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Hybrid Model Predictive Control for Stabilization of Wheeled Mobile Robots Subject to Wheel Slippage

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***Abstract*— This paper studies the problem of stabilizing wheeled mobile robots (WMRs) subject to wheel slippage to a predefined set. When slippage of the wheels can occur, WMRs can be modeled as hybrid systems. Model predictive control for such systems typically results in numerical methods of combinatorial complexity. We show that recently developed embedding techniques can be used to formulate numerical algorithms for the hybrid model predictive control (MPC) problem that have the same complexity as the MPC for smooth systems. We also discuss in detail the numerical techniques that lead to efficient and robust MPC algorithms. Examples are given to illustrate the effectiveness of the approach.**

I. INTRODUCTION

Modeling and control of wheeled mobile robots (WMRs) have been extensively studied in the robotics community (see [1]–[5] among many other). Most of the works assume that the contact between the wheels and the ground satisfies the conditions of pure rolling, which means that slipping can never happen. These conditions lead to a typical nonholonomic dynamic system. However, in reality slipping is by no means unusual. For example, wheel slippage can easily occur when the WMR drives on a slippery surface, when the WMR makes a high speed turn, or when the torques applied to the wheels are too large. As the WMR's dynamics change when the wheels switch between rolling and sliding, controllability fails, making stabilization to a predefined set [6] problematic.

Some studies (e.g. [7], [8]) have considered the skidding and slipping effects. But the proposed controllers are neither closed-loop nor robust. So the performance cannot be guaranteed in the presence of uncertain parameters, unmodeled friction and external disturbances. In [9], the authors model a Hilare type WMR with rolling and sliding as a hybrid system and develops a stabilizing switching controller. However, the proposed design methodology is difficult to generalize.

Model predictive control (MPC) [10] has been a popular approach for control of complex systems and is well known for its robustness. In [11] MPC is employed for tracking control of nonholonomic WMR, however only pure rolling (no wheel slippage) is considered.

In this paper, we use the techniques developed in [12] to solve the problem of stabilizing a 2-wheel WMR to a specified set. Both rolling and sliding motions of the wheels are considered. The WMR is modeled as a hybrid system exhibiting autonomous switches (no control input directly causes the switches) and controlled switches (a separate input directly controls the switches). The approach uses an extension of the embedding technique in [13] for the solution of the hybrid optimal control problem at each step of the MPC algorithm. An integral quadratic penalty function is utilized in the numerical method. Simulation results are given to demonstrate the effectiveness of the approach.

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II. MODEL

As in [9], we consider a 2 wheel differentially driven WMR moving on a horizontal plane (Fig. 1). The wheels of the WMR can either roll or slide (autonomous switches) and a regenerative brake can be switched on or off as necessary (controlled switches). The control (torque) inputs u_1 and u_2 drive wheels 1 and 2 respectively with power from a rechargeable battery. Regenerative braking reduces the amplitude of each wheel's angular velocity (w_1 and w_2) while recharging the battery.

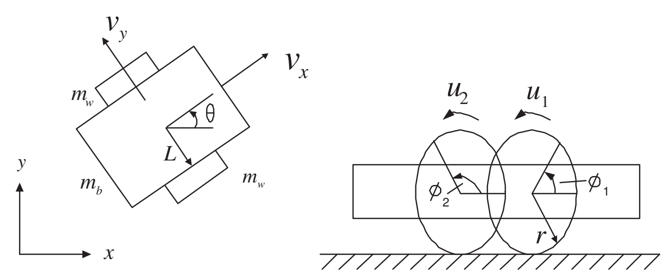


Fig. 1. A top view and a side view of the WMR.

The generalized coordinates of the WMR are its center of mass position x and y, body orientation relative to the x-axis θ, and the angles of the wheels ϕ_1 and ϕ_2. Since ϕ_1 and ϕ_2 themselves are unimportant, the state variables for the system are $z^T = [x, y, \theta, v_x, v_y, \omega, w_1, w_2] \in R^8$, where $[v_x, v_y]$ is the velocity of the center of mass of the WMR, expressed in the body frame; $\omega = \dot{\theta}$ is the turning velocity of the WMR; $w_1 = \dot{\phi}_1$ and $w_2 = \dot{\phi}_2$ are the rotational velocities of the wheel 1 and wheel 2, respectively. The equations of motion for the WMR are:

