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Dynamic Modeling and simulation of a 3-D Hybrid structure Eel-Like Robot*

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Abstract - **In this paper, we present the dynamic modeling of a 3D-Hybrid underwater eel-like robot using recursive algorithms based on the Newton-Euler equations. The robot is composed of a sequence of parallel modules connected in serie. The algorithm gives the head accelerations and the joint torques as a function of the cartesian positions, velocities and accelerations of the platform of the modules. The proposed algorithm can be considered as a generalization of the recursive Newton-Euler inverse dynamic algorithm of serial manipulators with fixed base. The proposed algorithm is easy to implement and to simulate whatever the number of degrees of freedom of the robot. An example with 12 parallel modules is presented.**

Index Terms – hybrid robots, parallel robots, dynamic modeling, mobile robot, autonomous structures, Eel-like robot.

I. INTRODUCTION

Recently, many projects are devoted for the design and control of anguilliform robots owing to their potential in specific underwater applications including inspection, and endoscope [1],[2],[3]. The work presented in this paper is realized in the framework of the project "Robot Anguille" supported by the French CNRS. On previous work [14], we have presented the modelling of a 3D serial eel-like robot using Newton-Euler algorithms taking into account the case where the base is mobile. In this paper, we treat the hybrid mechanical structure, which corresponds to the real prototype realized within the project "Robot Anguille".

The main characteristic of this system is that the acceleration of the base must be determined as a function of the modules motions. To develop the dynamic models for such a structure, we propose to use recursive Newton-Euler algorithms generalizing those proposed for rigid manipulators [4],[5],[7],[8], and makes use of a new method for the dynamic modelling of parallel robots [15][16]. The proposed algorithm is easy to implement using numerical calculation and its computational complexity can be improved by using some techniques of symbolic method [6],[7]. Assuming local controllers to realize the desired cartesian position, velocities and accelerations, this model can be used for the dynamic simulation of the eel-like robot.

II. GEOMETRIC MODELING OF THE STRUCTURE

The system treated in this paper is an eel-like robot whose structure is composed of a sequence of n parallel modules connected in serie. Each parallel module is equivalent to a spherical joint. It is designed based on the principle of extensor and flexor muscle [17]: see Figure 1:

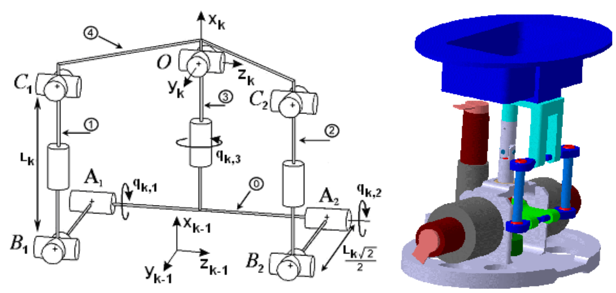

Figure 1: Structure and CAD model of the parallel module

The parallel modules have the same structure and dimension. Each module (see Figure 1) is composed of a base ⓪, 3 legs ①, ②, ③, and a platform ④. Each leg contains one actuated joint using DC motor.

The actuators are placed on the base, whereas the platform contains electronic circuits devoted to the control. For reasons of optimum space exploitation, each structure is connected in opposition compared to the previous one. The resulting robot is shown on Figure 2.

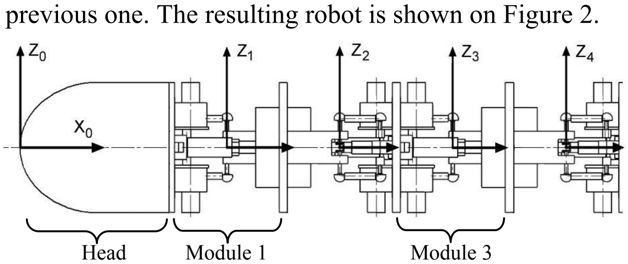

Figure 2: Assembly of the modules

III. KINEMATIC MODELING OF THE STRUCTURE

The modules are numbered from 1 to n, the head, numbered as link 0, is the base of the first module, whereas the tail is the platform of module n. We assign a frame Σ_k attached to the platform of each module k, such

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that the $\mathbf{x}_k$ axis is taken along the central line of the robot, and the $\mathbf{z}_k$ axis is along the yaw axis represented in Figure 1 by $\overrightarrow{\mathbf{A}_1\mathbf{A}_2}$, thus frame Σ_{k-1} is fixed with the base of module k.

The transformation matrix from frame Σ_{k-1} to frame Σ_k is expressed as a function of the following parameters :

- $\mathbf{\Theta}_k = \begin{bmatrix}\theta_k & \varphi_k & \psi_k\end{bmatrix}^T$ the roll, pitch and yaw angles,
- d_k: the distance between $\mathbf{z}_{k-1}$ and $\mathbf{z}_k$ along $\mathbf{x}_{k-1}$.

