2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

MIMO State Feedback Controller for a Flexible Joint Robot with Strong Joint Coupling

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***Abstract*— The paper describes the modeling and control of a robot with flexible joints (the DLR medical robot), which has strong mechanical couplings between pairs of joints realized with a differential gear-box. Because of this coupling, controllers developed before for the DLR light-weight robots cannot be directly applied. The previous control approach is extended in order to allow a multi-input-multi-output (MIMO) design for the strongly coupled joints. Asymptotic stability is shown for the MIMO controller. Finally, experimental results with the DLR medical robot are presented.**

***Index Terms*— flexible joint robots, MIMO design, modal decoupling based control**

I. Introduction

Joint flexibility becomes significant in most robot designs which are optimized for a light-weigh and for a high load/weight ratio. These features are essential for special robotics fieds such as medical robotics. In cooperation with industrial partners, a new light-weight robot (Fig.1) has been developed at DLR, specially customized for medical robotics applications [3]. The design is inspired from the DLR light-weight robot (LWRIII) design [16]. The DLR medical robot is a redundant robot with seven degrees of freedom and the joints are endowed, in addition to the motor position sensors, with torque sensors and link side position sensors which are mounted after the gear-box. In order to obtain an anthropomorphic robot design, wrist, elbow and shoulder are realized as coupled joints using a differential gear after a harmonic drive gear unit for each motor. In this way, the torque of both motors can be used in the principal directions of motion (e.g., the vertical plane), permitting there the increase of available torque for a given motor size and thus the reduction of total weight. However, the strong coupling between the axes do not permit independent controller design for each joint. The paper extends previous methods in order to enable and justify a simple MIMO design for these coupled joints. The method applies to an arbitrary number of coupled joints.

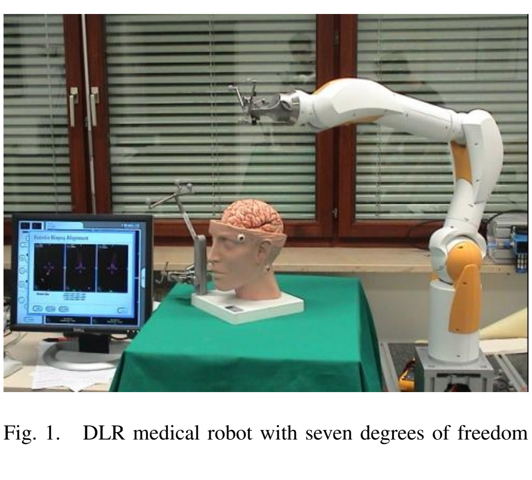

Fig. 1. DLR medical robot with seven degrees of freedom

The topics of flexible joint robot control has be treated extensively in literature. Some methods, such as feedback linearization [14], back-stepping [11], [12] or passivity based adaptive control [11], [6] belong to the standard reference and provide control solutions which apply for both regulation and tracking. However, due to their complexity and the requirement of high derivatives of the link side position, they have been applied so far only to small experimental systems with few degrees of freedom. Singular perturbation controllers [5], [2], [15] can be easily implemented, but are valid only for limited elasticity and lead to some limitations of the overall control bandwidth due to their cascaded nature. A very simple controller was proposed by [4], who showed that a PD-controller based on motor position and with additional gravity compensation at the desired position is globally asymptotically stable. Practically it turns out however that for robots with considerable elasticity such as the DLR medical robot, only limited performance can be achieved if the feedback is restricted only to motor state variables, without using link side information such as link position or link torque.

In [1] a regulation controller with full state feedback (motor position, link side torque, as well as their derivatives) and gravity and friction compensation was introduced. The asymptotic stability can be shown based on Lyapunov theory. This controller was already successfully applied to the DLR light-weight robots.

Corresponding to this method we introduce a MIMO controller with full state feedback and gravity compensation for the DLR medical robot, in order to deal with the strong joint coupling. The system stability is derived in analogy to [4] and [1] with a Lyapunov approach.

The content of this paper is structured as follows. In Sec.2, the model of the DLR medical robot with its coupled joints is described. Next, the design of a MIMO controller with gravity compensation based on modal decomposition is introduced in Sec. 3. In Sec. 4, the stability of the system is analyzed. Finally, in Sec. 5, identification results as well as experimental results with the MIMO controller are presented and discussed.

