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Energetic Effects of Adding Springs at the Passive Ankles of a Walking Biped Robot

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Abstract— **This paper investigates the energetic effects of adding springs at the passive ankles of a planar five-link, four-actuator walking biped robot. The energetic cost of walking with springs was determined by using a walking motion designed for the biped without springs added. The walking motion was then optimized for the presence of springs of various arbitrarily chosen stiffnesses and offset angles. The stability properties of the motions that resulted were checked. The energetic costs of walking and standing were then computed. It was found that standing with springs was more efficient, while walking was more costly than the same action without springs. Finally, the spring characteristics (stiffness and offset angle) and the motion were optimized simultaneously. The costs for walking and standing were computed, revealing that walking with springs was more efficient and standing was more costly than doing either without springs. A methodical approach to choosing the size of the feet based on this analysis of the spring characteristics is also presented.**

I. Introduction

The objective of the study presented in this paper is to analyze the effect of adding springs at the passive ankles of a planar biped robot that contacts the ground via a planar horizontal foot. The study was motivated by the hypotheses that bipeds with compliant ankles may be able to exhibit more 'natural-looking' gaits and gaits with better efficiency, as compared with bipeds without compliant joints. The use of compliant joints and springs at ankles has been studied by others, including [1], [5], [10], [12]. The work presented here differs from these studies in that a methodical approach to selecting the foot size and the spring characteristics is given.

The biped in question was previously studied as a five-link robot with point feet [3], [11] but now has massless feet and springs at the ankles. Since the ankles of the biped studied are passive, the biped is underactuated in single support. The control law must accommodate the underactuation; only the shape of the biped may be directly controlled. The control law used in this work uses a reference motion determined by optimization that ensures the existence of a stable gait [4].

The addition of feet and springs at the ankles has the potential of lowering the cost of walking and the cost of standing. In this paper it is assumed that all time spent in double support is spent standing still, and the biped's configuration while standing is identical to its configuration at the end of the step, when it enters double support. This study investigates the effects of feet and springs on these costs. Three studies were performed. In the first, the springs added to the biped were of arbitrarily chosen stiffness and offset angle. Costs were computed for this configuration. The second study optimized the walking motion for these springs and cost was computed. A third study was done in which the feet and springs and walking motion were optimized simultaneously.

The paper is structured as follows. Section II presents the dynamic model of the biped. Section III overviews the control strategy. Section IV summarizes the study of the existence and stability of periodic gaits of the biped. Section V proposes an optimization strategy for determining efficient motions for the robot with and without springs. Section VI presents a numerical study of the effects of springs and energy consumption for different reference motions with various average velocities. Section VII draws conclusions.

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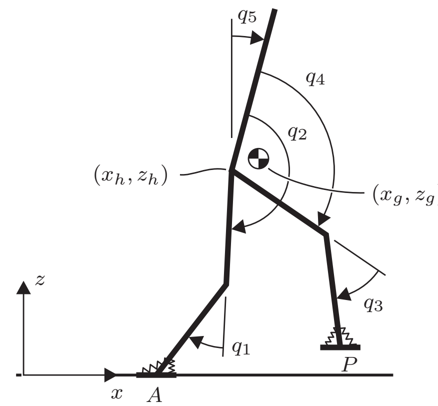

Fig. 1. Underactuated 5-link planar biped with feet and springs at the ankles. The compliance at the ankles follows a linear spring law.

TABLE I
Biped Parameters (detailed parameters may be found in [3]).

		Tibia	Femur	Torso
length	m	0.40	0.40	0.63
mass	kg	3.20	6.80	17.1

II. Model of the Biped

The biped under study consists of a torso and two identical legs. Each leg is composed of two massive links and a massless foot, all connected by frictionless joints; see Fig. 1. Apart from the feet, the biped is identical to the prototype RABBIT [3]. The biped's parameters are given in Table I.

The biped's gait is assumed to be composed of stance phases separated by instantaneous double support phases in which the legs swap their roles as stance and swing legs. The double support phase is assumed to be modeled by a rigid impact [9]. Since the steady-state gait is assumed to be symmetric with respect to consecutive steps, only one model for single support is needed.