The homogeneous transformation matrix, which defines frame Σ_k relative to frame Σ_{k-1} is given by the matrix:

$$^{k-1}\mathbf{T}_k = \mathbf{Trans}(\mathbf{x},d_k)\mathbf{Rot}(\mathbf{x},\theta_k)\mathbf{Rot}(\mathbf{y},\varphi_k)\mathbf{Rot}(\mathbf{z},\psi_k) =$$

$$\begin{bmatrix} C\varphi_k C\psi_k & -C\varphi_k S\psi_k & S\varphi_k & d_k \\ S\theta_k S\varphi_k C\psi_k + C\theta_k S\psi_k & -S\theta_k S\varphi_k S\psi_k + C\theta_k C\psi_k & -S\theta_k C\varphi_k & 0 \\ -C\theta_k S\varphi_k C\psi_k + S\theta_k S\psi_k & C\theta_k S\varphi_k S\psi_k + S\theta_k C\psi_k & C\theta_k C\varphi_k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

In the following the upper-left exponent indicates the projection frame. We note that the orientation matrix $^{k-1}\mathbf{R}_k$ of frame Σ_k with respect to frame Σ_{k-1} is the (3×3) upper-left sub-matrix of $^{k-1}\mathbf{T}_k$, whereas the position vector $^{k-1}\mathbf{P}_k$ is equal to the upper-right (3×1) sub-matrix.

The matrix $^{w}\mathbf{T}_0$ between the world fixed frame Σ_w and the frame fixed with the head frame Σ_0 is supposed known at $t=0$. It will be updated by integrating the head acceleration.

The Cartesian velocities and accelerations of the platform of the modules are calculated using the following recursive equations for k = 1,…,n:

$$^{k}\mathbb{T}_{k-1} = \begin{bmatrix} ^{k}\mathbf{R}_{k-1} & -{}^{k}\mathbf{R}_{k-1}\,{}^{k-1}\hat{\mathbf{P}}_k \\ \mathbf{0}_{3\times3} & ^{k}\mathbf{R}_{k-1} \end{bmatrix} \quad (2)$$

$$^{k}\mathbb{V}_k = {}^{k}\mathbb{T}_{k-1}\,{}^{k-1}\mathbb{V}_{k-1} + {}^{k}\mathrm{a}_k\,{}^{k}\mathbf{w}_k \quad (3)$$

$$^{k}\mathbf{w}_k = {}^{k}\mathbf{\Omega}_{RPY}\dot{\mathbf{\Theta}}_\kappa \quad (4)$$

$$^{k}\gamma_k = {}^{k}\mathrm{a}_k\,{}^{k}\dot{\mathbf{w}}_k + {}^{k}\xi_k \quad (5)$$

$$^{k}\dot{\mathbf{w}}_k = {}^{k}\dot{\mathbf{\Omega}}_{RPY}\dot{\mathbf{\Theta}}_k + {}^{k}\mathbf{\Omega}_{RPY}\ddot{\mathbf{\Theta}}_k \quad (6)$$

$$^{k}\xi_k = \begin{bmatrix} ^{k}\mathbf{R}_{k-1}\left[{}^{k-1}\hat{\boldsymbol{\omega}}_{k-1}\left({}^{k-1}\hat{\boldsymbol{\omega}}_{k-1}\,{}^{k-1}\mathbf{P}_k\right)\right] \\ ^{k}\hat{\boldsymbol{\omega}}_{k-1}\,{}^{k}\mathbf{w}_k \end{bmatrix} \quad (7)$$

$$^{k}\dot{\mathbb{V}}_k = {}^{k}\mathbb{T}_{k-1}\,{}^{k-1}\dot{\mathbb{V}}_{k-1} + {}^{k}\gamma_k \quad (8)$$

With:

$^{k-1}\hat{\mathbf{P}}_k$ (3×3) skew matrix associated with $^{k-1}\mathbf{P}_k$,

$\mathbb{V}_k$ (6×1) kinematic screw vector of link k, given by:

$$\mathbb{V}_k = \begin{bmatrix}\mathbf{V}_k^T & \boldsymbol{\omega}_k^T\end{bmatrix}^T \quad (9)$$

$\mathbf{V}_k$ linear velocity of the origin of frame Σ_k,

$\boldsymbol{\omega}_k$ angular velocity of frame Σ_k,

$\mathbf{w}_k$ (3×1) relative angular velocity of the platform k with respect to its base,

$^{k}\mathrm{a}_k$ a (6×3) projection matrix between the relative velocity of module k and the (6×1) relative screw vector such that:

$$^{k}\mathrm{v}_k = {}^{k}\mathrm{a}_k\,{}^{k}\mathbf{w}_k \quad (10)$$

since the parallel module is equivalent to a spherical joint:

$$^{k}\mathrm{a}_k = \begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{I}_{d3}\end{bmatrix}^T \quad (11)$$

$^{k}\mathbf{\Omega}_{RPY}$ (3×3) matrix giving $^{k}\mathbf{w}_k$ in terms of $\dot{\mathbf{\Theta}}_k$ defined by:

$$^{k}\mathbf{\Omega}_{RPY} = \begin{bmatrix} C\varphi_k C\psi_k & S\psi_k & 0 \\ -C\varphi_k S\psi_k & C\psi_k & 0 \\ S\varphi_k & 0 & 1 \end{bmatrix} = {}^{k}\mathbf{R}_{k-1}\,{}^{k-1}\mathbf{\Omega}_{RPY} \quad (12)$$

with:

$$^{k-1}\mathbf{\Omega}_{RPY} = \begin{bmatrix} 1 & 0 & S\varphi_k \\ 0 & C\theta_k & -S\theta_k C\varphi_k \\ 0 & S\theta_k & C\theta_k C\varphi_k \end{bmatrix} \quad (13)$$

IV. INVERSE DYNAMIC MODEL

The Inverse Dynamic Model gives the joint torques and the acceleration of the head as a function of platform positions, velocities and accelerations. In this section, we present the recursive Newton-Euler algorithm for computing the inverse dynamic model of eel like robot with hybrid structure. We will make use of the inverse dynamic modelling of one module as developed in [16].

