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Configuration Control and Recalibration of a New Reconfigurable Robot

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Abstract— **The advantages of reconfigurable robots have been discussed in the specialized literature. Conventionally, reconfigurability was a direct result of using modular joints. In this paper we discuss the configuration control and recalibration of a different class of reconfigurable robots, one which is equipped with lockable cylindrical joints with no actuators or sensors. Such a robot can be as versatile and agile as a hyper-redundant manipulator, but with a simpler, more compact, lighter design. A passive joint becomes controllable when the robot forms a closed kinematic chain and the joint lock is released. After reconfiguration, the values of the passive joints are computed from the value of the active joints using inverse kinematics of the closed chain. That problem is solved using a globally uniformly asymptotically stable scheme based on Closed-Loop Inverse Kinematics (CLIK). An asymptotically stable reconfiguration controller is also devised that takes the robot from one configuration to another by directly regulating the values of the passive joints. The controller has a rather simple structure, which only relies on the robot gravity and kinematics models. Conditions for the observability and the controllability of the passive joints are also derived in detail, and some numerical results are reported.**

I. Introduction

In space applications, systems are generally designed for minimum weight to reduce the launch cost. Another design constraint is that it should be compact enough to be accommodated within its designated space in the launch vehicle. Since the links of a space manipulator are usually long, they have to be folded before launch. For instance, CanadarmII has two long booms, each of which has a hinge at the middle, which allowed the booms to be folded before launch and then unfolded manually by astronauts in orbit. For on-orbit servicing missions whereby no human operator is present, the robot has to be able to deploy itself. Except for reconfigurable manipulators, there are two options: (i) not using long booms or (ii) using *hyper-redundant* manipulators. The former option may limit the types of operation possible as the robot would have a short reach, while the latter increases the complexity of the manipulation system.

Reconfigurable robot were originally introduced in [1] to increase the versatility of robotic manipulators. The concept was then developed further in [2]. Cellular robots based on hexagonal modules and those based on the concept of robot molecules were described in [3], [4] and [5], [6], respectively. Reconfigurable robots for space exploration were proposed in [7]. The design of Conro modules to build deployable modular robots that can be reconfigured to take different shapes such as snakes or hexapods were presented in [8], which has some similarities with earlier works on Tetrobot, [9]. All these reconfigurable robots are modular, hence needing an effective docking system for connecting and releasing the modules [10].

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Here we discuss the recalibration and configuration control of a new reconfigurable robot that does not rely on modular joints. To achieve reconfigurability, the new design uses *passive cylindrical joints* between adjacent *active joints* with noncoinciding axes [11]. Each cylindrical joint can be considered as the combination of two passive revolute and prismatic joints. The passive joints have no sensors or driving actuators, but they are equipped with a normally locked brake mechanism which can be unlocked simply by activating a solenoid. When locked, the entire cylindrical joint becomes a rigid link which connects the two neighboring active joints. The twist angle and the length of this link is determined by the value of the locked passive joints.

The modular robots have the great advantage of being able to change their both morphology and topology; more specifically, they can change their number of links, form a closed chain or break it, and create a tree-like structure, for example. All these capabilities, however, come at the expense of complexity in the joints and their docking system. The new design, on the other hand, can offer a simpler and more effective solution to the problem at hand as there is no need to detach any link or joint to reconfigure the robot. Thus, the entire reconfiguration operation can be performed autonomously with a higher level of reliability. This makes this new type of reconfigurable robots particularly attractive.

Fig. 1 illustrates the reconfiguration process of a typical reconfigurable arm with two cylindrical joints. In general, such an arm can change its configuration by completing the following steps: (i) The manipulator forms a closed kinematic chain by restricting the robot End-Effector (EE), Fig. 1(b); (ii) one or more of the brake mechanisms are released, thus increasing the degrees of freedom (DOF) of the constrained system; (iii) the closed-chain system is controlled in such a way that the desired active-joint positions are achieved, Figs. 1(c)–1(d); upon converging to these values, the brakes are locked again; (iv) the constraint on the motion of the EE is removed. A manipulator with a new configuration is now born, Fig. 1(h). Depending on the changes needed, steps (ii) and (iii) may need to be repeated with one or more of the passive joints unlocked at a time.