$$\begin{aligned}
\dot{x} &= v_x \cos\theta - v_y \sin\theta, & \dot{v}_x &= \omega v_y + \frac{F_x^1 + F_x^2}{m_b + 2m_w}, \\
\dot{y} &= v_x \sin\theta + v_y \cos\theta, & \dot{v}_y &= -\omega v_x + \frac{F_y^1 + F_y^2}{m_b + 2m_w}, \\
\dot{\theta} &= \omega, & \dot{\omega} &= L\frac{F_x^1 - F_x^2}{I_b + 2I_v}, \\
\dot{w}_1 &= \frac{F_x^1 r + u_1}{I_w}, & \dot{w}_2 &= \frac{F_x^2 r + u_2}{I_w},
\end{aligned} \tag{1}$$

where (i) m_b is the mass of the WMR's body; (ii) m_w is the mass of a wheel; (iii) I_b is the moment of inertia of the WMR's body about a vertical axis through the center of mass; (iv) I_w is the moment of inertia of a wheel around its axis; (v) I_v is the moment of inertia of a wheel around vertical axis through the center of mass; (vi) F_x^i is the force between the ground and the i-th wheel in the forward direction, expressed in the body frame; (vii) F_y^i is the force between the ground and the i-th wheel in the lateral direction, expressed in the body frame.

If wheel 1 is rolling, the relative velocity between the ground and the point of contact of wheel 1 is zero:

$$v_r^1 = [v_{rx}^1, v_{ry}^1]^T = [v_x + L\omega + rw_1, v_y]^T = 0. \quad (2)$$

Analogously, when wheel 2 is rolling,

$$v_r^2 = [v_{rx}^2, v_{ry}^2]^T = [v_x - L\omega + rw_2, v_y]^T = 0. \quad (3)$$

When wheel i is rolling, the forces F_x^i and F_y^i are ground reaction forces that prevent wheel slippage and they can be eliminated from Eq. (1) using Eq. (2) or Eq. (3). When wheel i is sliding, F_x^i and F_y^i are frictional forces arising from Coulomb's law:

$$[F_x^i, F_y^i]^T = -\mu_d \frac{v_r^i}{\|v_r^i\|}(\frac{m_b}{2} + m_w)g, \quad (4)$$

where μ_d is the coefficient of dynamic friction and g is the gravitational constant.

The autonomous switch from *rolling to sliding* occurs when the magnitude of the constraint force $F^i = [F_x^i, F_y^i]^T$ exceeds the maximum possible magnitude of the static friction,

$$\|F^i\| > \mu_s(\frac{m_b}{2} + m_w)g, \quad (5)$$

where μ_s is the coefficient of static friction. On the other hand, the switch from *sliding to rolling* occurs when (i) $v_r^i = [v_{rx}^i,\ v_{ry}^i]^T = 0$, and (ii) the maximum magnitude of the frictional force exceeds that of the constraint force F^i,

$$v_r^i = 0 \ \text{ and } \ \|F^i\| \leqslant \mu_s(\frac{m_b}{2} + m_w)g. \quad (6)$$

As mentioned, the system also has controlled switches where the regenerative brake can be switched on or off arbitrarily.

When the regenerative brake is off, the actuating torques of both wheels are

$$u_i = u_{i1} \in [-30, 30], \quad i = 1, 2. \quad (7)$$

When the regenerative brake is on,

$$u_i = u_{i2} = \begin{cases} -K_b w_i, & |w_i| \leqslant 3 \\ -30\,\mathrm{sgn}(w_i), & |w_i| > 3 \end{cases} \quad i = 1, 2, \quad (8)$$

where $K_b = 10$ is a fixed regenerative braking coefficient. Note that u_i saturates at 30 Nm.