A. Dynamic model of one parallel module

This model is calculated using the following equation:

$$\mathbf{\Gamma}_k = {}^{k}\mathbf{J}_k^T\,{}^{k}\mathrm{a}_k^T\left({}^{k}\mathbb{F}_k + {}^{k}\mathrm{f}_{ek}\right) + \sum_{i=1}^{3}\left(\frac{\partial\dot{\mathbf{q}}_{ik}}{\partial\dot{\mathbf{q}}_k}\right)^T \mathbf{H}_{ik} \quad (14)$$

with:

$\mathbf{\Gamma}_k$ vector of actuator torques,

$\mathbb{F}_k$ total external wrench on the platform of module k,

f_{ek} wrench (forces and moments) exerted by the module k on the environment,

$\mathbf{H}_{ik}$ the inverse dynamic model of leg i of module k,

$\dot{\mathbf{q}}_{ik}$ vector of the joint velocities of leg i of module k,

$\dot{\mathbf{q}}_k$ vector of the motorized joint velocities of module k:

$$\dot{\mathbf{q}}_k = \begin{bmatrix}\dot{q}_{k,1} & \dot{q}_{k,2} & \dot{q}_{k,3}\end{bmatrix}^T \quad (15)$$

Equation (14) can be rewritten in a simplified form:

$$\mathbf{\Gamma}_k = \mathbf{J}_k^T\mathrm{a}_k^T\left(\mathbb{F}_k + \mathrm{f}_{ek} + \mathbf{H}_{bk}\right) \quad (16)$$

where $\mathbf{H}_{bk}$ is the sum of the dynamic models of the legs expressed on the platform Cartesian space.

The joint positions, velocities and accelerations of the legs are calculated using the inverse kinematic models of the legs, which are given in appendix.

The total external wrench on the platform is calculated using Newton-Euler equation:

$$ {}^{k}\mathbb{F}_{k} = {}^{k}\mathbb{J}_{k}\,{}^{k}\dot{\mathbb{V}}_{k} + \begin{bmatrix} {}^{k}\boldsymbol{\omega}_{k} \times \left({}^{k}\boldsymbol{\omega}_{k} \times {}^{k}\mathbf{MS}_{k} \right) \\ {}^{k}\boldsymbol{\omega}_{k} \times \left({}^{k}\mathbf{I}_{k}\,{}^{k}\boldsymbol{\omega}_{k} \right) \end{bmatrix} \quad (17) $$

where :

$\mathbb{J}_k$ (6×6) is the spatial inertia matrix of the platform of module k:

$$ {}^{k}\mathbb{J}_{k} = \begin{bmatrix} M_k \mathbf{I}_{d3} & -{}^{k}\mathbf{M}\hat{\mathbf{S}}_{k} \\ {}^{k}\mathbf{M}\hat{\mathbf{S}}_{k} & {}^{k}\mathbf{I}_{k} \end{bmatrix} \quad (18) $$

${}^{k}\mathbf{I}_k$ inertia tensor of module k with respect to frame Σ_k,

$\mathbf{I}_{d3}$ (3×3) identity matrix,

M_k mass of module k,

$\mathbf{MS}_k$ first moments of module k.

When connecting the modules in serie the equilibrium equation of each module is given by (Figure 3):

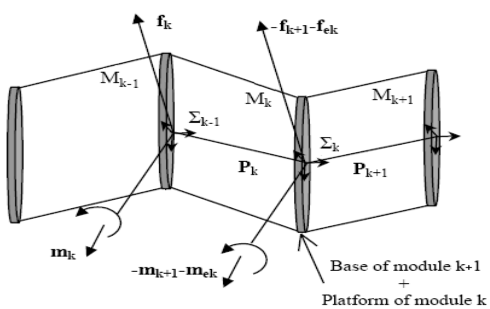

Figure 3: Forces and moments on module k

$$ {}^{k}\mathbb{f}_{k} = \left({}^{k}\mathbb{F}_{k} + {}^{k}\mathbb{F}_{bk} \right) + {}^{k+1}\mathbb{T}_{k}^{T}\,{}^{k+1}\mathbb{f}_{k+1} + {}^{k}\mathbb{f}_{ek} \quad (19) $$

where:

$\mathbb{f}_k$ (6×1) wrench exerted on module k by module $k-1$.

$\mathbb{F}_{bk}$ represent the resultant of total wrenches of the links of all the legs of module k. Details of its computation can be found on [15]. In fact, in our case, the inertial parameters of the legs of the modules are negligible compared to those of the base and the platform so we can consider that $\mathbf{H}_{bk} \simeq \mathbf{0}$ and $\mathbb{F}_{bk} \simeq \mathbf{0}$.

