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Model-Based Sensor Fault Detection and Isolation System for Unmanned Ground Vehicles: Theoretical Aspects (part I)

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Abstract— **This paper presents theoretical details of a model-based sensor fault detection and isolation system (SFDIS) applied to unmanned ground vehicles (UGVs). Structural analysis is applied to the nonlinear model of the vehicle for residual generation. Two different solutions have been proposed for developing the residual evaluation module. The vehicle sensor suite includes a Global Positioning System (GPS) antenna, an Inertial Measurement Unit (IMU), and two incremental optical encoders.**

I. Introduction

THIS paper has been motivated by the challenge to derive a model-based sensor fault detection and isolation (FDI) system for an unmanned ground vehicle (UGV). A complete theoretical study is presented here. The UGV, in this case, is supposed to be equipped with a sensor suite that includes a Global Positioning System (GPS) antenna measuring absolute position in the geodetic coordinates, an Inertial Measurement Unit (IMU) measuring robot linear accelerations and angular velocities, and incremental optical encoders mounted on motors measuring motor rotation.

Given a UGV (or any system), faults may be overcome by using robust "model-based fault diagnosis" defined as [1] "*...the determination of faults of a system from the comparison of available system measurements with a priori information represented by the mathematical model of the system through generation of residual quantities and their analysis*". Residuals are zero when no faults occur and non-zero otherwise, detecting the presence of a fault (faults). Residual quantities differ for each fault, allowing detection of the specific occurred fault (faults). From the actual implementation and testing point of view, model-based approaches do not require additional hardware components to realize an FDI algorithm since the algorithm may be implemented and tested in real-time via software (analytical redundancy).

A model-based fault diagnosis system consists in principle of a residual generation module and a residual evaluation module [2] that evaluates residuals deciding about the likelihood and/or presence of a fault. The decision process/rule applied to determine if any faults have occurred may be a threshold test on the instantaneous values or moving averages of the residuals, or it may follow statistical decision theory techniques.

The FDI problem has been widely investigated, and there exist many publications on the subject. Classification of fault types is presented in [3]. FDI approaches include diagnostic observers [4], parity equations [5], parameter estimation [6], [7] and state estimation [8], while surveys are provided in [3], [9] and [10]. Structural analysis based techniques are the topic of research presented in [11], [12], while [13] discusses how structural analysis may be used to find a minimal set of additional sensors to achieve full single fault isolation capability. In [14] a structural approach is investigated for complex systems, and applied to a ship propulsion benchmark that is presented in [15]. A structural analysis based method is also used in [16], [17] to extract system's inherent redundant information. Several approaches have been proposed to detect changes in signals or systems. They include likelihood ratio based approaches such as the Generalized Likelihood Ratio (GLR) test [18] or the marginalized likelihood ratio test [19], both effective whenever an accurate and tractable signal model exists and can be implemented. On-line versions based on statistical filtering have also shown good performance [18], [20] while other model-based approaches performing efficient off-line Bayesian segmentation include [21] and [22]. Other general and ad-hoc model-free methods have been designed to detect changes in signals with typical examples being time-frequency approaches [23] and wavelet approaches [24], [25].

In this work, additive and abrupt sensor faults have been considered describing changes in the system states interpreted as sensor faults. The residual generator module [14] generates the specific residuals to detect sensor faults following structural analysis [11], [12], [26], [27] of the (system) UGV nonlinear model, determining existing redundancies in sensors. Structural analysis depends not on analytical relations, but on relations between a variable and a constraint, allowing exploration of fundamental system properties using structure graphs. This is essentially a way to indicate which equations (or constraints) in a system (UGV in this case) are needed to find a solution for its variables. If there are more equations than needed, excess equations may be used to check the validity of observations. If such excess equation is not valid, the unmatched constraint called residual indicates the presence of a fault in the system (UGV) [28].

The residual evaluation module reduces to the problem of detecting a change in the mean of a random sequence. In this paper, two different solutions are also proposed for the residual evaluation module. The first solution includes a threshold test based on the minimum and maximum values of the generated residuals to decide if a fault has occurred or not. Thresholds are updated using a sliding window of a fixed number of sequential sample readings (the window size may vary). The second one is a particle filtering-based likelihood ratio decision solution.

Experimental validation of the proposed scheme has been performed on a differential drive mobile robot, the ATRV-Jr manufactured by iRobot, and it is presented in the second part of this work [29] (previous experimental results are also reported in [30]).

