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MODELING AND ANALYSIS OF BERNOULLI PRODUCTION SYSTEMS WITH SPLIT AND MERGE

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Abstract— **Many production systems have split and merge operations to increase production capacity and variety, improve product quality, and implement product control and scheduling policies. In this paper, we present analytical methods to model and analyze Bernoulli production systems with circulate and priority split/merge policies. The recursive procedures for performance analysis are derived, the convergence of the procedures and uniqueness of the solutions, along with the structural properties, are proved analytically, and the accuracy of the estimation is justified numerically with high precision.**

Keywords: Split, merge, production rate, Bernoulli reliability.

I. INTRODUCTION

Substantial amount of research effort has been devoted to performance analysis of production systems over the last fifty years. Most of the studies emphasize on serial production lines or assembly systems (see reviews [1]-[3] and monographs [4]-[7]). In modern manufacturing systems, split and merge operations are typically used to increase production capacity and variety, improve product quality, and implement product control and scheduling policies. For example, parallel operations/lines are used to increase production volumes, defective parts are separated from main line to be either repaired or scraped, dedicated operations may be carried out for specific products, etc. To implement such operations, different split and merge policies have been adopted to ensure desired system performance. In recent years, a few performance analysis methods have been developed to model such systems (see review [3] and representative papers [8]-[18]). Despite of these efforts, the split and merge systems with different policies have not been studied thoroughly. The goal of this paper is intended to contribute to this end.

In this paper, we consider Bernoulli production systems with split and merge operations. Two most widely used split and merge policies are considered: circulate and priority. In circulate policy, the split machine sends the part to downstream branches in circulation when it is not blocked by any branch. A branch will be ignored if it blocks the split machine. Similar scenario occurs in merge operations, where the merge station takes part from all upstream branches circularly ignoring the empty buffer branch. In priority policy, one branch has higher priority so that the split machine always dispatches parts to this branch unless it is blocked. Parts are sent to lower priority branch only when the split machine is blocked by the one with higher priority. Analogously, the merge station takes parts from higher priority upstream branch first. Similar policies have been studied for specific systems. For example, a flow line with split based on percentage is studied in [8]. Three station merge systems with shared merge buffer are discussed in [9]-[12]. Priority merge policy is used when blockage occurs. Papers [13] and [14] study multiple product systems where different products are processed at the dedicated machines or lines. Circulate or percentage merge policies are assumed, respectively. Rework and parallel lines are discussed in [15]-[18], where rework systems adopt priority merge policy, and equal probability split and merge policies are discussed in parallel lines with shared split/merge buffers. However, no comparisons among different policies and their impacts are discussed in the literature.

The remaining of the paper is structured as follows: Section II formulates the problem. The modeling and analysis methods for split and merge systems are introduced in Sections III and IV, respectively. Discussions and extensions to larger systems are presented in Section V. Finally, Section VI formulates the conclusions of the paper. Due to space limitation, all proofs are omitted and can be found in [19].

II. PROBLEM FORMULATION

The typical structures of split and merge systems are shown in Figures 1 and 2, respectively, where the circles represent the machines and the rectangles are the buffers. The following assumptions address the machines, the buffers, and their interactions.

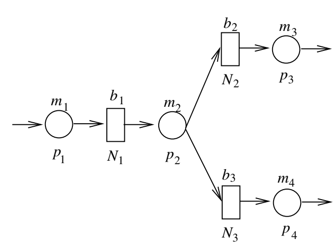

Fig. 1. Production system with split

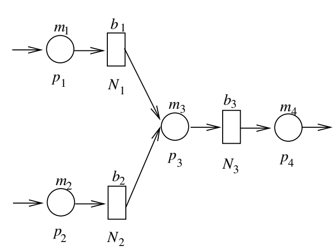

Fig. 2. Production system with merge

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1) All machines have identical processing times. The time is slotted as cycle time.
2) Each machine m_i, $i = 1, \ldots, 4$, is characterized by its reliability p_i, i.e., at each cycle, m_i has probability p_i to be up and $1 - p_i$ to be down. When it is up, it is capable of processing a part. When the machine is down, no production takes place.

