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Frequency Domain Stability Observer and Active Damping Control for Stable Haptic Interaction

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***Abstract*—Stable haptic interaction has been studied extensively by an energy-based approach. However, the energy in the haptic system is not directly measurable, but estimated from some measured quantities such as force and velocity; therefore, the estimated energy is occasionally inaccurate. To resolve this problem, a new observer, working in the frequency domain, is proposed in this research. The observer quantifies the degree of instability of a haptic system, and a proposed controller generates variable damping in proportion to this quantitative instability. Especially, for a double layered virtual wall, the proposed methods were much faster in detecting haptic instability than other schemes, and successfully reduced unstable behavior.**

I. INTRODUCTION

VARIOUS researches have been conducted to solve the haptic instability problem [1]-[3]. Based on the network theory, the components of a haptic system (e.g., a human, a haptic device, virtual environment) can be modeled as nodes. The nodes in the network interact mutually by exchanging energy [4]. If a node can create energy, it is called an *active node*. If not, it is called as a passive node. A haptic system usually becomes active due to the factors such as sampling, quantization, and time delays. Active haptic systems often show unstable behavior [5]. Colgate and Schenkel found that an active wall could make the haptic system unstable, especially when a rapid transition occurs repeatedly between low impedance and high impedance (e.g., the surface on a stiff virtual wall), and they presented the theoretical passivity condition for the haptic system [6]. In their research, they suggested that an increase in damping and sampling frequency of a haptic device could improve the stability of the haptic interface. The damping element, however, degrades the transparency of the haptic system. Therefore, only a marginal amount of damping should be added to the system, just enough to make the system stable. Hannaford and Ryu suggested the passivity controller (PC) based on the passivity observer (PO) in the time domain [3]. In this PO/PC scheme, the PO monitored the passivity level of the system and generated damping just enough to dissipate excessive energy.

Stable haptic interaction has been studied extensively by the energy-based approach. The observation of the energy level can provide information on the unstable behavior of a haptic system. However, the energy in the haptic system is not measured directly, but estimated from some measured quantities such as force and velocity. To estimate the power at some instant, therefore, all relevant quantities must be measured strictly at the same instant, which is often very difficult. For these reasons, the estimated energy (or passivity) in the haptic system is occasionally inaccurate. Furthermore, the physical quantities used for computation of energy are more rapidly and severely changing during the haptically unstable situation than during normal conditions, so the consequent energy value becomes more inaccurate. Therefore, the energy-based approach can fail to detect the haptic instability [7]. Hence, a more robust method is required to detect haptic instability.

One alternative to the passivity-based detection of the haptic instability is to directly detect it by analyzing the motion of a haptic interface. This approach is more reliable than the passivity-based approach because the haptic instability is directly reflected in the motion of a haptic device. Haptic instability is closely related to the human perception that gives rise to uneasiness. From the viewpoint of haptic sensation, when a human operator cannot feel the oscillation with high frequency and/or low magnitude beyond human perception, such haptic interaction can be regarded stable, although it can actually be theoretically unstable. Then, one logical question arises regarding which frequency or magnitude of the oscillation makes the user uneasy during the operation of a haptic device. In the motion of a haptic interface, there exist two types of motions: the motion driven by a user and the oscillation caused by the haptic instability. This paper presents how to separate these two types of motions by observing the motion of the haptic system. To achieve this, the characteristic of unstable haptic interaction is empirically investigated in the frequency domain, using the discrete Fourier transform (DFT).

In this research, we propose a novel method, called a haptic stability observer (HSO), for detecting the haptic instability. With the proposed HSO, haptic stability is described not by secondhand measures (i.e., energy and velocity) but by firsthand measures (e.g., position, force, and time). The HSO

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shows fast response and reliable results even for the multi-layered virtual wall where the passivity-based approach does not work properly. The HSO can also quantify degree of haptic instability, so that a new controller can be designed to provide a damping force in proportional to the quantified haptic instability.

