2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

Bounded attitude stabilization: Application on four-rotor helicopter

J. F. GUERRERO-CASTELLANOS, A. HABLY, N. MARCHAND, S. LESECQ

Abstract— **A quaternion based feedback is developed for attitude stabilization of rigid body. The control design takes into account the input bounds and is based on cascaded saturation approach. The global stability is guaranteed. A simulation study of the proposed scheme is illustrated for the four-rotor helicopter.**

I. INTRODUCTION

The problem of attitude control of a rigid body has attracted considerable amount of interest since the 1950's within the scientific communities of aeronautics, aerospace, control and robotics. This is due to the fact that many systems such as spacecrafts, satellites, helicopters, tactical missiles, coordinated robot manipulators, underwater vehicles and others enter within the framework of rigid body with a need for attitude control. Several approaches are applied such as optimal time control [13], Lyapunov design procedures [20], quaternion feedback [4], [6] and [22], predictive control (applied to spacecraft in [23] and to micro satellite in [5]), backstepping (quaternion-based in [8] and nonlinear adaptive in [15]), and robust control applied to tactical missiles [16]. This list is not exhaustive.

The above cited approaches do not consider the problem of attitude control which takes the input constraints into account. Few publications have treated this problem. In [21], the stabilization with non smooth control law of an under-actuated rigid spacecraft subject to input saturation is studied. In [1], a control law that drives a rigid underwater vehicle between arbitrary initial and final region of the state space while satisfying bounds on control and state is proposed. The authors in [2] have studied the robust sliding mode stabilization of the spacecraft attitude dynamics in presence of control input saturation based on the variable structure control (VSC) approach. Unfortunately, the stabilizing bounded control law that are applied are non smooth and this fact renders difficult the practical implementation. The approach proposed in the present paper is more in the spirit of the approach [24] where the problem of reorienting a rigid spacecraft within the physical limits of actuators has been investigated based on the cascaded saturation approach proposed by [19]. However, in [24] no formal stability proof is given. Although Teel's results is nice and founding, its performance in term of convergence speed is very poor for system of dimension $n \geq 3$ [9]. Nevertheless, as is mentioned in [12], for a double integrator plant, such as the presented in this work, the Teel's approach presents a good performance with regard to settling time stabilization.

The orientation of a rigid body can be parameterized by several methods: a rotation matrix, a unit quaternion (i.e. Euler parameters) and Euler angles. The unit quaternion is a four-parameter representation and is considered as a globally nonsingular parametrization. For more details on attitude representations, the reader can refer to the survey written by Shuster [14].

In this paper, the bounded attitude control of a rigid body is studied. The control scheme is applied on a four-rotor helicopter. The complete model of this special type of mini helicopter, also known as X-4 flyer or even quad-rotor, is developed in [10], [11]. This four-rotor helicopter has some advantages over conventional helicopters: owing to symmetry, this vehicle is dynamically elegant, inexpensive, and simple to design and construct. To our knowledge, the attitude stabilization of the four-rotor helicopter using quaternion feedback was firstly studied in [17] and more recently in [18]. In these papers, a quaternion-based feedback control scheme for attitude stabilization is applied without considering the boundedness of the control inputs .

The present paper is organized as follows. In section II, a rigid body quaternion-based orientation is given. The main problem is formulated in section III. The control law design is presented and its stability is proved in section IV. The application of this control law on a four-rotor helicopter is explained in section V. The simulation results are given in section VI. The paper ends with some conclusions given in section VII.

II. MATHEMATICAL BACKGROUND

As mentioned in the introduction, the attitude of a rigid body can be represented by a quaternion, consisting of a unit vector $\vec{e}$, known as the Euler axis, and a rotation angle β about this axis. The quaternion q is then defined as follows

$$q = \begin{pmatrix} \cos\frac{\beta}{2} \\ \vec{e}\sin\frac{\beta}{2} \end{pmatrix} = \begin{pmatrix} q_0 \\ \vec{q} \end{pmatrix} \in \mathbb{H} \tag{1}$$

where

$$\mathbb{H} = \{q \mid q_0^2 + \vec{q}^T\vec{q} = 1, q = [q_0 \quad \vec{q}]^T, q_0 \in \mathbb{R},\ \vec{q} \in \mathbb{R}^3\} \tag{2}$$

$\vec{q} = [q_1 \quad q_2 \quad q_3]^T$ and q_0 are known as the vector and scalar parts of the quaternion respectively. In attitude control applications, the unit quaternion represents the rotation from an inertial coordinate system $N(x_n, y_n, z_n)$ located at some point in the space (for instance, the earth NED frame), to

This work was partly supported by CONACYT-México and CNRS-Liban

The authors are with Control System Department, GIPSA-lab, CNRS UMR 5216, CNRS-INPG-UJF, ENSIEG BP 46, 38402 Saint Martin d'Hères Cedex, France fergue@lag.ensieg.inpg.fr, Ahmad.Hably@inpg.fr, Nicolas.Marchand@inpg.fr, Suzanne.Lesecq@inpg.fr

the body coordinate system $B(x_b, y_b, z_b)$ located at the center of mass of the rigid body.