A. The single support phase model

In the single support phase the ankle joints are assumed to have a linear spring in parallel with the joint. It is assumed that the massless feet are always parallel to the ground. For the supporting leg, the spring generates a torque that acts between the ground and the tibia. The potential energy of the supporting spring is $P_k = 1/2K(q_a - q_0)^2$ where $q_a = q_1+q_2+q_5$ is the absolute orientation of the supporting tibia, q_0 is the spring offset angle, and K is the spring's stiffness.

With the coordinates chosen as in Fig. 1, the first four joints are actuated. Hence, the dynamic model is written as

$$D(q)\ddot{q} + h(q,\dot{q}) + h_k(q) = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix}, \tag{1}$$

where $q \in \mathcal{Q} \subset \mathbb{R}^5$, $D(q)$ is the mass-inertia matrix, vector $h(q,\dot{q})$ contains the terms due to centrifugal, Coriolis, and gravitational forces, $\Gamma \in \mathbb{R}^4$ is the vector of the joint torques, and vector $h_k(q)$ contains the terms due to the added springs,

$$h_k(q) := \begin{bmatrix} K(q_a - q_0) \\ K(q_a - q_0) \\ 0 \\ 0 \\ K(q_a - q_0) \end{bmatrix}. \tag{2}$$

The state space is taken as $TQ = Q \times \mathbb{R}^5$, with $(q,\dot{q}) \in TQ$. The angular momentum of the biped around the point A is denoted σ_A and is given by

$$\sigma_A = d_5(q)\dot{q}, \tag{3}$$

where $d_5(q)$ is the fifth row of the inertia matrix, which corresponds to the unactuated coordinate. For simplicity it is assumed that $h_p = 0$. In single support the ankle is fixed to the ground and the externally-applied torque about the stance contact point A is equal to the time derivative of σ_A. Since the only external forces that generate a torque about A are gravity and the spring, it follows that

$$\dot{\sigma}_A = mg(x_g - x_A) - K(q_a - q_0), \tag{4}$$

where g is the gravity acceleration, x_g is horizontal position of the mass center, x_A is horizontal position of the contact point A.

1) Equilibrium of the supporting foot: The reaction force exerted by the ground, R, and the torque at the ankle between the tibia and the foot, $K(q_a - q_0)$, are considered when taking moments about the ankle of the supporting foot. Since the foot is stationary, it follows that (see Fig. 2)

$$K(q_a - q_0) - lR_z - h_pR_x = 0. \tag{5}$$

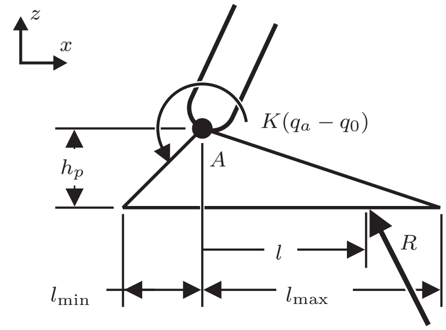

Fig. 2. The equilibrium of the supporting foot.

As defined in [7], the foot rotation indicator (FRI) is the location on the foot where the reaction force must act in order for the foot to remain still. Thus, since $h_p = 0$, the abscissa of the FRI, l, is

$$l = K(q_a - q_0)/R_z. \tag{6}$$

To avoid rotation of the feet, the FRI must be contained within the foot-ground contact segment, that is, l must satisfy

$$-l_{\min} < l < l_{\max}. \tag{7}$$

Since during the single support phase the stance foot is not moving with respect to the ground, it follows that

$$m \begin{bmatrix} \ddot{x}_g \\ \ddot{z}_g \end{bmatrix} + mg \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_x \\ R_z \end{bmatrix}. \tag{8}$$

In addition, the reaction force exerted by the ground (R_z) must be positive, and, to keep the biped from sliding, the reaction force must be inside the friction cone. These conditions may be written as $R_z > 0$ and $|R_x/R_z| < \mu$, where μ is the coefficient of static friction.

