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On the Performance of Multi-robot Target Tracking

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Abstract— **In this paper, we study the accuracy of Cooperative Localization and Target Tracking (CLATT) in a team of mobile robots, and derive analytical upper bounds for the position uncertainty. The obtained bounds provide a description of the asymptotic positioning performance of the robots and the targets as a function of the sensor characteristics and the structure of the graph of relative position measurements. By employing an Extended Kalman Filter (EKF) formulation for data fusion, two key asymptotic results are derived. The first provides the guaranteed worst-case positioning accuracy, whereas the second determines an upper bound on the *expected* covariance of the estimates. We investigate the effects of jointly estimating the targets' and the robots' position, and demonstrate that it results in *better* accuracy for the robots' position estimates. The theoretical results are confirmed both in simulation and experimentally.**

I. Introduction

When a team of robots is employed for tracking a number of targets, the position of the robots (*Localization*) and the position of the targets (*Tracking*) need to be concurrently estimated. In this paper, we study the problem of Cooperative Localization and Target Tracking (CLATT) in scenarios where teams of, possibly heterogeneous, mobile robots track the position of multiple targets. One of the main results of this paper is a proof that jointly estimating the position of the robots and targets results in *better* accuracy for the robots' position estimates, compared to when the robots localize ignoring the targets. Intuitively, this can be justified by considering that every time the robots measure range and bearing to the same targets, they indirectly perform observations of their relative positions.

Another significant contribution of this paper is a thorough investigation of *the robots' performance on average and in the worst case*. This is a common question that needs to be addressed before any investment in system development is made. In this work, performance refers to the accuracy of position estimation and is assessed by the state estimates' covariance for the robots and targets. *Analytical* upper bounds for the uncertainty of the robots' and the targets' localization are presented that are functions of the *sensors' characteristics* and of the structure of the *sensing graph* that connects the robots and targets. Furthermore, it is shown how *a priori* information about the distribution of the positions of the robots and the targets can be utilized to derive an upper bound for the *expected* value of the position estimates' covariance. The developed upper bounds can be employed in order to *predict* the position accuracy attained in a certain tracking application, and thus can facilitate the task of sensor selection, so as to meet user-imposed requirements.

In this paper, we consider the case where: (i) all robots are equipped with proprioceptive sensors that measure velocity, and are provided with orientation estimates of bounded uncertainty, (ii) one of the robots is equipped with a GPS receiver, enabling it to obtain absolute position estimates, and (iii) some of the robots are capable of measuring the relative positions of other robots and targets. These robot-to-robot and robot-to-target measurements are described by the *Relative Position Measurement Graph* (RPMG). This is a connected and directed graph, whose vertices represent the robots and targets, while the edges represent the relative-position measurements between them.

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II. Related Work

The problem of target tracking using multi-sensor networks has been the subject of extensive research in recent years [1], [2]. Most researchers address the problem of tracking with a network of *static* sensors [3], nevertheless, several approaches have been proposed for target tracking with mobile robots. For example, Parker [4] has developed a control strategy for multi-robot teams that minimizes the total time a target can evade being observed by the robots. Several region-based approaches to target tracking by mobile robots have been developed by Jung and Sukhatme [5], [6]. A hierarchical algorithm for localization and tracking using directional sensors is presented in [7]. Stroupe *et al.* [8] propose a distributed action-selection algorithm for optimizing the robots' trajectories with respect to the targets' position covariance. Despite their importance for practical applications, none of the aforementioned approaches addresses the problem of determining bounds on the performance (accuracy) of the robots' and targets' localization.

A number of approaches aimed at providing a description of the localization accuracy during target tracking have been developed for the case in which the sensors remain *static*. In [9], an optimal approach for fusing target tracking data is developed, and its performance is evaluated using Monte Carlo simulations. An analytical performance evaluation of this method is also provided in [10]. Zhang *et al.* [11] have studied the Cramer-Rao Lower Bound (CRLB) of the targets' position estimates. The CRLB for tracking manoeuvering targets is presented in [12]. In [13], the performance of a wireless sensor network in the presence of communication delays and false alarms is analyzed and compared to the CRLB. All aforementioned approaches focus on static sensor networks exclusively, and the results cannot be readily extended to networks of mobile sensors. Additionally, the CRLB cannot be employed to determine the *worst-case* performance of tracking, which is of interest before the deployment of a system in any application.

The main contribution of the work presented in this paper is the characterization of the steady-state accuracy of the position estimates in Cooperative Localization and Target Tracking (CLATT). This is achieved by deriving *analytical upper bounds* of the steady-state covariance matrix of the position estimates, for the worst-case scenario as well as for the *average* case (i.e., when a probabilistic description of the targets' and robots' positions is known in advance). Moreover, when the process noise in the target motion model in the limit becomes infinite (i.e., no prior information of the targets' motion model is available), a study of the localization accuracy provides a worst-case performance bound over all possible motion models. This analysis also demonstrates that the robots' position estimates are always *better* when, in addition to robot-to-robot measurements, the robots also process robot-to-target measurements.

III. Problem Formulation

Consider a group of M mobile robots, denoted as r_1, r_2, ..., r_M and N targets, denoted as T_1, T_2, ..., T_N, moving on a planar surface. A zero-velocity model is employed to describe the motion of the targets. The robots use proprioceptive measurements (e.g., from odometric or inertial sensors) to propagate their state (position) estimates, and are equipped with exteroceptive sensors (e.g., laser range finders) that enable them to measure the relative position of other robots and targets. Additionally, one of the robots is equipped with a GPS receiver, that enables it to measure its position in the global coordinate frame. In real situations, this absolute position is usually needed for coordinating actions by the command center.[1] All these measurements are fused using an Extended Kalman Filter (EKF) in order to produce estimates of the position of the robots and targets. In our formulation, we assume that each robot has access to measurements of its absolute orientation, and that an upper bound on the variance of these measurements can be determined *a priori*. This is the case, for example, when each robot is equipped with a heading sensor of limited accuracy (e.g., a compass [14] or a sun sensor [15]) that directly measures its orientation, or if the robots infer their orientation from measurements of the structure of the environment (e.g., based on the direction of the walls when this is known *a priori* [16]).

The variance of the absolute orientation measurements that each robot receives defines an upper bound on each robot's orientation uncertainty. The availability of such a bound enables us to decouple the task of position estimation from that of orientation estimation, for the purpose of determining upper bounds on the performance of CLATT. Specifically, the state vector includes only the *positions* of the robots and targets, and the orientation estimates are used as inputs to the system, of which noise-corrupted observations are available. Clearly, the resulting EKF-based estimator is a suboptimal one, because the correlations between the position and orientation estimates of the robots are discarded. Therefore, by deriving an upper bound on the covariance of the estimates produced with this suboptimal, "position-only" estimator, we simultaneously determine an upper bound on the covariance of the position estimates that would result from using a "full-state" EKF estimator.

In our formulation, which follows from those of [17], [18] the metric employed for describing the accuracy CLATT is the covariance matrix of the position estimates. The time evolution of the covariance matrix in the EKF framework is described by the propagation and update equations (cf. Eqs. (8) and (14)). Combining these equations yields the Riccati recursion (cf. Eq. (15)), whose solution is the covariance of the error in the state estimate at each time-step, immediately following the propagation phase of the EKF. In the case of CLATT, certain matrices in this recursion are time-varying and a general closed-form expression for the time evolution of the covariance matrix does not exist. We thus resort to deriving *upper bounds* for the covariance, by exploiting the convexity and monotonicity properties of the Riccati recursion (cf. Lemmas 4.1 and 4.2). These properties allow for the formulation of *constant coefficient* Riccati recursions, whose solutions provide upper bounds for the positioning uncertainty in CLATT.