The rotation matrix $C(q)$ corresponding to the attitude quaternion q, is computed as

$$C(q) = (q_0^2 - \vec{q}^T\vec{q})I_3 + 2(\vec{q}\vec{q}^T - q_0[\vec{q}^\times]) \quad (3)$$

where I_3 is the identity matrix and $[\xi^\times]$ is a skew symmetric tensor associated with the axial vector ξ

$$[\xi^\times] = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}^\times = \begin{pmatrix} 0 & \xi_3 & -\xi_2 \\ -\xi_3 & 0 & \xi_1 \\ \xi_2 & -\xi_1 & 0 \end{pmatrix} \quad (4)$$

Denoting by $\vec{\omega} = [\omega_1 \ \ \omega_2 \ \ \omega_3]^T$ the angular velocity vector of the body frame B relative to the inertial frame N, expressed in B, the kinematics equation is given by

$$\begin{aligned} \begin{pmatrix} \dot{q}_0 \\ \dot{\vec{q}} \end{pmatrix} &= \frac{1}{2}\begin{pmatrix} -\vec{q}^T \\ I_3 q_0 + [\vec{q}^\times] \end{pmatrix}\vec{\omega} \\ &= \frac{1}{2}\Xi(q)\vec{\omega} \end{aligned} \quad (5)$$

The attitude error is used to quantify the mismatch between two attitudes. If q defines the current attitude quaternion and q_d is the reference quaternion, i.e. the desired orientation, then the error quaternion that represents the attitude error between the current orientation and the desired one is given by

$$q_e = q \otimes q_d^{-1} \quad (6)$$

$\otimes$ denotes the quaternion multiplication and q^{-1} is the complementary rotation of the quaternion q, which is the quaternion conjugate [14].

The attitude dynamics for a rigid body is described by

$$I_f\dot{\vec{\omega}} = -\vec{\omega} \times I_f\vec{\omega} + \Gamma \quad (7)$$

where $I_f \in \mathbb{R}^{3\times3}$ is the symmetric positive definite constant inertial matrix of the rigid body expressed in the B frame and $\Gamma \in \mathbb{R}^3$ is the vector of control torques. Note that these torques also depend on the environmental disturbance torques (aerodynamic, gravity gradient, etc.).

III. Problem Statement

The objective is to design a control law that drives the rigid body attitude to a specified constant orientation and maintains this orientation. It follows that the angular velocity vector must be brought to zero and remains null. Let q_d denote the desired constant rigid body orientation, then the control objective is described by the following asymptotic condition

$$q \to q_d, \quad \vec{\omega} \to 0 \quad \text{as} \quad t \to \infty \quad (8)$$

As it has been mentioned above, if q_d denotes the desired constant rigid body orientation, then the error quaternion that represents the attitude error between the current orientation and the desired one is given by equation (6). If the inertial coordinate frame is selected and $q_d = [\pm1 \ \ 0 \ \ 0 \ \ 0]^T$, the error quaternion (6) coincides with the current attitude quaternion, that is, $q_e = q$. This control objective is then

$$q \to [\pm1 \ \ 0 \ \ 0 \ \ 0]^T, \quad \vec{\omega} \to 0 \quad \text{as} \quad t \to \infty \quad (9)$$

In this study, the case $q_d = [1 \ \ 0 \ \ 0 \ \ 0]^T$ that represents the attitude aligned up with the inertial coordinate system axes is considered. Nevertheless, the results can be applied to either desired orientation.

It is well known that actuator saturation reduces the benefits of the feedback. In the case where the controller continuously outputs infeasible control signal that will saturate the actuators, system instability will follow. Therefore, besides the asymptotic stability, the objective of the control law is to take into account the physical constraints of the control system in order to apply only feasible control signal to the actuator.

IV. Bounded Attitude Control Formulation

In this section, a control law that stabilizes the system described by (5) and (7) is proposed. The goal is to design a control torque that is bounded.

Definition 1: Given a positive constant M, a continuous, nondecreasing function $\sigma_M : \mathbb{R} \to \mathbb{R}$ is defined by

1) $\sigma_M(s) = s$ if $|s| < M$;
2) $\sigma(s) = \text{sign}(s)M$ elsewhere;

Theorem 1: Consider the rigid body rotational dynamics described by (5) and (7) with the following bounded control inputs $\Gamma = [\Gamma_1, \Gamma_2, \Gamma_3]^T$ such that

$$\Gamma_j = -\alpha\sigma_{M_2}(\lambda[\omega_j + \sigma_{M_1}(q_j)]) \quad (10)$$

where $j \in \{1,2,3\}$ with σ_{M_1} and σ_{M_2} are saturation functions with $M_1 \geq 1$, $M_2 \geq \lambda(2M_1 + \varepsilon)$ and $\varepsilon > 1$. α and λ are positive parameters. Then the inputs (10) globally asymptotically stabilize the rigid body to the origin ($q_0 = 1, \vec{q} = 0$ and $\vec{\omega} = 0$).