   Remark 1: Assumptions 1) and 2) formulate the Bernoulli reliability model of the machines. Many production systems can be characterized by this reliability model, where the machine downtime is comparable to machine cycle time. For example, in automotive assembly systems, the majority of the machine breakdowns is due to pallet jam, push button stop, etc., and only a short period of time is needed to correct these problems. In [7], an exp-B transformation is introduced to transform exponential machine reliability models, where machines may have different speeds, up- and downtimes, into Bernoulli models with acceptable accuracy. For instance, the slower machines in the split branches would be transformed into Bernoulli machine with a smaller p_i. Paper [20] shows that the differences in throughput using Bernoulli and other reliability models are typically small. Systems with exponential reliability machines will be addressed in future work.
3) Each buffer b_k, $k = 1, 2, 3$, has capacity N_k, $0 < N_k < \infty$.
4) A machine is blocked if it is up, downstream buffer is full and downstream machine does not take a part from the buffer at the beginning of the time slot. In split system, machines m_3 and m_4 are never blocked. In merge system, m_4 is never blocked.
5) A machine is starved if it is up, and upstream buffer is empty. Machine m_1 in split system is never starved, and m_1 and m_2 in merge system are never starved.
6) Machine m_2 in split system (correspondingly, m_3 in merge system) will send a part to downstream buffers b_2 and b_3 (respectively, take a part from upstream buffers b_1 and b_2) based on the following policies:
   - *Circulate policy.* m_2 will send a part to buffers b_2 and b_3 circularly if it is not blocked (respectively, m_3 takes part from b_1 and b_2 circularly when it is not starved). If it is blocked by one buffer, m_2 will send the part to another buffer (respectively, m_3 will take part from another buffer if it is starved by one).
   - *Priority policy.* m_2 will keep sending parts to buffer b_2 whenever it has space, i.e., b_2 has higher priority (respectively, m_3 takes part from b_1 if it has available parts). m_2 sends parts to b_3 only when it is blocked by b_2 (respectively, m_3 takes parts from b_2 only when it is starved by b_1).

   Remark 2: In practice, circulate and priority policies are used more often in production than other policies due to relatively easy implementation. For example, circulate policy is often used in parallel operations, and priority policy are typical in rework and re-entrant lines. Another policy based on percentage has also been studied in the literature, however, it is less popular due to implementation difficulty. Due to page limitation, the study of percentage policy is omitted in this paper. A detailed analysis on it can be found in [19].

The system under consideration is defined by assumptions 1)-6), which define a stationary, ergodic Markov chain in the time scale of the time slot. We consider the steady state of the chain in this paper and refer to this steady state as the normal system operations.

Let PR be the production rate of the system, i.e., the average number of parts produced by the last machines (m_3 and m_4 in split system and m_4 in merge case) per time slot. The problem addressed in this work is formulated as follows: *Given production system 1)-6), develop a method for evaluating the production rate as a function of the system parameters.*

Solutions to the problem are presented in Sections III and IV for split and merge systems, respectively.

III. MODELING AND ANALYSIS OF SPLIT SYSTEM

A. Idea of the Approach

The main difficulty of analyzing split system is that the split machine has to allocate capacity to different downstream branches and all machines and buffers are interfering with each other and impact such allocation. This makes the exact analysis all but impossible. Therefore, approximation is pursued. The idea of the approximation is based on overlapping decomposition ([18]), and is illustrated as follows (Figure 3):