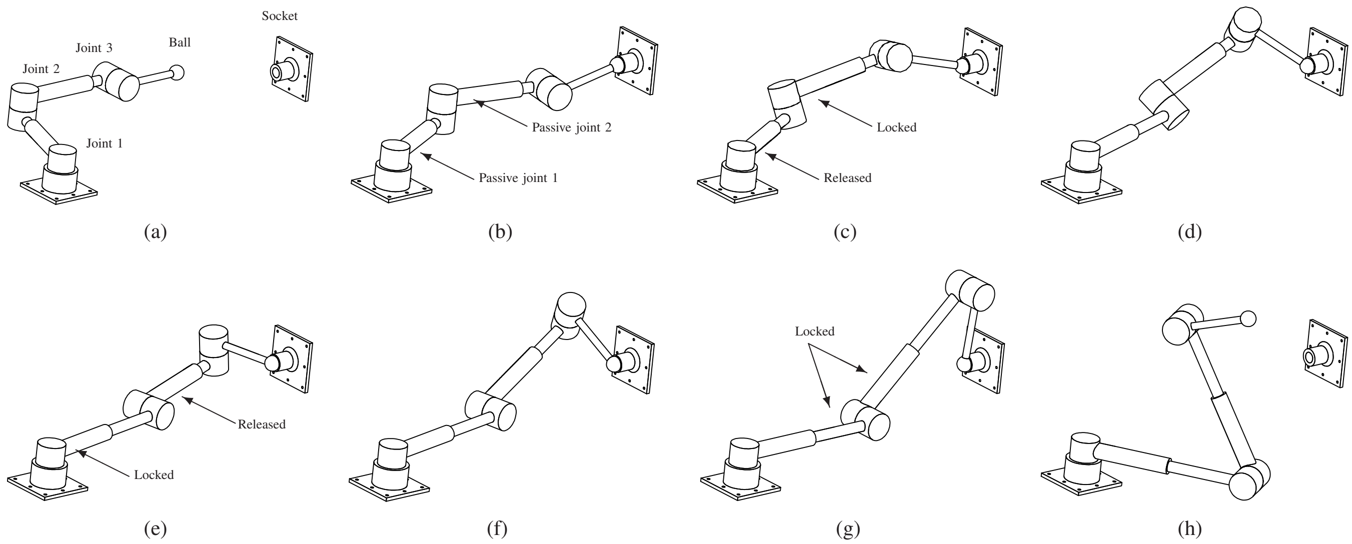

Fig. 1. Reconfiguration maneuver from a short, planar configuration to an anthropomorphic one; (a) the initial configuration; (b) constraining the EE motion with a ball joint; (c,d,e) unlocking and reconfiguring the first passive joint; (f,g) unlocking and reconfiguring the second passive joint; (h) the final configuration.

The kinematics of the reconfiguration process is the subject of Section II. Section III is devoted to the development of a Lyapunov-based reconfiguration control of the robot with passive joints; the control uses only the closed-chain kinematics and gravitational-force models of the robot. The controllability and singularity of the robot during reconfiguration are discussed in Section III-D. Finally, Section IV presents the numerical results pertaining to the reconfigurations shown in Fig. 1.

II. Reconfiguration Kinematics

As mentioned before, the reconfigurable mechanical manipulators proposed in this paper can only achieve reconfigurability by releasing one or more of their *passive* joints and simultaneously constraining the EE motion, the latter of which makes the originally serial manipulator parallel. The motion constraints are then used as a tool to change the joint variable of the released passive joints. The number of the parameters that can be changed at the same time depends on such kinematic properties as the manipulator degrees of freedom, the dimension of the EE constraint space, and the number of cylindrical joints released. The first parameter is known for each manipulator; the second can be considered a design parameter of the manipulator; the last, however, is case dependent and may need to be determined for each reconfiguration maneuver based on the other kinematic parameters of the manipulator.

A. Manipulator Recalibration:

The purpose of reconfiguring a manipulator is to change some of its DH parameters which are represented by the passive-joint variables. Since the passive joints are not instrumented, after any reconfiguration, the manipulator must be recalibrated; that can be done by computing the passive-joint variables from the constraint equations, given the sensor readings of the active joints. Assuming that there are n *active joints* and m released *passive, cylindrical joints*, we denote the active and the passive generalized coordinates of the manipulator by $\theta \in \mathbb{R}^n$ and $\psi \in \mathbb{R}^{2m}$, respectively. Note that ψ accounts only for the released passive joints; the total number of the locked and unlocked joints can be higher than m. The independent external motion constraints of the end-effector are then described by the r-dimensional vector function ϕ

$$\phi \equiv \phi(\theta, \psi) = 0. \tag{1}$$

Then, differentiating (1) with respect to time, we obtain

$$A_\theta \dot{\theta} + A_\psi \dot{\psi} = 0, \tag{2}$$

where $A_\theta = \partial\phi/\partial\theta$ and $A_\psi = \partial\phi/\partial\psi$. The above equation can be written in a more compact form as

$$A\dot{q} = 0, \tag{3}$$

where the vector of generalized coordinates q and the Jacobian matrix $A \in \mathbb{R}^{r\times(n+2m)}$ of the constraint equation (1) are defined as

$$q \triangleq \begin{bmatrix} \theta \\ \psi \end{bmatrix}, \tag{4}$$

$$A \equiv \begin{bmatrix} A_\theta & A_\psi \end{bmatrix} \triangleq \frac{\partial\phi}{\partial q}. \tag{5}$$

Now, let us assume the following holds:

Assumption 1: The Jacobian A_ψ remains full-rank during a reconfiguration maneuver.