Remark 1: Since a quaternion and its negative represent the same rotation, there exist two equilibrium point $[q_0 = \pm1 \ \vec{q} = 0]^T$. The equilibrium point ($q_0 = -1, \vec{q} = 0, \vec{\omega} = 0$) can be joined using $\Gamma_j = -\alpha\sigma_{M_2}(\lambda[\omega_j - \sigma_{M_1}(q_j)])$ with $j \in \{1,2,3\}$. Therefore, applying $\Gamma_j = -\alpha\sigma_{M_2}(\lambda[\omega_j + \text{sign}(q_0)\sigma_{M_1}(q_j)])$ ensures that of the two equivalent rotations of angle β and $2\pi - \beta$, the one of smaller angle is chosen. This can be demonstrated by adapting the following proof.

Proof: Consider the candidate Lyapunov function V, which is positive definite, radially unbounded and which belongs to the class C^2. V represents the total energy of the system

$$\begin{aligned} V &= \frac{1}{2}\vec{\omega}^T I_f\vec{\omega} + \kappa((1-q_0)^2 + \vec{q}^T\vec{q}) \\ &= \frac{1}{2}\vec{\omega}^T I_f\vec{\omega} + 2\kappa(1-q_0) \end{aligned} \quad (11)$$

where I_f is defined as before, and $\kappa > 0$ must be determined. The derivative of (11) after using (5) and (7) is given by

$$\begin{aligned} \dot{V} &= \vec{\omega}^T I_f\dot{\vec{\omega}} - 2\kappa\dot{q}_0 \\ &= \vec{\omega}^T(-\vec{\omega} \times I_f\vec{\omega} + \Gamma) + \kappa\vec{q}^T\vec{\omega} \\ &= \underbrace{\omega_1\Gamma_1 + \kappa q_1\omega_1}_{\dot{V}_1} + \underbrace{\omega_2\Gamma_2 + \kappa q_2\omega_2}_{\dot{V}_2} \\ &\quad + \underbrace{\omega_3\Gamma_3 + \kappa q_3\omega_3}_{\dot{V}_3} \end{aligned} \quad (12)$$

$\dot{V}$ is the sum of three terms ($\dot{V}_1$, $\dot{V}_2$, $\dot{V}_3$). First $\dot{V}_1$ is analyzed. From Γ_1 in (10) and equation (12), one gets

$$\dot{V}_1 = -\alpha\omega_1\sigma_{M_2}(\lambda[\omega_1 + \sigma_{M_1}(q_1)]) + \kappa q_1\omega_1 \qquad (13)$$

Assume that $|\omega_1| > |M_1 + \varepsilon|$, that is $|\omega_1| \in [M_1 + \varepsilon, +\infty[$. It follows that $|\omega_1 + \sigma_{M_1}(q_1)| \geq \varepsilon$ and $\omega_1 + \sigma_{M_1}(q_1)$ has the same sign as ω_1. From equation (13) and the norm condition on the quaternion, $\dot{V}_1$ takes the following form

$$\begin{aligned}\dot{V}_1 &= -\alpha\omega_1\sigma_{M_2}(\lambda[\omega_1 + \sigma_{M_1}(q_1)]) + \kappa\omega_1 q_1 \\ &\leq -\alpha|\omega_1|\sigma_{M_2}(\lambda\varepsilon) + \kappa|\omega_1|\end{aligned} \qquad (14)$$

For $M_2 > \lambda\varepsilon$ and κ chosen such that $\kappa < \alpha\lambda\varepsilon$, one can assure the decrease of V_1 i.e. $\dot{V}_1 < 0$. Consequently, ω_1 enters $\Phi_1 = \{\omega_1 : |\omega_1| \leq M_1 + \varepsilon\}$ in finite time t_1 and remains in it thereafter. In this case, $\omega_1 + \sigma_{M_1}(q_1) \in [-2M_1 - \varepsilon, 2M_1 + \varepsilon]$. Let M_2 verify the following inequality $M_2 > \lambda(2M_1 + \varepsilon)$. For time t_2 such that $t_2 > t_1$, the argument of σ_{M_2} will be bounded as follows

$$|\lambda(\omega_1 + \sigma_{M_1})| \leq \lambda(2M_1 + \varepsilon) < M_2 \qquad (15)$$

Consequently, σ_{M_2} operates in a linear region

$$\Gamma_1 = -\alpha\lambda[\omega_1 + \sigma_{M_1}(q_1)] \qquad (16)$$

As a result, (13) becomes

$$\dot{V}_1 = -\alpha\lambda\omega_1^2 - \alpha\lambda\omega_1\sigma_{M_1}(q_1) + \kappa q_1\omega_1 \qquad (17)$$