II. Modelling of the medical robot

The DLR medical robot has seven degrees of freedom and its kinematics was specially designed for the workspace of a surgical application. While the first joint is very similar to a LWRIII joint, a coupled design was chosen for the joints 2-3, 4-5, 6-7. As a consequence of the coupling, a movement of one robot joint has to be realized by the coordinated movement of two actuators. A schematic view for the coupling gears is shown in Fig. 2. The differential gear-box is composed of three conical gears. Ignoring the elasticity, the effect of the gear-box can be described by the following transformations for the positions

$$\theta_i = T_i \theta_{m,i} \ . \tag{1}$$

For the torques, one has:

$$\tau_{m,i} = T_i^T \tau_i \tag{2}$$

with

$$T_i = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \ . \tag{3}$$

The motor position is denoted herein by $\theta_{m,i}$, while θ_i is the same position expressed in *link coordinates*. It is important to note the difference between the motor position expressed in link coordinates θ_i and the link side position, which will be denoted by q_i. While θ_i represents the same system state as $\theta_{m,i}$ only written in another coordinate system, the second value q_i is a different state variable, representing the position of the link after the joint elasticity. It can be also expressed in motor or link coordinates. The coordinate system will always be denoted by a subscript, a missing subscript denotes link coordinates. Accordingly, τ_i and $\tau_{m,i}$ are the joint torques expressed in link and motor coordinates respectively. While the elasticity of joint 1 stems merely from the harmonic drive gear, for the coupled joint one has the additional elasticity of the differential gear.

For the modelling of the entire medical robot, the following flexible joint model is used:

$$\mathbf{u}_m = \mathbf{J_m}\ddot{\theta}_\mathbf{m} + \mathbf{T^T}(\tau + \mathbf{DK^{-1}}\dot{\tau}) + \tau_{\mathbf{fric,m}} \tag{4}$$

$$\tau + \mathbf{DK^{-1}}\dot{\tau} = \mathbf{M(q)\ddot{q}} + \mathbf{C(q,\dot{q})\dot{q}} + \mathbf{g(q)} \tag{5}$$

$$\tau = \mathbf{K}(\mathbf{T}\theta_\mathbf{m} - \mathbf{q}) \tag{6}$$

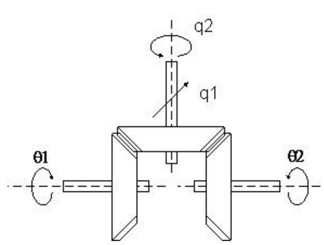

Fig. 2. Structure of the differential gear-box.

The transformation matrix for the entire robot is expressed as

$$T = \begin{bmatrix} 1 & \dots & & 0 \\ & T_i & & \\ \vdots & & T_i & \vdots \\ 0 & \dots & & T_i \end{bmatrix} \in R^{7x7}, \forall T_i \in R^{2x2} \tag{7}$$

In these equations all quantities are expressed in those coordinates, in which they are measured or determined by identification. Therefore the first, motor side equation is expressed mainly in motor coordinates, while the link side dynamics is written in link coordinates. The joint torque vector $\tau \in \mathbf{R^n}$ is defined by the linear relation $\tau = \mathbf{K}(\mathbf{T}\theta_\mathbf{m} - \mathbf{q}) = \mathbf{K}(\theta - \mathbf{q})$. The matrices $\mathbf{K} \in \mathbf{R^{nxn}}$ and $\mathbf{D} \in \mathbf{R^{nxn}}$ are positive definite matrices which in a quite general form have a block diagonal structure for the medical robot.

$$P = \begin{bmatrix} P_1 & \dots & & 0 \\ & P_2 & & \\ \vdots & & P_4 & \vdots \\ 0 & \dots & & P_6 \end{bmatrix} \tag{8}$$

$$P_1 \in R, P_i \in R^{2x2} \ \ with \ i = \{2,4,6\}, \ P = \{K, D\}$$

$\mathbf{J_m}$ is a diagonal matrix containing the motor inertias. Further on, $\mathbf{M(q)}$, $\mathbf{C(q,\dot{q})}$ and $\mathbf{g(q)}$ are the mass matrix, the vector of Coriolis and centrifugal terms and the gravity term of the rigid robot dynamics. The motor torque vector $\mathbf{u_m}$ is the input quantity for the controller. $\tau_{\mathbf{fric,m}}$ is the motor side friction vector.

The following properties of the robot model will be used in this paper:

E.1 The mass matrix is symmetric and positive definite

$$M(q) = M(q)^T > 0 \quad \forall q \in R^n \tag{9}$$

and the eigenvalues λ satisfy:

$$\lambda_m \leq \ \|M(q)\| \ \leq \lambda_M \tag{10}$$

E.2 The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric and satisfies:

$$x^T(\dot{M}(q) - 2C(q,\dot{q}))x = 0 \quad \forall x, q, \dot{q} \in R^n \tag{11}$$

E.3 The gravity torque $g(q)$ is given as the gradient of a potential function $U_g(q)$ so that $g(q) = \partial U_q(q)/\partial q$ and there exists a real number $\alpha > 0$, such that:

$$\|g(q_1) - g(q_2)\| \leq \alpha\|q_1 - q_2\| \ \ , \forall \ q_1, q_2 \in R^n \tag{12}$$

holds, implying

$$\|U_g(q_d) - U_g(q) + (q - q_d)^T g(q_d)\| \leq \frac{1}{2}\alpha\|q - q_d\|^2 \tag{13}$$