B. Recursive calculation of the inverse dynamic model of Eel-like robot

The inverse dynamic algorithm presented here consists of three recursive equations (A forward, then a backward, then a forward).

i) Forward recursive calculation

In this first step we calculate for each module the screw transformation matrices, the kinematic screw vector, and the elements of the accelerations and wrenches on the modules that are independent of the acceleration of the robot head ($\dot{\mathbf{V}}_0, \dot{\boldsymbol{\omega}}_0$). Thus we calculate for k = 1,…,n: ${}^{k}\mathbb{T}_{k-1}$, ${}^{k}\mathbb{V}_{k}$ and ${}^{k}\boldsymbol{\gamma}_{k}$ using (2), (3) and (5)respectively. We calculate also ${}^{k}\boldsymbol{\beta}_{k}$ representing the elements of the Newton-Euler equations, which are independent of the module accelerations:

$$ {}^{k}\boldsymbol{\beta}_{k} = {}^{k}\mathbb{f}_{ek} + \begin{bmatrix} {}^{k}\boldsymbol{\omega}_{k} \times \left({}^{k}\boldsymbol{\omega}_{k} \times {}^{k}\mathbf{MS}_{k} \right) \\ {}^{k}\boldsymbol{\omega}_{k} \times \left({}^{k}\mathbf{I}_{k}\,{}^{k}\boldsymbol{\omega}_{k} \right) \end{bmatrix} \quad (20) $$

ii) Backward recursive equations

In this second step we obtain the head acceleration using the inertial parameters of the composite body, where the composite body k consists of the inertial parameters of the platforms of modules k, …, n. Note that the platform of module k-1 is the base of module k.

Using (17) and (20), the equilibrium equation (19) can be rewritten as:

$$ {}^{k}\mathbb{f}_{k} = {}^{k}\mathbb{J}_{k}\,{}^{k}\dot{\mathbb{V}}_{k} + {}^{k}\boldsymbol{\beta}_{k} + {}^{k+1}\mathbb{T}_{k}^{T}\,{}^{k+1}\mathbb{f}_{k+1} \quad (21) $$

Applying the Newton-Euler equations on the composite module k and since ${}^{n+1}\mathbb{f}_{n+1} = 0$, (21) can be rewritten as:

$$ {}^{k}\mathbb{f}_{k} = {}^{k}\mathbb{J}_{k}^{c}\,{}^{k}\dot{\mathbb{V}}_{k} + {}^{k}\boldsymbol{\beta}_{k}^{c} \quad (22) $$

With:

$$ {}^{k}\mathbb{J}_{k}^{c} = {}^{k}\mathbb{J}_{k} + {}^{k+1}\mathbb{T}_{k}^{T}\,{}^{k+1}\mathbb{J}_{k+1}^{c}\,{}^{k+1}\mathbb{T}_{k} \quad (23) $$

$$ {}^{k}\boldsymbol{\beta}_{k}^{c} = {}^{k}\boldsymbol{\beta}_{k} + {}^{k+1}\mathbb{T}_{k}^{T}\,{}^{k+1}\boldsymbol{\beta}_{k+1}^{c} + {}^{k+1}\mathbb{T}_{k}^{T}\,{}^{k+1}\mathbb{J}_{k+1}^{c}\,{}^{k+1}\gamma_{k+1} \quad (24) $$

${}^{k}\mathbb{J}_{k}^{c}$ is the spatial inertial matrix of the composite link k.

For k = 0, since ${}^{0}\mathbb{f}_{0}$ is equal to zero, we obtain using (22):

$$ {}^{0}\dot{\mathbb{V}}_{0} = -\left({}^{0}\mathbb{J}_{0}^{c} \right)^{-1}\,{}^{0}\boldsymbol{\beta}_{0}^{c} \quad (25) $$

iii) Forward recursive equations: for k=1,…,n

In this last step, we calculate the acceleration of module k, then the wrench ${}^{k}\mathbb{f}_{k}$ applied onto the module k and finally the motor torques:

$$ {}^{k}\dot{\mathbb{V}}_{k} = {}^{k}\mathbb{T}_{k-1}\,{}^{k-1}\dot{\mathbb{V}}_{k-1} + {}^{k}\gamma_{k} \quad (26) $$

$$ {}^{k}\mathbb{f}_{k} = {}^{k}\mathbb{J}_{k}^{c}\,{}^{k}\dot{\mathbb{V}}_{k} + {}^{k}\boldsymbol{\beta}_{k}^{c} \quad (27) $$

The motor torques are calculated by projecting ${}^{k}\mathbb{f}_{k}$ on the joint space using the Jacobian matrix of the module. By taking into account the coulomb friction parameters ($\mathbf{F}_{sk}$) and viscous friction parameters ($\mathbf{F}_{vk}$) as well as the rotor inertia ($\mathbf{I}_{ak}$) of the motors of module k, the joint torques is given by:

$$ \boldsymbol{\Gamma}_{k} = {}^{k}\mathbf{J}_{k}^{T}\,{}^{k}\mathbb{a}_{k}^{T}\,{}^{k}\mathbb{f}_{k} + \mathbf{F}_{sk}\,\mathrm{sign}\left(\dot{\mathbf{q}}_{k} \right) + \mathbf{F}_{vk}\,\dot{\mathbf{q}}_{k} + \mathbf{I}_{ak}\,\ddot{\mathbf{q}}_{k} \quad (28) $$

where : $\mathbf{F}_{sk}$, $\mathbf{F}_{vk}$, $\mathbf{I}_{ak}$ are (3×3) diagonal matrices.