Since $M_1 \geq 1$, $\sigma_{M_1}(q_1)$ is not saturated, it comes that

$$\dot{V}_1 = -\alpha\lambda\omega_1^2 - \alpha\lambda q_1\omega_1 + \kappa q_1\omega_1 \qquad (18)$$

Choosing $\kappa = \alpha\lambda$ which satisfies the inequality $\kappa < \lambda\varepsilon$ since $\varepsilon > 1$, one obtains

$$\dot{V}_1 = -\lambda\alpha\omega_1^2 \leq 0 \qquad (19)$$

The same argument is applied to $\dot{V}_2$ and $\dot{V}_3$, and (12) becomes

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 && (20)\\ &= -\lambda\alpha\omega_1^2 - \lambda\alpha\omega_2^2 - \lambda\alpha\omega_3^2 && (21)\\ &= -\vec{\omega}^T A\vec{\omega} \leq 0 && (22)\end{aligned}$$

where

$$A = \alpha\lambda I_3 \qquad (23)$$

with I_3 the identity matrix of size 3×3. In order to complete the proof, the LaSalle Invariance Principle is invoked. All the trajectories converge to the largest invariant set $\bar{\Omega}$ in $\Omega = \{(\vec{q}, \vec{\omega}) : \dot{V} = 0\} = \{(\vec{q}, \vec{\omega}) : \vec{\omega} = 0\}$. In the invariant set, $I_f\dot{\vec{\omega}} = -\alpha\lambda[\sigma_{M_1}(q_1) \quad \sigma_{M_1}(q_2) \quad \sigma_{M_1}(q_3)]^T = 0$, that is, $\bar{\Omega}$ is reduced to the origin. This ends the demonstration of the asymptotic stability of the closed loop system. ■

V. Application to Four-rotor Helicopter

The control attitude strategy presented in the previous section is applied to the attitude regulation of a four-rotor helicopter as the one shown in Fig.1.

A. Four-rotor Helicopter Dynamics

This mini helicopter has four fixed-pitch rotors mounted at the four ends of a simple cross frame. On this platform (under development), given that the front and rear motors rotate counter-clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend to cancel in trimmed flight. The collective input (or throttle input) is the sum of the thrusts of each rotor ($f_1 + f_2 + f_3 + f_4$). Pitch movement (θ) is obtained by increasing (reducing) the speed of the rear motor while reducing (increasing) the speed of the front motor. The roll movement (ϕ) is obtained similarly using the lateral motors. The yaw movement (ψ) is obtained by increasing (decreasing) the speed of the front and rear motors while decreasing (increasing) the speed of the lateral motors. This should be done while keeping the total thrust constant. In order to model the system dynamics, two frames are defined: the inertial frame $N(x_n, y_n, z_n)$ and the body-fixed frame $B(x_b, y_b, z_b)$ as shown in Fig.2.

Fig. 1. Four-Rotor helicopter prototype of GIPSA-Lab

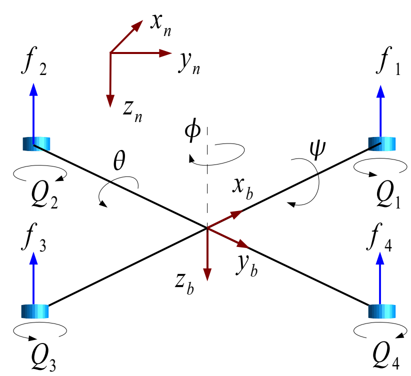

Fig. 2. Four-Rotor helicopter configuration: the inertial frame $N(x_n, y_n, z_n)$ and the body-fixed frame $B(x_b, y_b, z_b)$

According to [11] and section II, the four-rotor helicopter model may be expressed in terms of quaternions

$$\begin{aligned}\dot{p} &= v && (24)\\ \dot{v} &= \vec{g}^N - \frac{1}{m}C^T(q)\vec{T} && (25)\\ \dot{q} &= \frac{1}{2}\Xi(q)\vec{\omega} && (26)\\ I_f\dot{\vec{\omega}} &= -\vec{\omega} \times I_f\vec{\omega} - \Gamma_G + \Gamma && (27)\end{aligned}$$

m denotes the mass of the helicopter, $\vec{g}$ is the vector of the gravity acceleration and $\times$ is the cross product. $p = (x,y,z)^T$ represents the position of the origin of the B-frame with respect to the N-frame, $v = (v_x, v_y, v_z)^T$ is the linear velocity of the origin of the B-frame expressed in the N-frame, and $\vec{\omega}$ denotes the angular velocity of the helicopter expressed in the B-frame. $\Gamma_G \in \mathbb{R}^3$ contains gyroscopic torques, due to the rotational motion of the mini helicopter and the four rotors, $\Gamma \in \mathbb{R}^3$ is the vector of the control torques and $\vec{T} = [0\,0\,T]^T$ is the total thrust expressed in the B-frame. The attitude model of the four rotor aircraft differs from the general model (5)-(7) in the gyroscopic torques Γ_G. However, it will be proved that the approach of section IV can still be applied.