B. The impact model

The rigid impact at double support is modeled as an algebraic map [9]

$$\dot{q}^+ = E\triangle(q)\dot{q}^-, \tag{9}$$

where $\triangle(q) \in \mathbb{R}^{5\times5}$ and $E \in \mathbb{R}^{5\times5}$ is a permutation matrix that permutes the coordinates at leg exchange. Since the torque produced by the spring is finite, the torque does not affect the impact. Since the foot is massless and its height is assumed to be zero, the reaction force is applied at the ankle, which is denoted P. Thus, the angular momentum around the ankle of the impacting foot is conserved.

At impact, the supporting leg switches, thus the contact point switches from A to P. At the moment of impact, the angular momenta about each of the two points is related by

$$\sigma_P = \sigma_A^- + m(x_P - x_A)\dot{z}_g^-, \tag{10}$$

where σ_A^- is the angular momentum about A before the impact, $\dot{z}_g^-$ is the vertical velocity of the mass center before the impact, σ_P is the angular momentum around P (before and after the impact), and $(x_P - x_A)$ is the distance between the two feet. Since P is located at the ankle of the supporting foot after impact, $\sigma_A^+ = \sigma_P$. Thus, it follows that

$$\sigma_A^+ = \sigma_A^- + m(x_P - x_A)\dot{z}_g^-. \quad (11)$$

The overall model is described by (1) and (2) in the single support phase, separated by impacts described by (9).

III. The Control Law

The objective of the control law is to track a reference motion. This is accomplished using the time scaling control law of [6]. The key feature of the control is that the reference motion $q^d \in \mathbb{R}^5$ of the biped is enforced as a function of a time-scaling parameter s.

A. Constraints on reference motion choice

The evolution of the scalar s is assumed to be strictly monotonically increasing over a step from 0 to 1 with respect to time. For $0 < s_k < 1$, where s_k is the time scaling parameter s for the k^{th} step, the robot configuration $q^d(s_k)$ is such that the swing leg end touches the ground at $s_k = 0, 1$ and is otherwise above the ground. To correspond to a periodic gait, the reference motion and its derivative must be periodic. Since the legs swap their roles from one step to the following step, the desired configurations must be such that $q^d(1) = E\, q^d(0)$.

If the motion is tracked perfectly, the joint velocities before impact, $\dot{q}^- = (dq^d(1)/ds)\dot{s}_k(1)$, and after impact, $\dot{q}^+ = (dq^d(0)/ds)\dot{s}_{k+1}(0)$ must satisfy

$$\frac{dq^d(0)}{ds} = E\triangle(q^d(1))\frac{dq^d(1)}{ds}\,\alpha, \quad (12)$$

where $\alpha := \dot{s}_k(1)/\dot{s}_{k+1}(0)$.

B. Definition of the control law

The control law allows a classic tracking controller (i.e., one for a fully-actuated robot) to be used despite the robot's underactuation. Use of a classic tracking controller is possible by treating the parameterizing scalar s as a virtual input.

Define the output $y := q(t) - q^d(s(t))$ on (1). When $y \equiv 0$, the velocity and acceleration of the joint variables are

$$\begin{aligned}\dot{q}^d(t) &= \frac{dq^d(s(t))}{ds}\dot{s}\\ \ddot{q}^d(t) &= \frac{dq^d(s(t))}{ds}\ddot{s} + \frac{d^2q^d(s(t))}{ds^2}\dot{s}^2.\end{aligned} \quad (13)$$

The desired behavior in closed loop is

$$\ddot{q} = \frac{dq^d(s)}{ds}\ddot{s} + v(s,\dot{s},q,\dot{q}) \quad (14)$$

with $v(s,\dot{s},q,\dot{q}) = d^2q^d(s)/ds^2\dot{s}^2 + \psi$, where ψ is chosen to obtain a finite-time stabilization of the desired trajectories using the control law given in [2]. Define

$$L(q,\dot{q}) := \frac{\partial d_5(q)}{\partial q}\dot{q}^2 - mg(x_g - x_s) + K(q_a - q_0). \quad (15)$$