A. Position propagation

The discrete-time kinematic equations for the i-th robot are

$$x_{r_i}(k+1) = x_{r_i}(k) + V_i(k)\delta t\cos(\phi_i(k)) \quad (1)$$
$$y_{r_i}(k+1) = y_{r_i}(k) + V_i(k)\delta t\sin(\phi_i(k)) \quad (2)$$

where $V_i(k)$ denotes the robot's translational velocity at time-step k and δt is the sampling period. In the EKF, the estimates of the robot's position are propagated using the measurements of the its velocity, $V_{m_i}(k)$, and estimates of the its orientation, $\hat{\phi}_i(k)$.

By linearizing Eqs. (1) and (2), the error propagation equation for the robot's position is derived:

$$\begin{bmatrix} \widetilde{x}_{r_{i_{k+1|k}}} \\ \widetilde{y}_{r_{i_{k+1|k}}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \widetilde{x}_{r_{i_{k|k}}} \\ \widetilde{y}_{r_{i_{k|k}}} \end{bmatrix} + \begin{bmatrix} \delta t\cos(\hat{\phi}_i(k)) & -V_{m_i}(k)\delta t\sin(\hat{\phi}_i(k)) \\ \delta t\sin(\hat{\phi}_i(k)) & V_{m_i}(k)\delta t\cos(\hat{\phi}_i(k)) \end{bmatrix} \begin{bmatrix} w_{V_i}(k) \\ \widetilde{\phi}_i(k) \end{bmatrix}$$
$$\Leftrightarrow \widetilde{X}_{r_{i_{k+1|k}}} = I_2\widetilde{X}_{r_{i_{k|k}}} + G_{r_i}(k)W_i(k), \; i = 1\dots M \quad (3)$$

where[2] $w_{V_i}(k)$ is a zero-mean white Gaussian noise sequence of variance $\sigma^2_{V_i}$ affecting the velocity measurements, and $\widetilde{\phi}_i(k)$ is the error in the robot's orientation estimate at time k. This is modeled as a zero-mean white Gaussian noise sequence of variance $\sigma^2_{\phi_i}$.

From Eq. (3), the covariance matrix of the system noise affecting the i-th robot is:

$$Q_{r_i}(k) = C(\hat{\phi}_i(k)) \begin{bmatrix} \delta t^2\sigma^2_{V_i} & 0 \\ 0 & \delta t^2 V^2_{m_i}(k)\sigma^2_{\phi_i} \end{bmatrix} C^T(\hat{\phi}_i(k)) \quad (4)$$

where $C(\hat{\phi}_i)$ is the 2×2 rotation matrix associated with $\hat{\phi}_i$.

[1] The problem can be reformulated for relative position measurements only, however this case is outside the scope of this paper.

[2] Throughout this paper, $\mathbf{0}_{m\times n}$ denotes the $m\times n$ matrix of zeros, $\mathbf{1}_{m\times n}$ denotes the $m\times n$ matrix of ones, and I_n denotes the $n\times n$ identity matrix. The symbol $\otimes$ represents the Kronecker product for matrices.

A zero-velocity model is used to model the targets' motion [1], hence, the state propagation equations for the i-th target are

$$X_{T_{i_{k+1|k}}} = X_{T_{i_{k|k}}} + \delta t W_{T_i}(k)$$

where $W_{T_i} = [w_{T_{x_i}} \quad w_{T_{y_i}}]^T$ is the white noise process, introduced in the target's motion model to express the uncertainty in the actual motion of the target.

The error propagation equation is given by:

$$\widetilde{X}_{T_{i_{k+1|k}}} = I_2 \widetilde{X}_{T_{i_{k|k}}} + \delta t W_{T_i}(k) \tag{5}$$

and the covariance of the system noise of the i-th target is

$$Q_{T_i} = E\left\{\delta t^2 W_{T_i}(k) W_{T_i}^T(k)\right\} = \delta t^2 \sigma_T^2 I_2 = Q_T \tag{6}$$

where σ_T^2 is the variance of the targets' system noise along each axis, assumed to be identical for all targets. The state vector for the entire system is defined as the stacked vector comprising the positions of the robots and the targets, i.e.,

$$X = \begin{bmatrix} X_{r_1}^T & \cdots & X_{r_M}^T & X_{T_1}^T & \cdots & X_{T_N}^T \end{bmatrix}^T$$

Hence, the state transition matrix for the entire system at time-step k is $\mathbf{\Phi}_k = I_{2M+2N}$, and the covariance matrix of the system noise is:

$$\mathbf{Q}(k) = \begin{bmatrix} \mathbf{Q}_r(k) & \mathbf{0}_{2M\times 2N} \\ \mathbf{0}_{2N\times 2M} & \mathbf{Q}_T \end{bmatrix} \tag{7}$$

where $\mathbf{Q}_r(k) = \mathbf{Diag}(Q_{r_i}(k))$, and $\mathbf{Q}_T = I_N \otimes (Q_T)$ are block diagonal matrices describing the system noise covariance for the robots and targets, respectively.

The equation for propagating the covariance matrix of the state error is written as:

$$\mathbf{P}_{k+1|k} = \mathbf{P}_{k|k} + \mathbf{Q}(k) \tag{8}$$

where $\mathbf{P}_{\ell|k} = E\{\widetilde{X}_{\ell|k}\widetilde{X}_{\ell|k}^T\}$ is the covariance of the error in the estimate of $X(\ell)$, after the measurements up to time k have been processed.

B. Measurement model

At every time-step, the robots perform robot-to-robot and robot-to-target relative position measurements:

$$z_{ij} = C^T(\phi_i)\left(X_{S_{ij}} - X_{r_i}\right) + n_{z_{ij}} \tag{9}$$

where r_i is the observing robot, and $n_{z_{ij}}$ is the noise affecting this measurement. S_{ij} denotes the subject of the j-th measurement of robot i, i.e.,

$$S_{ij} \in \{r_1, r_2, \ldots, r_M, T_1, T_2, \ldots, T_N\} \setminus \{r_i\}$$

If the i-th robot performs M_i relative position measurements, the index j assumes integer values in the range $[1, M_i]$ to describe these measurements. By linearizing the last expression, the measurement error equation is obtained:

$$\widetilde{z}_{ij}(k+1) = H_{ij}(k+1)\widetilde{X}_{k+1|k} + \Gamma_{ij}(k+1)n_{ij}(k+1)$$

where

$$H_{ij}(k+1) = C^T(\hat{\phi}_i(k+1))\ H_{o_{ij}} \tag{10}$$

$$H_{o_{ij}} = \begin{bmatrix} 0_{2\times2} & \cdots & \underbrace{-I_2}_{r_i} & \cdots & \underbrace{I_2}_{S_{ij}} & \cdots & 0_{2\times2} \end{bmatrix}$$

$$\widetilde{X}_{k+1|k} = \begin{bmatrix} \cdots & \widetilde{X}_{r_i}^T & \cdots & \widetilde{X}_{S_{ij}}^T & \cdots \end{bmatrix}_{k+1|k}^T$$

$$\Gamma_{ij}(k) = \begin{bmatrix} I_2 & -C^T(\hat{\phi}_i(k+1))J\widehat{\Delta p}_{ij}(k+1) \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad n_{ij}(k) = \begin{bmatrix} n_{z_{ij}}(k+1) \\ \widetilde{\phi}_i(k+1) \end{bmatrix}$$

$$\widehat{\Delta p}_{ij}(k+1) = \widehat{X}_{S_{ij}\ k+1|k} - \widehat{X}_{r_i\ k+1|k}$$