Equations (24)-(27) describe the 6 degrees of freedom of the system and can be separated into translational (24)-(25) and rotational (26)-(27) motions.

In this application, the speed of the rotors may reach high values (more than 200 rad/sec), therefore, the reactive torque generated in the free air, by rotor i due to rotor drag can be approximated by $Q_i = k\varpi_i^2$ as in [7] and the total thrust generated by the four rotors can be approximated by

$$T = b\sum_{i=1}^{4}\varpi_i^2 \tag{28}$$

where ϖ_i is the rotational speed of rotor i. $k > 0$ and $b > 0$ are two parameters depending on the density of air, the radius, the shape, the pitch angle of the blade and other factors [3]. The vector Γ_G is given by

$$\Gamma_G = \sum_{i=1}^{4} I_r(\vec{\omega} \times z_n)(-1)^{i+1}\varpi_i \tag{29}$$

I_r is the inertia of the so-called rotor (composed of the motor rotor itself and of the shape and of the gears). The components of the control torque $\Gamma \in \mathbb{R}^3$ generated by the rotors are given by $\Gamma = (\Gamma_1 \;\; \Gamma_2 \;\; \Gamma_3)^T$, with

$$\Gamma_1 = db(\varpi_2^2 - \varpi_4^2) \tag{30}$$
$$\Gamma_2 = db(\varpi_1^2 - \varpi_3^2) \tag{31}$$
$$\Gamma_3 = k(\varpi_1^2 - \varpi_2^2 + \varpi_3^2 - \varpi_4^2) \tag{32}$$

where d represents the distance from one rotor to the center of mass of the four-rotor helicopter. Combining (28) with (30)-(32), the forces applied to the helicopter are written in vector form

$$\begin{aligned}\begin{pmatrix}\Gamma \\ T\end{pmatrix} &= \begin{pmatrix} 0 & db & 0 & -db \\ db & 0 & -db & 0 \\ k & -k & k & -k \\ b & b & b & b \end{pmatrix}\begin{pmatrix}\varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2\end{pmatrix} \\ &= N\varpi_r \end{aligned} \tag{33}$$

with $\varpi_r = [\varpi_1^2 \;\; \varpi_2^2 \;\; \varpi_3^2 \;\; \varpi_4^2]^T$ the rotor speeds of the four motors.

B. Four-rotor Torque Control Design

In order to stabilize the attitude of the four-rotor helicopter, equations (26)-(27) are used. The rotational motion of the helicopter responds to control torques arising from the linear combination of the rotation speed of the rotors (33). Hence, the maximum airframe control torque depends of the much higher rotation speed capability of the motors that are used. The rotors are driven by DC permanent magnet motors which support a maximum voltage of 9 V as in [18]. When this voltage is applied to the motor the rotation speed reaches $\varpi_{i,max} = 260\ rad/sec$. Consequently the maximum torque that is applied to influence the helicopter rotational motion is given by

$$\bar{\Gamma}_1 = 0.40\ Nm \quad \bar{\Gamma}_2 = 0.40\ Nm \quad \bar{\Gamma}_3 = 0.15\ Nm$$

Note that these torques are not identical about the three axis. In order to avoid the actuator saturation, the bounded attitude control presented in the previous section is applied to the subsystem (26)-(27).

Lemma 1: Consider the four-rotor helicopter rotational dynamics described by (26) and (27) with the following bounded control inputs

$$\begin{aligned}\Gamma_1 &= -\alpha\sigma_{M_\phi}(\lambda_1[\omega_1 + \sigma_M(q_1)]) \\ \Gamma_2 &= -\alpha\sigma_{M_\theta}(\lambda_2[\omega_2 + \sigma_M(q_2)]) \\ \Gamma_3 &= -\alpha\sigma_{M_\psi}(\lambda_3[\omega_3 + \sigma_M(q_3)])\end{aligned} \tag{34}$$

where σ_M and $\sigma_{M_{\phi,\theta,\psi}}$ are saturation functions where $M \geq 1$ and

$$M_\phi \geq \lambda_1(2M+\varepsilon), \quad M_\theta \geq \lambda_2(2M+\varepsilon), \quad M_\psi \geq \lambda_3(2M+\varepsilon)$$

with $\varepsilon > 1$, α and $\lambda_{1,2,3}$ three positive parameters. Then the inputs (34) globally stabilize the four-rotor helicopter to the origin ($q_0 = 1, \vec{q} = 0$ and $\vec{\omega} = 0$).