II. Empirical Examination of Haptic Stability in Frequency Domain

This section discusses how to estimate haptic stability by observing the actual motion of a haptic interface in the frequency domain, and the frequency separation between human motion and haptic unstable behavior is investigated in actual haptic systems. A physical motion can be described either by the time function in the time domain or by the spectrum in the frequency domain. These two functions can be related by the Fourier transform, and the discrete Fourier transform (DFT) can be similarly applied for a discrete system. An important feature of the DFT is that it has an extremely fast algorithm, called the Fast Fourier Transform (FFT), for its calculation [8].

To analyze the motion of a haptic interface in the frequency domain, the DFT parameters should be properly determined. Basically, the sampling period T is selected in consideration of the Nyquist frequency f_c. According to the sampling theorem, the Nyquist frequency is half of the sampling frequency f_s, so it is given by

$$f_c = \frac{f_s}{2} = \frac{1}{2T} \quad (1)$$

Because the frequency of interest of a haptic interface is in the range of about 10 to 100 Hz [9], the sampling frequency f_s should be over 200 Hz. To investigate the frequency of interest, the related parameters with the FFT are adjusted. However, the sampling frequency is limited by the controller performance. Although actual haptic systems usually have a single unified sampling frequency for signal acquisition process and actuator control process, the sampling frequency for the signal acquisition process should be much higher than that for the actuator control process.

When the sampling period T is selected for a given system, the number of data N determines the length of a signal for the DFT analysis. That is, the record length T_o is defined by

$$T_o = T \times N \quad (2)$$

The frequency resolution Δf is then given by

$$\Delta f = \frac{1}{T_o} \quad (3)$$

A trade-off between the frequency resolution and computational speed can be achieved by adjusting the number of data N. The smaller value of N reduces the computational load significantly, thus resulting in real-time operation. Furthermore, the unstable behavior of a haptic interface can be detected faster due to the reduced record length, and thus the signal of a motion can be immediately reflected in the DFT analysis. On the other hand, the larger value of N achieves a finer frequency resolution for the DFT analysis. That is, the frequency resolution should be fine enough to distinguish the characteristic of a haptic interface from that of a human operator. Therefore, the frequency resolution should be below 10Hz because the frequency components related to human motion is relatively low. Table 1 lists some DFT parameters in terms of the sample period of a haptic system. The number of input data N in the table is selected as a power of two in consideration of a general FFT algorithm.

In order to investigate the characteristics of a haptic interaction, experiments for the two different situations were conducted. In the first experiment, the human operator generated oscillating motion as fast as possible to mimic the haptic instability. In the second experiment, the real unstable oscillation was generated by the contact with the stiff virtual wall. The PHANToM, one of the most famous haptic devices, was used for these experiments together with the PC-based controller with a sampling period of 1 msec. With 256 samples obtained for the FFT operation, the frequency resolution was about 3.9 Hz as shown in Table 1, and the record length was 256 msec. To achieve a faster response, the FFT algorithm was executed every 10 msec. As a result, most of the record length was overlapped, and thus most input data used at one step were reused for the following step. With this overlapped use of data, unstable oscillation was detected faster at the cost of a rather lengthy computation time.

TABLE I
RECOMMENDED DFT PARAMETERS
FOR ANALYSIS OF HAPTIC STABILITY IN FREQUENCY DOMAIN

Sampling period	DFT parameters	N			
		64	128	256	512
T = 0.5 msec (f_c= 1000 Hz)	Δf (Hz)	31.25	15.63	7.81	**3.91**
	T_0 (msec)	32	64	128	**256**
T = 1 msec (f_c= 500 Hz)	Δf (Hz)	15.63	7.81	**3.91**	1.95
	T_0 (msec)	64	128	**256**	512
T = 2 msec (f_c= 250 Hz)	Δf (Hz)	7.81	**3.91**	1.95	0.98
	T_0 (msec)	128	**256**	512	1024
T = 4 msec (f_c= 125 Hz)	Δf (Hz)	**3.91**	1.95	0.98	0.49
	T_0 (msec)	**256**	512	1024	2048

Recommended parameters are denoted in bold.