The error, $n_{ij}(k+1)$, in each relative-position measurement comprises two independent components. Firstly, each observation is corrupted by noise in the measurement of the range and bearing. This noise is described by two independent, zero-mean, Gaussian processes, with variance $\sigma_{\rho_i}^2$ and $\sigma_{\theta_i}^2$, affecting the range and bearing measurements, respectively. A second source of error is the uncertainty in the measuring robot's orientation estimates, $\widetilde{\phi}_i(k+1)$. This error affects *all* relative position measurements performed by robot i, thus rendering them correlated. The covariance matrix of the relative position measurements recorded by robot i is given by [19]:

$$\boldsymbol{R}_i(k+1) = \boldsymbol{\Xi}_{\hat{\phi}_i}^T(k+1)\boldsymbol{R}_{o_i}(k+1)\boldsymbol{\Xi}_{\hat{\phi}_i}(k+1) \tag{11}$$

where $\boldsymbol{\Xi}_{\hat{\phi}_i}(k+1) = I_{M_i} \otimes C(\hat{\phi}_i(k+1))$, and

$$\begin{aligned}\boldsymbol{R}_{o_i}(k+1) =& \sigma_{\rho_i}^2 I_{2M_i} - D_i \operatorname{diag}\left(\frac{\sigma_{\rho_i}^2}{\hat{\rho}_{ij}^2}\right) D_i^T + \sigma_{\theta_i}^2 D_i D_i^T \\ &+ \sigma_{\phi_i}^2 D_i \mathbf{1}_{M_i\times M_i} D_i^T \end{aligned} \tag{12}$$

In this last expression, $\hat{\rho}_{ij}$ is the estimated range of the relative position measurement z_{ij}, and $D_i = \mathbf{Diag}\left(J\widehat{\Delta p}_{ij}\right)$ is the block-diagonal matrix with elements $J\widehat{\Delta p}_{ij}$, $j = 1 \ldots M_i$.

The measurement matrix describing all the relative-position measurements performed by robot i at each time-step is a matrix whose block rows are H_{ij}, $j = 1 \ldots M_i$, i.e.:

$$\boldsymbol{H}_i(k+1) = \boldsymbol{\Xi}_{\hat{\phi}_i}^T(k+1)\boldsymbol{H}_{o_i} \tag{13}$$

where $\boldsymbol{H}_{o_i}$ is a *constant* matrix with block rows $H_{o_{ij}}$, $j = 1 \ldots M_i$ (cf. Eq. (10)).

In addition to relative-position measurements, one of the robots receives measurements of its position in the global coordinate frame. Assuming that robot r_1 is equipped with a GPS receiver, the corresponding measurement matrix is:

$$\boldsymbol{H}_{o_0} = \begin{bmatrix} I_2 & \mathbf{0}_{2\times(2M+2N-2)} \end{bmatrix}$$

Accordingly, we define $\boldsymbol{R}_{o_0}$ as the covariance matrix of the GPS measurement, given by $\boldsymbol{R}_{o_0} = \sigma_{GPS}^2 I_2$.

The measurement matrix for the entire system, $\mathbf{H}(k+1)$, is defined as the block matrix with block rows $\boldsymbol{H}_i(k+1), i = 0 \ldots M$. Since the measurements performed by different robots are independent, the measurement covariance matrix, $\mathbf{R}(k+1)$, is a block-diagonal matrix with elements $\boldsymbol{R}_i(k+1), i = 0 \ldots M$ (cf Eq. (11)). The covariance update equation of the EKF is written as

$$\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{P}_{k+1|k}\mathbf{H}^T(k+1)\mathbf{S}^{-1}\mathbf{H}(k+1)\mathbf{P}_{k+1|k}$$

with $\mathbf{S} = \mathbf{H}(k+1)\mathbf{P}_{k+1|k}\mathbf{H}^T(k+1) + \mathbf{R}(k+1)$. Substitution from Eqs. (11) and (13) and simple algebraic manipulation cancels out the orientation-dependent terms, and yields the expression

$$\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{P}_{k+1|k}\mathbf{H}_o^T\mathbf{S}_o^{-1}\mathbf{H}_o\mathbf{P}_{k+1|k} \tag{14}$$

with $\mathbf{S}_o = \mathbf{H}_o\mathbf{P}_{k+1|k}\mathbf{H}_o^T + \mathbf{R}_o(k+1)$. In these equations

$\mathbf{H}_o$ is a matrix whose block rows are $\boldsymbol{H}_{o_i}$, while $\mathbf{R}_o$ is a block-diagonal matrix with elements $\boldsymbol{R}_{o_i}$.

IV. CLATT Positioning Accuracy Characterization

The time evolution of the covariance matrix of the position estimates in CLATT is described by the Riccati recursion, which can be derived by substituting the expression from Eq. (14) into Eq. (8). The resulting expression is:

$$\begin{aligned}\mathbf{P}_{k+1} &= \mathbf{P}_k - \mathbf{P}_k\mathbf{H}_o^T\left(\mathbf{H}_o\mathbf{P}_k\mathbf{H}_o^T + \mathbf{R}_o(k+1)\right)^{-1}\mathbf{H}_o\mathbf{P}_k \\ &+ \mathbf{Q}(k+1) \end{aligned} \quad (15)$$

where we use the substitutions $\mathbf{P}_k = \mathbf{P}_{k+1|k}$ and $\mathbf{P}_{k+1} = \mathbf{P}_{k+2|k+1}$ to simplify the notation. The initial value of this recursion is given by the position estimates' covariance matrix at the onset of the robots' tracking task, and is assumed to be an arbitrary positive semidefinite matrix, $\mathbf{P}_0$. We note that the matrices $\mathbf{Q}(k+1)$ and $\mathbf{R}_o(k+1)$ in this Riccati recursion are *time-varying*, and thus a closed-form expression for $\mathbf{P}_k$ cannot be derived for the general case. However, by exploiting the monotonicity and concavity properties of the Riccati recursion, we are able to derive *upper bounds* for the worst-case value, as well as for the average value of the covariance matrix at steady state.

