta_0)] =$$
$$= T\sum_{j=0}^{n-1}\cos(\theta_0 + ja\omega T) - \int_0^{nT}\cos(\theta_0 + a\omega\tau)d\tau$$

which clearly tends to zero as $T \to 0$ and $n \to \infty$ so that $\mathcal{T} = n(a+1)T$ remains constant, since the first term is -when $T \to 0, n \to \inf$ just the definition of the definite integral.On the same line of thought, we can show that $\hat{y}_n - \hat{y} \to 0$ when $T \to 0$,and therefore when the switching frequency tends to infinity, the motion of the agent is described by the smooth vector field $[\cos\theta\ \sin\theta\ \frac{a\omega}{1+a}]^T$

In the case of switching among vector fields $-g_2$ and g_4 we have that -using the same notation-

$$x_n = x_0 - TU\sum_{j=0}^{n}\cos(\theta_0 + aj\omega T) \\ + \frac{U}{\omega}(\sin(\theta_0 + na\omega T) - \sin\theta_0)$$

,while

$$\hat{x}_n = \tfrac{\hat{U}}{\hat{\omega}}(\sin(\theta_0 + n(a+1)T\hat{\omega}) - \sin(\theta_0))$$

We will show that when $\hat{U} = \frac{a-1}{1+a}U$ and $\hat{\omega} = \frac{a}{1+a}\omega$ then

$$x_n - \hat{x}_n \to 0, T \to 0$$

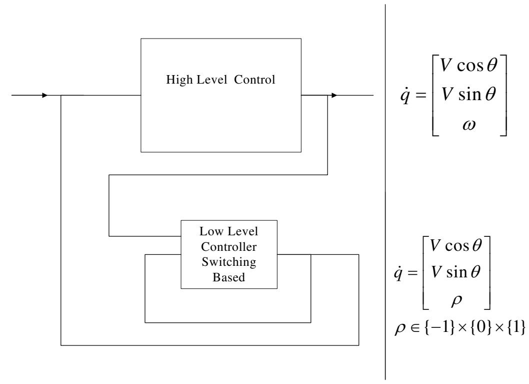

Fig. 4. Multilevel Control Scheme

Indeed we have that

$$x_n - \hat{x}_n = U[\frac{1}{a\omega}(\sin(\theta_0 + na\omega T) - \sin\theta_0] \\ - T\sum_{j=0}^{n}cos(\theta_0 + aj\omega T)]$$

But

$$T\sum_{j=0}^{n}\cos(\theta_0 + aj\omega T) \approx \int_0^{nT}cos(\theta_0 + a\omega T) = \\ \frac{1}{a\omega}(\sin(\theta_0 + na\omega T) - \sin\theta_0)$$

Therefore,

$$T \to 0, x_n - \hat{x}_n \to .0$$

VI. Controller Synthesis

The control scheme for a single micro robot is a depicted on Fig. 4. We implemented a multilevel controller design, with the high level controller considering the robot as unicycle, without turning radius constraints. This controller, produces a control pair (linear and angular velocity (V, ω)) which is then fed to lower level, and is implemented by the actual robot. Of course, it is obvious that the lower level controller cannot execute an arbitrary velocity pair. Nevertheless, the lower level controller can execute an arbitrary turning radius trajectory, which -in geometric terms- means that any trajectories followed by a non holonomic unicycle can be executed by the micro-robot, with a different -perhaps- timing.

Mathematically, the interactions between the high level, and low level controller is twofold. First, the upper level controller, chooses among the possible combinations of vector fields that the low-level controller will execute.Table I is used, or to be more exact, the complete version of it, including all possible motion directions.

Then, the width of the pulses is appropriately chosen, so that the robotic agent moves along the same path that would move, had it actually been able to move non-holonomically without further constrains. The width of the pulse is calculated based on Table I, so that the micro-robot trajectory matches the desired.

For every pair $\{\omega, V\}$ issued as commands from the high level controller, there is a pair of vector fields, and a time step width, that renders the micro-robot trajectory equivalent (moving on the same trajectory) as if it was moving with $\{\omega, V\}$.