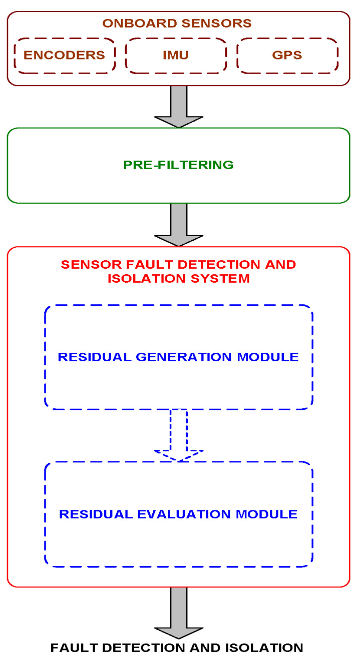

Fig. 1 Developed fault detection and isolation

Section II of this paper provides formulation of the problem and mathematical model of the considered UGV. Pre-filtering technique of the inertial sensors is described in Section III. Section IV presents the developed model-based fault detection and isolation system, explaining theoretical aspects of the proposed residual generation module and the residual evaluation module. In Section V, several remarks conclude this paper.

II. PROBLEM FORMULATION

Due to their navigation capabilities, unmanned ground vehicles are becoming every day more important for a wide variety of applications; for instance, they are able to autonomously navigate for long time and in different environment situations. The full development of a navigation system for UGVs requires solving a number of difficult technical challenges. Different and heterogeneous sensors have to be fused together to permit an accurate positioning of the vehicle, and a sensor fault detection and isolation system is needed to monitor the operation status of the navigation sensors.

This paper presents theoretical aspects for developing a sensor fault detection and isolation system (SFDIS) applied to unmanned ground vehicles. The considered vehicle is equipped with a Global Positioning System (GPS) antenna, an Inertial Measurement Unit (IMU), and two incremental optical encoders. The SFDIS has to be able to detect every single and multiple sensor fault in the presence of noise-corrupted measurements, and when possible to isolate them. Model-based FDI techniques are a feasible solution to this problem. In order to successfully apply these techniques, mathematical modeling of the system is required. In the following, the mathematical model of the vehicle is introduced.

The differential drive UGV is assumed to move in an area devoid of slopes (here the altitude $h(t)$ is considered constant) and without wheel slippage.

For such a vehicle, the NE tangent plane is considered with the *X*-direction coincident with *N*-direction and, the *E*-direction with *Y*-direction, respectively [31]. In this frame $\psi(t)$ describes the angle between the main axis of the robot and the *X*-direction. The kinematics equations have the following forms [31]:

$$\dot{x}(t) = \dot{f}(t)\cos\psi(t) \tag{1}$$

$$\dot{y}(t) = \dot{f}(t)\sin\psi(t) \tag{2}$$

$$\dot{\psi}(t) = r(t) \tag{3}$$

where $\dot{f}(t)$ and $r(t)$ are, respectively, the forward and angular velocities of the robot, expressed by:

$$\dot{\psi}(t) = \frac{d}{2L}\left(\omega_R(t) - \omega_L(t)\right) \tag{4}$$

$$\dot{f}(t) = \frac{d}{4}\left(\omega_R(t) + \omega_L(t)\right) \tag{5}$$

where $\omega_R(t)$ and $\omega_L(t)$ are the angular velocities of the right and left wheels, respectively, d is the wheel diameter and L is the distance between the wheels. Localization of the UGV in a two-dimensional space requires knowledge of coordinates x and y of the midpoint between the two driving wheels and of angle $\psi(t)$.

The set of measurement devices, used for the robot localization, consists of: two incremental optical encoders mounted on wheel motors which provide right and left wheel angle velocities $\omega_R(t)$ and $\omega_L(t)$, respectively; an Inertial Measurement Unit which provides the forward linear acceleration $\ddot{f}(t)$ and the angular velocity $r(t)$ of the vehicle; a Global Positioning System antenna measuring the latitude $\lambda(t)$, the longitude $\Phi(t)$ and altitude $h(t)$ with respect to the Earth-Centered-Earth-Fixed (ECEF) frame. The last set of measurements is related to the vehicle variables by the set of following equations:

$$\dot{x}(t) = \dot{\lambda}(t) R_{\lambda_0} \quad (6)$$

$$\dot{y}(t) = \dot{\Phi}(t) R_{\Phi_0} \cos\lambda(t) \quad (7)$$

and the constants R_{λ_0} and R_{Φ_0} depend on the altitude $h(t)$, on the radius of curvature and on the transverse radius of curvature [31], which are all assumed constant for the developed experiments without loss of generality.