E.4 The friction torque $\tau_{fric,m}(\dot{\theta})$ is expressed by the following relation [1], [13]

$$\tau_{fric,m} = \begin{cases} Min[u_m, (f_c + \mu \mid \tau_m \mid) sign(u_m)], \\ \qquad if \ \mid \dot{\theta}_m \mid \leq \varepsilon \\ (f_c + \mu \mid \tau_m \mid) sign(\dot{\theta}_m) + f_v \dot{\theta}_m, \\ \qquad if \ \mid \dot{\theta}_m \mid > \varepsilon \end{cases} \tag{14}$$

Herein f_c is the Coulomb friction and f_v the viscous friction component. μ is the coefficient of the load dependent friction. ε is a small constant.

For compensation of the friction, a disturbance observe is used, which is however not topic of this paper. Exact friction compensation will be assumed in the following analysis. The analysis of the friction effects on the convergence can be found in [1].

III. Controller design

In this section, a MIMO controller for the coupled joints will be designed based on the modal analysis approach. This approach makes use of the following lemma regarding the double diagonalization of two matrices:

Lemma (Decomposition of symmetric matrices): Given a symmetric matrix A, and a symmetric, positive definite matrix B, there exists an invertible matrix Q, such that $A = QQ^T$ and $B = QCQ^T$ holds, with the matrix C being diagonal.

As known from modal analysis, a mechanical system of the form

$$f = M\ddot{x} + Kx \tag{15}$$

can be transformed using previous lemma into so called modal coordinates, in which the system is decoupled. The mass matrix $M = QM_QQ^T$ and the stiffness $K = QQ^T$ are p.d. and symmetric, with M_Q diagonal. The vector f represents a generalized force acting on the system and x is the state. In modal coordinates we have

$$f_Q = M_Q\ddot{x}_Q + x_Q \ . \tag{16}$$

with $x_Q = Q^Tx$ and $f_Q = Q^{-1}f$ A SISO design can now be done for each decoupled sub-system.

In order to apply the idea to the flexible joint robot, let us rewrite (4) in link coordinates.

$$u = J\ddot{\theta} + \tau + DK^{-1}\dot{\tau} \tag{17}$$

$$\begin{aligned} with \quad \theta &= T\theta_m && (18)\\ u &= T^{-T}(u_m - \tau_{fric,m}) && (19)\\ J &= T^{-T}J_mT^{-1} && (20) \end{aligned}$$

Notice that transformations of the type (20) are congruence transformations, preserving symmetry and positive definiteness of the matrices. In general, however, J is not diagonal any more.

Let us consider for a linear control design the linearized model of a coupled joint i around a worst case position:

$$\begin{aligned} u_i &= J_i\ddot{\theta}_i + \tau_i + D_iK_i^{-1}\dot{\tau}_i \\ \tau_i + D_iK_i^{-1}\dot{\tau}_i &= M_i\ddot{q}_i \end{aligned} \tag{21}$$

Again $K_i \in \mathbf{R}^{2x2}$, $D_i \in \mathbf{R}^{2x2}$, $J_i \in \mathbf{R}^{2x2}$ and $M_i \in \mathbf{R}^{2x2}$ are symmetric, positive definite matrices, with i={2,4,6}[1]. It follows that a coupled, 8th order systems with two inputs is obtained for a coupled joint. The coupling is given by the matrices M_i, K_i, D_i and J_i.

[1] i={2,4,6} for coupled joint {2-3}, {4-5}, {6-7}.

Notice at this point that the model contains four matrices, namely J_i, K_i, D_i, M_i, out of which only two can be diagonalized simultaneously, e.g, K_i and M_i. In order to be able to diagonalize the entire system, we first make the assumption $D_i = \lambda_D K_i$, with λ_D being a scalar, depending on material properties. As it will become clear in the stability analysis, stability is preserved however also for different p.d. matrix D_i, so the error related to this approximation may affect only the transient performance. Furthermore, using the following torque controller[2]:

$$u_i = J_i(\lambda_J K_i)^{-1}w_i + (I - J_i(\lambda_J K_i)^{-1})(\tau_i + D_iK_i^{-1}\dot{\tau}_i), \tag{22}$$

one can bring the motor inertia J_i to the form $\lambda_J K_i$ with λ_J being a scalar. The vector w_i is a new control input and $I \in R^{2x2}$ is the unit matrix. One obtains with this first controller the following system equations:

$$\begin{aligned} w_i &= \lambda_J K_i\ddot{\theta}_i + \tau_i + \lambda_D\dot{\tau}_i \\ \tau_i + \lambda_D\dot{\tau}_i &= M_i\ddot{q}_i \end{aligned} \tag{23}$$