The characteristics of two motions explained above were analyzed in the frequency domain and the results are shown in Fig. 1 and Fig. 2. Figure 1 shows the position and frequency spectrum with time when the user mimicked the unstable haptic display. The wall was assumed to be located at x = -0.05m. In the spectrum, a dark color means the dominant frequency components of the motion at the moment. Thus, the spectrum indicates that the measured motion data are mainly composed of slow motions of frequency below 20Hz. The user generates fast oscillation during region S_1, so the dominant frequency increases slightly more than that in other

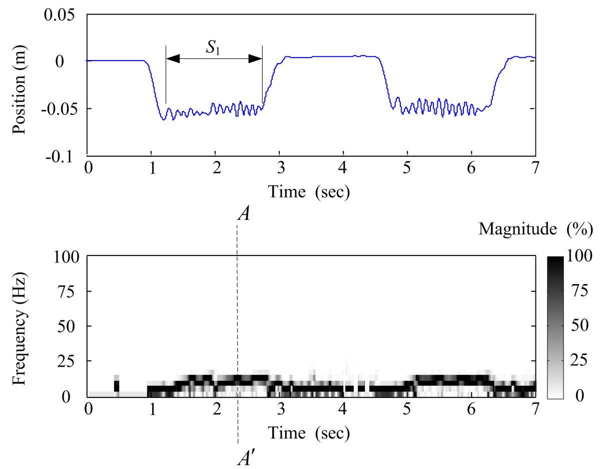


Fig. 1. Characteristics of human motion in haptic interaction. When the user mimics haptic instability during S_1, dominant frequency slightly increases but is limited below 20Hz.

regions, but there is not a large difference. Although the user generates some oscillation in S_1, the system is considered stable with respect to the haptic interface.

Fig. 2 illustrates the motion generated by the contact of the haptic device with an active stiff wall, which causes haptic instability (i.e., oscillation). The virtual wall with stiffness of 10kN/m is located at $x = -0.05$m. The user moves the handle of the haptic device toward the wall, but it is blocked by the stiff wall. The surface of the wall then becomes unstable while the device is in region U_1 in Fig. 2. The user pulls the handle back and pushes it again in region S_2; consequently, the unstable oscillation appears again in region U_2.

Fig. 1 and Fig. 2 show the different frequency spectrums. The frequency of human motion in Fig. 1 is limited to under 20 Hz (centered around 10Hz), although the user shakes the handle as fast as possible. However, the dominant frequency in the unstable behavior of the haptic interface is observed in the frequency range over 30Hz. A small fraction is seen even above 70Hz. This result is well illustrated in Fig. 3 which shows the section of the frequency spectrum at AA' in Fig. 1 and BB' in Fig. 2. Note that the magnitude at each frequency is normalized by the maximum value of each case.

The haptic instability of most haptic systems is commonly observed at higher frequency than that associated with the

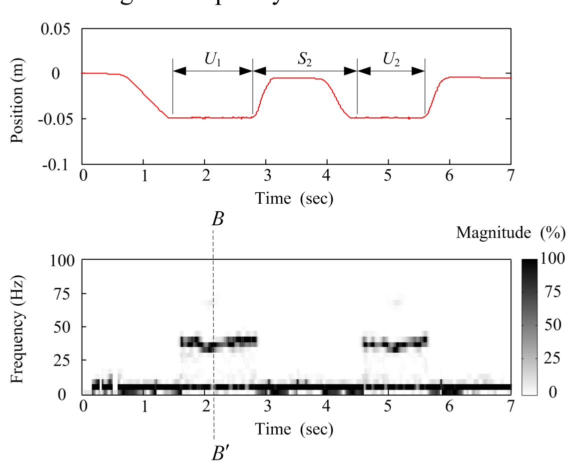


Fig. 2. Characteristic of unstable motion on active wall. When unstable behavior occurs during U_1 and U_2, dominant frequency is over 30Hz.