A. Bound on worst-case steady-state covariance

In this section, we derive an upper bound for the position estimates' covariance matrix in CLATT. It can be shown [19] that the right-hand side of Eq. (15) is a matrix-increasing function of the covariance matrices $\mathbf{Q}(k+1)$ and $\mathbf{R}_o(k+1)$, as well as of the state covariance matrix $\mathbf{P}_k$. These properties allow us to prove the following lemma [19]:

Lemma 4.1: If $\mathbf{R}_u$ and $\mathbf{Q}_u$ are matrices such that $\mathbf{R}_u \succeq \mathbf{R}_o(k)$ and $\mathbf{Q}_u \succeq \mathbf{Q}(k)$, for all $k \geq 0$, then the solution to the Riccati recursion

$$\mathbf{P}^u_{k+1} = \mathbf{P}^u_k - \mathbf{P}^u_k\mathbf{H}_o^T\left(\mathbf{H}_o\mathbf{P}^u_k\mathbf{H}_o^T + \mathbf{R}_u\right)^{-1}\mathbf{H}_o\mathbf{P}^u_k + \mathbf{Q}_u \quad (16)$$

with the initial condition $\mathbf{P}^u_0 = \mathbf{P}_0$, satisfies $\mathbf{P}^u_k \succeq \mathbf{P}_k$ for all $k \geq 0$. □

Since $\mathbf{Q}_T$ is a constant matrix (cf. Eq. (7)), an upper bound for $\mathbf{Q}(k)$ can be found as:

$$\mathbf{Q}_u = \begin{bmatrix} \mathbf{Q}_{r_u} & \mathbf{0}_{2M\times 2N} \\ \mathbf{0}_{2N\times 2M} & \mathbf{Q}_T \end{bmatrix} \text{ where } \mathbf{Q}_{r_u} = \mathbf{Diag}(q_i I_2),$$
$$q_i \simeq \max(\delta t^2\sigma^2_{V_i}, \delta t^2 V_i^2\sigma^2_{\phi_i}) \quad (17)$$

For deriving this equation we have assumed that the velocity of each robot is approximately constant and equal to V_i [19].

An upper bound for $\mathbf{R}_{o_i}(k+1)$, $i = 1\ldots M$, can be derived by considering the maximum distance, ρ_{o_i}, at which relative position measurements can be recorded by robot i. This distance can, for example, be determined by the maximum range of the robots' relative position sensors. It can be shown that [19]

$$\boldsymbol{R}_i(k+1) \preceq \left(\sigma^2_{\rho_i} + M_i\sigma^2_{\phi_i}\rho_o^2 + \sigma^2_{\theta_i}\rho_o^2\right) I_{2M_i} = r_i I_{2M_i} = \boldsymbol{R}^u_i$$

The GPS measurement covariance is constant (i.e., its upper bound is trivially $\mathbf{R}^u_0 = \mathbf{R}_{o_0}$), and hence, an upper bound on $\boldsymbol{R}_o(k+1)$ is computed as

$$\mathbf{R}_o(k+1) = \mathbf{Diag}(\boldsymbol{R}_{o_i}(k+1)) \preceq \mathbf{Diag}(\boldsymbol{R}^u_i) = \mathbf{R}_u \quad (18)$$

Having derived upper bounds for $\mathbf{Q}(k+1)$ and $\mathbf{R}_o(k+1)$, mere substitution in Eq. (16) and numerical evaluation of the solution to the resulting recursion, yields an upper bound on the maximum possible uncertainty of the position estimates in CLATT, at any time instant after the deployment of the robot team. However, since the system is observable [19], after sufficient time the covariance matrix of the position estimates will reach a steady state, during which the covariance fluctuates around a mean value. Thus it is interesting to evaluate the upper bound $\mathbf{P}^u_k$ after sufficient time, i.e., as $k \to \infty$. This derivation is presented in Section V. In the remainder of this section, we study the time evolution of the *expected* covariance of the position estimates in CLATT.

B. Bound on average steady-state covariance

The expression in Eq. (16) determines the time evolution of the upper bound of the CLATT covariance matrix for a robot team with a given set of sensors and a known RPMG. This bound holds under any possible spatial configuration of the robots and the targets, regardless of their trajectories within their operational area. However, in realistic scenarios, the robots' and targets' relative positions can often be described by known probability distribution functions (e.g., when the robots follow the targets at given a distance). Such *a priori* knowledge of the distribution of the relative positions between robots and targets allows us to compute the *average* value of the matrix $\mathbf{R}_o(k+1)$, for example, by Monte Carlo simulations. This information can be exploited in order to compute a *tighter* upper bound for the *expected* covariance of the position estimates.

Specifically, it can be shown [19] that the right-hand side of Eq. (15) is a concave function of the matrices $\mathbf{P}_k$, $\mathbf{R}_o(k+1)$ and $\mathbf{Q}(k+1)$. This property enables us to employ Jensen's inequality [20] to prove, by induction, the following lemma [19]:

Lemma 4.2: If $\bar{\mathbf{R}} = E\{\mathbf{R}_o(k+1)\}$ and $\bar{\mathbf{Q}} = E\{\mathbf{Q}(k+1)\}$ then the solution to the following Riccati recursion

$$\bar{\mathbf{P}}_{k+1} = \bar{\mathbf{P}}_k - \bar{\mathbf{P}}_k\mathbf{H}_o^T\left(\mathbf{H}_o\bar{\mathbf{P}}_k\mathbf{H}_o^T + \bar{\mathbf{R}}\right)^{-1}\mathbf{H}_o\bar{\mathbf{P}}_k + \bar{\mathbf{Q}} \quad (19)$$

with initial condition $\bar{\mathbf{P}}_0 = \mathbf{P}_0$, satisfies $\bar{\mathbf{P}}_k \succeq E\{\mathbf{P}_k\}$ for all $k \geq 0$. □

Assuming that the robots' heading is uniformly distributed, the average value of the system noise covariance matrix is computed [19] by averaging over all values of orientation:

$$E\{Q_{r_i}\} = \frac{\delta t^2\sigma^2_{V_i} + \delta t^2 V_i^2\sigma^2_{\phi_i}}{2} I_2 = \bar{q}_i I_2 \quad (20)$$

and thus

$$\bar{\mathbf{Q}} = \begin{bmatrix} \bar{\mathbf{Q}}_r & \mathbf{0}_{2M\times 2N} \\ \mathbf{0}_{2N\times 2M} & \mathbf{Q}_T \end{bmatrix}, \text{ with } \bar{\mathbf{Q}}_r = \mathbf{Diag}\left(\bar{q}_i I_2\right) \quad (21)$$

In order to simplify the presentation, we assume a uniform distribution for the positions of the robots and targets in a square area of side α. Using the definition of $\boldsymbol{R}_{o_i}(k+1)$ in

Eq. (12), it can be shown that [19]

$$\bar{\mathbf{R}}_i = E\{\boldsymbol{R}_{o_i}\} = \left(\frac{1}{2}\sigma_{\rho_i}^2 + \frac{\alpha^2}{12}\sigma_{\phi_i}^2 + \frac{\alpha^2}{6}\sigma_{\theta_i}^2\right) I_{2M_i} + \frac{\alpha^2}{12}\sigma_{\phi_i}^2 \left(\mathbf{1}_{M_i\times M_i}\otimes I_2\right) \quad (22)$$

The covariance of the GPS measurement noise is constant, so its expected value is simply $\bar{\mathbf{R}}_0 = \boldsymbol{R}_{o_0}$, and thus the average value of $\mathbf{R}_o(k+1)$ is given by (cf. Eq. (11))

$$\bar{\mathbf{R}} = E\{\mathbf{R}_o(k+1)\} = \mathbf{Diag}(\bar{\mathbf{R}}_i) \quad (23)$$

Instead of uniform distributions, other distributions may be used for the derivation of $\bar{\mathbf{R}}$ and $\bar{\mathbf{Q}}$, but the results will not be necessarily in closed form. Nevertheless, Lemma 4.2 holds for *any* pdf.