III. Filtering of inertial sensors

Any sensor system is affected by errors, thus a pre-filtering of sensor measurements is required. In particular, an inertial system is characterized by position errors that increase with time and distance [33]. Thus, the motivation to build error models for inertial sensors is to reduce the effect of unbounded position and orientation errors. On the UGV under consideration, only one gyroscope is used for yaw rate measurements, and one accelerometer is used for forward acceleration measurements.

A Kalman filter (KF) is derived to estimate the true values of orientation, angular rate, linear acceleration, velocity, position and errors associated with them. The error model has been derived using the Levenberg-Marquardt iterative least squares fit method [33], [34] to fit data from the gyroscope and the accelerometer.

The yaw rate readings $z_1(k)$ of the gyro and the forward acceleration readings $z_2(k)$ have the following forms:

$$\begin{aligned} z_1(k) &= \dot{\psi}(k) + \varepsilon_{\dot{\psi}}(k) + v_1(k) \\ z_2(k) &= \ddot{f}(k) + \varepsilon_{\ddot{f}}(k) + v_2(k) \end{aligned} \quad (8)$$

where $\ddot{f}$ is the acceleration of the robot in the robot coordinate frame, ψ is the Euler angle around z-axis, $\varepsilon_{\dot{\psi}}(k)$, $\varepsilon_{\ddot{f}}(k)$ are additive drifts and $v_1(k)$, $v_2(k)$ are additive zero-mean white noise. Defining

$$\mathbf{x}(k) := \big[\psi(k) \quad \dot{\psi}(k) \quad \ddot{\psi}(k) \quad \dddot{\psi}(k) \quad \varepsilon_{\psi}(k) \quad \varepsilon_{\dot{\psi}}(k) \quad f(k) \quad \dot{f}(k) \quad \ddot{f}(k) \quad \varepsilon_{\ddot{f}}(k)\big]^T$$

and $z(k) := \big[z_1(k) \quad z_2(k)\big]^T$, $v(k) := \big[v_1(k) \quad v_2(k)\big]^T$ the output readings model (8) has the following form:

$$\mathbf{z}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) \quad (9)$$

where

$$\mathbf{C} := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Making use of Levenberg-Marquardt iterative least squares fit method for deducing a linear parametric model to fit data from the gyroscope and the accelerometer [33], [34], the following linear model can be derived:

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{u}(k) + \mathbf{w}(k) \quad (10)$$

with zero-mean white noise $\mathbf{w}(k)$ and

$$\mathbf{F} := \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 & \frac{1}{6}T_s^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & T_s & \frac{1}{2}T_s^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T_s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{T_{\dot{\psi}}}{T_{\dot{\psi}}+T_s} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T_s & \frac{1}{2}T_s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T_s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{T_{\ddot{f}}}{T_{\ddot{f}}+T_s} \end{bmatrix}$$

$$\mathbf{u}(k) := \left[0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{T_s(C_{1\dot{\psi}}+C_{2\dot{\psi}})}{T_{\dot{\psi}}+T_s} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{T_s(C_{1\ddot{f}}+C_{2\ddot{f}})}{T_{\ddot{f}}+T_s}\right]^T$$

where T_s is the sampling interval and $C_{1\dot{\psi}}$, $C_{2\dot{\psi}}$, $T_{\dot{\psi}}$, $C_{1\ddot{f}}$, $C_{2\ddot{f}}$, $T_{\ddot{f}}$ are the drift model parameters deduced by the Levenberg-Marquardt iterative least squares fit method as reported in TABLE I.

TABLE I
DRIFT MODEL PARAMETERS FOR THE INERTIAL SENSORS

	C_1	C_2	T
$\dot{\psi}$	-0.00826 °/*s*	0.0594 °/*s*	121.1276 *s*
$\ddot{f}$	0.001 m/s^2	0.0083 m/s^2	128.8318 *s*

Under the assumption that accelerometer and gyroscope are independent sensors, the covariance matrix $\mathbf{Q}$ of the zero-mean white noise $\mathbf{w}(k)$ has the form [35], [36]:

$$\mathbf{Q} = E\left\{\mathbf{w}(k)\mathbf{w}^T(k)\right\} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix}$$