user's hand motion. In other words, the frequency separation occurs between human motion and haptic instability. Different results would be obtained for various empirical parameters (i.e., characteristics of the user and/or environment, inherent damping, sampling period to control, etc.). Especially, the bandwidth of the haptic interface mainly affects the frequency separation. If the bandwidth of a haptic system were lower than that of human motion at the body part in contact, the phenomenon would become unclear. In this research, it is assumed that most actual haptic systems have considerably higher bandwidth than the human motion. The above experiments show that haptic stability can be directly observed in the frequency domain unlike the energy that is estimated from the other measured quantities.

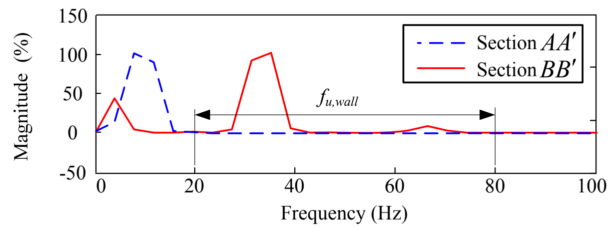


Fig. 3. Frequency spectrum of human hand motion and haptic instability.

III. Active Damping Control Using HSO

One solution to reduce haptic instability is the addition of a damping element to the system, but excessive damping is likely to degrade the transparency of a haptic system. Therefore, it is important to determine the exact time and the amount of required damping. To detect whether the haptic interface is stable or not, we propose a novel haptic stability observer (HSO) in this research. The stability index of HSO, R_{HSO}, is defined as the ratio of the sum of the amplitudes H at the frequency range f_{unstable} over which high frequency oscillations are observed to the sum of amplitudes at all frequencies f_{all} as follows:

$$R_{HSO} = \frac{\sum_{f_{unstable}} H}{\sum_{f_{all}} H} \tag{4}$$

The R_{HSO} is a dimensionless index in the range of 0 to 1. In (4), the frequency range f_{unstable} for haptic instability needs to be specified, which is determined differently depending on the application. In the previous section, the unstable behavior of a given haptic interface was observed in the frequency range $f_{u,wall}$ in Fig. 3, so $f_{unstable}$ is specified as 20 to 80 Hz.

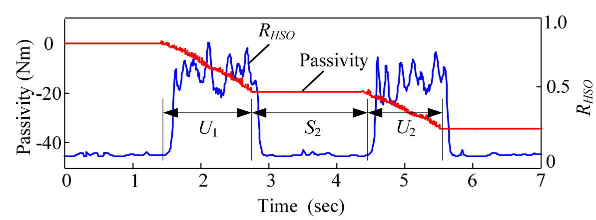


Fig. 4. Observation of haptic instability by passivity observer and proposed observer.

Figure 4 shows the energy level of the passivity observer and the stability index R_{HSO} of the proposed observer HSO during the previous experiments on the active wall. As long as a system is passive, which means the system dissipates energy, the energy level of the system cannot become below zero. The passivity observer suggested in [9] regards the negative energy level as an indicator of unstable behavior. In Fig. 4, the passivity observer (PO) can successfully detect unstable behavior in region U_1 by showing the negative energy level. It fails, however, to detect stable region S_2 because the energy level is not reset to zero after unstable region U_1 and still remains negative. The passivity observer needs an additional process to reset the energy level for proper operation. A passivity controller (PC) was proposed to dissipate excessive energy for this reset process [2]. The PO/PC successfully works for a simple virtual wall, but it is still difficult to determine when to reset the energy level for some critical cases, such as a multi-layered virtual wall.