Now it is possible to decouple the flexible joint using a modal transformation. One obtains for this worst case design constant control parameters for a linear controller. However, stability can be ensured for the complete nonlinear system which provides a good performance in the entire workspace. Following the double diagonalization lemma, there exists a matrix $Q_i \in R^{2x2}$, such that

$$\begin{cases} K_i = Q_iQ_i^T \\ M_i = Q_iM_{Q,i}Q_i^T \end{cases} \tag{24}$$

holds, with $M_{Q,i}$ p.d. and diagonal. By substituting (24) into (23) one obtains:

$$\begin{cases} w_{Q,i} = \lambda_J\ddot{\theta}_{Q,i} + (\theta_{Q,i} - q_{Q,i}) + \lambda_D(\dot{\theta}_{Q,i} - \dot{q}_{Q,i}) \\ (\theta_{Q,i} - q_{Q,i}) + \lambda_D(\dot{\theta}_{Q,i} - \dot{q}_{Q,i}) = M_{Q,i}\ddot{q}_{Q,i} \end{cases} \tag{25}$$

with

$$\begin{cases} \theta_{Q,i} = Q_i^T\theta_i \\ q_{Q,i} = Q_i^Tq_i \\ w_{Q,i} = Q_i^{-1}w_i \end{cases} \tag{26}$$

The system (25) is decoupled, since the matrix $M_{Q,i}$ is diagonal. For the decoupled subsystems the controller is chosen in the following form:

$$\begin{aligned} w_{Q,i} = K_{PQ,i}\widetilde{\theta}_{Q,i} - K_{QD,i}\dot{\theta}_{Q,i} - K_{QT,i}(\theta_{Q,i} - q_{Q,i}) \\ -K_{QS,i}(\dot{\theta}_{Q,i} - \dot{q}_{Q,i}) \end{aligned} \tag{27}$$

with $\widetilde{\theta}_{Q,i} = \theta_{Qd,i} - \theta_{Q,i}$. The matrices $K_{PQ,i} \in R^{2x2}, K_{DQ,i} \in R^{2x2}, K_{TQ,i} \in R^{2x2}, K_{SQ,i} \in R^{2x2}$ are diagonal. From (26) one obtains by transforming back into link coordinates:

$$w_i = K_{P,i}\widetilde{\theta}_i - K_{D,i}\dot{\theta}_i - K_{T,i}K_i^{-1}\tau_i - K_{S,i}K_i^{-1}\dot{\tau}_i \tag{28}$$

with

$$\begin{cases} K_{P,i} = Q_iK_{PQ,i}Q_i^T \\ K_{D,i} = Q_iK_{DQ,i}Q_i^T \\ K_{T,i} = Q_iK_{TQ,i}Q_i^T \\ K_{S,i} = Q_iK_{SQ,i}Q_i^T \end{cases} \tag{29}$$

[2] Torque feedback can generally be interpreted as scaling of the kinetic energy of the rotor [17].

All the involved matrices are now symmetric. This is required for the stability analysis in next section.
With the controller (22) applied to all joints (21) leads to the following new system equations for the entire medical robot:

$$w = \lambda_J K\ddot{\theta} + \tau + DK^{-1}\dot{\tau} \quad (30)$$
$$\tau + DK^{-1}\dot{\tau} = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) \quad (31)$$
$$\tau = K(\theta - q) \quad (32)$$

Finally, the complete controller for the robot has the following form in link coordinates:

$$\begin{aligned} w &= K_P\widetilde{\theta} - K_D\dot{\theta} - K_T K^{-1}\tau - K_S K^{-1}\dot{\tau} \\ &\quad +(K+K_T)K^{-1}g(q_d) \end{aligned} \quad (33)$$

with $\widetilde{\theta} = \theta_d - \theta$. The control gain matrices K_C are positive definite and symmetric

$$K_C = \begin{bmatrix} K_{C,1} & \dots & & 0 \\ & K_{C,2} & & \\ \vdots & & K_{C,4} & \vdots \\ 0 & \dots & & K_{C,6} \end{bmatrix} \quad (34)$$

$$K_{C,1} \in R,\ K_{C,i} \in R^{2x2}\ with\ i = \{2,4,6\}$$
$$C \in \{P,D,T,S\}$$

IV. Stability analysis

A. Choice of the Lyapunov function

Following lemma will be used for the analysis:
Lemma (positive definite symmetric Matrix): Given is a symmetric matrix A:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix} \quad (35)$$

such that every submatrix $\mathbf{A_{ij}}$ *is quadratic. Matrix A is positive definite, if* $\mathbf{A_{11}}$ *is positive definite and* $\mathbf{A_{22}} \geq \mathbf{A_{12}^T A_{11}^{-1} A_{12}}$ *holds.*