The second derivative of s and the torques required to achieve the desired behavior, (14), may be obtained using (1), (3), (4), and (15),

$$\ddot{s} = (-d_5(q)v - L(q,\dot{q}))\left(d_5(q)\frac{dq^d(s)}{ds}\right)^{-1} \quad (16)$$

$$\Gamma = [I_{4\times 4}\ 0]\left[D(q)\left[\frac{dq^d(s)}{ds}\ddot{s} + v\right] + h(q,\dot{q}) + h_k(q)\right]. \quad (17)$$

If $d_5(q)(dq^d(s)/ds) \neq 0$, the control law (16) and (17) ensures that $q(t)$ converges to $q^d(s(t))$ in a finite time, which can be chosen as less than the duration of one step [8]. After the output is driven to zero, the control law guarantees perfect tracking (i.e., $y \equiv 0$).

IV. Periodic motion and Stability study

The control law (16) and (17) ensures that after one step the motion of the biped will match the reference motion. To determine stability of a periodic motion, the behavior of the evolution of $\dot{s}_k(t)$ is studied. The dynamics of s may be deduced from (3) and (4). The unconstrained dynamics are

$$\sigma_A = d_5(q(s))\frac{dq^d(s)}{ds}\dot{s}(s) =: I(s)\dot{s}(s) \quad (18a)$$

$$\dot{\sigma}_A = mg(x_g(q^d(s)) - x_A) - K(q_a(q^d(s)) - q_0). \quad (18b)$$

Combination of (18a) and (18b) results in

$$\frac{1}{2}\sigma_A(0)^2 = \frac{1}{2}\sigma_A(s)^2 + V(s), \quad (19)$$

$$V(s) = \int_0^s I(\xi)(K(q_a(\xi) - q_0) - mg(x_g(\xi) - x_A))d\xi \quad (20)$$

for $0 \le s \le 1$; see [4].

Assume that the function $I(s)$ has a constant sign and define $\zeta := 1/2\sigma_A^2$. Since $V(s)$ can be calculated for any given $q^d(s)$, it follows that

$$\zeta_k(s) = \zeta_k(0) - V(s). \quad (21)$$

The reference motion $q^d(s)$ is defined with the assumption that s is an increasing function. Thus, $\zeta_k(0)$ must be such that ζ_k does not vanish:

$$\zeta_k(0) > \max_s V(s). \quad (22)$$

Since the evolution of the angular momentum is given by (11) and $\dot{z}_g$ and σ_A are proportional to $\dot{s}$ at impact, (11) can be rewritten as $\sigma_A^+ = \delta\sigma_A^-$, where

$$\delta := 1 + m(x_P - x_A)\frac{dz_g(1)}{dq}\frac{dq(1)}{ds}(I(1))^{-1}. \quad (23)$$

The value of $\zeta_{k+1}(0)$ can be written as a function of $\zeta_k(1)$,

$$\zeta_{k+1}(0) = \delta^2\zeta_k(1). \quad (24)$$

Combining (21) and (24), define $\varphi : \mathbb{R} \to \mathbb{R}$ as

$$\varphi := \delta^2(\zeta_k(0) - V(1)). \quad (25)$$

The stability of a periodic motion of the model (1) may be studied via the function φ as it is classically done using the technique introduced by H. Poincaré. The method of Poincaré is applied by first defining a Poincaré section, a hypersurface that is transverse to the orbit corresponding to the periodic motion. The discrete-time (first) return map of φ to the section may be studied to discern the stability properties of the orbit.

Here, the section is taken at $s = 0$ so that the fixed point $\zeta^*(0)$ of the Poincaré return map corresponds to the intersection between φ and the identity function. Equation (25) implies that the fixed-point is

$$\zeta^*(0) = \frac{-\delta^2 V(1)}{1-\delta^2}. \tag{26}$$

The corresponding value of $\dot{s}$ is

$$\dot{s}^*(0) = \sqrt{\frac{-2V(1)}{\alpha^2 I(1)^2 - I(0)^2}}. \tag{27}$$

Note that if $-2V(1)/(\alpha^2 I(1)^2 - I(0)^2)$ is negative, no periodic motion corresponds to the periodic trajectory $q^d(s)$. A gait will exist if $I(s)$ has a constant sign for $0 \le s \le 1$ and if $\zeta^*(0)$ is defined and greater than $\max_s V(s)$ (for a smaller value of $\zeta^*(0)$ the biped will not complete the step). These two conditions imply that the angular momentum σ_A has constant sign during a step. Note that $\zeta(0)$ is also upper-bounded because of ground reaction force constraints [4].