V. FLUID-STRUCTURE INTERACTION MODEL

To simulate the hydrodynamic behavior of the robot, we have use in previous work [14] a simple model to express the contact forces between the fluid and the modules of the eel. In our study the three dimensional fluid forces are modelled by a density of wrenches applied onto each cross section of the modules, which only depends of the transverse modules' motion.

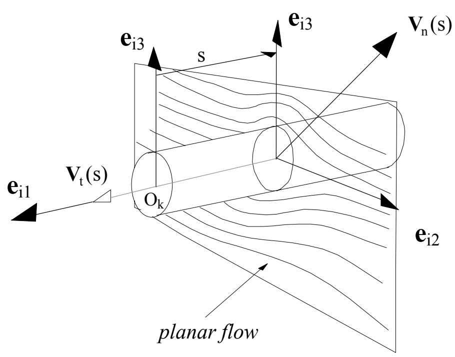

Figure 4: Decomposition into planar flow for fluid wrench calculation

To simplify the writing, we assume that the mass per unit of volume of the robot is equal to that of water such that the robot is neutrally buoyant. Moreover ${}^k\mathbf{V}_i(s)$ denotes the velocity of a cross section of the module k positioned at the distance s along the **x** axis of the platform from the point O_k (see Figure 4). This velocity can be decomposed in the local frame $(\mathbf{e}_{i1},\mathbf{e}_{i2},\mathbf{e}_{i3})$ as:

$$ {}^k\mathbf{V}_i(s) = \mathrm{V}_{ti}(s)\mathbf{e}_{i1} + \mathrm{V}_{ni2}(s)\mathbf{e}_{i2} + \mathrm{V}_{ni3}(s)\mathbf{e}_{i3} \tag{29} $$

And:

$$ \mathbf{V}_{ni} = \mathrm{V}_{ni2}\mathbf{e}_{i2} + \mathrm{V}_{ni3}\mathbf{e}_{i3} $$

We also define $\|\mathbf{V}_{ni}\| = \sqrt{\mathrm{V}_{ni2}^2 + \mathrm{V}_{ni3}^2}$. Similar relations to (29) can be written for ${}^k\dot{\mathbf{V}}_i(s)$, ${}^k\boldsymbol{\omega}_i(s)$ and ${}^k\dot{\boldsymbol{\omega}}_i(s)$.

With the previous assumptions, the model of the contact forces between the fluid and the modules of the eel is that of Morison [9], and can be defined as a hydraulic wrenches densities per unit along the module axial length (w.r.t. the S cross section center):

$$ {}^k\boldsymbol{f}_{hk}(s) = \begin{bmatrix} {}^k\boldsymbol{f}_{hk}(s) \\ {}^k\boldsymbol{m}_{hk}(s) \end{bmatrix} = \begin{bmatrix} {}^k\boldsymbol{f}_{drag}(s) \\ {}^k\boldsymbol{m}_{drag}(s) \end{bmatrix} + \begin{bmatrix} {}^k\boldsymbol{f}_{am}(s) \\ {}^k\boldsymbol{m}_{am}(s) \end{bmatrix} \tag{30} $$

where, forces and moments are given respectively:

$$ {}^k\boldsymbol{f}_{drag}(s) = \mathrm{C}_{ld1}|\mathrm{V}_{ti}(s)|\mathrm{V}_{ti}(s)\mathbf{e}_{i1} + \sum_{j=2}^{3}\mathrm{C}_{ldj}\|\mathbf{V}_{ni}(s)\|\mathrm{V}_{nij}(s)\mathbf{e}_{ij} \tag{31} $$

$$ {}^k\boldsymbol{m}_{drag}(s) = \mathrm{C}_{ad1}|\omega_{ti}|\omega_{ti}\mathbf{e}_{i1} \tag{32} $$

$$ {}^k\boldsymbol{f}_{am}(s) = \sum_{j=2}^{3}\mathrm{C}_{lmj}\dot{\mathrm{V}}_{nij}(s)\mathbf{e}_{ij} \tag{33} $$

$$ {}^k\boldsymbol{m}_{am}(s) = \mathrm{C}_{am1}\dot{\omega}_{ti}\mathbf{e}_{i1} \tag{34} $$