III. METHODOLOGY

In this subsection we will describe the method presented in [12]. Four quantities are used to describe the evolution of a control system subject to autonomous and controlled switches: (i) the discrete state $\xi(t) \in D_\xi = \{1, 2, \ldots, d_\xi\}$, (ii) the continuous state $x(t) \in \mathbb{R}^n$, (iii) the discrete control input $\alpha(t) \in D_\alpha = \{1, 2, \ldots, d_\alpha\}$, and (iv) the continuous control input $u(t) \in \mathbb{R}^m$. The discrete state of the system describes the autonomous switches. We only consider systems for which the autonomous switches depend on the continuous state $x(t)$ and the input $u(t)$, they do not depend on the current discrete state $\xi(t)$. Such systems are usually called *memoryless*. Formally, the evolution of the discrete state of the memoryless system is defined by a piecewise continuous function $\eta : \mathbb{R}^n \times \mathbb{R}^m \to D_\xi$, that for each continuous state x and continuous control input u selects the discrete state ξ of the system:

$$\xi^+(t) = \eta(x, u). \quad (9)$$

Let $M_i \subseteq \mathbb{R}^n \times \mathbb{R}^m, i \in D_\xi$ be the set of pairs (x, u) corresponding to the discrete state $i \in D_\xi$:

$$M_i = \{(x, u) \in \mathbb{R}^n \times \mathbb{R}^m \mid \eta(x, u) = i\},$$

and let $f_{(i,j)}, i \in D_\xi, j \in D_\alpha$ be a collection of $\mathcal{C}^1$ vector fields

$$f_{(i,j)} : M_i \to \mathbb{R}^n.$$

The evolution of the continuous state $x(t)$ is then described by:

$$\dot{x}(t) = f_{(\eta(x(t),u(t)),\alpha(t))}(x(t), u(t)), \quad x(t_0) = x_0. \quad (10)$$

At each $t \geq t_0$ and for each discrete state $\xi(t) \in D_\xi$, the switching control input $\alpha(t) \in D_\alpha$ thus selects the d_α vector field that governs the evolution of the continuous state. We assume that the continuous control input $u(t)$ is constrained to the convex and compact set $\Omega \subseteq \mathbb{R}^m$ and that the switching control input $\alpha(t)$ and the continuous control input $u(t)$ are both measurable functions.

Given that the discrete state $\xi(t)$ is completely determined by $x(t)$ and $u(t)$ through Eq. (9), we can define for each $j \in D_\alpha$ a piecewise $\mathcal{C}^1$ vector field:

$$f_j(x(t), u(t)) \triangleq f_{(\eta(x(t),u(t)),j)}(x(t), u(t)), \quad (11)$$

and rewrite Eq. (10) simply as:

$$\dot{x}(t) = f_{\alpha(t)}(x(t), u(t)), \quad x(t_0) = x_0. \quad (12)$$

Consider the system described by Eq. (12). Both $\alpha(t)$ and $u(t)$ are control variables and for the optimal control problem we require that they are chosen on the interval $[t_0, t_f]$ so that the following initial and terminal constraints are satisfied: $(t_0, x(t_0)) \in \mathcal{T}_0 \times \mathcal{B}_0$ and $(t_f, x(t_f)) \in \mathcal{T}_f \times \mathcal{B}_f$. We will assume that the endpoint constraint set $\mathcal{B} = \mathcal{T}_0 \times \mathcal{B}_0 \times \mathcal{T}_f \times \mathcal{B}_f$ is contained in a compact set in $\mathbb{R}^{2n+2}$. The optimization functional is defined as

$$J_C(t_0, x_0, u, \alpha) = g(t_0, x_0, t_f, x_f) + \int_{t_0}^{t_f} f^0_{(\eta(x(t),u(t)),\alpha(t))}(x(t), u(t))dt, \quad (13)$$

where g is a real-valued $\mathcal{C}^1$ function defined on a neighborhood of $\mathcal{B}$, and the functions $f^0_{(i,j)} : M_i \to \mathbb{R}, i \in D_\xi, j \in D_\alpha$ are of class $\mathcal{C}^1$. Given that the evolution of the discrete state $\xi(t)$ is governed by Eq. (9), we can define similarly as in Eq. (11) for each $j \in D_\alpha$ a new piecewise $\mathcal{C}^1$ function

$$f^0_j(x(t), u(t)) \triangleq f^0_{(\eta(x(t),u(t)),j)}(x(t), u(t)), \tag{14}$$

and rewrite the cost functional as:

$$J_C(t_0, x_0, u, \alpha) = g(t_0, x_0, t_f, x_f) + \int_{t_0}^{t_f} f^0_{\alpha(t)}(x(t), u(t))dt. \tag{15}$$

The hybrid optimal control problem (HOC) is defined:

$$\min_{\alpha,u} J_C(t_0, x_0, u, \alpha),$$

subject to the constraints: (i) $x(\cdot)$ satisfies Eq. (12); (ii) $(t_0, x(t_0), t_f, x(t_f)) \in \mathcal{B}$; (iii) for each $t \in [t_0, t_f]$, $\alpha(t) \in D_\alpha$ and $u(t) \in \Omega$.