V. Asymptotic Solutions to The Riccati Equation

Given the constant-coefficient Riccati recursions of Eq. (16) and Eq. (19), the upper bounds for the positioning performance of CLATT can be evaluated, by numerical integration of the respective recursions. As explained in Section IV, the asymptotic solutions to these equations are of particular interest, since they provide a characterization of the steady-state localization and tracking performance. In order to obtain the steady-state solutions of the Riccati recursions of Lemmas 4.1 and 4.2, we first note that by employing the substitutions

$$\mathbf{R}_s \rightarrow \mathbf{R}_u,\ \mathbf{Q}_s \rightarrow \mathbf{Q}_u \text{ and } \mathbf{R}_s \rightarrow \bar{\mathbf{R}}\ \ ,\ \mathbf{Q}_s \rightarrow \bar{\mathbf{Q}} \quad (24)$$

Eq. (16) and Eq. (19)) can be written as

$$\mathbf{P}_{k+1}^s = \mathbf{P}_k^s - \mathbf{P}_k^s\mathbf{H}_o^T\left(\mathbf{H}_o\mathbf{P}_k^s\mathbf{H}_o^T + \mathbf{R}_s\right)^{-1}\mathbf{H}_o\mathbf{P}_k^s + \mathbf{Q}_s \quad (25)$$

In [19], the steady-state solution to this Riccati recursion is shown to be:

$$\mathbf{P}_{ss}^s = \mathbf{Q}_s^{1/2}\mathbf{U}_s \operatorname{diag}\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{\lambda_i}}\right)\mathbf{U}_s^T\mathbf{Q}_s^{1/2} \quad (26)$$

where $\mathbf{C}_s = \mathbf{U}_s\Lambda\mathbf{U}_s^T$, $\Lambda = \operatorname{diag}(\lambda_i)$ is the SVD of:

$$\mathbf{C}_s = \mathbf{Q}_s^{1/2}\mathbf{H}_o^T\mathbf{R}_s^{-1}\mathbf{H}_o\mathbf{Q}_s^{1/2} \quad (27)$$

The upper bounds on the worst-case and the expected steady-state positioning uncertainty of CLATT can be evaluated by employing the substitutions shown in Eq. (24).

At this point, we should note that the upper bounds on the steady-state uncertainty depend on the topology of the RPMG, the accuracy of the proprioceptive and exteroceptive sensors of the robots, and the target noise covariance. However, the steady-state uncertainty is independent of the initial covariance of the robots and targets, which comes as no surprise, since the system is observable.

VI. Infinite Target Noise Covariance

When employing any of the common motion models (e.g., zero-velocity, zero-acceleration, zero-jerk [1]) for target tracking, the implicit assumption made is that *some information* about the target's motion characteristics is available. Our confidence in this information determines the selection of the variance of the process noise included in the motion model. When a low variance value is selected, this implies that our *a priori* knowledge is deemed very reliable. In general, the use of prior information in estimation results in increased accuracy. In order to study the most general case (i.e., when *no knowledge* about the motion of the targets is available), we now examine the positioning performance of CLATT in the limit as the process noise, in the zero-velocity model we have used, approaches infinity (i.e., we let $\sigma_T \rightarrow \infty$). This implies that after each EKF propagation step the position of the targets is completely unknown and corresponds to a *non-informative* prior for the targets' position before each update. By evaluating an upper bound for the accuracy of the position estimates for the robots and the targets in this scenario, we essentially determine the worst-case performance of CLATT when no target model is assumed.

Specifically, application of the matrix inversion lemma to the Riccati recursion in Eq. (25) leads to:

$$\mathbf{P}_{k+1}^s = \left((\mathbf{P}_k^s)^{-1} + \boldsymbol{\Psi}\right)^{-1} + \mathbf{Q}_s$$

where the term

$$\boldsymbol{\Psi} = \mathbf{H}_o^T\mathbf{R}_s^{-1}\mathbf{H}_o = \begin{bmatrix}\boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12}\\ \boldsymbol{\Psi}_{21} & \boldsymbol{\Psi}_{22}\end{bmatrix} \quad (28)$$

represents the information of the exteroceptive measurements at every time-step.

To represent the absence of an informative motion model for the target, we set

$$\mathbf{Q}_s = \begin{bmatrix}\mathbf{Q}_{rr} & \mathbf{0}_{2M\times 2N}\\ \mathbf{0}_{2N\times 2M} & \mu I_{2N}\end{bmatrix} \quad , \quad \mu \rightarrow \infty \quad (29)$$

Thus, after each propagation step, the covariance matrix $\mathbf{P}_k^s$ can be expressed in the form:

$$\mathbf{P}_k^s = \begin{bmatrix}\mathbf{P}_{rr_k}^s & \mathbf{P}_{rT_k}^s\\ \mathbf{P}_{Tr_k}^s & \mathbf{P}_{TT_k}^s + \mu I_{2N}\end{bmatrix} \quad (30)$$

where $\mathbf{P}_{rr_k}^s$ is the covariance matrix corresponding to the robots' position estimates, $\mathbf{P}_{TT_k}^s + \mu I_{2N}$ is the covariance matrix of the targets' position estimates, and $\mathbf{P}_{rT_k}^s$ is the correlation between the robots and targets. Eq. (30) thus yields

$$\begin{aligned}\mathbf{P}_{k+1}^s &= \begin{bmatrix}\mathbf{P}_{rr_{k+1}}^s & \mathbf{P}_{rT_{k+1}}^s\\ \mathbf{P}_{Tr_{k+1}}^s & \mathbf{P}_{TT_{k+1}}^s\end{bmatrix}\\ &= \lim_{\mu\rightarrow\infty}\left(\begin{bmatrix}\mathbf{P}_{rr_k}^s & \mathbf{P}_{rT_k}^s\\ \mathbf{P}_{Tr_k}^s & \mathbf{P}_{TT_k}^s + \mu I_2\end{bmatrix}^{-1} + \begin{bmatrix}\boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12}\\ \boldsymbol{\Psi}_{21} & \boldsymbol{\Psi}_{22}\end{bmatrix}\right)^{-1} + \mathbf{Q}_s\\ &= \left(\begin{bmatrix}(\mathbf{P}_{rr_k}^s)^{-1} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} + \begin{bmatrix}\boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12}\\ \boldsymbol{\Psi}_{21} & \boldsymbol{\Psi}_{22}\end{bmatrix}\right)^{-1} + \mathbf{Q}_s \quad (31)\end{aligned}$$

Considering the submatrix of $\mathbf{P}_{k+1}^s$ corresponding to the robots separately, we obtain the following recursion:

$$\mathbf{P}_{rr_{k+1}}^s = \mathbf{P}_{rr_k}^s\left(I_{2M} + \boldsymbol{\Psi}_{rr}\mathbf{P}_{rr_k}^s\right)^{-1} + \mathbf{Q}_{rr} \quad (32)$$

where

$$\boldsymbol{\Psi}_{rr} = \boldsymbol{\Psi}_{11} - \boldsymbol{\Psi}_{12}\boldsymbol{\Psi}_{22}^{-1}\boldsymbol{\Psi}_{21} \quad (33)$$

At this point, we should note that all the eigenvalues of $\boldsymbol{\Psi}_{rr}$ are positive, since $\boldsymbol{\Psi}_{rr}$ is the Schur complement of $\boldsymbol{\Psi}_{11}$ in the positive definite matrix $\mathbf{H}_o^T\mathbf{R}_s^{-1}\mathbf{H}_o$.