More formally, a mapping function is defined, that essentially is the low level of the multi-level controller. Function

$$\Phi : (V, \omega) \rightarrow \mathcal{V} \times \mathcal{V} \times R^{+},$$

where $\mathcal{V} = \{0, 1, 2, 3, 4, -1, -2, -3, -4\}$ represents the set of all possible vector fields, and R represents the set in which the parameter a takes values. The selection of the pair of vector fields is done using Table I, and the computation of the parameter a by solving the w.r.t to a the appropriate relation of the same table (i.e. the relation connecting the vector fields choose, the parameter a with the characteristics of the trajectory). This function selects which pair of vector fields will be activated and computes the parameter.Thus, for example, when the high-level controller issues a command corresponding to a right turn ($V > 0, \omega > 0$), with curvature more than one $\omega > V$, function Φ outputs as vector fields v.f. $-2, 3$, and the pulse widths is calculated as following:

$$k^{-1}\frac{\omega}{V} = \frac{a-1}{a} \rightarrow a = (1 - k^{-1}\frac{V}{\omega})^{-1},$$

where $k = [L]^{-1}$ is a unity constant, necessary since the curvature is a physical quantity, and k represents the dimensions of curvature.

The higher level controller used can be any controller designed for a non-holonomic robot,as for example the stabilization controllers found in [5] or the obstacle avoidance controller found on [10].

Given a closed loop control law for the non-holonomic system of the form $[\omega, V] = F(q)$, the switching control law the actual micro-robot will follow, will be given by $\Phi(F(q))$. Obviously, the stability of the overall system depends greatly -a well known fact in multilevel control- on the relative bandwidths of the high level control, and the low level drive.

Finally, we are interested on utilizing this control paradigm, for controlling a swarm of micro-agents, by reducing their kinematics to unicycle kinematics and using techniques developed for swarms of unicycles, like for example the controller found in [3]. A interesting problem pops-up in utilizing this driving scheme for a multi-agent system. Even for infinite frequency, the low-level drive is guaranteed to follow the trajectory of the high-level drive, but not in the same time window,as the mean robot velocity is curvature specific.

This does not play any role in a single robot setup, where convergence is time-independent, but is crucial in a multi-agent setup.

VII. Simulations

To verify the usefulness of this control scheme, we present a number of simulated trajectories.

For our simulations, and since we are mainly interested in checking the feasibility of our low level pwm control , we will use -as high level controller- the controller found on [1]: The velocity and angular velocity are given by

$$\begin{aligned} \upsilon &= k_\rho\sqrt{x^2+y^2} \\ \omega &= k_\alpha(\arctan(y/x)-\theta) - k_\phi(\pi/2-\theta) \\ \omega|_{(0,0,\theta)} &= (k_\alpha + k_\phi)(\pi/2-\theta) \end{aligned}$$

It can be shown that as long as

$$k_\rho > 0, k_\phi < 0, k_\alpha + k_\phi - k_\rho > 0$$
$$k_\alpha + 2k_\phi - \frac{2}{\pi}k_\rho > 0$$

the controller stabilizes the system to the origin. As a result, using infinite switching frequency, the micro-robot will converge to the origin.

The micro-robot is assumed to be moving with constant velocity (which can be either positive or negative) and with a turning radius set to 1. $V = \pm 1, \omega\pm = 1, \rho\pm = 1$. These values represent the actual linear and angular velocity of the robot, and are chosen arbitrarily.

Fig.5 depicts a simulated trajectory tracking experiment. A non-holonomic trajectory with steep turns serves as input trajectory,and the robot has to track it. A simple non-nolonomic controller is used as high level controller, and the low level drive is to execute this path.The switching mode drive works well, even though the switching frequency is relatively low. The drive's characteristic chattering-like motion is evident, when comparing the nominal trajectory with the actual trajectory. In Fig.6, the control effort associated with this simulated experiment is depicted, using the actual angular velocity of the micro-robot, switching from $\omega = -1$, to $\omega = 0$ and finally to $\omega = 1$.