with

$$\mathbf{Q}_1 := \begin{bmatrix} \frac{T_s^7}{252}\sigma_1^2 & \frac{T_s^6}{72}\sigma_1^2 & \frac{T_s^5}{30}\sigma_1^2 & \frac{T_s^4}{24}\sigma_1^2 & 0 & 0 \\ \frac{T_s^6}{72}\sigma_1^2 & \frac{T_s^5}{30}\sigma_1^2 & \frac{T_s^4}{24}\sigma_1^2 & \frac{T_s^3}{6}\sigma_1^2 & 0 & 0 \\ \frac{T_s^5}{30}\sigma_1^2 & \frac{T_s^4}{24}\sigma_1^2 & \frac{T_s^3}{6}\sigma_1^2 & \frac{T_s^2}{2}\sigma_1^2 & 0 & 0 \\ \frac{T_s^4}{24}\sigma_1^2 & \frac{T_s^3}{6}\sigma_1^2 & \frac{T_s^2}{2}\sigma_1^2 & T_s\sigma_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{T_s^3}{3}\sigma_2^2 & \frac{T_s^2}{2}\sigma_2^2 \\ 0 & 0 & 0 & 0 & \frac{T_s^2}{2}\sigma_2^2 & T_s\sigma_2^2 \end{bmatrix}$$

and

$$\mathbf{Q}_2 := \begin{bmatrix} \frac{T_s^5}{20}\sigma_3^2 & \frac{T_s^4}{8}\sigma_3^2 & \frac{T_s^3}{6}\sigma_3^2 & 0 \\ \frac{T_s^4}{8}\sigma_3^2 & \frac{T_s^3}{6}\sigma_3^2 & \frac{T_s^2}{2}\sigma_3^2 & 0 \\ \frac{T_s^3}{6}\sigma_3^2 & \frac{T_s^2}{2}\sigma_3^2 & T_s\sigma_3^2 & 0 \\ 0 & 0 & 0 & T_s\sigma_4^2 \end{bmatrix}$$

where σ_1, σ_2, σ_3 and σ_4 being the experimentally determined standard deviations of the residuals of the fitted models (for further details, see [33]).

The state vector estimated by the Kalman filter is given by the standard recursive estimator [35]:

$$\hat{\mathbf{x}}(k+1|k+1) = \mathbf{F}\hat{\mathbf{x}}(k|k) + \mathbf{u}(k) + \mathbf{G}(k+1)\mathbf{v}(k+1)$$

where $\hat{\mathbf{x}}(k+1|k+1)$ is the estimate of the state vector at time $(k+1)T_s$ based on all observations up to this time, $\mathbf{G}(k+1)$ is the filter gain, and $\mathbf{v}(k+1) = \mathbf{z}(k+1) - \mathbf{C}\hat{\mathbf{x}}(k+1|k)$ is the innovation vector provided by the new observations at time $(k+1)T_s$. All states, including drift parameters, are estimated at every sample time. The KF has been implemented in real-time on the UGV with a sampling interval of $T_s = 200$ ms (for details, see the second part of this work [29]). Initial estimates of the bias errors initialize the filter by averaging the output of each inertial sensor over a large number of samples when the robot is not in motion. As data is collected by the inertial sensors, the parallel running KF filters the measurements and provides estimates of the quantities of interest for the mobile robot.

IV. Proposed Sensor FDI System

A model-based fault diagnosis system consists of a residual generation module and a residual evaluation module. As mentioned, residuals are signals that, in the absence of faults deviate from zero only due to modeling uncertainties, with nominal value being zero, or close to zero under actual working conditions. If a fault occurs, residuals deviate from zero and the faulty conditions can be distinguished from the fault free ones. This section describes the theoretical aspects for developing of the proposed residual generation module and residual evaluation module.

A. The residual generation module

In the past, the residual generation problem has attracted a good deal of attention, mainly focused on linear systems [1]-[3], [8]-[10]. Recently, the residual generation problem for nonlinear systems is becoming an active research topic [37], [38]. Structural analysis is a feasible solution [11], [27] to this problem and here it is applied to the UGV nonlinear model.

The UGV nonlinear model is considered as a set of constraints $C = \{c_1, c_2, \ldots, c_9\}$ applied to a set of variables $Z = X \cup K$, where X denotes the subset of the unknown variables, K denotes the subset of the known ones (sensor measurements, variables with known values), and reference variables. The set of unknown variables is: $X = \{\psi, \dot{\lambda}, \dot{\Phi}, \dot{f}, \dot{\psi}\}$ and the set of known variables is $K = \{\lambda, \Phi, \ddot{f}, r, \omega_L, \omega_R\}$ where the time dependency has been omitted for simplicity. The set of constraints for the UGV is:

$$c_1:\quad \dot{\lambda} = \frac{\dot{f}\cos\psi}{R_{\lambda_0}} \tag{11}$$

$$c_2:\quad \dot{\Phi} = \frac{\dot{f}\sin\psi}{R_{\Phi_0}\cos\lambda} \tag{12}$$