In [19] the steady-state solution of Eq. (32) is shown to be:

$$\mathbf{P}_{rr}^s = \mathbf{Q}_{rr}^{1/2}\mathbf{U}_r^{'}\operatorname{diag}\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{\lambda_i^{'}}}\right)\mathbf{U}_r^{'T}\mathbf{Q}_{rr}^{1/2} \quad (34)$$

where

$$\mathbf{C}_r = \mathbf{Q}_{rr}^{1/2}\boldsymbol{\Psi}_{rr}\mathbf{Q}_{rr}^{1/2} = \mathbf{U}_r^{'}\operatorname{diag}(\lambda_i^{'})\mathbf{U}_r^{'T} \quad (35)$$

Since the covariance of the targets' motion model noise is infinite, the location of the target immediately after each propagation step approaches infinity (cf. Eq. (30)). However, once robot-to-target measurements are received and processed (i.e., after each update step), the covariance for the position of the targets can be computed as follows (cf. Eqs. (31), (28)):

$$\mathbf{P}^s_{TT} = \left(\mathbf{\Psi}_{22} - \mathbf{\Psi}_{21}\left((\mathbf{P}^s_{rr})^{-1} + \mathbf{\Psi}_{11}\right)^{-1}\mathbf{\Psi}_{12}\right)^{-1} \quad (36)$$

A. Improved localization due to target measurements

We now show that when robots localize while simultaneously estimating the position of targets, the resulting accuracy for the robots' position is improved. A complete proof for this result cannot be included in this paper due to space limitations. A detailed derivation is presented in [19].

For simplicity of presentation, we show this result for the constant-coefficient system model represented by the Riccati recursion in Eq. (25), assuming that $\mathbf{R}_s$ is a diagonal matrix. As previously mentioned, infinite process-noise variance for the targets defines the *worst-case* scenario in terms of localization accuracy. Thus it suffices to show that even in this case the covariance for the robots' position estimates of Eq. (34) is smaller than the covariance computed when the targets are not included in the state vector of the system (i.e., no target observations are incorporated).

We note that the block rows of the measurement matrix $\mathbf{H}_o$ can be permuted, so that the following partitioning holds:

$$\mathbf{H}_o = \begin{bmatrix} \mathbf{H}_{rr} & \mathbf{0} \\ \mathbf{H}_{rT} & \mathbf{H}_{TT} \end{bmatrix} \quad (37)$$

where $\mathbf{H}_{rr}$ is the measurement matrix that would arise if *only* robot-to-robot and GPS measurements were employed for localization, and the rest of the block rows describe the robot-to-target measurements.

The information matrix for the entire system, $\mathbf{\Psi}$, can be expressed as:

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{H}^T_{rr} & \mathbf{H}^T_{rT} \\ \mathbf{0} & \mathbf{H}^T_{TT} \end{bmatrix} \begin{bmatrix} \mathbf{R}^{-1}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^{-1}_T \end{bmatrix} \begin{bmatrix} \mathbf{H}_{rr} & \mathbf{0} \\ \mathbf{H}_{rTT} & \mathbf{H}_{TT} \end{bmatrix} =$$

$$\begin{bmatrix} \mathbf{H}^T_{rr}\mathbf{R}^{-1}_r\mathbf{H}_{rr} + \mathbf{H}^T_{rT}\mathbf{R}^{-1}_T\mathbf{H}_{rT} & \mathbf{H}^T_{rT}\mathbf{R}^{-1}_T\mathbf{H}_{TT} \\ \mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{rT} & \mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{TT} \end{bmatrix} \quad (38)$$

Thus the information matrix for the robots, $\mathbf{\Psi}_{rr}$, is (cf. (33)):

$$\mathbf{\Psi}_{rr} = \mathbf{H}^T_{rr}\mathbf{R}^{-1}_r\mathbf{H}_{rr} + \mathbf{H}^T_{rT}\mathbf{R}^{-1}_T\mathbf{H}_{rT}$$
$$- \mathbf{H}^T_{rT}\mathbf{R}^{-1}_T\mathbf{H}_{TT}\left(\mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{TT}\right)^{-1}\mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{rT} \quad (39)$$

When the targets are ignored (the case of Cooperative Localization (CL) only), the robots' positioning uncertainty is described by the Riccati recursion

$$\mathbf{P}_{rr_{k+1}} = \mathbf{P}_{rr_k}\left(I_{2M} + \mathbf{\Psi}_{CL}\mathbf{P}_{rr_k}\right)^{-1} + \mathbf{Q}_{rr} \quad (40)$$

where the information matrix is now $\mathbf{\Psi}_{CL} = \mathbf{H}^T_{rr}\mathbf{R}^{-1}_r\mathbf{H}_{rr}$, and we obtain

$$\mathbf{\Psi}_{rr} - \mathbf{\Psi}_{CL} =$$
$$\mathbf{H}^T_{rT}\mathbf{R}^{-1}_T\left(\mathbf{R}_T - \mathbf{H}_{TT}\left(\mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{TT}\right)^{-1}\mathbf{H}^T_{TT}\right)\mathbf{R}^{-1}_T\mathbf{H}_{rT}$$

The term $\mathbf{R}_T - \mathbf{H}_{TT}\left(\mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{TT}\right)^{-1}\mathbf{H}^T_{TT}$ is the Schur complement of $\mathbf{R}_T$ in the positive semidefinite matrix:

$$\begin{bmatrix} \mathbf{R}_T & \mathbf{H}_{TT} \\ \mathbf{H}^T_{TT} & \mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{TT} \end{bmatrix} \quad (41)$$

and therefore

$$\mathbf{R}_T - \mathbf{H}_{TT}\left(\mathbf{H}^T_{TT}\mathbf{R}^{-1}_T\mathbf{H}_{TT}\right)^{-1}\mathbf{H}^T_{TT} \succeq \mathbf{0} \Rightarrow \mathbf{\Psi}_{rr} - \mathbf{\Psi}_{CL} \succeq \mathbf{0}$$

This implies that the information matrix for the robots' position in the case of CLATT is larger (in the positive semi-definite sense) than the corresponding information matrix during CL. It is straightforward to show that the solution of the Riccati recursion is a matrix-decreasing function of the information matrix, and therefore, the covariance matrix of the robots' position estimates in the case of CLATT is *smaller*.

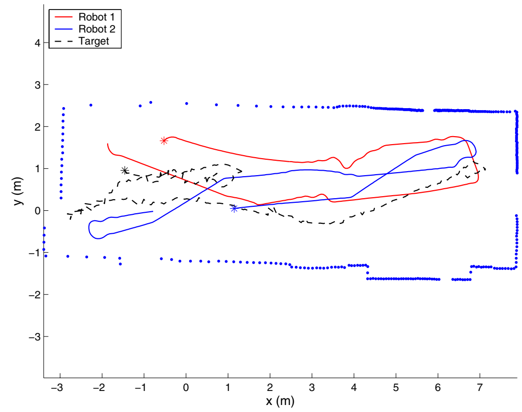


Fig. 1. Trajectories of the robots and the target during the experiment. A sample laser scan acquired by robot 1 is superimposed (after being transformed to the global frame), in order to illustrate the geometry of the area where the robots operate.