Then, by making use of (2), one can *uniquely* obtain an estimate of $\dot{\psi}$ from the measured value of $\dot{\theta}$ through

$$\dot{\psi} = Q\dot{\theta}, \quad \text{where} \quad Q \triangleq -A_\psi^{+} A_\theta, \tag{6}$$

with "+" denoting the pseudo-inverse. Note that the matrix $A_\psi \in \mathbb{R}^{r\times 2m}$ being full-rank requires that

$$r \geq 2m. \tag{7}$$

This means that, in order to be able to observe the value of ψ, the number of passive DOFs should not exceed the number of the constraints.

1) Dynamic Estimator: Having computed $\dot{\psi}$, one can then integrate it to obtain the time-history of the passive-joint variables during a reconfiguration maneuver. The integration, however, will inevitably lead to a drift in the position error. In order to suppress this drift, we can use a dynamic estimator which employs the constraint equations ϕ as a measure of the estimation error. For convenience, it would be easier to use the same number of "measurements" as the number of variables to be estimated. Therefore, we choose a combination $\phi' \in \mathbb{R}^{2m}$ of the constraint equations defined by

$$\phi' \triangleq W\phi,$$

where W is a full-rank constant matrix. Then, premultiplying the velocity constraint (2) by W, we obtain

$$A'_\theta\dot{\theta} + A'_\psi\dot{\psi} = 0,$$

in which $A'_\theta \triangleq WA_\theta$ and $A'_\psi \triangleq WA_\psi$. Equation (6) can then be simplified as

$$\dot{\psi} = Q'\dot{\theta}, \quad \text{where} \quad Q' \triangleq A'^{-1}_\psi A'_\theta. \tag{8}$$

As shown in Fig. 2, we realize the dynamic estimator by closing the loop using an $A'^T_\psi K_O\,\phi'$ feedback; This feedback in addition to the feedforward given by (8) results in

$$\dot{\hat{\psi}} = Q'(\theta,\hat{\psi})\dot{\theta} - A'^T_\psi(\theta,\hat{\psi})K_O\,W\phi(\theta,\hat{\psi}). \tag{9}$$

In the above equation, $\hat{\psi}$ denotes the estimate of the value of the passive joints, and $K_O \in \mathbb{R}^{2m\times 2m}$ is the positive-definite estimator gain matrix. The inputs and outputs of the estimator loop are $\{\theta,\dot{\theta}\}$ and $\{\hat{\psi},\dot{\hat{\psi}}\}$, respectively.

Proposition 1: Let us assume that A_ψ remains a full-rank matrix during the estimation precess, and that an estimate of ψ is obtained from system (9). Then, the constraint equation $\phi(\theta,\hat{\psi})$ as a function of the estimated values of the passive joints globally uniformly asymptotically converges to zero. The estimator will also be globally exponentially stable.

Proof: Consider the positive-definite Lyapunov function candidate

$$V = \frac{1}{2}\phi'^T K_O\,\phi',$$

which satisfies the following bounds

$$\lambda_{\min}(K_O)\|\phi'\|^2 \le V \le \lambda_{\max}(K_O)\|\phi'\|^2. \tag{10}$$

Differentiating V with respect to time along the trajectories of (9) yields

$$\begin{aligned}\dot{V} &= \phi'^T K_O A'_\theta\dot{\theta} + \phi'^T K_O A'_\psi\dot{\hat{\psi}}\\ &= \phi'^T K_O A'_\theta\dot{\theta} - \phi'^T K_O A'_\psi\big(A'^T_\psi K_O\phi' + A'^{-1}_\psi A'_\theta\dot{\theta}\big)\\ &= -\phi'^T K_O A'_\psi A'^T_\psi K_O\phi'\\ &\le -\lambda_{\min}(A'^T_\psi A'_\psi)\lambda^2_{\min}(K_O)\|\phi'\|^2. \end{aligned}\tag{11}$$

Therefore, based on the Lyapunov stability theory for non-autonomous system [12, p. 138], it can be inferred from (10) and (11) that $\phi'(\hat{\psi},\theta) = 0$ must be a globally uniformly asymptotically stable equilibrium point of the system (9). As $\phi' \to 0$ when $t \to \infty$, $\hat{\psi} \to \psi$ and $\phi \to 0$. Furthermore, because the bounding functions of V and $\dot{V}$ are of the form $a\|\phi'\|^b$ where a and b are strictly positive constants, the system is also globally exponentially stable [12, p. 140]. ■

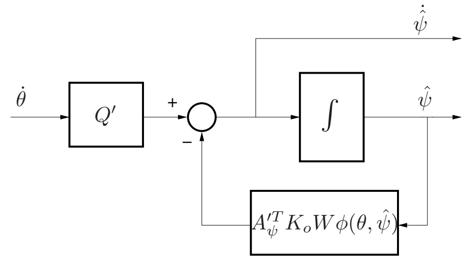

Fig. 2. Estimating the states of the passive joints

Apparently, the estimator (9) is similar to the closed-loop inverse kinematics (CLIK) scheme [13], [14], where the inverse kinematics problem is solved by reformulating it in terms of the convergence of an equivalent feedback control system. CLIK ensures that the constraint error remains inside a small ball. Our estimator, however, *eliminates* the constraint error.