For proof of stability, we slightly rewrite the equations (30,31,32):

$$w = \lambda_J K\ddot{\theta} + K\Delta + D\dot{\Delta} \quad (36)$$
$$K\Delta + D\dot{\Delta} = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) \quad (37)$$

Herein Δ is used for $\Delta = \theta - q$. The controller (33) is now given by

$$\begin{aligned} w &= K_P\widetilde{\theta} - K_D\dot{\theta} - K_T\Delta - K_S\dot{\Delta} + \\ &\quad (K+K_T)K^{-1}g(q_d) \end{aligned} \quad (38)$$

For a given desired link position q_d the corresponding motor position θ_d is given by

$$\theta_d = q_d + K^{-1}g(q_d) \quad (39)$$

By choosing $\mathbf{P} = [\theta, \dot{\theta}, \mathbf{q}, \dot{\mathbf{q}}]^{\mathbf{T}}$ as a state vector of the system (36,37) with the controller (38), then an equilibrium point $\mathbf{P} = [\theta_\mathbf{o}, \mathbf{0}, \mathbf{q_o}, \mathbf{0}]^{\mathbf{T}}$ must satisfy the equilibrium equations:

$$\begin{cases} K_P(\theta_d - \theta_o) - (K+K_T)(\theta_o - q_o) \\ +(K+K_T)K^{-1}g(q_d) = 0 \\ K(\theta_o - q_o) = g(q_o) \end{cases} \quad (40)$$

Obviously, $P = P_d = [\theta_d, 0, q_d, 0]^T$ satisfies these equations. The following Lyapunov function candidate is chosen:

$$\begin{aligned} V(\theta,\dot{\theta},q,\dot{q}) &= \frac{1}{2}\dot{\theta}^T K(K+K_T)^{-1}\lambda_J K\dot{\theta} + \frac{1}{2}\dot{q}^T M(q)\dot{q} \\ &\quad + \frac{1}{2}(\widetilde{\theta} - \widetilde{q})^T K(\widetilde{\theta} - \widetilde{q}) \\ &\quad + \frac{1}{2}\widetilde{\theta}^T K(K+K_T)^{-1}K_P\widetilde{\theta} \\ &\quad + U_g(q) - U_g(q_d) + \widetilde{q}^T g(q_d) \end{aligned} \quad (41)$$

with $\widetilde{q} = q_d - q$. This function contains in addition to the motor and link side kinetic energy also the potential energy related to the gravitational vector and to the joint elasticity. Furthermore, the potential energy of the controller is considered.

B. Stability of the SISO-controller

If it is assumed that all matrices J, M, K, D for parameter design in Sec. III are diagonal in link coordinates, then these coordinates are the modal coordinates already, leading to diagonal gain matrices K_P, K_D, K_T, K_S. Then a SISO design can be done for all joints in link coordinates. The stability conditions [1] are given by

$$K_{P,i} > 0 \quad (42)$$
$$K_i + K_{Ti} > 0 \quad (43)$$
$$\alpha < K_i \quad (44)$$
$$\alpha < \frac{K_i K_{P,i}}{K_{P,i} + K_{T,i} + K_i} \quad (45)$$
$$K_{D,i} > \frac{(K_{S,i}K_i - K_{T,i})^2}{4K_i D_i (K_{T,i} + K_i)} \quad (46)$$

The values $K_{P,i}$, $K_{D,i}$, $K_{T,i}$, $K_{S,i}$, α, K_i and D_i are scalars with i=1..n. The equations (42), (43) are obviously fulfilled. Condition (44) requires that the joint stiffness is high enough for sustaining the robot in the gravity field when motors are fixed and is certainly fulfilled for any practically useful robot. Similarly, condition (45) requires that the stiffness of the controlled system can sustain the robot against gravity when a fixed desired position is commanded [1]. This is also generally fulfilled for a position controller. Finally, the last condition (46) requires enough controller damping, corresponding to the other controller and plant parameters.

C. Stability of the MIMO-controller

For the coupled joints we consider $K_{P,i}$, $K_{D,i}$, $K_{T,i}$, $K_{S,i} \in R^{2x2}$ with i={2,4,6}. In Sec. III it was shown that the matrices $K_{P,i}$, $K_{D,i}$, $K_{T,i}$, $K_{S,i}$ are positive definite and that there exist a matrix $Q_i \in R^{2x2}$ such that the conditions (29) are fulfilled and

$$\begin{cases} K_i = Q_i Q_i^T \\ D_i = \lambda_D K_i = \lambda_D Q_i Q_i^T \end{cases} \quad (47)$$