The next step is to embed system (12) into a larger set of systems. For the HOC, $\alpha(t) \in D_\alpha = \{1, 2, \ldots, d_\alpha\}$. Introduce d_α new variables $\alpha_i \in [0, 1]$, $i \in D_\alpha$, that satisfy

$$\sum_{i=1}^{d_\alpha} \alpha_i(t) = 1. \tag{16}$$

Let u_i be the control input for each vector field f_i, $i \in D_\alpha$, in (10). Now define a new system:

$$\dot{x}(t) = \sum_{i=1}^{d_\alpha} \alpha_i(t) f_i(x(t), u_i(t)), \quad x(t_0) = x_0, \tag{17}$$

and the associated cost functional

$$J_E(t_0, x_0, u, \alpha) = g(t_0, x_0, t_f, x_f) + \int_{t_0}^{t_f} \sum_{i=1}^{d_\alpha} \alpha_i(t) f^0_i(x(t), u_i(t))dt. \tag{18}$$

The HOC has become an embedded optimal control problem (EOC):

$$\min_{\alpha_i,u_i} J_E(t_0, x_0, u, \alpha),$$

subject to the following constraints: (i) $x(\cdot)$ satisfies Eq. (17); (ii) $(t_0, x(t_0), t_f, x(t_f)) \in \mathcal{B}$; (iii) for each $t \in [t_0, t_f]$ and each $i \in D_\alpha$, $\alpha_i(t) \in [0, 1]$ and $u_i(t) \in \Omega$; (iv) for each $t \in [t_0, t_f]$, $\sum_{i=1}^{d_\alpha} \alpha_i(t) = 1$.

The EOC is amenable to the classical necessary and sufficient conditions of optimal control theory. Moreover, it was shown in [13] that the set of trajectories of the hybrid system (10) is dense (in the L^∞ sense) in the set of trajectories of the embedded system (17). It is also shown that if one first solves the EOC and obtains a solution, either the solution is of the switched type, or suboptimal trajectories of the HOC can be constructed that can approach the value of the cost for the EOC arbitrarily closely and satisfy the boundary conditions within ϵ for arbitrary $\epsilon > 0$.

A variation of direct collocation [14] is used to numerically solve the EOC. In this case, $u(t)$ and $x(t)$ are chosen from finite-dimensional spaces. Given basis functions $\{\phi^j\}_{j=0}^N$ and $\{\psi^j\}_{j=0}^M$,

$$x_i = \sum_{j=0}^{N} p_i^j \phi^j(t), \quad p_i^j \in \mathbb{R}, \quad i = 1, \ldots, n,$$
$$u_i = \sum_{j=0}^{M} q_i^j \psi^j(t), \quad q_i^j \in \mathbb{R}, \quad i = 1, \ldots, m.$$

Since f is only piecewise $\mathcal{C}^1$ and as a result $x(t)$ can be nonsmooth, the basis functions $\{\phi^j\}_{j=0}^N$ are chosen to be nonsmooth. Similarly, since the control $u(t)$ can be discontinuous, the basis functions $\{\psi^j\}_{j=0}^M$ are chosen to be discontinuous. Partition the time interval $[t_0, t_f]$ into N subintervals with the endpoints $t_0 < t_1 < \ldots < t_{N-1} < t_N = t_f$.

The state trajectory is approximated by a piecewise-linear function:

$$\widehat{x}_i(t) = x_i(t_j) + \frac{t - t_j}{t_{j+1} - t_j} \left(x_i(t_{j+1}) - x_i(t_j) \right),$$
$$t_j \le t < t_{j+1}, \quad i = 1, \ldots, n.$$

This approximation corresponds to $p_i^j = x_i(t_j)$ and the triangular basis functions:

$$\phi^j(t) = \begin{cases} \frac{t - t_{j-1}}{t_j - t_{j-1}} & t_{j-1} \le t < t_j, \\ \frac{t_{j+1} - t}{t_{j+1} - t_j} & t_j \le t < t_{j+1}, \\ 0 & \text{otherwise.} \end{cases}$$

The control input is chosen to be piecewise constant so that

$$\widehat{u}_i(t) = u_i(t_j), \quad t_j \le t < t_{j+1}, \quad i = 1, \ldots, m.$$

This approximation corresponds to $q_i^j = u_i(t_j)$ and the square basis functions:

$$\psi^j(t) = \begin{cases} 1 & t_j \le t < t_{j+1}, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the above choice of basis functions implies $M = N - 1$.