III. Reconfiguration Control

During any reconfiguration maneuver, the robot will have some unactuated joints. Then, the main challenge will be to develop a dedicated controller for reconfiguration.

A. The Projection Method

Consider that the reconfigurable robot forms a closed chain with some of its passive joints unlocked. The dynamics equations of the constrained mechanical system can be derived as

$$M\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau - \tau^c \tag{12}$$

subject to $\phi(q) = 0$, where $\tau \in \mathbb{R}^{n+2m}$ is the vector of generalized forces; τ^c represents the constraint force; $q \in \mathbb{R}^{n+2m}$ contains the active joints together with the released passive joints; $M(q) \in \mathbb{R}^{(n+2m)\times(n+2m)}$ is the inertia matrix; $C(q,\dot{q}) \in \mathbb{R}^{(n+2m)\times(n+2m)}$ contains the Coriolis and centrifugal terms; and $g(q) \in \mathbb{R}^{n+2m}$ is the gravity torque.

Now, given the constraint Jacobian matrix $A \in \mathbb{R}^{r\times(n+2m)}$, we can uniquely define *symmetric* matrix P [15] given below as its *null-space orthogonal projector*:

$$P \triangleq I - A^+A \tag{13}$$

where I is the $(n+2m)\times(n+2m)$ identity matrix. Because P is an orthogonal projection onto the null-space of the Jacobian—a.k.a. the *tangent space*—any vector in the null-space of A is projected onto itself, whereas any vector perpendicular to the tangent space lies in the null-space of P. The vector of augmented generalized coordinates $\dot{q}$ belongs to the former group, as $A\dot{q} = 0$, and the vector of constraint generalized forces τ^c belongs to the latter, as $\forall\, \dot{q} \in \mathcal{R}(P) = \mathcal{N} \triangleq \mathcal{N}(A),\ \dot{q}^T\tau^c \equiv 0$. In other words, these two relations hold:

$$P\dot{q} = P^T\dot{q} = \dot{q}, \quad \text{and} \quad P\tau^c = 0. \tag{14}$$

Hence, premultiplying (12) by P, one can eliminate τ^c from the set of equations:

$$PM\ddot{q} = P\big(\tau - C(q,\dot{q})\dot{q} - g(q)\big). \tag{15}$$

Moreover, the generalized force can be decomposed into two components denoted by subscripts $\|$ and $\perp$, lying in the orthogonal subspaces the tangent space $\mathcal{N}$ and the null-space $\mathcal{N}^\perp$ of P, respectively:

$$\tau = \tau_\| + \tau_\perp. \tag{16}$$

Because $\tau_\perp \in \mathcal{N}^\perp$ and the constrained motion occurs in $\mathcal{N}$, by definition, this component of the actuation generalized forces does not contribute to the motion of the system [16].

B. The Control Law

The control objective is to regulate the passive joints of the system to their desired values ψ_d. The number of independent generalized coordinates of the system is $d = n + 2m - r$, the DOF of the system. This means that one can control the constrained mechanical system by only controlling an independent set $\chi(q) \in \mathbb{R}^d$ of the generalized coordinates.

Assumption 2: Assume that the elements of the passive joint vector ψ be a set of independent variables.
Then, $\chi(q)$ can be selected as

$$\chi \triangleq \begin{bmatrix} \psi \\ \eta(\theta) \end{bmatrix},$$

where the elements of vector $\eta(\theta) \in \mathbb{R}^{n-r}$ can be any independent functions of θ. Therefore, the addition of the auxiliary variables η makes χ a complete set of independent variables. Clearly, in the case that the number of EE constraints is equal to that of active joints, i.e., $n = r$, the auxiliary variable η vanishes and hence $\chi = \psi$. The time derivative of χ, which constitutes an independent set of generalized velocities, can then be obtained from

$$\dot{\chi} \equiv D\dot{q} = DP\dot{q} \tag{17}$$

where

$$D \triangleq \frac{\partial \chi}{\partial q} = \begin{bmatrix} 0_{2m\times n} & I_{2m\times 2m} \\ \partial\eta/\partial\theta & 0_{(n-k)\times 2m} \end{bmatrix}$$

is the Jacobian of the independent variables.

Now, let us consider the following control law:

$$\tau_\| = -PD^T\big(K_D\dot{\chi} + K_P(\chi - \chi_\mathrm{d})\big) + Pg(q) \tag{18}$$

where K_D and K_P are $d\times d$, positive-definite feedback gains, and vector $\chi_\mathrm{d}^T = \begin{bmatrix}\psi_\mathrm{d}^T & \eta_\mathrm{d}^T\end{bmatrix}$ contains the desired variables.

Theorem 1: The constrained mechanical system (12) under the control law (18) asymptotically converges to the desired position χ_d.