Where $\boldsymbol{f}_{drag}$ and $\boldsymbol{m}_{drag}$ are due to the friction viscosity and pressure difference whereas $\boldsymbol{f}_{am}$ and $\boldsymbol{m}_{am}$ are in relation with the quantity of fluid displaced during the movement (call "added mass"). Finally the coefficients C_{ldj}, C_{lmj}, C_{ad1} and C_{am1} are depending on the density of the fluid, the shape and size of the profile (here elliptic) and the Reynolds number of the moving profile in the fluid (approximately 10^5). Their expressions are given in section VI. Then by superimposing all the "slice-by-slice" contributions from $s = 0$ to $s = d_{k+1}$ (the axial length of the k^{th} module), we find the global wrench exerted by module k on the fluid, expressed at O_k:

$$ {}^k\mathbb{f}_{hk} = \begin{bmatrix} {}^k\mathbf{f}_{hk} \\ {}^k\mathbf{m}_{hk} \end{bmatrix} = \begin{bmatrix} {}^k\mathbf{f}_{dragk} \\ {}^k\mathbf{m}_{dragk} \end{bmatrix} + \begin{bmatrix} {}^k\mathbf{f}_{amk} \\ {}^k\mathbf{m}_{amk} \end{bmatrix} = \int_0^{d_{k+1}} {}^k\boldsymbol{f}_{hi}(s)ds \tag{35} $$

The first term (drag and viscous wrench) of (35) is integrated numerically at each sample time of the algorithm, while the second contribution (added mass) can be explicitly computed in the local frame $(\mathbf{O}_k,\mathbf{e}_{k1},\mathbf{e}_{k2},\mathbf{e}_{k3})$ as:

$$ {}^k\mathbb{f}_{amk} = \begin{bmatrix} {}^k\mathbf{f}_{amk} \\ {}^k\mathbf{m}_{amk} \end{bmatrix} = {}^k\mathbb{J}_k^{ak}\,{}^k\dot{\mathbb{V}}_k + {}^k\boldsymbol{\beta}_{amk} \tag{36} $$

Where ${}^k\mathbb{J}_k^{ak}$ is the (6×6) added inertia matrix and ${}^k\boldsymbol{\beta}_{amk}$ the (6×1) matrix of Coriolis-centrifugal forces, both produced by the added fluid masses.

Equation (36) shows that some of the elements corresponds to ${}^k\mathbb{f}_{ek}$ due to the fluid contact forces and other to a constant term to be added to the (6×6) inertia matrix. Thus using previous results [14], we can write:

$$ {}^k\mathbb{f}_{ek} = {}^k\mathbb{f}_{dragk} + {}^k\boldsymbol{\beta}_{amk} \tag{37} $$

$$ {}^k\mathbb{J}_k = {}^k\mathbb{J}_k + \mathbb{J}_k^{ak} \tag{38} $$

with:

$$ {}^k\mathbb{J}_k^{ak} = \begin{bmatrix} {}^k\mathbf{m}_{aGk} & -{}^k\mathbf{m}_{aGk}\,{}^k\hat{\mathbf{S}}_k \\ {}^k\hat{\mathbf{S}}_k\,{}^k\mathbf{m}_{aGk} & {}^k\mathbf{I}_{aGk} - {}^k\hat{\mathbf{S}}_k\,{}^k\mathbf{m}_{aGk}\,{}^k\hat{\mathbf{S}}_k \end{bmatrix} \tag{39} $$

and:

$$ {}^k\boldsymbol{\beta}_{amk} = \begin{bmatrix} {}^k\mathbf{m}_{aGk}\left({}^k\hat{\boldsymbol{\omega}}_k\,{}^k\hat{\boldsymbol{\omega}}_k\,{}^k\mathbf{S}_k\right) \\ 4/3\,{}^k\hat{\mathbf{S}}_k\,{}^k\mathbf{m}_{aGk}\left({}^k\hat{\boldsymbol{\omega}}_k\,{}^k\hat{\boldsymbol{\omega}}_k\,{}^k\mathbf{S}_k\right) \end{bmatrix} \tag{40} $$

With:

${}^k\mathbf{S}_k$ the position of the center of mass of platform k with respect to the origin of Σ_k.

Furthermore, with the expressions of the local (slice-by-slice) added mass coefficients (33) and (34), other terms can be detailed as:

- ${}^k\mathbf{m}_{aGk} = \mathrm{diag}(0,\mathrm{C}_{lm2}d_{k+1},\mathrm{C}_{lm3}d_{k+1})$ the (3×3) matrix of added linear inertia,
- ${}^k\mathbf{m}\hat{\mathbf{s}}_{aGk} = \mathbf{0}_{3\times3}$ the (3×3) matrix of added linear-angular coupled inertia,
- ${}^k\mathbf{I}_{aGk} = \mathrm{diag}(\mathrm{C}_{am1}d_{k+1},\mathrm{C}_{lm3}d_{k+1}^3/12,\mathrm{C}_{lm2}d_{k+1}^3/12)$, the (3×3) matrix of added angular inertia,

VI. SIMULATION EXAMPLE

In this section, we present some simulation results obtained for an eel-like robot using Matlab and Simulink. The robot is composed of 12 identical modules (Figure 2).