In Fig. 7 we present a stabilization experiment, using the afore mentioned controller. The robot is controlled towards $(0, 0, 0)$, and the controls are implemented using the switching mode drive. The switching frequency was set to 2 times the integration frequency. When the curvature of the trajectory becomes very large low level drive noise corrupts the stabilizing controller, making the system not robust.

This phenomenon is more vividly depicted in Fig. 8, where the effects of the switching frequency are depicted. Each curve represents the evolution of the robot position, under the same high level control, and starting from the same initial conditions, but with the switching frequency increasing from being equal to the integration frequency ($f = 1$) to 16nth times the integration frequency $f = 16$. It should be noted that the high and low level controller are synchronized, in the sense that the implemented velocity pair given by the high level controller remains the same until the low level drive finishes a cycle. This figure clearly depicts the effects of a low frequency, especially near the origin, were the system undergoes very tight turns. When the switching frequency increases, the net result of the motion tends to the nominal, smooth unicycle.

VIII. Conclusions

We have presented a kinematic switching motion drive, for a micro-robot moving on curves with bounded curvature.

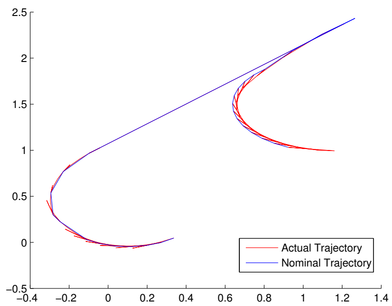

Fig. 5. X-Y Micro Robot Trajectory.

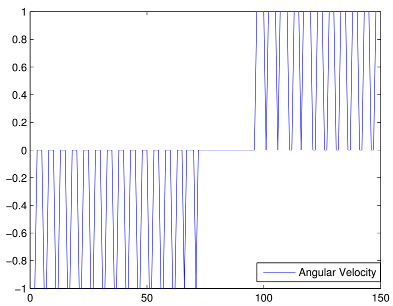

Fig. 6. Control Effort.

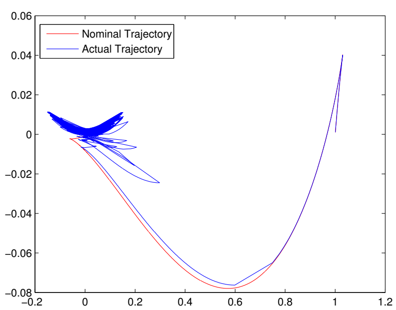

Fig. 7. Stabilization to (0,0)

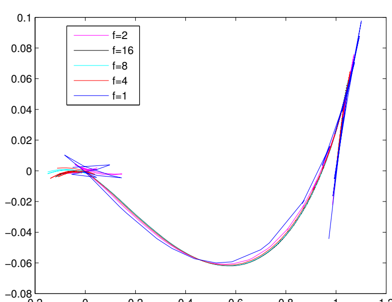

Fig. 8. Stabilization to (0,0)

We designed appropriate switching schemes, so that the robot tracks any desired non-holonomic trajectory, with arbitrary curvature, independent from the curvature of the vehicle self motion curve. A number of simulations was presented to verify the use of this driving scheme.

The main advantage of our driving scheme -besides its simplicity- is the fact that it is a low level controller, effectively screening the idiosyncracies of the micro-robot from the high level controller, and making the micro-robot appear as a non holonomic integrator. With this way, any of the vast number of non-holonomic control schemes can be implemented on a micro-robot.

As a future research plan, we plan to focus on how to use this control scheme in the case of a swarm of micro robots,taking into account the necessary synchronization.

Moreover, an analysis of the high level control scheme is necessary, in order to formally assess the time scales of the problem, and how the interactions of the high level controller with the low level drive, focusing on the question of how high must be the frequency of the low-level drive in order to retain the convergence properties of the high-level controller.

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