Proof: Substituting control law (18) in the dynamics equation (15), we obtain

$$PM\ddot{q} = -PC(q,\dot{q})\dot{q} - PD^TK_D\dot{\chi} - PD^TK_Pe, \tag{19}$$

where $e = \chi - \chi_\mathrm{d}$ is the position error. Now consider the following candidate Lyapunov function:

$$V = \frac{1}{2}\dot{q}^TM\dot{q} + \frac{1}{2}e^TK_Pe, \quad \forall\, \dot{q} \in \mathcal{N}. \tag{20}$$

Then, using (17) and the first of (14), one can compute the time-derivative of the above function along the solution of (19):

$$\begin{aligned}\dot{V} =& \frac{1}{2}\dot{q}^T\dot{M}\dot{q} + \dot{q}^TM\ddot{q} + \dot{\chi}^TK_Pe \\ =& \frac{1}{2}\dot{q}^T\dot{M}\dot{q} + \dot{q}^TPM\ddot{q} + \dot{\chi}^TK_Pe \\ =& \dot{q}^T(\frac{1}{2}\dot{M} - C)\dot{q} - \dot{\chi}^TK_D\dot{\chi}\end{aligned}$$

However, since $\dot{M} - 2C$ is a skew-symmetric matrix, we will have

$$\dot{V} = -\dot{\chi}^TK_D\dot{\chi} \le 0 \tag{21}$$

which is negative-semidefinite. Clearly, we have $\dot{V} = 0$ only if $\dot{\chi} = 0$, namely, $\dot{q} = 0$, in which case we can find the largest invariant set with respect to system (19) as

$$\Omega = \{\chi, \dot{\chi} : \dot{\chi} = 0, \quad PD^TK_p(\chi - \chi_\mathrm{d}) = 0\} \tag{22}$$

On the other hand, from (17), one can see that DP—and thus its transpose PD^T—must be a full-rank matrix as $\dot{\chi}$ are selected to be a complete set of independent generalized velocities. Therefore, the vector equation inside (22) can only hold if $e = \chi - \chi_\mathrm{d}$ vanishes. Then, $\Omega = \{\chi = \chi_\mathrm{d}, \dot{\chi} = 0\}$ is the largest invariant set which satisfies $V = 0$. Therefore, according to LaSalle's Global Invariant Set Theorem [12, p. 115], the solution of (19) asymptotically converges to the invariant set Ω. Consequently, as the time progresses, χ and thus ψ asymptotically approach their desired values χ_d and ψ_d, respectively. ■

C. Constrained Systems with Passive Joints

Because the passive joints are unactuated, the vector of the generalized forces should contain as many zeros as twice the number of the released passive joints:

$$\tau \equiv \tau_\| + \tau_\perp = \begin{bmatrix}\tau_\mathrm{act} \\ 0_{2m}\end{bmatrix} \tag{23}$$

where vector $\tau_\mathrm{act} \in \mathbb{R}^n$ represents the actuation torque, applied at the active joints. The problem here is that there is no guarantee that $\tau_\|$ is directly realizable. Therefore, we will have to find the $\tau_\perp$ that can be added to the control torque to produce an actuation torque in the form of (23). It should be noted that this will not change the dynamics behavior of the system because $\tau_\perp$ lies in the null-space of P.

Premultiplying both sides of (23) by P, we arrive at

$$\tau_\| = \begin{bmatrix}P_1 & P_2\end{bmatrix}\begin{bmatrix}\tau_\mathrm{act} \\ 0_{2m}\end{bmatrix} = P_1\tau_\mathrm{act}, \tag{24}$$

in which the projection matrix has been partitioned into submatrices $P_1 \in \mathbb{R}^{(n+2m)\times n}$ and $P_2 \in \mathbb{R}^{(n+2m)\times 2m}$. Given $\tau_{\|}$, the above equation will have at least one solution for $\tau_{\rm act}$ if

$$\mathcal{N} \subseteq \mathcal{R}(P_1). \tag{25}$$

In that case, there is a $\tau_{\rm act}$ that can produce the generalized torque control $\tau_{\|}$. The minimum-norm solution, i.e., $\|\tau_{\rm act}\| \rightarrow \min$, can be obtained using the pseudo-inverse of P_1:

$$\tau_{\rm act} = P_1^{+}\tau_{\|}. \tag{26}$$

Finally, substituting $\tau_{\|}$ from (18) into (26), we can derive the motor-torque control law as

$$\tau_{\rm act} = -P_1^{+}PD^T\big(K_D\dot{\chi} + K_P(\chi - \chi_{\rm d})\big) + P_1^{+}Pg(q). \tag{27}$$

Remark 1: The two submatrices P_1 and P_2 of the projection matrix P can be used to select the $\dot{\theta}$ and $\dot{\psi}$ parts of $\dot{q}$:

$$\dot{\theta} = P_1^T\dot{q}, \qquad \dot{\psi} = P_2^T\dot{q} \tag{28}$$

Remark 2: From the second of (28), it is clear that Assumption 2, i.e., the requirement for the entries of $\dot{\psi}$ to be independent, implies that P_2 should be a full-rank matrix.