$$c_3:\quad \dot{\psi} = r \tag{13}$$

$$c_4:\quad \dot{\lambda} = \frac{d\lambda}{dt} \tag{14}$$

$$c_5:\quad \dot{\Phi} = \frac{d\Phi}{dt} \tag{15}$$

$$c_6:\quad \ddot{f} = \frac{d\dot{f}}{dt} \tag{16}$$

$$c_7:\quad \dot{\psi} = \frac{d\psi}{dt} \tag{17}$$

$$c_8:\quad \omega_L = \frac{2}{D}\left(\dot{f} - \frac{L}{2}r\right) \tag{18}$$

$$c_9:\quad \omega_R = \frac{2}{D}\left(\dot{f} + \frac{L}{2}r\right) \tag{19}$$

The structure is described by the following binary relation:

$$S:\ C \times Z \rightarrow \{0,1\}$$

$$(c_i, z_j) \rightarrow \begin{cases} S(c_i, z_j) = 1 & \text{iff } c_i \text{ applies to } z_j \\ S(c_i, z_j) = 0 & \text{otherwise} \end{cases}$$

This system structure may be also represented by the incidence matrix illustrated in TABLE II. More details about structural analysis and the matching algorithm may be found in [27].

TABLE II
INCIDENCE MATRIX

	KNOWN						UNKNOWN				
	λ	Φ	$\ddot{f}$	r	ω_L	ω_R	ψ	$\dot{\lambda}$	$\dot{\Phi}$	$\dot{f}$	$\dot{\psi}$
c_1							1	1		1	
c_2	1						(1)		1	1	
c_3			1								(1)
c_4	1							(1)			
c_5		1							(1)		
c_6			1							1	
c_7							×				1
c_8				1	1					1	
c_9				1		1				(1)	

The matching algorithm identifies the over-determined parts of the system [14], [27], resulting in the corresponding lists of matched and unmatched constraints. The matched constraints are $M = \{c_2, c_3, c_4, c_5, c_9\}$ while the unmatched constraints are $U = \{c_1, c_6, c_7, c_8\}$. Each of the unmatched constraints gives a parity equation. In this way, the detection equations are derived by back tracing the matching of the unknown variables involved in the unmatched constraints until only known variables are part of the expression. The resulting parity equations are:

$$c_1\left(\dot{f}, \psi, \dot{\lambda}\right) = 0 \tag{20}$$

$$c_6\left(\dot{f}, \ddot{f}\right) = 0 \tag{21}$$

$$c_7\left(\psi, \dot{\psi}\right) = 0 \tag{22}$$

$$c_8\left(\dot{f}, r, \omega_L\right) = 0 \tag{23}$$

Back tracing unknown variables to known variables gives the following five residuals r_1, r_2, r_3, r_4 and r_5:

$$r_1 = R_{\lambda_0}\left(\frac{d\lambda}{dt} - \frac{1}{R_{\lambda_0}}\left(\frac{d}{2}\omega_R - \frac{L}{2}r\right)\cdot \cos\left(\arcsin\left(\frac{d\Phi}{dt}\cdot\frac{2R_{\Phi_0}\cos\lambda}{d\omega_R - Lr}\right)\right)\right) \tag{24}$$

$$r_2 = \ddot{f} - \frac{d}{dt}\left(\frac{d}{2}\omega_R - \frac{L}{2}r\right) \tag{25}$$

$$r_3 = r - \frac{d}{dt}\left(\arcsin\left(\frac{d\Phi}{dt}\cdot\frac{2R_{\Phi_0}\cos\lambda}{d\omega_R - Lr}\right)\right) \tag{26}$$

$$r_4 = \omega_L - \omega_R + \frac{2L}{d}r \tag{27}$$

$$r_5 = \int_0^t\left(\ddot{f} - \frac{d}{dt}\left(\frac{d}{2}\omega_R - \frac{L}{2}r\right)\right)d\tau \tag{28}$$

It is essential to clarify that re-running the matching algorithm may result in different residuals, but they do not add any information to the FDI task. Further, the first residual has been multiplied by R_{λ_0} to avoid computational rounding errors and, in order to have a strong detectability of the faults and thus more isolability, it has been integrated. For the same reason, the second residual has been also integrated and it has been added to the residual set as an extra one (the fifth one). After this, all residuals have been discretized for on-line implementation. Because of each measure is affected by sensor noise, numeric integrations have been re-initialized in each time-window, such that to avoid increasing of computational errors.

B. *The residual evaluation module*

The *residual evaluation* module detects a change in the mean of an observed and distributed random sequence achieved by sequential change detection algorithms.