If the norm of the slope of the function φ is less than 1,

$$\delta^2 < 1, \tag{28}$$

then for an initial state close to the periodic motion, the biped's state will converge towards the periodic motion.

V. Definition of optimal periodic motion

The design of a valid reference motion for the biped is not a trivial task. The motion has to satisfy constraints on the limits of the actuators and the ground contact reaction forces, among others. To make the design systematic, constrained parameter optimization is used for reference motion design.

Unlike the approach of [4], where the desired reference internal motions were chosen to be polynomials in time, the desired joint motions are chosen to be polynomials in s. The polynomial coefficients are chosen to enforce periodicity of the desired walking motion.

A. The optimized parameters

To ensure periodicity and avoid contact between the swing foot and the ground, an intermediate configuration is defined for the biped at $s = 0.5$. Thus, each of the five joint variables is defined by a fourth-degree polynomial function of s,

$$q_j(s) = \sum_{k=0}^{4} a_{jk} s^k, \quad j = 1, \ldots, 5, \tag{29}$$

where j is the joint number. The polynomial functions $q_j(s)$, $j = 1, \ldots, 5$, are uniquely defined by q_i, q_f, q_{int}, dq_i/ds, and dq_f/ds. The subscripts i, f and int correspond to the initial (at $s = 0$), final (at $s = 1$) and intermediate (at $s = 0.5$) states of the robot, respectively.

Since the initial and final configurations for the stance phase are double support configurations, only four independent variables are necessary to define these configurations. Here, the step length, $(x_P - x_A)$, the position of the hips, (x_h, z_h), and angle q_5 are used (see Fig. 1).

Since the position of the robot is constant during the impact and the legs swap their roles from one step to the next, it follows that

$$q_{1i} = q_{3f}, q_{2i} = q_{4f}, q_{3i} = q_{1f}, q_{4i} = q_{2f}, q_{5i} = q_{5f}. \tag{30}$$

Using (30), the polynomial functions $q(s)$ can be defined as function of q_{int}, $(x_P - x_A)$, x_{hf}, z_{hf}, q_{5f}, dq_f/ds, and α. These 15 parameters are the parameters of optimization.

B. The optimization setup

Note that the reference motion must be compatible with the dynamic model. The corresponding gait will only be stable if $\zeta^*(0)$, given by (26), satisfies (22) and if (28) holds.

Since the Joule effect in the motors causes the greatest energetic loss when walking, the cost function is chosen to be proportional to this loss of energy. The cost function is defined as follows

$$\mathcal{C}_w = \frac{1}{(x_P - x_A)} \int_0^T |\Gamma(s)\,\dot{q}|dt. \tag{31}$$

To avoid the stance foot lifting from the ground, the minimal vertical reaction force is constrained to be greater than 100 N. The required friction coefficient is constrained to be less than $2/3$. The initial orientation of the torso is constrained to be between 0 and 15 degrees. To ensure a gait with feasible final hip height and step length, the values of z_{hf} and $(x_P - x_A)$ are constrained to be such that $0.6 < z_{hf} < 0.78$ m and $0.1 < (x_P - x_A) < 0.8$ m.

C. The cost of standing

In addition to walking, most bipeds also stand. Hence, it is important to assess the added cost associated with standing on the robot's design. Here, the cost of standing is assumed to be the static power dissipated by supplying the torque required to maintain quiescent standing.