D. Kinematic Conditions for Controllability

1) Equivalent Conditions: Condition (25) may seem too restrictive or difficult to satisfy, especially that one cannot easily manipulate either of the two subspaces involved to satisfy the condition. However, this concern is a nonissue. In fact, we can show that, if Assumption 1 holds, (25) is automatically satisfied. In other words, if the values of the passive joints can be uniquely determined from those of the active joints, then they can also be changed to their desired values.

Proposition 2: If the Jacobian matrix A_ψ is full-rank, then

i) there is no nonzero vector that lies in both $\mathcal{N}$ and $\mathcal{N}(P_1^T)$:

$$\mathcal{N}(P_1^T) \cap \mathcal{N} = \emptyset \tag{29}$$

ii) the range of P_1 is the same as the null-space of the constraint Jacobian:

$$\mathcal{R}(P_1) = \mathcal{N}. \tag{30}$$

Proof: We prove the first part of the proposition by contradiction. To this end, let us consider a vector $\xi \neq 0$ that lies in both $\mathcal{N}$ and $\mathcal{N}(P_1^T)$. Then, by definition, ξ must satisfy both

$$A\xi = 0 \quad \text{and} \quad P_1^T\xi = 0. \tag{31}$$

The first relation is the same as (2); as such, one can divide ξ into two sub-arrays $u \in \mathbb{R}^n$ and $v \in \mathbb{R}^{2m}$—corresponding to $\dot{\theta}$ and $\dot{\psi}$, respectively—such that $\xi^T = \begin{bmatrix}u^T & v^T\end{bmatrix}$. Moreover, if A_ψ is full-rank, one can compute v from u using a relation similar to (6): $v = Qu$. Then, comparing the second of (31) with (28), we can see that

$$u = P_1^T\xi = 0 \Rightarrow v = Qu = 0 \quad \Rightarrow \quad \xi \equiv \begin{bmatrix}u^T & v^T\end{bmatrix}^T = 0,$$

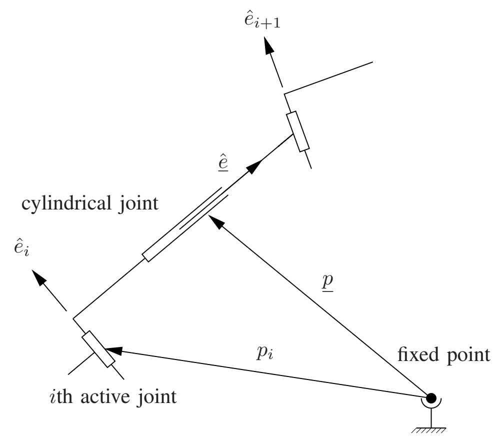

Fig. 3. Axes of the manipulator joints

which is a contradiction, i.e., the only vector ξ that satisfies (31) is the zero vector, the trivial solution. That completes the proof of the first part of the proposition.

For the second part, we notice that (29) amounts to

$$\mathcal{N} \subseteq \mathcal{N}^{\perp}(P_1^T). \tag{32}$$

To relate the above relation to the range of P_1, we resort to the fundamental theorem of linear operator transformation [15], which states that the range of a linear operator is the same as the null-space orthogonal of its transpose. Then,

$$\mathcal{R}(P_1) = \mathcal{N}^{\perp}(P_1^T) \tag{33}$$

which combined with (32) results in $\mathcal{N} \subseteq \mathcal{R}(P_1)$. However, $\mathcal{R}(P_1)$ is evidently a subset of $\mathcal{R}(P) \equiv \mathcal{N}$. Hence, we must have

$$\mathcal{N} \subseteq \mathcal{R}(P_1) \subseteq \mathcal{N} \quad \Rightarrow \quad \mathcal{R}(P_1) = \mathcal{N},$$

which completes the proof. ∎

The results of the above development can be briefly stated as follows:

Corollary 1: Let both Assumptions 1 and 2 hold. Then,

i) the states of the passive joints can be uniquely obtained from those of the active joints, e.g., equations (6) or (9), and

ii) the torque-control law (27) forces the actual positions of the passive joints, ψ, and the auxiliary variable, η, asymptotically converges to their desired values $\psi_{\rm d}$ and $\eta_{\rm d}$, while demanding minimum actuation force.

2) Singularity Analysis: The condition that A_ψ should remain non-singular during the configuration change is critical for both observability and controllability of the passive joints. Therefore, the robot should properly be positioned, so that the initial and final configurations are not in the vicinity of singularities. For the general case of a reconfigurable robot, it is difficult to identify all postures that render the constraint Jacobian singular. Nevertheless, we can find the singular postures for a simple, yet important case, namely, when only one cylindrical joint is released, and all translational motions of the EE is constrained using a spherical constraint, as shown in Fig. 3. In that case, the constraint Jacobian A