Two different solutions have been developed and analysed: the first is an "ad hoc solution" consisting of an adaptive/moving threshold test on the instantaneous values of the obtained residuals; the second is a "particle filtering-based likelihood ratio decision solution".

B.1 Adaptive/moving threshold test solution

An ad-hoc decision method developed and tested is presented in this subsection.

Let $\left\{r_i\left(kT_s\right) : k \in \left[\left(\left(j-1\right)\cdot n + 1\right), j \cdot n\right]\right\}$ be an observed sequence (randomly distributed) of the i-th residual $r_i\left(kT_s\right)$ in a generic j-th sliding window of size n (number of readings in the sliding window), with conditional density $p_\theta\left(r_i\left(kT_s\right) \middle| r_i\left(\left(k-1\right)T_s\right), \ldots, r_i\left(T_s\right)\right)$. Before the unknown change time t_0, the conditional density parameter θ is constant and equal to θ_0, after a positive change, the parameter is equal to θ_1, and after a negative change, it is equal to $\theta_2 = -\theta_1$. The hypotheses are:

$$\mathrm{H}_0 : \theta = \theta_0 \ \text{ for } \ \left(\left(j-1\right)\cdot n + 1\right) \le k \le j \cdot n$$

$$\mathrm{H}_1 : \theta = \theta_0 \ \text{ for } \ \left(\left(j-1\right)\cdot n + 1\right) \le k \le t_0 - 1 \ \text{ and } \ \theta = \theta_1 \ \text{ for } \ t_0 \le k \le j \cdot n$$

$$\mathrm{H}_2 : \theta = \theta_0 \ \text{ for } \ \left(\left(j-1\right)\cdot n + 1\right) \le k \le t_0 - 1 \ \text{ and } \ \theta = \theta_2 \ \text{ for } \ t_0 \le k \le j \cdot n$$

The on-line problem is to detect the occurrence of the change as soon as possible.

The *decision test* is based on the following decisions:

if $r_i\left(kT_s\right) < H_{i,j}$ or $r_i\left(kT_s\right) > h_{i,j}$ accept H_0 and set $g_i\left(kT_s\right) = 0$

if $r_i\left(kT_s\right) \ge H_{i,j}$ accept H_1 and set $g_i\left(kT_s\right) = +1$

if $r_i\left(kT_s\right) \le h_{i,j}$ accept H_2 and set $g_i\left(kT_s\right) = -1$

where $g_i(kT_s)$ is the decision function of the *i*-th residual, $H_{i,j}$ and $h_{i,j}$ are upper and lower thresholds, respectively. This is useful to detect deviations from θ_0 in both directions, namely increases (θ_1) and decreases (θ_2). For each specific residual, $H_{i,j}$ and $h_{i,j}$ have been properly chosen. In particular, statistical investigation of fault free residuals has resulted in fixing the thresholds of the first and fifth residual as shown in TABLE III (different experimental tests have validated this choice).

TABLE III
THRESHOLD VALUES FOR THE FIRST AND LAST RESIDUAL

RESIDUAL	$H_{i,j}$	$h_{i,j}$
$r_1(kT_s)$	10 *m/s*	-10 *m/s*
$r_5(kT_s)$	0.0042 *m/s*	-0.0042 *m/s*

Because of high noise, for the remaining three residuals, it is not possible to set constant values of the thresholds $H_{i,j}$ and $h_{i,j}$. In order to set these threshold values, an ad-hoc algorithm has been applied to the *i*-th residual $r_i(kT_s)$ with *i*=2,3,4. The proposed algorithm may be summarized as follows:

1) Let *j*=1. In the first sliding window *j*=1 of size *n*, assume that the system operates in a fault free working mode, that is, H_0 is accepted. Let determine the absolute minimum $m_{i,j}$ and absolute maximum $M_{i,j}$ values of the observed sequence of the *i*-th residual $r_i(kT_s)$. Set upper and lower thresholds for the next sliding window as follows:

$$H_{i,j+1} = M_{i,j} + \Delta^{+}_{i,j} \qquad \text{(a)}$$

$$h_{i,j+1} = m_{i,j} - \Delta^{-}_{i,j} \qquad \text{(b)}$$

with $\Delta^{+}_{i,j}$ and $\Delta^{-}_{i,j}$ constant values resulting from statistical investigations of faultless operation residuals.