The torques may be found by manipulating the single-support model (1), adding a term to represent the vertical reaction force at the swing foot and setting $\dot{q} = 0$ and $\ddot{q} = 0$. For simplicity, the horizontal reaction force is chosen to be zero since it does not effect the rotational equilibrium of the biped. The configuration q at the end of each step (when the biped is in double support) was used. The result is

$$\begin{bmatrix} \Gamma_s \\ F_{sw} \end{bmatrix} = \left[\begin{bmatrix} I \\ 0 \end{bmatrix} \middle| -J_{sw}^T \right]^{-1} (h(q,0) + h_k(q)), \tag{32}$$

where $\Gamma_s \in \mathbb{R}^4$ is the vector of the static joint torques, $J_{sw} \in \mathbb{R}^{5\times 1}$ is the manipulator Jacobian for the swing foot and $F_{sw} \in \mathbb{R}$ is the (vertical) reaction force at the swing foot.

The cost of walking is given by (31), and the cost of standing by $\mathcal{C}_s = V_m \Gamma_s' \Gamma_s / K_t$, where V_m is the voltage of

TABLE II

CASE 2: OPTIMIZED MOTION WITH ARBITRARILY CHOSEN SPRINGS FOR AN AVERAGE VELOCITY OF 1.25 M/S.

Stiffness		Walking Cost	Standing Cost	Cost Change Walking 2 m	Cost Change Standing 2 s	Cost Change Walking 2 m and Standing 2 s
K Nm/rad	q_0 rad	$\mathcal{C}_w$ J/m	$\mathcal{C}_s$ J/s	$\delta\mathcal{C}$ J	$\delta\mathcal{C}$ J	$\delta\mathcal{C}$ J
0	-	184	563	-	-	-
25	3.3	187	460	6	-205	-199
50	3.3	223	359	78	-407	-329
75	3.3	257	278	145	-570	-425
50	3.2	295	307	223	-511	-288
50	3.3	223	359	78	-407	-329
50	3.4	208	409	48	-308	-261

the motor, and K_t is the motor torque constant. The total cost of walking for k steps and standing for T_s seconds is

$$\mathcal{C} = k\,(x_P - x_A)\mathcal{C}_w + T_s\mathcal{C}_s. \tag{33}$$

VI. THE EFFECTS OF ADDING THE SPRINGS

The effects of ankles with compliance provided by springs were analyzed. The study was motivated by the hypothesis that bipeds with compliant ankles may be able to exhibit more 'natural-looking' gaits. The addition of springs can also increase the efficiency of the resulting gait. Three cases were studied.

In the first case, a motion designed for the biped with point feet (and no springs) was used. Massless feet and springs at the ankles were added. Three spring stiffnesses were investigated:[1] 25 Nm/rad, 50 Nm/rad, and 75 Nm/rad. The biped was simulated at the steady-state walking gait for one step. The required foot size and the changes in walking cost with, compared to without, these springs was computed.

In the second case, the same spring stiffness and offset angles were used, but the motion was optimized using the model with springs included. The foot size and cost were computed and found to be lower than in the previous case.

In the third case, both the motion and the springs were optimized concurrently, which resulted in walking motions that were more efficient than those in the first two cases.

In each case, four motions with different average walking velocities were used to explore how cost changes with velocity. Due to space constraints, the results presented here are for only one of these velocities, 1.25 m/s. The results for the other velocities follow similar trends.

To choose the spring offset angles, the range of q_a, the absolute angle of the stance tibia, was investigated. For an average walking velocity of 0.75 m/s, q_a, varies in the range (3.04, 3.25) rad, for 1 m/s, (3.12, 3.31) rad, for 1.25 m/s, (3.22, 3.39) rad, and for 1.5 m/s, (3.34, 3.47) rad. Based on these ranges for q_a and the fact that in double support the legs are near the equilibrium position of the spring, the spring offset angles considered were $q_0 = 3.2$ rad, 3.3 rad, and 3.4 rad.

A. Case 1: Reference motion with arbitrarily chosen springs

The average velocity and energetic cost both increase (resp. decrease) as the spring offset angle increases (resp. decreases).[2] This effect is more pronounced as stiffness increases. Additionally, increases in stiffness also increase the required size of the feet. Finally, as the spring offset angle increases, the mean position of the FRI generally moves backward.