On the contrary, the HSO properly illustrates the unstable phenomenon in Fig. 4. It can clearly distinguish the stable behavior in region S_2 from the unstable behavior in regions U_1 and U_2. As unstable behavior occurs, the stability index of HSO rises to the value of 0.5 in Fig. 4, which means that half of the motion is associated with the frequency range f_{unstable} for the period of unstable behavior. This index becomes almost zero during stable haptic interaction. Accordingly, the HSO can be used to determine haptic instability with an adequate threshold. A time delay of about 90 msec occurs in detecting the haptic instability in Fig. 4 because it takes time to update data in the record length of the FFT algorithm. Thus, this delay can be decreased by reducing the record length.

During haptic interaction, a human operator exerts force, F_H, just enough to overcome the inertia of the haptic device, which is characterized by the mass, m, and the inherent damping coefficient, b, as shown in Fig. 5. When the environment generates a force, F_E, this generated force is balanced by the human force, F_H, and the user gets moderate haptic emotions by means of the haptic device. If the forces F_H and/or F_E rapidly and repeatedly change, the equilibrium

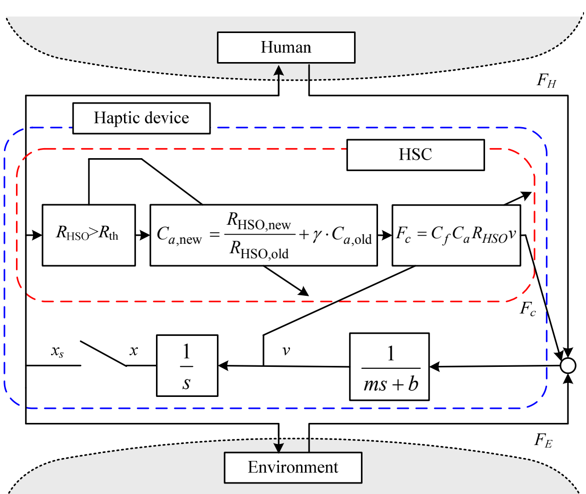


Fig. 5. Block diagram of the haptic stable frequency controller

cannot be maintained further, and finally unstable behavior arises.

An active damping control scheme is proposed in this research as illustrated in Fig. 5. The proposed controller, called a haptic stability controller (HSC), uses the HSO to determine the time and the amount of damping required to stabilize the haptic interface. The main idea is the setting of the damping coefficient B_{HSC} in proportion to the stability index R_{HSO} as follows:

$$B_{HSC} = C_p \cdot R_{HSO} \tag{5}$$

where C_p is the proportional gain. The damping coefficient B_{HSC} is set to a positive value when unstable haptic interaction is detected, but to zero during stable haptic interaction. As a result, the control damping force, F_c, is described by

$$F_c = B_{HSC} \cdot v \tag{6}$$

where v is the velocity of the device. Therefore, the control force F_c is applied when unstable interaction occurs; otherwise, it vanishes. If the haptic instability grows, then the stability index also increases and more control force is exerted on the system. The HSC updates the proportional gain C_p of a given system so that it can ensure the stability and also conserve the transparency of a system. In the HSC law, the gain C_p is defined by

$$C_p = C_f \cdot C_a \tag{7}$$

where C_f is the fixed gain which determines the basic magnitude of the gain C_p, and C_a is the adaptive gain computed depending on the degree of haptic instability.