2) Let *j*=*j*+1. In the sliding window *j* of size *n* at each sample time $k \in [((j-1)\cdot n+1), j\cdot n]$, the *decision test* runs on-line and the decision function $g_i(kT_s)$ of the *i*-th residual is updated:

$$\text{if } \mathrm{H}_0 \text{ is accepted, set } g_i(kT_s) = 0$$
$$\text{if } \mathrm{H}_1 \text{ is accepted, set } g_i(kT_s) = +1$$
$$\text{if } \mathrm{H}_2 \text{ is accepted, set } g_i(kT_s) = -1$$

If H_0 is accepted, determine the absolute minimum $m_{i,j}$ and absolute maximum $M_{i,j}$ of the *i*-th residual $r_i(kT_s)$ in the considered sliding window. Set the lower and upper thresholds for the next sliding window as in (a) and (b), respectively. Otherwise, if H_1 or H_2 are accepted, also only in one sample time $k \in [((j-1)\cdot n+1), j\cdot n]$, then $H_{i,j+1}$ and $h_{i,j+1}$ keep the predetermined values, i.e. $H_{i,j+1} = H_{i,j}$, $h_{i,j+1} = h_{i,j}$.

3) Go to step 2).

Experimentation resulted in $\Delta^{+}_{i,j}$ and $\Delta^{-}_{i,j}$ values as shown in TABLE IV.

TABLE IV
DELTA VALUES FOR THE SECOND, THIRD AND FOURTH RESIDUAL

RESIDUAL	$H_{i,j}$	$h_{i,j}$
$r_2(kT_s)$	$5\lvert M_{2,j}\rvert$	$5\lvert m_{2,j}\rvert$
$r_3(kT_s)$	$\lvert M_{3,j}\rvert + \lvert m_{3,j}\rvert / 2$	$\lvert M_{3,j}\rvert + \lvert m_{3,j}\rvert / 2$
$r_4(kT_s)$	$2\lvert M_{4,j}\rvert$	$2\lvert m_{4,j}\rvert$

Note that, in the *j*-th sliding window, if H_0 is accepted, then upper and lower thresholds are based on maximum and minimum values of $r_i(kT_s)$ determined in the previous ((*j*-1)-th) sliding window, otherwise they keep the previous values. This allows for updated thresholds in every new sliding window. However, if a fault occurs in a sliding window, the residual changes its mean and its maximum value, but the upper and lower thresholds do not change. Hence, the thresholds are independent from the occurrence of the faults, permitting to detect them. This has been validated through a large set of experimental tests, as shown in the part II of this work.

B.2 Particle filtering-based likelihood ratio decision solution

A particle filtering-based decision module is proposed to estimate the pdfs (probability density functions). As probabilistic method, is also integrated for analysing the performance of the developed SFDIS.

Let $r_i(kT_s)$ be the *i*-th residual with pdf $p_\theta(r_i)$ depending upon one scalar parameter θ, which is the mean value of the residual. Before an unknown change time, k_0T_s, θ is equal to θ_0. At time k_0T_s, it changes to $\theta = \theta_1 \neq \theta_0$. The hypotheses are:

$$\mathrm{H}_0: \ \theta = \theta_0 \text{ for } \forall k$$
$$\mathrm{H}_1: \ \theta = \theta_0 \text{ for } k \le k_0\text{-1 and } \theta = \theta_1 \text{ for } k \ge k_0$$

Consider the logarithm of the likelihood ratio of an observation $r_i(kT_s)$, which is a function of random variable $r_i(kT_s)$, defined by:

$$s(r_i(kT_s)) = \ln\frac{p_{\theta_1}(r_i(kT_s))}{p_{\theta_0}(r_i(kT_s))} \tag{29}$$

where $p_{\theta_b}(r_i(kT_s))$ $(b=0,1)$ is a pdf parameterized by θ_b. The key statistical property of this ratio [18] with E_{θ_0} and E_{θ_1} denoting the expectations of the random variables with distributions $p_{\theta_0}(r_i)$ and $p_{\theta_1}(r_i)$ respectively, is reflected through:

$$E_{\theta_0} = \int_{-\infty}^{+\infty} s(r_i)\, p_{\theta_0}(r_i)\, dr_i < 0 \,,$$

$$E_{\theta_1} = \int_{-\infty}^{+\infty} s(r_i)\, p_{\theta_1}(r_i)\, dr_i > 0 \,.$$

Therefore, any change in parameter θ is reflected as a change in the sign of the mean value of the LLR [18]. Moreover, observations $r_i(kT_s)$ are independent of each other; the joint LLR for observations from $r_i(lT_s)$ to $r_i(kT_s)$ may be expressed as:

$$S_l^k = \sum_{j=l}^{k} \ln\frac{p_{\theta_1}(r_i(jT_s))}{p_{\theta_0}(r_i(jT_s))} \tag{30}$$

This joint LLR S_1^k shows a negative drift before change, and a positive drift after change. This behavior is used for detecting any change between two known pdfs $p_{\theta_0}(r_i)$ and $p_{\theta_1}(r_i)$. Note that the pdfs $p_{\theta_0}(r_i(jT_s))$ and $p_{\theta_1}(r_i(jT_s))$ are non-Gaussian and have to be estimated on-line ([18], [39], [40]). The particle filter based approach found in [41] and [42] is used to estimate both pdfs.