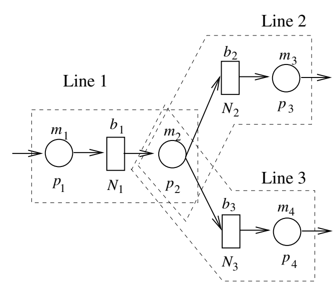

Fig. 3. Overlapping decomposition of split system

Consider the split system depicted in Figure 3. Assume the probabilities that m_2 is blocked by b_2 and b_3 are known, machine m_2 can be modified as m_2' to take into account these effects. Denote this line as Line 1 (m_1, b_1 and m_2'). Then the probability that m_2 is starved by b_1 can be calculated. Now consider machine m_2 with capacity allocated only to buffer b_2 and m_3, modify m_2 into m_2'' to include only such capacity and its starvation probability by b_1, we obtain Line 2 (m_2'', b_2 and m_3). Thus, the probability that m_2 is blocked by b_2 can be calculated. Analogously, m_2 again can be modified into m_2''' to take into account the starvation probability and the only capacity allocated to b_3 and m_4, Line 3 (m_2''', b_3 and m_4) is composed and the probability that m_2 is blocked by b_3 can be obtained. Using these probabilities, we carry out the analysis for Line 1 again, and the procedure is repeated anew.

When the procedure is convergent, we obtain the production rates of Lines 1-3. The specific split policies (priority or circulate) will be taken into account when modifications of m_2 are carried out.

B. Recursive Procedures

Introduce operator $PR(p_1, p_2, N_1)$ to denote the production rate calculation of a two-machine serial line, and p_i and N_1 represent the machine reliability and buffer capacity, respectively (see [7] for details). Using this operator, the recursive procedures to analyze split systems with different policies are developed.

1) Circulate policy: Consider the split system in Figure 1. The rationale behind the modification of m_2 is that, in Line 1, m_2 is available to b_1 if it is neither blocked by b_2 nor b_3. In Line 2, when m_2 is not starved, it is available to b_2 50% of time if b_3 is not full, and 100% of time otherwise. Similar argument applies to Line 3. Thus, the recursive procedure is introduced as follows:

Procedure 1:

$$
\begin{aligned}
&\text{Line 1} && \\
p_2'(s+1) &= p_2(1 - X_{2N_2}(s)X_{3N_3}(s)), \\
pr_1(s+1) &= PR(p_1, p_2'(s+1), N_1), \qquad (1)\\
X_{10}(s+1) &= 1 - \frac{pr_1(s+1)}{p_2'(s+1)}, \\
&\text{Line 2} && \\
p_2''(s+1) &= p_2(0.5(1 - X_{3N_3}(s)) + X_{3N_3}(s)) \\
&\quad (1 - X_{10}(s+1)), \\
pr_2(s+1) &= PR(p_2''(s+1), p_3, N_2), \qquad (2)\\
X_{2N_2}(s+1) &= 1 - \frac{pr_2(s+1)}{p_2''(s+1)}, \\
&\text{Line 3} && \\
p_2'''(s+1) &= p_2(0.5(1 - X_{2N_2}(s+1)) + X_{2N_2}(s+1)) \\
&\quad (1 - X_{10}(s+1)), \\
pr_3(s+1) &= PR(p_2'''(s+1), p_4, N_3), \qquad (3)\\
X_{3N_3}(s+1) &= 1 - \frac{pr_3(s+1)}{p_2'''(s+1)}, \\
s &= 0, 1, 2, \ldots, \\
X_{2N_2}(0) &= X_{3N_3}(0) = 0,
\end{aligned}
$$

where X_{10}, X_{2N_2}, X_{3N_3} denote the probabilities that b_1 is empty, b_2 and b_3 are full, respectively, s is the iteration number and

$$
\begin{aligned}
PR(p_1, p_2, N) &= p_2[1 - Q(p_1, p_2, N)], \qquad (4)\\
Q(p_1, p_2, N) &= \begin{cases} \frac{(1-p_1)(1-\alpha(p_1,p_2))}{1-\frac{p_1}{p_2}\alpha^N(p_1,p_2)}, & \text{if } p_1 \neq p_2 \\ \frac{1-p_1}{N+1-p_1}, & \text{if } p_1 = p_2, \end{cases} \qquad (5)\\
\alpha(p_1, p_2) &= \frac{p_1(1-p_2)}{p_2(1-p_1)}. \qquad (6)
\end{aligned}
$$