The HSC updates the adaptive gain C_a by the method used in reinforcement learning. After application of the control force given by (6) at time i-1, the haptic stability at time i is examined to evaluate whether the magnitude of the control force is appropriate or not. For this purpose, the evaluation value E_i at time i is defined as

$$E_i = \frac{R_{HSO,i}}{R_{HSO,i-1}} \tag{8}$$

The adaptive gain $C_{a,i}$ in (7) is then updated by

$$C_{a,i} = E_i + \gamma \cdot C_{a,i-1} \tag{9}$$

where γ is the discount factor in the range between 0 and 1. Equation (11) can be rewritten recursively by

$$\begin{aligned} C_{a,i} &= E_i + \gamma \cdot C_{a,i-1} \\ &= E_i + \gamma \cdot E_{i-1} + \gamma^2 \cdot C_{a,i-2} \\ &\;\;\vdots \\ &= E_i + \gamma \cdot E_{i-1} + \cdots + \gamma^{i-2} \cdot E_2 + \gamma^{i-1} \cdot E_1 \end{aligned} \tag{10}$$

That is, the current adaptive gain $C_{a,i}$ is a discounted sum of the previous evaluation values, and thus all past evaluation values affect the current adaptive gain. The discount factor γ allows the payoffs distant in time to be weighted less than more immediate payoffs. This update rule is commonly used in reinforcement learning [10].

In (10), the discount factor guarantees that the adaptive gain $C_{a,i}$ is bounded if the evaluation values are bounded. Since the value E becomes very large (i.e., unbounded) as the stability index $R_{HSO,i\text{-}1}$ in (8) approaches zero, the HSC updates the adaptive gain only when the stability index is greater than the prescribed threshold. Finally, the update rule is achieved by

$$C_{a,i} = \begin{cases} \dfrac{R_{HSO,i}}{R_{HSO,i-1}} + \gamma \cdot C_{a,i-1} & \text{if } R_{HSO,i-1} \geq R_{th} \\ C_{a,i-1} & \text{otherwise} \end{cases} \tag{11}$$

where R_{th} is the threshold above which the haptic interface is regarded unstable.

As shown in Fig. 5, the gain C_p is updated while the haptic instability continues, but it is kept constant if the instability disappears. When the unstable behavior occurs at the beginning of the haptic interaction, only a small damping force is provided to the system because the initial proportional gain is small. As the instability grows, however, the proportional gain becomes larger by (11), thereby resulting in the larger control damping force. As the system becomes stabilized, the proportional gain is kept constant, as shown in Fig. 8(d).

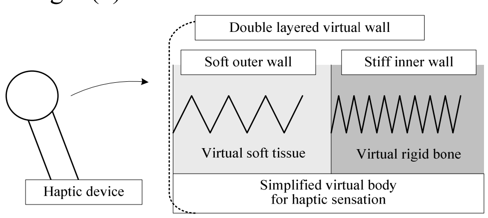


Fig. 6. double layered wall for modeling tissue and bone.