Let's assume that the normal behavior and all possible sensor faults can be described by a given finite set of linear stochastic state space models indexed by $m = 0,1,\ldots,M$

$$\begin{aligned} x_i^{(m)}((k+1)T_s) &= x_i^{(m)}(kT_s) \\ r_i(kT_s) &= a_i^{(m)} x^{(m)}(kT_s) + v_i^{(m)}(kT_s) \end{aligned} \tag{31}$$

where state $x_i^{(m)}(\cdot)$ is the normalized sensor fault magnitude, $v_i^{(m)}(\cdot)$ is the nominal residual without faults independent of past and present states and $a_i^{(m)}(\cdot)$ is the estimation of the sensor fault magnitude obtained from experimental results.

The central idea of the proposed method is to compute the joint likelihood of the observations conditioned on each hypothesized model through Monte-Carlo estimation that uses the complete sample-based pdf information provided by the particle filter, and then activating in parallel M LLR tests for $\mathrm{H_m}$ $(m=1,2,\ldots,M)$ versus H_0. Specifically, the joint LLR to be computed in this case is:

$$S_l^k(m) = \sum_{j=l}^{k} \ln\frac{p_{\theta_1}(r_i(jT_s)|\mathrm{H}_m, Z_{j-1})}{p_{\theta_0}(r_i(jT_s)|\mathrm{H}_m, Z_{j-1})} \tag{32}$$

where the likelihood of the observation $r_i(jT_s)$ gives its past values $Z_{j-1} = \{r_i(T_s), r_i(2T_s), \ldots, r_i((j-1)T_s)\}$, i.e. $p(r_i(jT_s)|\mathrm{H}_m, Z_{j-1})$ $(m=0,1,\ldots,M)$ is the one step output prediction density based on H_m defined by the m-th measurement model and the known statistics of $v^{(m)}(jT_s)$. Hence, the decision function $g_i(kT_s)$ may be obtained as:

if $\max(0, S_1^k(m)) = 0$ accept H_0 and set $g_i(kT_s) = 0$

if $\max(0, S_1^k(m)) > 0$ accept H_1 and set $g_i(kT_s) = +1$.

V. Concluding Remarks

The paper presents a model-based Sensor Fault Detection Isolation System for a mobile robot platform. Theoretical aspects for developing both the residual generation module and evaluation module are detailed and analyzed. Structural Analysis is the key model-based technique for residual generation. Two different solutions have been proposed for developing the residual evaluation module. The first is an "ad hoc solution" consisting of an adaptive/moving threshold test on the instantaneous values of the obtained residuals; the second is a "particle filtering-based likelihood ratio decision solution". The result is a SFDIS which is able to detect every single/multiple faults of the considered sensor equipment. Sensor faults are detected and isolated in all situations where a single sensor fault is occurred at a time. The situation is slightly different when multiple sensor faults occur simultaneously. In this case, fault isolability cannot be guaranteed in all situations, as it is resumed in the fault signature table (TABLE V)

TABLE V
Effects of the Sensor Faults on the Residuals

	f_{GPS}	f_{ACC}	f_{GYRO}	f_{EL}	f_{ER}
r_1	×	0	0	0	0
r_2	0	×	0	0	0
r_3	0	0	×	0	0
r_4	0	0	×	×	×
r_5	0	×	0	0	×

where f_{GPS}, f_{ACC}, f_{GYRO}, f_{EL} and f_{ER} denote "GPS antenna fault", "accelerometer fault", "gyroscope fault", "left optical encoder fault" and "right optical encoder fault", respectively, while "×" ("0") indicates that the fault in the corresponding column affects (does not affect) the residual of the corresponding row (for further details, see [27], [29]).

In conclusion, the proposed sensor FDI system could be easily integrated with any UGV navigation system. If a sensor fault is detected, it is isolated and the faulty sensor is identified. The accuracy, reliability and robustness of a SFDIS is a challenge problem. A possible solution of this problem is to increase the sensitivity of the residuals as much as possible.

Abrupt faults have been considered in this paper, but also incipient faults are important. Different residual generation and evaluation modules are under investigation to permit detection and isolation of this kind of faults.

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