TABLE I
PARAMETERS OF THE RECONFIGURABLE ROBOT

Link 1 & 2	Link 3
$d = 0.2$ m	-
$1 < L < 2$ m	$L = 0.7$ m
$-180 < \alpha^{\dagger} < 180$ deg	$\alpha^{\dagger} = 0$ deg
$\rho_x^{\ddagger} = 0.5$ m	$\rho_x = 0.15$ m
$\rho_y^{\ddagger} = \rho_y^{\ddagger} = 0$	$\rho_y = \rho_z = 0$
$m^{\ddagger} = 1$ kg	$m = 0.3$ kg
$I_{xx}^{\ddagger} = 0.0025$ kgm^2	$I_{xx} = 0.00075$ kgm^2
$I_{yy}^{\ddagger} = I_{zz}^{\ddagger} = 0.0833$ kgm^2	$I_{yy} = I_{zz} = 0.0022$ kgm^2

† Twist angle
‡ Quantities associated with each segment of cylindrical links

is of this form:

$$A = [\underbrace{p_1 \times \hat{e}_1 \quad \cdots \quad p_n \times \hat{e}_n}_{A_\theta} \quad \underbrace{\underline{\hat{e}} \quad \underline{p} \times \underline{\hat{e}}}_{A_\psi}] \tag{34}$$

where $\hat{e}_i$'s are unit vectors along the active-joint axes, and p_i's are the vectors connecting the fixed point to the joints; similarly, $\underline{e}$ and $\underline{p}$ represent the unit and the position vectors associated with the passive joint. The two submatrices A_θ and A_ψ are essentially blocks of the Jacobian matrix of a serial manipulator pertaining to the manipulator translational motion. A detailed discussion on the structure of the Jacobian matrix of serial robots can be found in [17], among others.

It is apparent from (34) that the two columns of A_ψ will be independent unless $\underline{p} \times \underline{\hat{e}}$ vanishes.

Remark 3: For the case of the constrained robot where $m = 1$, $n = 3$, and only the translational motions of the EE are constrained (e.g., Fig.1), Assumption 1 holds if the fixed point or the robot EE does not lie on the axis of the released cylindrical joint.

IV. SIMULATION

In this section, we resort to a simulation to demonstrate that the proposed controller can indeed realize the configuration change of a robot as depicted in Fig. 1. The robot has three active joints and two lockable cylindrical joints, of which only one is released at a time. The kinematic and inertial parameters of the robot used in the simulation are given in Table I, and the controller gains are set to $K_p = \mathrm{diag}([5\ 0.8])$ and $K_d = \mathrm{diag}([6\ 1])$. The configuration change takes place in two stages:

i) the first link is twisted -90 deg and its length is increased by 0.5 m, as shown in Figs. 1(b)–1(e), and
ii) the second link is twisted by 90 deg and its length is increased by 0.5 m, as shown in Figs. 1(f)–1(g).

The two above stages can be described by

$$\begin{matrix} 1^{\text{st}}\ \text{CJ} \\ \\ 2^{\text{nd}}\text{CJ} \end{matrix} \begin{bmatrix} 1.1\ \text{m} \\ 0\ \text{deg} \\ 1.1\ \text{m} \\ 90\ \text{deg} \end{bmatrix} \xrightarrow[\text{stage}]{1^{\text{st}}} \begin{bmatrix} 1.6\ \text{m} \\ -90\ \text{deg} \\ 1.1\ \text{m} \\ 90\ \text{deg} \end{bmatrix} \xrightarrow[\text{stage}]{2^{\text{nd}}} \begin{bmatrix} 1.6\ \text{m} \\ -90\ \text{deg} \\ 1.5\ \text{m} \\ 180\ \text{deg} \end{bmatrix}.$$

Therefore, the desired values of the passive joints are set to

$$\psi_{\mathrm{d}_1} = \begin{bmatrix} 1.6\ \text{m} \\ -90\ \text{deg} \end{bmatrix} \quad \text{and} \quad \psi_{\mathrm{d}_2} = \begin{bmatrix} 1.6\ \text{m} \\ 180\ \text{deg} \end{bmatrix}.$$

In free space, when all cylindrical joints are locked, a conventional controller drives the active joints to achieve the desired values $\theta_{\mathrm{d}} = \begin{bmatrix} 26.32 & -35.76 & 16.60 \end{bmatrix}^T$ deg in order to grasp a point at $r_0 = \begin{bmatrix} 2.7 & 0.0 & 0.4 \end{bmatrix}^T$ m. It turns out that forming a closed chain at this point allows both configuration changes to be performed consecutively without running into any singularity. Also, assume that the function $r(\theta, \psi) \in \mathbb{R}^3$ represents the position of the robot EE. Then, the constraint can be expressed by

$$\phi(\theta, \psi) = r(\theta, \psi) - r_0 = 0$$

The projection matrix required to implement the controller is calculated from the above kinematic function.

The time histories of the passive joints, active joints, joint velocities, and joint torques during the configuration change are illustrated in Figs. 4 and 5. Note that the transition from the first stage of the reconfiguration to the second one takes place at $t = 10$ sec. As can be seen from the time histories of the link lengths and the twist angles in Figs. 4(a) and 4(b), the controllers was able to reconfigure the robot as was intended, and the designated kinematic parameters are changed to their desired values. The robot reconfiguration procedure is illustrated in Figs. 1(b)–1(g) at $t = 0, 0.8, 1.8, 10, 10.8, 20$ seconds, respectively; Figs. 1(a) and 1(h) show the manipulator before and after its reconfiguration, respectively.