$M_{\phi,\theta,\psi}$ and α are chosen to satisfy the following equations

$$\bar{\Gamma}_1 = \alpha M_\phi \quad \bar{\Gamma}_2 = \alpha M_\theta \quad \bar{\Gamma}_3 = \alpha M_\psi$$

Proof: The steps of the proof are identical to the ones of Theorem 1. Indeed, the only difference lies in the vector Γ_G that adds a term canceled because of the relation:

$$\vec{\omega}^T\Gamma_G = \vec{\omega}^T(\vec{\omega} \times z_n)\sum_{i=1}^{4} I_r(-1)^{i+1}\omega_{ri} = 0$$

■

C. Rotor Speed Control

Actually, the control inputs of the four-rotor helicopter are the four rotor torques τ_i, $i \in \{1,2,3,4\}$. Hence, the rotor speed control task is to force the actual speed ϖ_i to track a desired smooth reference profile ϖ_i^*, corresponding to the desired torques applied in the four-rotor frame provided by (34). In this application, four DC motors are used. The motor dynamics are governed by two coupled first-order equation with respect to armature current and shaft speed. The mechanics model for the motor can be expressed by

$$I_r\dot{\varpi}_i = \tau_i - Q_i, \quad i = \{1,2,3,4\} \tag{35}$$

where ϖ_i and I_r are defined above.

The desired speed for the four rotors is obtained from (33). Therefore, $\varpi_{r_d} = N^{-1}\Upsilon$, with $\Upsilon = [\Gamma \;\; T]^T$. It follows that the total thrust T must be specify respecting the constraint $T = mg$, because one is interested in hover flight. After that

the attitude stabilization is achieved, the total thrust can be varied ($T \leq mg$ or $T \geq mg$) in order to stabilise the altitude. However, this problem is not discussed here.
Assume that the motor parameters are well known, then an exponential stability of the speed tracking error is achieved using the following controller [18]:

$$\tau_i = Q_i + I_r \dot{\varpi}_{i_d} - \gamma_i \tilde{\varpi} \qquad (36)$$

where γ_i is a positive parameter, and $\tilde{\varpi}_i = \varpi_i - \varpi_{d_i}$ is the tracking error.

VI. SIMULATION

In order to show the performance of the proposed controller, two simulations studies are carried out. In both studies, the desired thrust is given $T = mg = 4.59N$. The maximum four-rotor frame torque that can be applied is $\bar{\Gamma} = [0.40 \;\; 0.40 \;\; 0.15]^T \; Nm$.
The first case is performed without adding external disturbances. The objective is to show the attitude stabilization capabilities from some given initial attitude sufficiently far from the origin to show the efficiency of the bound on the control torques to avoid unwanted damages. The initial conditions are set to $\phi = -45°, \quad \theta = 50°, \quad \psi = -175°$. The convergence of the roll, pitch and yaw angles is plotted in Fig.3. The four-rotor helicopter angular velocity and the applied control torques are shown respectively in Fig.4 and Fig.5. As expected, the desired attitude is reached in suitable time for practical implementation while the control stays in the pre-required limits. For the second case, the initial conditions are set to be $\phi = -25°, \quad \theta = 30°, \quad \psi = -10°$ as in [17]. In this simulation the robustness of the proposed controller with respect to external disturbances is studied. The disturbances are introduced into the system after the attitude stabilization of the four-rotor is achieved. This causes the angular velocity to reach a value of 4 rad/sec around the three axes as is shown in Fig. 7. It can be seen in Fig.8 the control reaches its limit and takes action on the system to overcome the perturbations. This second case shows that the controller formulated in this paper is robust with respect to external disturbances. This property is essential for real time implantation where aerodynamic disturbance torques are non trivial. The robustness of the proposed approach with respect to the inertial parameters or other model errors remains to be checked. However, the robustness study carried in [9] for a similar class of saturated control let us glimpse good results in this direction.

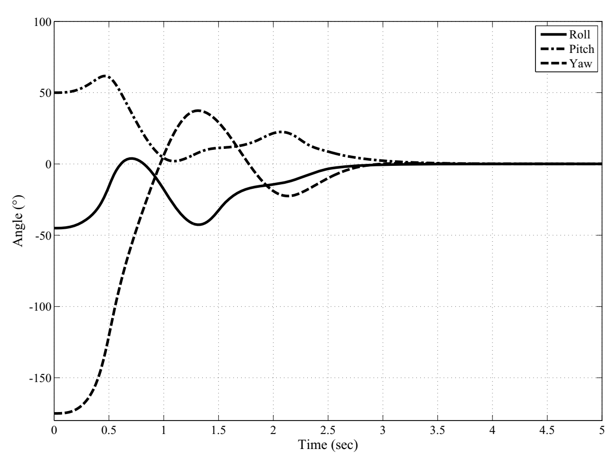

Fig. 3. First case: The convergence of the roll, pitch and yaw angles of the four-rotor helicopter with initial conditions $\phi = -45°, \quad \theta = 50°, \quad \psi = -175°$