Furthermore, it follows for the coupled joint that

$$\begin{cases} K_i(K_i + K_{T,i})^{-1}\lambda_J K_i = \lambda_J Q_i (I + K_{TQ,i})^{-1} Q_i^T \\ K_i(K_i + K_{T,i})^{-1}K_{P,i} = Q_i (I + K_{TQ,i})^{-1} K_{PQ,i} Q_i^T \end{cases} \quad (48)$$

are symmetric and positive definite. From property (E.3) it follows:

$$\begin{aligned} V(\theta,\dot\theta,q,\dot q) \;\geq\;& \frac{1}{2}\dot\theta^T K(K+K_T)^{-1}\lambda_J K\dot\theta + \frac{1}{2}\dot q^T M(q)\dot q \\ &+\frac{1}{2}(\widetilde\theta-\widetilde q)^T K(\widetilde\theta-\widetilde q) \\ &+\frac{1}{2}\widetilde\theta^T K(K+K_T)^{-1}K_P\widetilde\theta - \frac{1}{2}\widetilde q^T\alpha\widetilde q \end{aligned} \tag{49}$$

Herein M(q) and $K(K+K_T)^{-1}\lambda_J K$ are positive definite from (E.1) and (48). In order to show that V is positive for $P \neq P_d$, one has to show the positive definiteness of following function:

$$\begin{aligned} V_1 \;=\;& \frac{1}{2}(\widetilde\theta-\widetilde q)^T K(\widetilde\theta-\widetilde q) \\ &+\frac{1}{2}\widetilde\theta^T K(K+K_T)^{-1}K_P\widetilde\theta - \frac{1}{2}\widetilde q^T\alpha\widetilde q > 0 \end{aligned} \tag{50}$$

For the DLR medical robot $\mathbf{V_1}$ can be divided into a sum of independent sub-functions for each simple and coupled joint:

$$V_1 = \sum_{i=1,2,4,6} V_{1i} > 0 \tag{51}$$

If (42,43,44,45) hold, then $\mathbf{V_{11}}$ is positive definite for the simple joint. For the coupled joints i with i={2,4,6} $\mathbf{V_{1i}}$ is positive definite, if the Hessian of $\mathbf{V_{1i}}$ is positive definite. The Hessian is given by

$$H(V_{1i}) = \frac{1}{2}\begin{bmatrix} K_i + K_i(K_i+K_{T,i})^{-1}K_{P,i} & K_i \\ -K_i & K_i-\alpha I \end{bmatrix} > 0 \tag{52}$$

Since $K_i + K_i(K_i+K_{T,i})^{-1}K_{P,i}$ are positive definite from (48), it follows that H($\mathbf{V_{1i}}$) is positive definite, if the condition

$$K_i - \alpha I > K_i[K_i + K_i(K_i+K_{T,i})^{-1}K_{P,i}]^{-1}K_i \tag{53}$$

or, equivalently,

$$\begin{aligned} \alpha I \;<\;& K_i(K_i+K_{T,i}+K_{P,i})^{-1}K_{P,i} \\ =\;& Q_i(I+K_{TQ,i}+K_{PQ,i})^{-1}K_{PQ,i}Q_i^T \end{aligned} \tag{54}$$

(See Appendix 1) is fulfilled.

The derivative of the Lyapunov functions along the system trajectory is:

$$\begin{aligned} \dot V \;=\;& \dot\theta^T K(K+K_T)^{-1}\lambda_J K\ddot\theta + \dot q^T M(q)\ddot q \\ &+\frac{1}{2}\dot q^T\dot M(q)\dot q + (\widetilde\theta-\widetilde q)^T K(-\dot\theta+\dot q) \\ &-\widetilde\theta^T K(K+K_T)^{-1}K_P\dot\theta \\ &+\dot q^T g(q) - \dot q^T g(q_d) \end{aligned} \tag{55}$$

With (36,37) and (38) substituted in (55), one obtains:

$$\begin{aligned} \dot V \;=\;& \dot\theta^T K(K+K_T)^{-1}[K_P\widetilde\theta - K_D\dot\theta - K_T\Delta - K_S\dot\Delta \\ &+(K+K_T)K^{-1}g(q_d) - K\Delta - D\dot\Delta] \\ &+\dot q^T[K\Delta + D\dot\Delta - C(q,\dot q)\dot q - g(q)] + \frac{1}{2}\dot q^T\dot M(q)\dot q \\ &-(\Delta_d-\Delta)^T K\dot\Delta - \widetilde\theta^T K(K+K_T)^{-1}K_P\dot\theta \\ &+\dot q^T g(q) - \dot q^T g(q_d) \end{aligned} \tag{56}$$

From (E.2) it follows:

$$\begin{aligned} \dot V \;=\;& -\dot\theta^T K(K+K_T)^{-1}K_D\dot\theta - \dot\theta^T K\Delta \\ &-\dot\theta^T K(K+K_T)^{-1}(K_S+D)\dot\Delta \\ &+\dot\theta^T g(q_d) + \dot q^T K\Delta + \dot q^T D\dot\Delta - \Delta_d^T K\dot\Delta \\ &+\Delta^T K\dot\Delta - \dot q^T g(q_d) \end{aligned} \tag{57}$$