VII. Experiments and Simulations

In order to demonstrate the validity of the theoretical analysis in a realistic setting, we have conducted a real-world experiment. A team comprising two Pioneer 3 robots, each equipped with two opposite-facing SICK LMS200 laser scanners to provide a 360° field of view, was employed. Additionally, one Pioneer 1 robot is used as the target. The robots move randomly at a constant velocity of 0.1m/sec, while performing CLATT in an area of approximate dimensions 10m×4m. The estimated trajectories of the robots and the target are shown in Fig. 1. In this experiment, absolute position and orientation measurements are obtained by employing line-fitting on the laser scans. The upper bound for the standard deviation of the position measurements is $\sigma_{GPS} = 0.2$m and for the absolute orientation measurements is $\sigma_\phi = 1°$. Only robot 1 records absolute position measurements, while both robots measure the relative position of the target, and robot 1 measures the relative position of robot 2 at every time step. The robots are equipped with wheel encoders that provide velocity measurements with standard deviation $\sigma_V = 5\times10^{-3}$m/sec. The odometry measurements are available at a rate of 10Hz, while the laser scanners provide measurements at a frequency of 2Hz.

In Fig. 2, the time evolution of the diagonal elements of the position estimates' covariance matrix is shown, and compared to the worst-case upper bounds. We note at this

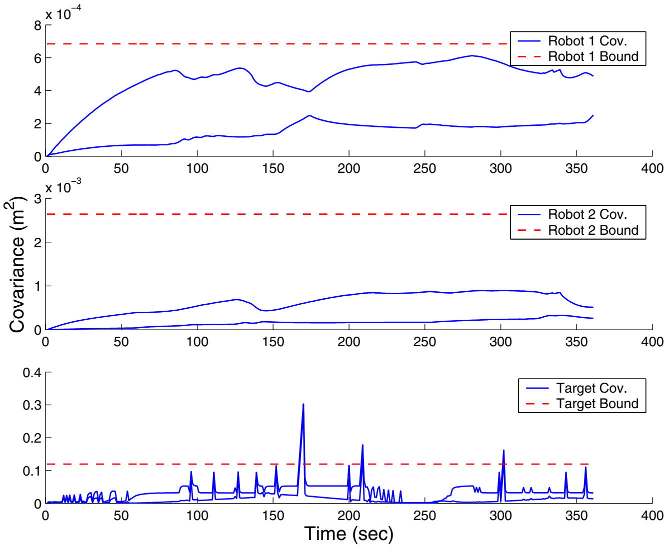


Fig. 2. Comparison of the robots' and target's position covariance computed during the experiment (solid blue lines) against the analytically-computed worst-case bound (dashed red lines). In each plot, the diagonal elements of the covariance matrix corresponding to both the x- and y- axes are plotted. The intervals that the covariance of the target position estimate exceeds the bound correspond to periods when robot-to-target measurements were not available due to occlusions.

point that, due primarily to the existence of occlusions and data association failures, the robot-to-target measurements were not possible at every time instant. As a result, the RPMG did not remain constant for the entire duration of the experiment. The spikes in the target covariance occur at the occasions when none of the robots was able to measure the position of the target. However, once the robot-to-target measurements become available again, the uncertainty falls to levels that are correctly predicted by the bounds. This is not a surprising result, as the system is observable, and thus its covariance converges to the steady-state value independent of possible temporary reconfigurations of the RPMG.

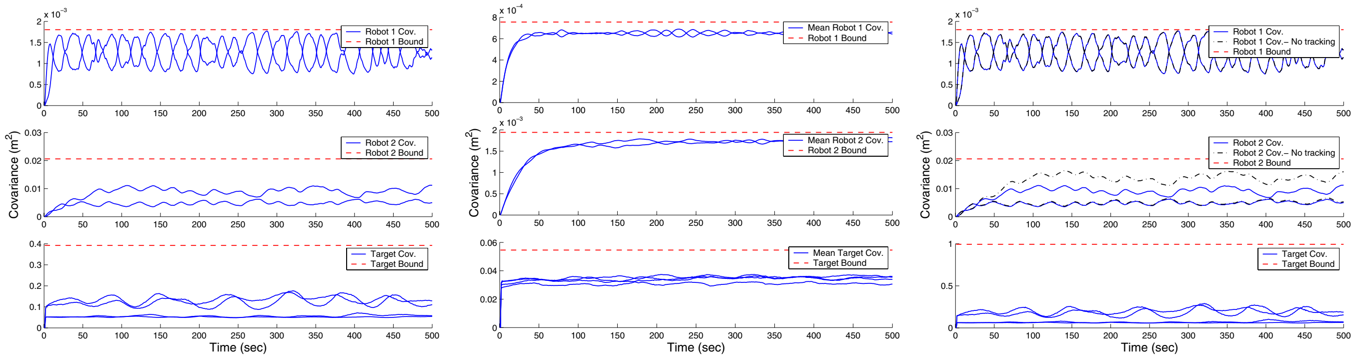
In order to further illustrate the properties of the derived bounds, simulation results are also presented. In the simulation setup, two robots and two targets are moving in a square of dimensions 30×30m. The velocity of the robots is kept constant at $V = 0.25$m/s, while their orientation changes randomly, using samples drawn from a uniform distribution. The standard deviation of the robots' velocity measurement noise is $\sigma_V = 0.05V$ and the standard deviation of the errors in the orientation measurements is $\sigma_\phi = 1°$, for both robots. The values selected for the standard deviations of the exteroceptive measurements are $\sigma_\theta = 2°$ for the bearing measurements, and $\sigma_\rho = 0.15$m for the range measurements. In addition, one of the robots is equipped with GPS, providing absolute position measurements with a standard deviation of $\sigma_{GPS} = 0.25$m. For the results presented in this section, both robots measure relative position of both targets, and the GPS-bearing robot measures the relative position of the other robot. The targets being tracked in this experiment are two robots, identical to the tracking ones. For the motion model of the targets we have used $\sigma_T = 0.5$m.

In order to demonstrate the validity of the bound on the worst-case steady-state covariance of CLATT, presented in Section V, a particularly adverse scenario for the initial position of the robots and the targets is considered. Specifically, the robot-to-robot and robot-to-target distances are large, and thus the relative position measurements carry limited positioning information, since the error in these measurements increases with distance. In Fig. 3(a), the time evolution of the position estimates' covariance for the robots and the targets are shown, and compared to the theoretical steady-state worst-case performance bound. Clearly, the upper bound is indeed larger than the steady-state covariance of the targets and robots. It can also be seen that the covariance bound of the robot which is equipped with GPS is tighter. This is attributed to the fact that the GPS accuracy is the dominant factor in determining this robot's position accuracy, and the uncertainty in the GPS measurements is exactly known.

Although the bound of Lemma 4.1 accounts for the worst-case accuracy of CLATT, it does not yield a sufficient performance description when the robots and targets are more evenly distributed in space. In such cases, employing the steady-state performance bound on the *expected* uncertainty, results in a closer description of the positioning performance, as demonstrated in Fig. 3(b). In these plots, the average values (over 100 runs) of the covariance in CLATT, are compared against the theoretically derived bounds on the expected uncertainty. For each run of the algorithm, the initial positions of the targets and the robots were selected using samples from a uniform distribution. Comparison of the covariance of the robots' and targets' position estimates with the corresponding bounds demonstrates that when available information about the distribution of the robots' and targets' positions is exploited, (by employing the expressions of Lemma 4.2), a better characterization of the expected accuracy of the position estimates is achieved, which results in tighter bounds.