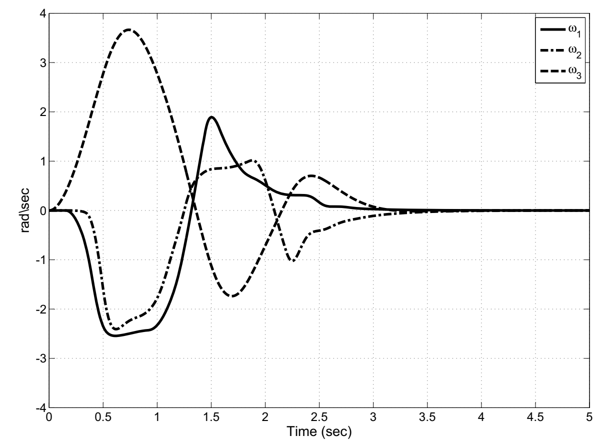

Fig. 4. First case: The evolution of the angular velocity of the four-rotor helicopter

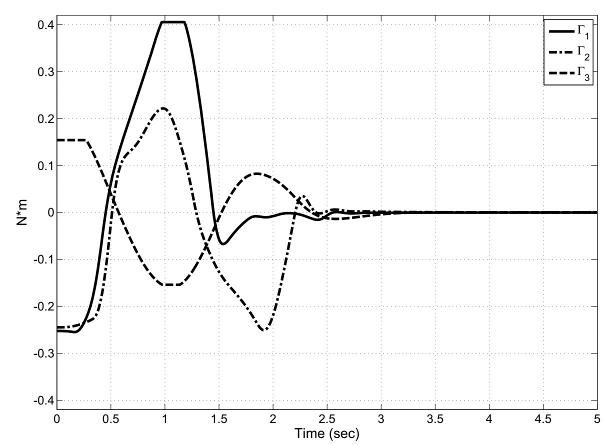

Fig. 5. First case: The bounded control torques applied on the four-rotor helicopter

VII. CONCLUSIONS AND FUTURE WORKS

This paper developed a bounded global stabilizing control law for rigid body. The controller is based on a cascaded-saturation design and the quaternion representation of the rigid body. The complexity of the approach is very closed to the one of unconstrained linear control. This control law is applied to a four-rotor helicopter. The numerical simulations have showed the effectiveness of the proposed controller and its robustness with respect to external disturbances.
The proposed approach is currently being implemented on a plate-form to check the practical applicability of the control law. This approach will be compared with other control

schemes such as the nonlinear model predictive control, backstepping and others. However, as far as the authors know, the proposed approach is by far the simplest and the more suitable for embedded implementation.

REFERENCES

[1] C. Belta. On controlling aircraft and underwater vehicles. In *IEEE International Conference on Robotics and Automation*, 2004.

[2] J.D. Boskovic, S.-M. Li, and R.K. Mehra. Robust stabilization of spacecraft in the presence of control input saturation using sliding mode control. In *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 1999.

[3] P. Castillo, A. Dzul, and R. Lozano. Real-time stabilization and tracking of a four-rotor mini rotorcraft. *IEEE Transactions on Control Sytems Technology*, 12(4):510–516, 2004.

[4] O. Fjellstad and T. Fossen. Quaternion feedback regulation of underwater vehicles. In *3rd IEEE Conference on Control Application*, pages 24–26, 1994.

[5] O. Hegrenas, J.T. Gravdahl, and P. Tondel. Attitude control by means of explicit model predictive control, via multi-parametric quadratic programming. In *American Control Conference*, volume 2, pages 901–906, 2005.

[6] S.M. Joshi, A.G. Kelkar, and J.T. Wen. Robust attitude stabilization of spacecraft using nonlinear quaternion feedback. *IEEE Transactions on Automatic Control*, 40(10):1800–1803, 1995.

[7] F. Kendoul, I. Fantoni, and R. Lozano. Modeling and control of a small aircraft having two tilting rotors. In 44^{th} *IEEE conference on Decision and Control and European Control Conference, CDC-ECC'05*, 2005.

[8] R. Kristiansen and P. J. Nicklasson. Satellite attitude control by quaternion-based backstepping. In *American Control Conference (ACC)*, 2005.

[9] N. Marchand and A. Hably. Global stabilization of multiple integrators with bounded controls. *Automatica*, 41(12):2147–2152, 2005.

[10] P. McKerrow. Modelling the dragonflyer four-rotor helicopter. In *IEEE International Conference on Robotics and Automation*, 2004.

[11] P. Pounds, R. Mahony, P. Hynes, and J. Roberts. Design of a four-rotor aerial robot. In *Australian Conference on Robotics and Automation*, 2002.

[12] V. G. Rao and D. S. Bernstein. Naive control of the double integrator. *IEEE Control Systems Magazine*, 21:86–97, Oct. 2001.

[13] S.L. Scrivener and R.C. Thompson. Survey of time-optimal attitude maneuvers. *Journal of Guidance, Control and Dynamics*, 17(2):225–233, 1994.

[14] M.D. Shuster. A survey of attitude representations. *Journal of the astronautical sciences*, 41(4):439–517, 1993.