With the choice (39) it follows $\Delta_d = K^{-1}g(q_d)$. By substituting in (57) it follows:

$$\begin{aligned} \dot V \;=\;& -\dot\theta^T K(K+K_T)^{-1}(K_D+K_S+D)\dot\theta - \dot q^T D\dot q \\ &+\dot q^T D\dot\theta + \dot\theta^T K(K+K_T)^{-1}(K_S+D)\dot q \end{aligned} \tag{58}$$

$\dot V$ can be decomposed in terms related to each subsystem.

$$\dot V = \sum_{i=1,2,3,4} \dot V_i < 0 \tag{59}$$

The condition for the negative definiteness of the term related to the first joint was derived in (46). In order to have negative definiteness of $\dot V_i$ with i={2,4,6} it is required that the Hessian H($-\dot V_i$) is positive definite.

$$H(-\dot V_i) = \begin{bmatrix} dv_{11} & dv_{12} \\ dv_{12} & dv_{22} \end{bmatrix} \tag{60}$$

with

$$\begin{cases} dv_{11} = K_i(K+K_{T,i})^{-1}(K_{D,i}+K_{S,i}+D_i) \\ dv_{12} = -\frac{1}{2}[K_i(K+K_{T,i})^{-1}(K_{S,i}+D_i)+D_i] \\ dv_{22} = D_i \end{cases} \tag{61}$$

or

$$\begin{cases} dv_{11} = Q_i(I+K_{QT,i})^{-1}(K_{QD,i}+K_{QS,i}+\lambda_D I)Q^T \\ dv_{12} = -\frac{1}{2}Q_i[(I+K_{QT,i})^{-1}(K_{QS,i}+\lambda_D I)+\lambda_D I]Q^T \\ dv_{22} = \lambda_D Q_i Q^T \end{cases} \tag{62}$$

Since matrix $D_i = \lambda_D K_i$ was assumed in (47). In order to ensure positive definiteness of $H(-\dot V_i)$ one needs

$$dv_{2,2} > dv_{1,2} * (dv_{1,1})^{-1} * dv_{1,2} \tag{63}$$

or, equivalently,

$$\begin{aligned} 4\lambda_D I \;>\;& [(I+K_{QT,i})^{-1}(K_{QS,i}+\lambda_D I)+\lambda_D I] \\ & (K_{QD,i}+K_{QS,i}+\lambda_D I)^{-1}[K_{QS,i}+2\lambda_D I+\lambda_D K_{QT,i}] \end{aligned} \tag{64}$$

It can be easily verified that the conditions (54) and (64) correspond to (45,46) when reduced to the scalar case, thus having similar interpretation. The asymptotic stability follows from the invariance principle of Krasovskii-LaSalle. The system converges to the largest invariant set contained in the subspace $P = [\theta, 0, q, 0]^T$. This set is given by

$$K_P\widetilde\theta - (K_T+K)(\theta-q) + (K+K^T)K^{-1}g(q_d) = 0 \tag{65}$$

$$K(\theta-q) = g(q) \tag{66}$$

With θ_d from (39) and θ from (66) substituted in (65), one obtains the equilibrium equations:

$$g(q_d) - g(q) = K[K_P+K_T+K]^{-1}K_P(q-q_d) \tag{67}$$

For $P \neq P_d$ and from (12) the following relations follow

$$\begin{aligned} \| g(q_d)-g(q) \| \;=\;& \| K[K_P+K_T+K]^{-1}K_P(q_d-q) \| \\ \leq\;& \alpha \| q_d - q \| \end{aligned} \tag{68}$$

TABLE I
PARAMETERS FOR THE SISO EXPERIMENT IN JOINT 4-5

	Achse 4	Achse 5	
J	0.2984	0.2984	Kgm^2
M	0.6807	0.0057	Kgm^2
$Max_{-}g(q)$	18.076	0.0	Nm
k	2976	2815	Nm/rad
d	3.89	1.0	sNm/rad
f_c	4.4459	3.3386	Nm
f_v	3.8197	3.1624	sNm/rad

Regarding (54) it follows that the equality is fulfilled only if $q = q_d$. Consequently, $P = P_d$, i.e. the point $[\theta_d, 0, q_d, 0]$ is globally asymptotically stable.

V. EXPERIMENTS

In this section two experiments are described. The first plots show the identification results for motor constant, the friction and the elasticity. Each coupled joint was identified separately on a joint testbed. The parameters for joint 4-5 in the SISO approximation are given in table I.