The system equations are enforced at the midpoints:

$$\dot{\widehat{x}}(t) - f(\widehat{x}(t), \widehat{u}(t)) = 0, \quad \text{for } t = \frac{t_j + t_{j+1}}{2}, \quad j = 0, \ldots, N-1. \tag{19}$$

If the original problem contains additional equality and inequality constraints they can be easily added in a similar way. With the chosen representation of x and u, approximation of the integral (18) with a finite sum (using e.g. trapezoidal rule), and together with the equality constraints represented by Eq. (19) the optimal control problem thus becomes a nonlinear programming problem in the unknowns p_i^j and q_i^j.

IV. Application to WMR

To apply the above methodology to WMR, observe that from Eq. (16), $\alpha_2(t) = 1 - \alpha_1(t)$. The embedded control is thus

$$u_i(t) = \alpha_1(t)\, u_{i1}(t) + (1 - \alpha_1(t))\, u_{i2}(t). \qquad (20)$$

Note that in the embedded problem which will be solved, $\alpha_1(t) \in [0, 1]$. The embedded formulation is given by Eqs. (1) and (20), and the forces F_x^i and F_y^i in rolling and sliding mode.

The objective of the control design is to stabilize the WMR from a given initial state to a predefined set within an allotted time while minimizing energy usage. Suppose the initial state is $z_0 = z(0)$ and the target set can be described by $z_f = z(T)$. Note that some state variables in z_f may be free. For example, if the task is to command the WMR to drive along the line $y = 1$ with a constant forward velocity v_0, then $z_f^T = [x(T), 1, 0, v_0, 0, 0, -\frac{v_0}{r}, -\frac{v_0}{r}]$ (the values for $w_1(T)$ and $w_2(T)$ follow from the rolling condition). As uncontrolled sliding is undesirable, we penalize sliding motion in our performance index.

There are two problems associated with numerically solving the optimization problem. One is that the terminal constraints cannot be imposed as hard constraints because the system is stabilizable but not controllable in the sliding regime. Using hard constraints could make the optimal control problem unfeasible. Therefore, the terminal state constraints are enforced through the cost functional as soft constraints. The other problem is that when the state equations are imposed as hard constraints the numerical solution becomes difficult. To avoid this, we enforce the state equations as soft constraints and use the quadratic penalty function method [15] to numerically solve the optimization problem. We refer the reader to [15] for the details of the quadratic penalty function method.

Consider the problem

$$\min_{x \in X} f(x) \quad \text{subject to} \quad h(x) = 0, \qquad (21)$$

where $f : R^n \to R, h : R^n \to R^m$ are given functions and X is a given subset of R^n. For any scalar c, define the augmented Lagrangian function $L_c : R^n \times R^m \to R$ by

$$L_c(x) = f(x) + \frac{1}{2} c \left| h(x) \right|^2, \qquad (22)$$

where c is the penalty parameter.

Instead of solving the original problem (21), we would like to solve a sequence of problems of the form

$$\min_{x \in X} L_{c_k}(x), \qquad (23)$$

where $\{c_k\}$ is a penalty parameter sequence satisfying

$$0 < c_k < c_{k+1}, \quad c_k \to \infty. \qquad (24)$$

It can be shown in [15] that, if we can construct a sequence of approximate problems which converges in a well-defined sense to the original problem, then the corresponding sequence of the approximate solutions will yield in the limit a solution of the original problem.