B. Case 2: Optimized motion for arbitrarily chosen springs

Considering only walking, adding a spring to the passive ankle increases the energetic cost slightly. When only standing is considered, adding springs reduces the cost. As spring stiffness increases, additional cost of standing with springs decreases; that is, $K = 75$ Nm/rad is more efficient than $K = 50$ Nm/rad, which is more efficient than $K = 25$ Nm/rad. As the spring offset angle increases, the additional cost of standing with springs increases that is, $q_0 = 3.2$ rad is more efficient than $q_0 = 3.3$ rad, which is more efficient than $q_0 = 3.4$ rad. These results can be seen in Table II.

When both walking and standing are considered, the cost is reduced; see Table II. For example, using a spring stiffness of $K = 50$ Nm/rad and an offset angle of $q_0 = 3.3$ rad, the cost of walking for two meters is 446 J, whereas the nominal cost, the cost of the same motion without added springs, is only 368 J. Thus, the addition of springs increased the cost of walking by 78 J. The cost of standing for two seconds in this configuration is 719 J, whereas the nominal cost is 1126 J, so the springs decrease the cost of the motion by 407 J. Combining these two motions yields a total cost of 1165 J. Compared to a total nominal cost of 1494 J, the addition of springs results in a savings of 329 J. Thus, for the case in which arbitrarily chosen springs are added to a specific walking motion, the greatest energy savings occurs when the biped does not walk but simply stands still.

C. Case 3: Simultaneously optimized springs and motion

Since the optimal motion may require an evolution of the FRI that is incompatible with the design of the robot, the size of the feet were taken into account in the optimization process, and optimizations were run for feet ranging in size from 0 cm to 30 cm. The stiffness and equilibrium position were also limited to $0 < K < 100$ Nm/rad and $3 < q_a < 5$ rad.

[1] The stiffness were chosen to obtain reasonable size of the feet; see (7).

[2] The data corresponding to this case are not included due to space limitations.

TABLE III

CASE 3: SIMULTANEOUSLY OPTIMIZED MOTION AND SPRING FOR AN AVERAGE VELOCITY OF 1.25 M/S.

Maximum %cline6-8 Foot Size	Stiffness		Walking Cost	Standing Cost	Cost Change Walking 2 m	Cost Change Standing 2 s	Cost Change Walking 2 m and Standing 2 s
$l_{\max}$ cm	K Nm/rad	q_0 rad	$\mathcal{C}_w$ J/m	$\mathcal{C}_s$ J/s	$\delta\mathcal{C}$ J	$\delta\mathcal{C}$ J	$\delta\mathcal{C}$ J
0	0	-	184	563	-	-	-
5	4.79	4.41	171	591	-27	56	30
10	8.64	4.43	167	622	-34	118	83
15	11.74	4.45	159	653	-50	181	131
20	14.35	4.47	151	684	-67	242	176
25	17.12	4.55	144	730	-80	334	255
30	18.92	4.67	138	776	-93	425	333

In single support (see Table III), the spring offset angle must be large for low velocities and small feet.[3] For a walking rate of 0.75 m/s, the offset angle reaches the maximum (5 rad). As foot size and velocity increase, the offset angle becomes smaller, but never approaches the minimum. The optimal stiffness increases when the size of the feet increases. In general, the optimized spring at the ankle reduces the energetic cost of walking but may require large feet.

Standing with springs optimized for walking has a higher cost than standing in the same configuration without springs (see Table III). When the spring is optimized with the walking motion, as foot size and spring stiffness increase, the additional cost of standing with springs generally increases. For example, a maximum foot size of 10 cm increases the cost of standing for two seconds with springs by 118 J, whereas a maximum foot size of 20 cm yields a cost increase of 242 J, and a maximum foot size of 30 cm yields a cost increase of 425 J. For unreasonably large feet, the cost of standing decreases with increases in spring stiffness but is generally still more costly than standing with no springs.