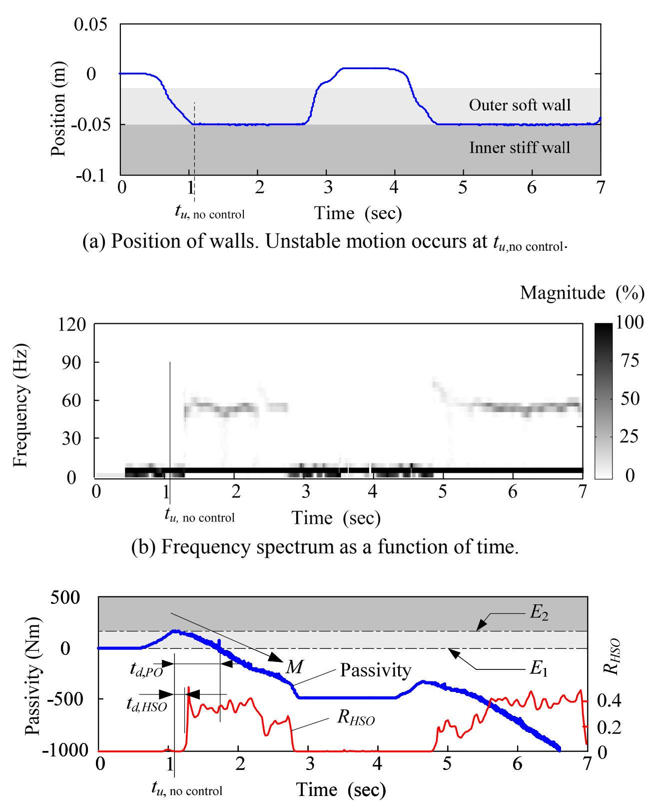
IV. Experiments and Discussion

A virtual body for the medical application is considered here. A virtual body is composed of complex virtual objects, such as virtual skin, virtual tissue, virtual organ, and virtual bone. For haptic sensation, the virtual body is simply modeled as a double layered wall composed of a rigid bone covered with soft tissues as shown in Fig. 6.

It consists of two different walls, and a stiff wall is located inside a soft wall. In this multi-layered virtual wall, it is very difficult to observe and remove unstable interaction by employing existing energy-based approaches, although haptic stability is more critical in medical applications. To evaluate the proposed method in this research, a double layered wall was provided in the virtual space, and the experiments on haptic stability were conducted.

A double layered virtual wall was implemented by the haptic interface. The soft wall (0.2kN/m) and stiff wall (10kN/m) were located at x = -0.01m and at x = -0.05m, respectively. The sampling period was set to 1msec and 256 samples were used for the FFT operation which is performed every 10msec. The user can push the handle of the haptic master into the soft wall with relative ease, but is blocked by the stiff wall.

Figure 7 presents the experimental results when no control is used. The user starts to move the handle into the wall at t = 0.5 sec, easily penetrates into the soft wall, and touches the inner stiff wall. The motion, however, is blocked at the surface of the inner wall, and unstable haptic interaction occurs at the surface at $t_{u,\text{ no control}}$, as shown in Fig. 7(a). The frequency spectrum in Fig. 7(b) shows that the unstable interaction signal is composed of the frequency components around 50Hz. Figure 7(c) shows the energy level with time. While the user moves the device stably in the soft wall, the energy level varies between E_1 and E_2 depending on the motion of the device. The passivity observer initialized to energy level E_1 (= 0) continues to rise to level E_2 until the handle contacts the inner stiff wall. The energy level starts to decrease when the system becomes unstable at the surface of the inner stiff wall. The passivity observer, however, cannot detect the early stage of haptic instability because the energy level is still positive at this stage. As the instability continues, the energy level becomes negative, so the passivity observer detects the instability, but it takes the time $t_{d,PO}$ until the energy level reaches zero in Fig. 7(c). In this critical case, the proposed approach is much faster than the energy-based



(a) Position of walls. Unstable motion occurs at $t_{u,\text{no control}}$.

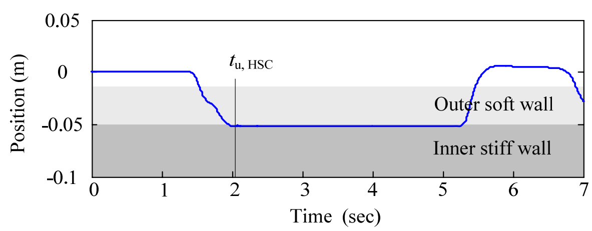
(b) Frequency spectrum as a function of time.

(c) Performance of proposed HSO and passivity observer.

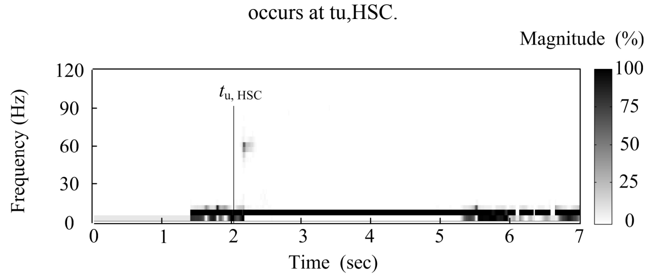
Fig. 7. Characteristics of motion at doubled layered wall without control.

approach, because the proposed observer only takes $t_{d,\mathrm{HSC}}$ to detect instability, as shown in Fig. 7(c).