The results of the identification for the coupled joints 4-5 are shown by comparing the measurements with the simulation in Fig. 3 and Fig. 4. One can see that the desired current as well as the measured torques, motor positions and motor velocities fit well to the simulated values.

The figures 5 and 6 compare the results of the SISO and the MIMO controller. In Fig. 5 it can be seen that the torque of joint 5 has less noise and the torque of joint 4 has a faster response time for the MIMO controller. The position and velocity errors can be seen in Fig 6. Especially for motor 4, the error of the MIMO controller is considerably lower.

VI. CONCLUSIONS

In this paper, a MIMO state feedback controller has been designed trough modal decomposition for the DLR medical robot in order to deal with the high coupling between the robot joints. The asymptotic stability was shown based on Lyapunov theory. The experiments validated the performance of the MIMO controller.

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VII. APPENDIX

From the positive definiteness condition of the Hessian V_{2i} (53) one reaches (54) through the following steps:

$$\begin{aligned} & K - \alpha I - K[I + (K + K_T)^{-1} K_P]^{-1} \\ = \; & K - \alpha I - K[K + K_T + K_P]^{-1}(K + K_T \pm K_P) \\ = \; & -\alpha I + K[K + K_T + K_P]^{-1} K_P \end{aligned} \quad (69)$$

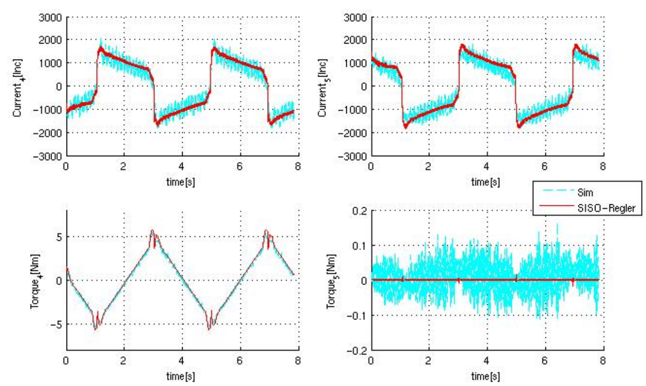

Fig. 3. Comparison of current and torque between simulation and SISO controller

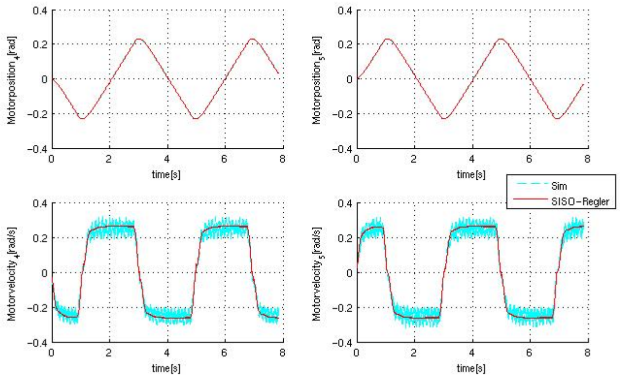

Fig. 4. Comparison of position and velocity between simulation and SISO controller

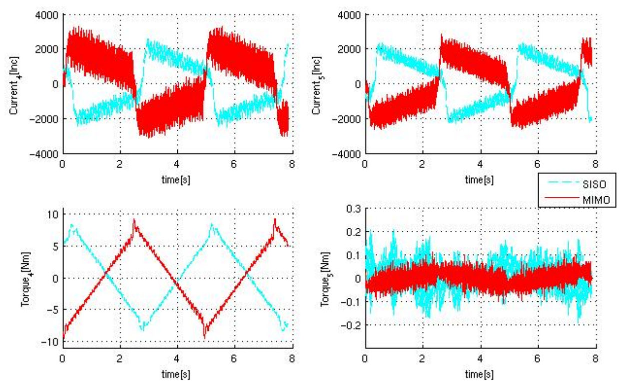

Fig. 5. Comparison of current and torque between MIMO controller and SISO controller

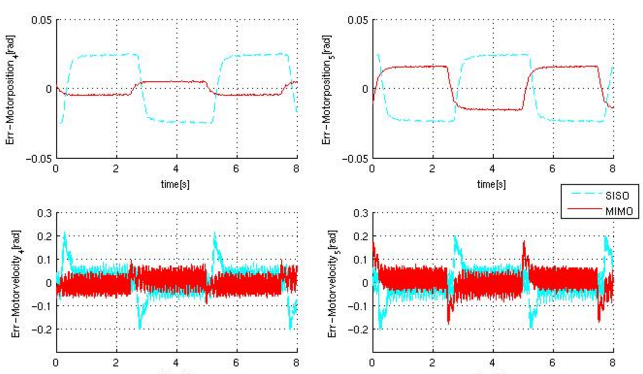

Fig. 6. Comparison of position error and velocity error between MIMO controller and SISO controller.