The head (the base of module 1), is composed of a half of a spheroid and an elliptic cylinder. The other modules k, for k =1,…,12 are elliptic cylinders. The total length of the robot is equal to 2.08 meter. The cross section is of elliptic shape whose great axis length (2b) is equal to 18cm and its small axis length (2a) is equal to 13cm. Finally, the coefficients of our fluid model are given by:

$$C_{ld1} = (1/2)\rho C_1 \pi(a+b)/2, C_{ld2} = (1/2)\rho C_2 2b,$$

$$C_{ld3} = (1/2)\rho C_3 2a, C_{ad1} = (1/2)\rho C_4 (b^2 - a^2)^2,$$

$$C_{lm2} = \rho\pi b^2 C_5, C_{lm3} = \rho\pi a^2 C_6, C_{am1} = \rho\pi C_7 (b^2 - a^2)^2/8$$

where ρ is the robot volume density taken equal to one. with from [11]:

$$C_1 = 0.01, C_2 = C_3 = C_4 = 1 \text{ and } C_5 = C_6 = C_7 = 1.$$

In this example, we study the planar forward propulsion. Such a motion in the plane, is produced by a motion law of the following form along the x axis of the robot [12],[13]:

$$Q(s,t) = f(t).A.e^{\alpha.s}.\sin\left[2\pi\left(\frac{s}{\lambda}-\frac{t}{T}\right)\right] \quad (41)$$

Where A is the amplitude of the motion, α is introduced to increase the amplitude when going from the head to the tail, $1/T$ is the frequency of the wave, λ represents the length of the wave, and s the curvilinear coordinate along eel's backbone. The function $f(t)$ is a polynomial function of the fifth order where the coefficients satisfy the following conditions:

$$f(0) = 0, f(t_f) = 1, \dot{f}(0) = \ddot{f}(0) = 0, \dot{f}(t_f) = \ddot{f}(t_f) = 0$$

t_f is the ending time of $f(t)$.

To apply this continuous motion law to our model, we have to discretize equation (41) to have the corresponding value on each joint. As the motion is along the z axis, θ_k and φ_k are set equal to zero, whereas Ψ_k is defined by:

$$\psi_k(t) = Q(X_{k+1},t) - Q(X_k,t) \quad (42)$$

and $X_k = \sum_{j=1}^{k} d_j$ the distance of each module from the head

Figures 5 and 6 show the simulation result with the following numerical values for the joint evolutions:

$$A = 0.2, \alpha = 0.8, t_f = 4s, \lambda = 1.5m, T = 3s$$

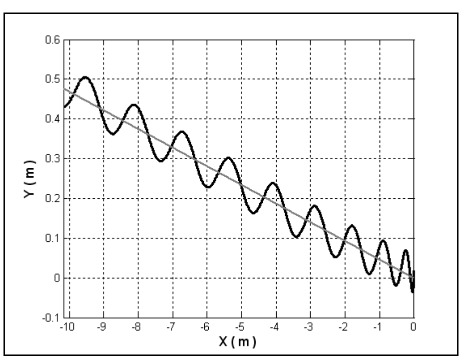

Figure 5: Head's trajectory in the $\mathbf{x}_w$-$\mathbf{x}_w$ plane of the fixed frame.

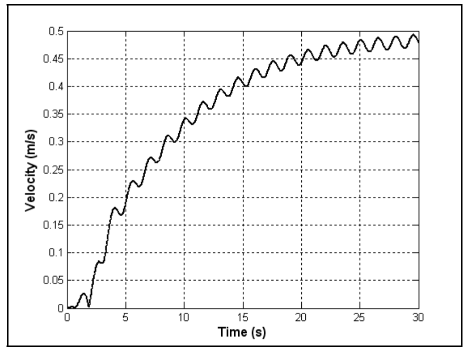

Figure 6: Velocity of the head with respect to time in world frame.

VII. CONCLUSION

This paper presents the inverse dynamic modeling of a swimming eel like robot composed of a hybrid mechanical structure composed of parallel modules connected in serie and whose base is free to move in all directions. The proposed algorithm, allows to obtain a three dimensional dynamic model and is a generalization of the recursive Newton-Euler computed torque approach of articulated manipulators to the case of hybrid structure with a free base. This dynamic model is developed using the recursive Newton-Euler formalism, as it is well known in the robotics community. Moreover, based on the literature of fluid mechanics, we have adopted a simplified model of fluid-structure contact.

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APPENDIX

In this appendix, we calculate the joint positions and velocities as a function of the cartesian position and velocities.

A Inverse geometric model

To calculate the inverse geometric model of leg 1 and 2 (for leg 3 we can see immediately that $q_{k,3} = \theta_k$), we calculate the Cartesian positions of points C_i and B_i in frame Σ_{k-1} and then use the geometric constraint in distance between these two points. Thus we can write:

$$ {}^{k-1}\mathbf{P}_{Ci} = \begin{bmatrix} P_{Cix} & P_{Ciy} & P_{Ciz} \end{bmatrix}^T = {}^{k-1}\mathbf{R}_k \, {}^k\mathbf{P}_{Ci} \tag{43} $$

$$ {}^{k-1}\mathbf{B}_1 = 1/2\begin{bmatrix} -Sq_{k,1}L_k\sqrt{2} - 2L_k & Cq_{k,1}L_k\sqrt{2} & -L_k\sqrt{2} \end{bmatrix}^T $$

$$ {}^{k-1}\mathbf{B}_2 = 1/2\begin{bmatrix} Sq_{k,2}L_k\sqrt{2} - 2L_k & Cq_{k,2}L_k\sqrt{2} & L_k\sqrt{2} \end{bmatrix}^T $$

Where $Cq_{k,i} = \cos(q_{k,i})$, $Sq_{k,i} = \sin(q_{k,i})$ and L_k the length of vector $\overrightarrow{B_iC_i}$.