Hence the performance index for this study is

$$J = c_0 \left\| z(T) - z_f \right\|^2 + \int_0^T [c_1\, \alpha_1(t)\, u_1^2 + c_2\, \alpha_1(t)\, u_2^2 + c_3 \|v_r\|^2 + c_4 \|DE_1 - DE_2\|^2] dt. \qquad (25)$$

where the positive weights c_i (for $i = 0, \ldots, 4$) are constant. The term (i) $c_0 \left\| z(T) - z_f \right\|^2$ drives the final states $z(T)$ of the WMR toward the desired set (free state variables are excluded from this term); (ii) $c_1\, \alpha_1(t)\, u_1^2$ penalizes the actuating power usage of wheel 1; (iii) $c_2\, \alpha_1(t)\, u_2^2$ penalizes the actuating power usage of wheel 2; (iv) $c_3 \|v_r\|^2$ is to limit sliding motion; (v) $c_4 \|DE_1 - DE_2\|^2$ enforces the constraints of the state equations by gradually increasing c_4, where DE_1 and DE_2 are the left-hand sides and the right-hand sides of Eq. (1), respectively. There is no penalty for regenerative braking usage.

A. MPC Design

To cope with disturbances and modeling uncertainties, an MPC-type controller is adopted to drive the WMR from a given initial state z_0 to the target state z_f at a pre-specified final time. The MPC approach can be summarized as follows:

1) Given z_0, partition the time interval T into N equal subintervals of length $h = \frac{T}{N}$, to compute a (backward) piecewise constant control sequence $\{\widehat{u}_1, \ldots, \widehat{u}_N\}$, where $\widehat{u}_i^T = [u_1((i-1)h), u_2((i-1)h)]$, and the state values $\{z_1, \ldots, z_N\}$.
2) For $k = 1, \ldots, N$, solve the embedded problem over the receding horizon $[t_{k-1}, t_N]$ by minimizing the performance index of Eq. (25) with the initial state z_{k-1} and obtain the (look ahead) control sequence $\{\widehat{u}_k, \ldots, \widehat{u}_N\}$.
3) Apply the control input $\widehat{u}_k$ for the time interval $t_{k-1} \le t < t_k$ to the model. The value of the state of the model at the end of the interval becomes z_k, the initial condition for the next iteration.
4) Repeat steps 2 and 3 until $k = N$.

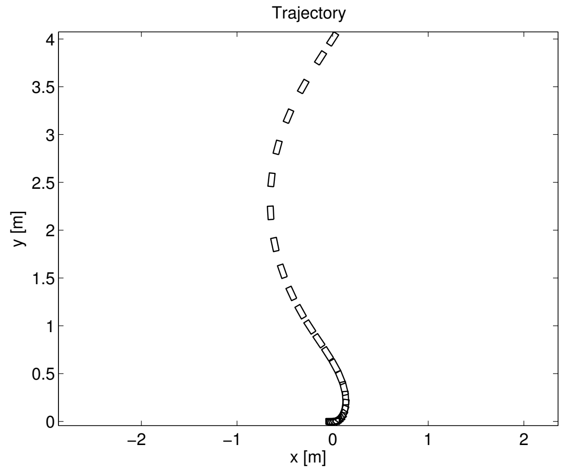
B. Numerical Method

A variation of direct collocation [14] is used to numerically solve the EOC at each step of the MPC algorithm. However, instead of enforcing the state equations explicitly, the above penalty method is used.

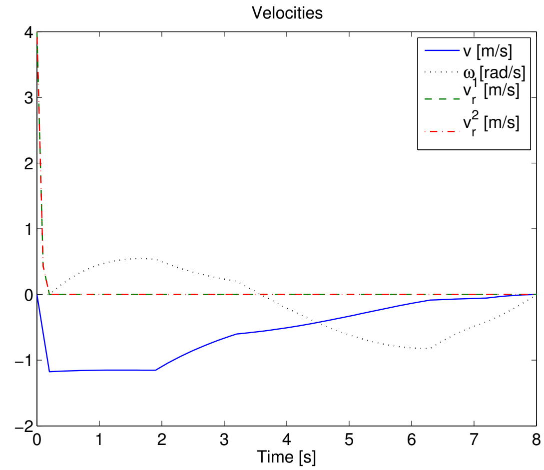
We start with a zero initial guess and compute the optimal solution for $c_4 = 1$. This solution is subsequently used as the initial guess for $c_4 = 10, 10^2, 10^3$, and 10^4, the solution for each step serving as the initial guess for the next step. Finally term (v) of the performance index is removed and the state equations are enforced as hard constraints. We used the solution for $c_4 = 10^4$ as initial guess and do the optimization again. In this way it can be guaranteed that the final optimal solution satisfies the state equations.