For example, when the maximum foot size is chosen to be 15 cm, the cost of standing for two seconds in this configuration is 1307 J, whereas the nominal cost is only 1126 J, so the springs increase the cost of standing by 181 J. Combining walking and standing yields a total cost of 1625 J. Compared to a total nominal cost of 1494 J, the addition of springs increase the cost by 131 J. The cost of walking for two meters is 318 J, whereas the nominal cost is 368 J. Thus, the addition of springs decreased the cost of walking by 50 J, making this case, walking with springs simultaneously optimized for the motion is the most efficient scenario. Thus, for the case in which the springs and walking motion are simultaneously optimized, the greatest energy savings occurs when the biped walks without standing still.

VII. CONCLUSION

Massless feet and springs were added to the leg ends of a passive biped, and the associated additional costs of walking and standing with these springs were calculated. The size of the feet was chosen methodically using (6) and (7). For the case in which arbitrarily chosen springs were added to a motion designed for the biped without feet or springs, it was found that walking with additional springs is significantly more costly than walking without the springs. When the motion is optimized for the arbitrarily chosen springs, standing with springs in double support is more efficient, but walking with springs is somewhat less efficient compared to doing so without. Simultaneously optimizing the springs and motions results in walking that is the most efficient of any of the previous cases, while standing in double support is the least efficient.

[3]The data corresponding to the results presented in this paragraph are not included due to space limitations.

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REFERENCES

[1] S. Anderson, M. Wisse, C. Atkeson, J. Hodgins, G. Zeglin, and B. Moyer, "Powered bipeds based on passive dynamic principles," in *Humanoid Robots, 2005 5th IEEE-RAS International Conference on*, 2005, pp. 110–116.

[2] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators," *IEEE Transactions on Automatic Control*, vol. 43, no. 5, pp. 678–682, 1998.

[3] C. Chevallereau, G. Abba, Y. Aoustin, F. Plestan, E. R. Westervelt, C. Canudas, and J. W. Grizzle, "RABBIT: a testbed for advanced control theory," *IEEE Control Systems Magazine*, vol. 23, no. 5, pp. 57–79, Oct. 2003.

[4] C. Chevallereau, A. Formal'sky, and D. Djoudi, "Tracking of a joint path for the walking of an underactuated biped," *Robotica*, vol. 22, pp. 15–28, 2004.

[5] S. H. Collins and A. Ruina, "A bipedal walking robot with efficient and human-like gait," in *Proc. of the 2005 IEEE International Conference on Robotics and Automation, Barcelona, Spain*, 2005, pp. 1983–8.

[6] O. Dahl and L. Nielsen, "Torque-limited path following by online trajectory time scaling," *IEEE Transactions on Robotics and Automation*, vol. 6, no. 5, pp. 554–561, 1990.

[7] A. Goswami, "Postural stability of biped robots and the foot-rotation indicator (FRI) point," *International Journal of Robotics Research*, vol. 18, no. 6, pp. 523–33, June 1999.

[8] J. W. Grizzle, G. Abba, and F. Plestan, "Asymptotically stable walking for biped robots: Analysis via systems with impulse effects," *IEEE Transactions on Automatic Control*, vol. 46, pp. 51–64, Jan. 2001.

[9] Y. Hürmüzlü and D. B. Marghitu, "Rigid body collisions of planar kinematic chains with multiple contact points," *International Journal of Robotics Research*, vol. 13, no. 1, pp. 82–92, 1994.

[10] J. E. Pratt and G. A. Pratt, "Exploiting natural dynamics in the control of 3D bipedal walking simulation," in *Proc. of the 1999 Int. Conf. on Climbing and Walking Robots*, 1999.

[11] E. R. Westervelt, J. W. Grizzle, C. Chevallereau, J.-H. Choi, and B. Morris, *Feedback Control of Dynamic Bipedal Robot Locomotion*. Taylor & Francis/CRC Press, 2007.

[12] J. Yamaguchi, D. Nishino, and A. Takanishi, "Realization of dynamic biped walking varying joint stiffness using antagonistic driven joints," in *Proc. of the 1998 IEEE International Conference on Robotics and Automation, Leuven, Belgium*, vol. 3, 1998, pp. 2022–9.