By considering the geometric condition $\|\mathbf{B}_i\mathbf{C}_i\| = L_k$ and assuming that $Sq_{k,i} = \dfrac{2Q_i}{1+Q_i^2}$ and $Cq_{k,i} = \dfrac{1-Q_i^2}{1+Q_i^2}$, where $Q_i = \tan(q_{k,i}/2)$, we can write the inverse geometric model as the solution of a second degree equation:

$$ \alpha_{i1}Q_i^2 + \alpha_{i2}Q_i + \alpha_{i3} = 0 \tag{44} $$

with:

$$ \alpha_{i1} = \sqrt{2}\left(L_k\sqrt{2} - P_{Ciy} + e_iP_{Ciz} + \sqrt{2}P_{Cix}\right)/L_k $$

$$ \alpha_{i2} = 2\sqrt{2}e_i\left(1 + P_{Cix}/L_k\right) $$

$$ \alpha_{i3} = \sqrt{2}\left(L_k\sqrt{2} + P_{Ciy} + e_iP_{Ciz} + \sqrt{2}P_{Cix}\right)/L_k $$

$e_1 = 1$ and $e_2 = -1$

We can then deduce $q_{k,i}$ (for i = 1,2) from (44):

$$ Q_i = \left(-\alpha_{i2} + e_i\sqrt{\alpha_{i2}^2 - 4\alpha_{i1}\alpha_{i3}}\right)/\left(2\alpha_{i1}\right) \tag{45} $$

then:

$$ q_{k,i} = 2\operatorname{atan}\left(Q_i\right) $$

B Inverse Kinematic model

To calculate the Inverse kinematic model, we make use of the method proposed in [16]. The idea is to transform the module as three serial manipulators connected to the same platform. This transformation is done by virtually isolating the platform, so that leg 1 and 2 can be reduced to a 3 dof serial manipulator (joint A_i and universal joint on B_i) and leg 3 is reduced to a 1dof manipulator. The rows of the inverse Jacobian matrix are obtained by calculating the motorized joint velocities from the leg Jacobian matrices, For the first two rows, we first calculate the velocity of the points Ci in terms of $\mathbf{w}_k$:

$$ \mathbf{V}_{k,Ci} = \mathbf{J}_{vi}\mathbf{w}_k \tag{46} $$

with:

$$ \mathbf{J}_{vi} = -\hat{\mathbf{P}}_{Ci},\ i = 1,2 $$

Then we can write the kinematic model of one leg:

$$ \mathbf{V}_{ik} = \mathbf{J}_{ik}\dot{\mathbf{q}}_{ik} \tag{47} $$

Where ${}^k\mathbf{J}_{ik}$ is the (3×3) jacobian matrix of leg i.

We can finally use kinematic constraints by writing that equations (46) and (47) are equal:

$$ \mathbf{J}_{vi}\mathbf{w}_k = \mathbf{J}_{ik}\dot{\mathbf{q}}_{ik} \tag{48} $$

We can deduce the i-th row of the inverse kinematic model of the module from the relation:

$$ \dot{q}_{k,i} = \mathbf{J}_{ik}^{-1}(1,:)\left(\mathbf{J}_{vi}\mathbf{w}_k\right) \tag{49} $$

where A(i,:) gives the i-th row of matrix A

The calculation of $\mathbf{J}_{ik}^{-1}$ give the following result for i=1,2:

$$ {}^{k-1}\mathbf{J}_k^{-1}(i,:) = \begin{bmatrix} e_iN_i/D_i & -e_i\mathbf{C}q_{k,i}/D_i & 1/D_i \end{bmatrix} {}^{k-1}\hat{\mathbf{P}}_{Ci} $$

With:

$$ N_i = \sqrt{2} + e_iSq_{k,i} $$

$$ D_i = Cq_{k,i}\left(L_k + P_{Cix}\right) + e_iSq_{k,i}P_{Ciy} $$

For leg 3, the kinematic model can be written as the equality of the projection of the angular velocity of both the platform and the link of leg 3 along the axis defined by the common perpendicular of the two axis of the universal joint:

$$ {}^{k-1}\mathbf{h}_k^T\begin{bmatrix}1 & 0 & 0\end{bmatrix}^T \dot{q}_{k,3} = {}^{k-1}\mathbf{h}_k^T\,{}^{k-1}\mathbf{w}_k \tag{50} $$

where $\mathbf{h}_k$ is the unit vector along the vector product of the two axis of the universal joint (in our case, it corresponds to the last two columns of matrix $\mathbf{\Omega}_{RPY}$). Thus, using (50) we have:

$$ {}^{k-1}\mathbf{h}_k = \begin{bmatrix} C\varphi_k & S\theta_kS\varphi_k & C\theta_kS\varphi_k \end{bmatrix}^T \tag{51} $$

Thus the third row of the inverse Jacobian matrix of module k is:

$$ {}^{k-1}\mathbf{J}_k^{-1}(3,:) = \begin{bmatrix} 1 & S\theta_k\tan(\varphi_k) & C\theta_k\tan(\varphi_k) \end{bmatrix} \tag{52} $$