V. CONCLUSION

The calibration and reconfiguration control of a new class of reconfigurable robots with passive cylindrical joints was discussed. These robots have a simpler design compared to the conventional reconfigurable robots, which are based on modular joints. Moreover, since one does not need to connect or disconnect the joints and the links, the configuration change can be performed reliably and autonomously.

For reconfiguration, the end-effector of the robot is constrained and then one or more passive joints are released. The Lyapunov-based controller then drives the active joints so that the released passive joints reach their desired value. The controller requires only the models of the constraint kinematics and the links gravity, thus making the controller implementation easy. The conditions for observability and controllability of the passive joints were also derived. We proved that these conditions would require that Jacobian of the constraint with respect to the passive joints be full-rank. Finally, some simulation results for a system with three active revolute joints and two passive cylindrical joints were presented.

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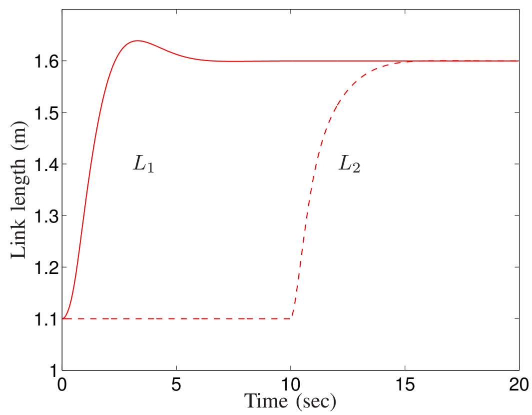
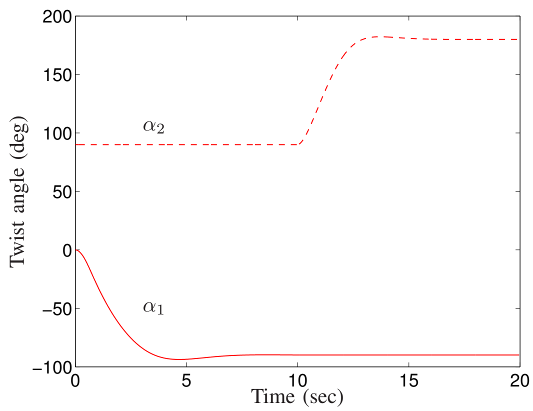
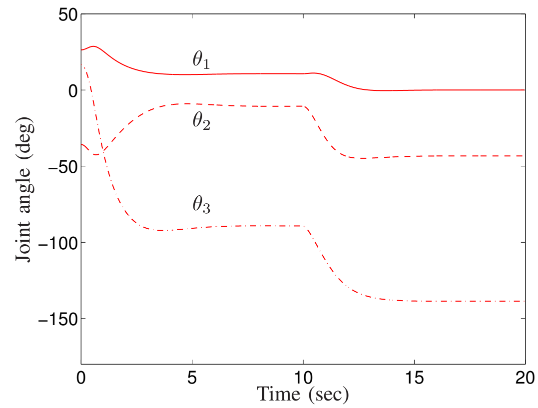


(a)

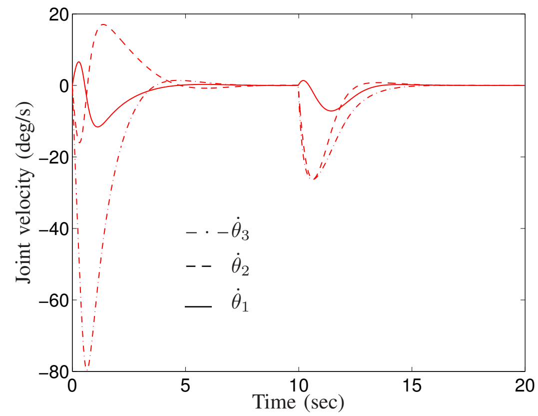


(b)

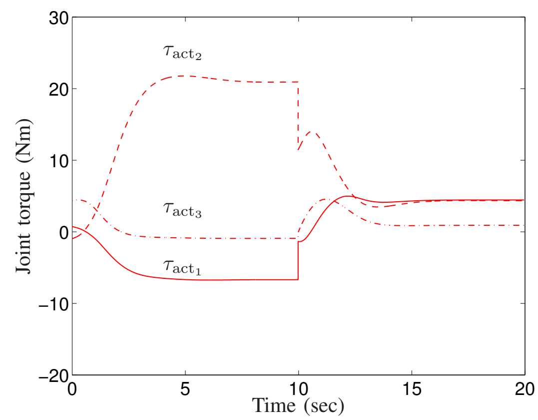
Fig. 4. Trajectories of quantities associated with the passive joints during the configuration change



(a)



(b)



(c)

Fig. 5. Trajectories of quantities associated with the active joint during the configuration change

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