[15] S.N. Singh and W. Yim. Nonlinear adaptive backstepping design for spacecraft attitude control using solar radiation pressure. In 41^{sd} *IEEE conference on Decision and Control, CDC'02*, 2002.

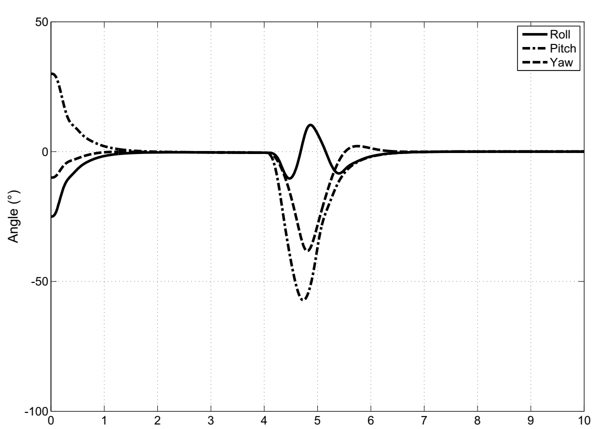

Fig. 6. Second case: The convergence of the roll, pitch and yaw angles of the four-rotor helicopter with initial conditions $\phi = -25°, \quad \theta = 30°, \quad \psi = -10°$

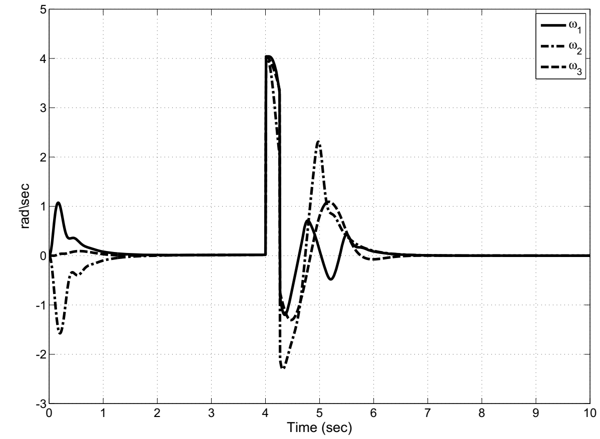

Fig. 7. Second case: The evolution of the angular velocity of the four-rotor helicopter

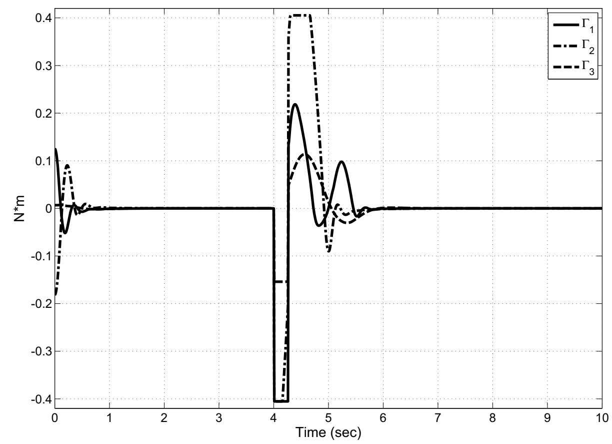

Fig. 8. Second case: The bounded control torques applied on the four-rotor helicopter

[16] C Song, S.-J. Kim, S.-H. Kim, and H.S. Nam. Robust control of the missile attitude based on quaternion feedback. *Control Engineering Practice*, 14:811–818, 2005.

[17] A. Tayebi and S. McGilvray. Attitude stabilization of a four-rotor aerial robot. In 43^{rd} *IEEE conference on Decision and Control, CDC'04*, 2004.

[18] A. Tayebi and S. McGlivary. Attitude stabilization of a vtol quadrotor aircraft. *IEEE Transactions on Control Systems Technology*, 14(3):562–571, 2006.

[19] A.R. Teel. Global stabilization and restricted tracking for multiple integrators with bounded controls. *Systems & Control Letters*, 18:165–171, 1992.

[20] P. Tsiotras. New control laws for the attitude stabilization of rigid bodies. In *13th IFAC Symposium on Automatic Control in Aerospace*, pages 316–321, 1994.

[21] P. Tsiotras and J. Luo. Control of underactuated spacecraft with bounded inputs. *Automatica*, 36(8):1153–1169, 2000.

[22] J.T. Wen and K. Kreutz-Delgado. The attitude control problem. *IEEE Transactions on Automatic Control*, 36(11):1148–1162, 1991.

[23] J.T. Wen, S. Seereeram, and D.S. Bayard. Nonlinear predictive control applied to spacecraft attitude control. In *American Control Conference*, volume 3, pages 1899–1903, 1997.

[24] B. Wie and J. Lu. Feedback control logic for spacecraft eigenaxis rotations under slew rate and control constraints. *Journal of Guidance, Control and Dynamics*, 18(6):1372–1379, 1995.