Finally, simulation experiments were conducted, to study the effects of using an infinite target noise covariance. The solid lines in the plots of Fig. 3(c) present the time evolution of the covariance of the robots' and targets' position estimates when performing CLATT with no prior knowledge about the targets' motion characteristics (i.e., infinite target noise covariance). For comparison, the dash-dotted lines show the covariance of the robots' position estimates when the robots localize ignoring the presence of the targets. This figure confirms that when performing CLATT, the localization information obtained by the simultaneous observation of a target by multiple robots results in substantial improvement of the robots' position accuracy. As expected, the amount of improvement for the robot that is equipped with a GPS (and therefore mostly relies on absolute measurements for position estimation) is almost negligible.

An additional interesting observation can be made by comparing the covariance upper bound for the case of a non-informative motion model (dashed lines in Fig. 3(c)) with the bound previously computed with $\sigma_T = 0.5$m (cf. Fig. 3(a)). The numerical values of the upper bounds for the robots differ by less than 0.1%. This indicates that, in this particular setup, using *a priori* information for the



(a) (b) (c)

Fig. 3. (a) Comparison of the robots' and targets' actual position covariance (solid blue lines) against the worst-case bound (dashed red lines). In each plot, the diagonal elements of the covariance matrix corresponding to both the x- and y- axes are plotted. Note the different scales in the plots' vertical axes. (b) The mean covariance values for the robots' and targets' position uncertainty (solid blue lines), computed over 100 runs, vs. the upper bounds on the expected covariance (dashed red lines). In each plot, the diagonal elements of the mean covariance matrix, corresponding to both the x- and y- axes are plotted. (c) The top two plots compare the robots' position covariance in the case of CLATT with an infinite target noise covariance (solid blue lines), with the covariance when the targets are ignored (dash-dotted black lines). The third plot shows the covariance of the targets' position estimate, when a non-informative model is employed for tracking. The dashed red lines represent the corresponding upper bounds for the latter case.

targets' motion results in a negligible gain for the robot's localization accuracy over using a non-informative target model. However, this information significantly affects the accuracy with which the targets' positions can be estimated, resulting in 50% smaller covariance values, as shown in the corresponding figures.

VIII. Conclusion

In this paper we studied the problem of determining upper bounds for the covariance of the position estimates in Cooperative Localization and Target Tracking (CLATT). The developed analytical expressions enable us to *predict* the worst-case performance of a team of robots with a given set of sensors in a particular tracking scenario. Moreover, when *a priori* knowledge about the distribution of the relative positions of the robots and targets is available, this can be utilized in order to derive a tighter upper bound for the *expected* value of the covariance of the position estimates. In our work, we employed a zero-velocity motion model for the targets. By evaluating the upper bound of the steady-state covariance of the position estimates, in the limit as the targets' motion model becomes non-informative, we computed the worst-case performance of tracking when no target motion model is assumed. The effect of jointly estimating the positions of the targets and robots on the performance of localization is studied, and it is shown that CLATT outperforms Cooperative Localization, in terms of the robots' positioning accuracy.

References

[1] Y. Bar-Shalom, *Multitarget-multisensor tracking: Principles and techniques*. YBS Publishing, 1995.
[2] A. W. Stroupe, "Collaborative execution of exploration and tracking using move value estimation for robot teams (MVERT)," Ph.D. dissertation, Carnegie Mellon University, September 2003.
[3] A. Ribeiro, G. B. Giannakis, and S. I. Roumeliotis, "SOI-KF: Distributed Kalman filtering with low-cost communications using the sign of innovations," *IEEE Trans. Signal Processing*, vol. 54, no. 12, pp. 4782–4795, Dec 2006.
[4] L. E. Parker, "Distributed algorithms for multi-robot observation of multiple moving targets," *Autonomous Robots*, vol. 12, no. 3, pp. 231–255, 2002.
[5] B. Jung and G. S. Sukhatme, "Tracking targets using multiple robots: The effect of environment occlusion," *Autonomous Robots*, vol. 13, no. 3, pp. 191–205, 2002.
[6] ——, "A generalized region-based approach for multi-target tracking in outdoor environments," in *IEEE Int. Conf. on Robotics and Automation*, New Orleans, LA, April 26 - May 1 2004, pp. 2189–2195.
[7] M. Mazo, A. Speranzon, K. Johansson, and X. Hu, "Multi-robot tracking of a moving object using directional sensors," in *IEEE International Conference on Robotics and Automation*, New Orleans, LA, April 26 - May 1 2004, pp. 1103–1108.
[8] A. W. Stroupe, R. Ravichandran, and T. Balch, "Value-based action selection for exploration and mapping with robot teams," in *IEEE International Conference on Robotics and Automation*, New Orleans, LA, April 26 - May 1 2004, pp. 4190– 4197.
[9] K. Chang, R. Saha, and Y. Bar-Shalom, "On optimal track-to-track fusion," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, no. 4, pp. 1271–1276, Oct 1997.
[10] K. Chang, T. Zhi, and R. Saha, "Performance evaluation of track fusion with information matrix filter," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 38, no. 2, pp. 455–466, Apr 2002.
[11] X. Zhang, P. Willett, and Y. Bar-Shalom, "The Cramer-Rao bound for dynamic target tracking with measurement origin uncertainty," in *IEEE Conf. on Decision and Control*, Las Vegas, NV, Dec 2002.
[12] A. Bessell, B. Ristic, A. Farina, X. Wang, and M. Arulampalam, "Error performance bounds for tracking a maneuvering target," in *Proceedings of the Sixth International Conference on Information Fusion*, Cairns, Australia, 8-11 July 2003, pp. 903–910.
[13] M. Hernandez, A. Marrs, S. Maskell, and M. Orton, "Tracking and fusion for wireless sensor networks," in *Proc. of the Fifth Int. Conf. on Information Fusion*, Annapolis, MD, 8-11 July 2002, pp. 1023–1029.
[14] A. Georgiev and P. K. Allen, "Localization methods for a mobile robot in urban environments," *IEEE Transactions on Robotics*, vol. 20, no. 5, pp. 851–864, October 2004.
[15] R. Volpe, "Mars rover navigation results using sun sensor heading determination," in *Proc. of the IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Kyingju, Korea, Oct. 17-21 1999, pp. 469–467.
[16] S. Pfister, S. Roumeliotis, and J. Burdick, "Weighted line fitting algorithms for mobile robot map building and efficient data representation," in *Proceedings of the IEEE International Conference on Robotics and Automation*, Taipei, Taiwan, Sep. 14-19 2003, pp. 1304–1311.
[17] A. I. Mourikis and S. I. Roumeliotis, "Performance bounds for cooperative simultaneous localization and mapping (C-SLAM)," in *Proc. of Robotics: Science and Systems*, Cambridge, MA, June 2005, pp. 73–80.
[18] ——, "Performance analysis of multirobot cooperative localization," *IEEE Transactions on Robotics*, vol. 22, no. 4, pp. 666–681, Aug. 2006.
[19] F. M. Mirzaei, A. I. Mourikis, and S. I. Roumeliotis, "Analysis of positioning uncertainty in cooperative localization and target tracking (CLATT)," Dept. of Comp. Sci., Univ. of Minnesota, Tech. Rep., 2005.
[20] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.