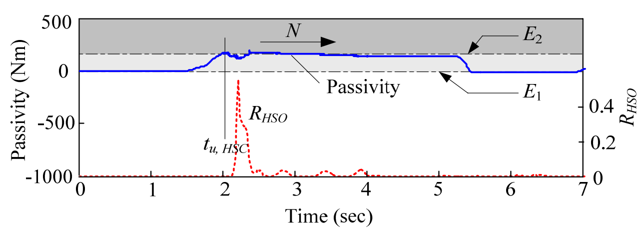
Figure 8 shows the experimental results obtained with the use of the HSC scheme. The handle moves through the soft wall and contacts the stiff wall, and then unstable haptic interaction occurs at time $t_{u,HSC}$, as shown in Fig. 8(a). When unstable haptic interaction begins, high frequency components appear as shown in Fig. 8(b), which is similar to Fig. 7(b). The HSO detects this instability with a short delay much smaller than $t_{d,PO}$ as shown in Fig. 8(c). Then the HSC is activated and the gain C_a is properly updated by (11). The discount factor γ in (11) is set to 0.8, and the fixed gain C_f in (7) is chosen to be 10^2 in this experiment. Consequently, the damping coefficient B_{HSC} varies as shown in Fig. 8(d). At the start of control, the generated damping force is too small to regulate the unstable interaction. The effect of HSC is fed back to the controller according to the update rule given by Eq. (9). The proportional gain C_p then increases, while the stability index, R_{HSO}, exceeds the prescribed threshold. Then



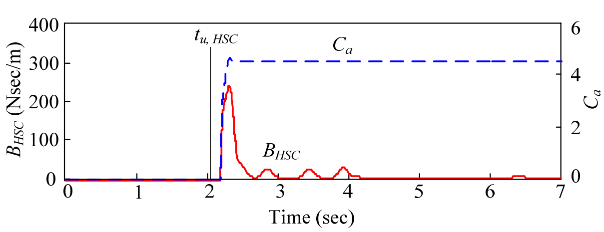
(a) Position. Stiff wall is located inside the soft wall. Unstable motion occurs at tu,HSC.



(b) Frequency spectrum.



(c) Passivity observer. Estimated energy is maintained horizontally.



(d) Damping coefficient and proportional gain.

Fig. 8. Characteristics of motion in doubled layered wall with HSC.

the control force grows, and the unstable interaction settles down in a short period, and finally R_{HSO} starts to decrease. Therefore, the proportional gain C_p converges as shown in Fig. 8(d).

The energy level maintains its level E_1 if the system remains stable at the surface of the soft wall, as shown in Fig. 8(c). The energy level keeps the level E_2 while the user stably contacts the surface of the inner stiff wall. The HSC regulates the energy to the level E_2 horizontally.

V. Conclusion

In this research, the frequency separation between human motion and haptic instability was reported, and the practical availability of the phenomenon was showed.

The haptic stability observer (HSO) was proposed to detect unstable haptic interaction by analyzing the motion in the frequency domain. It can quantify the degree of haptic instability of a haptic system. Since the HSO directly analyzes the motion, it is more reliable than the passivity observer which represents only the necessary condition for the unstable haptic interaction.

To remove or alleviate unstable haptic interaction, the haptic stability controller (HSC) was proposed in this research to provide variable damping in proportion to the quantitative haptic instability. The proposed controller was designed to adapt itself to the given system, autonomously updating the variable damping coefficient. A double layered wall was implemented in the virtual environment to verify the validity of the HSC. Experimental results show that the HSO is much faster than the passivity observer in detecting haptic instability for the multi-layered wall, and the HSC successfully reduces unstable haptic interaction.

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