V. Simulations

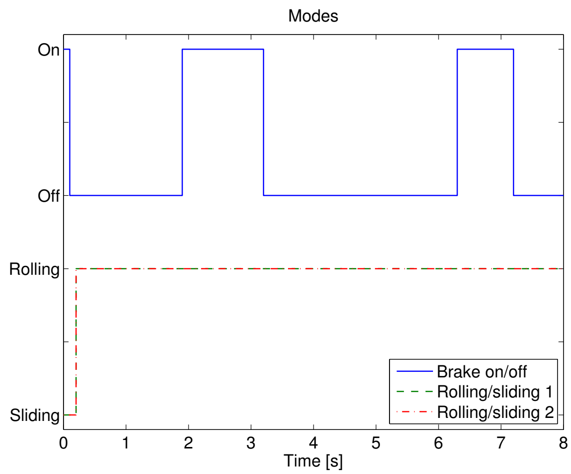
The parameters used in simulations were $m_b = 1$, $m_w = 0.5$, $L = 1$, $r = 4$, $\mu_d = 0.6$, $\mu_s = 0.7$, and $g = 9.8$. Simulation has been done for the WMR performing two



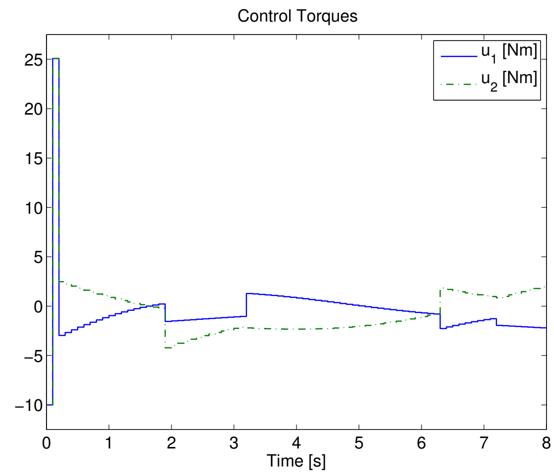
(a) Trajectory of the WMR.



(b) Forward (v) and angular (ω) velocities of the WMR; relative velocities of the wheels (v_r^1 and v_r^2).



(c) Switching behavior of the WMR.



(d) Control torques of the WMR.

Fig. 2. Simulation results for stabilization to a point.

different tasks. The first task is to drive the WMR from the initial state $z_0^T = [0, 4, 1, 0, 0, 0, 1, 1]$ to the origin (the final state is $z_f = 0$) within $T = 8$ sec. Fig. 2 shows the trajectory, forward velocity, relative velocities of both wheels, angular velocity of the body, modes and control inputs of the WMR in task 1. Note that the number of grid points for this simulation is $N = 80$. The figures show that both wheels slide initially and they are both driven to rolling mode after about 0.2 sec. The WMR reaches the origin within $T = 8$ sec successfully.

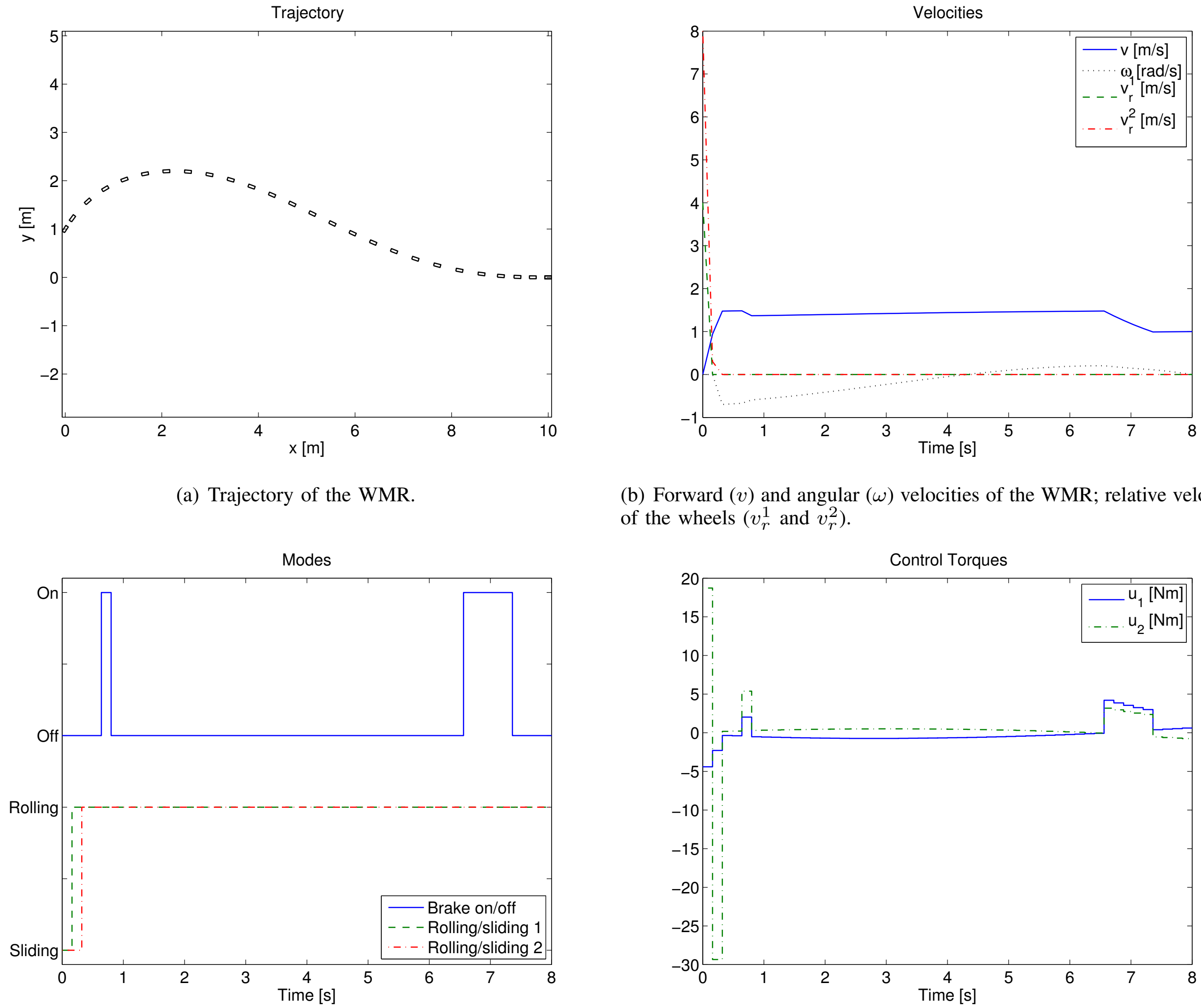
The second task is to stabilize the WMR from the initial state $z_0^T = [0, 1, 1, 0, 0, 0, -1, -2]$ to the y axis with a constant forward velocity 1 (which means that the final state is $z_f^T = [x(T), 0, 0, 1, 0, 0, -0.25, -0.25])$ within $T = 8$ sec. See Fig. 3 for the trajectory, forward velocity, relative velocities of both wheels, angular velocity of the body, modes and control inputs of the WMR in task 2. The number of grid points for this simulation is $N = 50$. From the figures we can see that both wheels slide initially. Wheel 1 switches to rolling at about 0.15 sec while wheel 2 switches to rolling at about 0.3 sec. Again, the WMR is stabilized to the desired trajectory within $T = 8$ sec without any difficulty.

From the simulation results for these two examples we can see that the WMR is successfully stabilized to the predefined set within an allotted time in spite of the initial sliding condition. The approach achieves good performance.

VI. Conclusions

The paper studies application of hybrid model predictive control (MPC) to stabilization of wheeled mobile robots (WMRs) subject to wheel slippage. The approach is based on the theoretical insight from [13] and numerical techniques from [12]. Wheel slippage in the WMR results in a hybrid system, but using the results from [13] the hybrid optimal control resulting from the application of MPC can be formulated as a smooth MPC problem and thus effectively solved using the numerical methods developed in [12]. To improve the convergence of the MPC we use the quadratic penalty function. Simulation results show that the performance of the approach is good.

Although the WMR studied in the paper is a simple 2-wheel mobile robot, the approach can be easily extended to



(a) Trajectory of the WMR.

(b) Forward (v) and angular (ω) velocities of the WMR; relative velocities of the wheels (v_r^1 and v_r^2).

(c) Switching behavior of the WMR.

(d) Control torques of the WMR.

Fig. 3. Simulation results for stabilization to a line.

other WMRs, or other complex robotic systems. Also, the set to which the WMR is stabilized can be generalized from a point or a line to more complex trajectories.

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