2) Priority policy: Assuming buffer b_2 has higher priority than b_3. Then, in Line 2, m_2 is always available to b_2 when it is not starved. m_2 is available to b_3 only when it is not starved, but blocked by b_2. The recursive procedure is modified as follows:

Procedure 2:

$$
\begin{aligned}
&\text{Line 1} && \\
p_2'(s+1) &= p_2(1 - X_{2N_2}(s)X_{3N_3}(s)), \\
pr_1(s+1) &= PR(p_1, p_2'(s+1), N_1), \qquad (7)\\
X_{10}(s+1) &= 1 - \frac{pr_1(s+1)}{p_2'(s+1)}, \\
&\text{Line 2} && \\
p_2''(s+1) &= p_2(1 - X_{10}(s+1)), \\
pr_2(s+1) &= PR(p_2''(s+1), p_3, N_2), \qquad (8)\\
X_{2N_2}(s+1) &= 1 - \frac{pr_2(s+1)}{p_2''(s+1)}, \\
&\text{Line 3} && \\
p_2'''(s+1) &= p_2(1 - X_{10}(s+1))X_{2N_2}(s+1), \\
pr_3(s+1) &= PR(p_2'''(s+1), p_4, N_3), \qquad (9)\\
X_{3N_3}(s+1) &= 1 - \frac{pr_3(s+1)}{p_2'''(s+1)}, \\
s &= 0, 1, 2, \ldots, \\
X_{2N_2}(0) &= X_{3N_3}(0) = 0.
\end{aligned}
$$

C. Convergence

Let $\widehat{PR}_i$, $i = 1, 2, 3$, denote the production rates obtained for Line i if Procedures 1 and 2 are convergent. It is shown below that these procedures lead to convergent results.

Theorem 1: Under assumptions 1)-6), Procedures 1 and 2 are convergent, therefore, the following limits exist:

$$
\lim_{s\to\infty} pr_i(s) := \widehat{PR}_i, \qquad i = 1, 2, 3. \qquad (10)
$$

Corollary 1: Under assumptions 1)-6), the steady state equations of Procedures 1 and 2 have unique solutions.

Therefore, we obtain an estimate of the production rates, $\widehat{PR}_{s,c}$ for circulate policy, $\widehat{PR}_{s,p}$ for priority policy, of the split systems in steady state. Such estimates equal to $\widehat{PR}_2 + \widehat{PR}_3$ in their corresponding procedures.

D. Accuracy

The accuracy of the estimation is investigated numerically. Specifically, we randomly and equiprobably select machine and buffer parameters from the following sets, and construct 50 split systems.

$$
\begin{aligned}
p_1, p_2 &\in [0.75, 0.95], \\
p_3, p_4 &\in [0.4, 0.6], \qquad (11)\\
N_i &\in \{1, 2, 3\}.
\end{aligned}
$$

Both circulate and priority split policies are applied to these systems. For each of these lines, both analytical method using Procedures 1 and 2 and simulation approach using *Simul8* ([21]) are pursued to evaluate system production rates. In each simulation, 10,000 cycles of warmup time are assumed, and the next 100,000 cycles are used for collecting steady state statistics. 20 replications are carried out to obtain the average production rate, with 95% confidence intervals consistently ranging within ± 0.0006. Typically, the computation time for Procedures 1 and 2 is within a fraction of second, and is around 5 minutes for simulation on a

PC with 3.4GHz processor and 2GB RAM. The differences between analytical and simulation results are evaluated as

$$\begin{aligned}\epsilon_{s,c} &= \frac{\widehat{PR}_{s,c} - PR_{s,c}}{PR_{s,c}} \cdot 100\%,\\ \epsilon_{s,p} &= \frac{\widehat{PR}_{s,p} - PR_{s,p}}{PR_{s,p}} \cdot 100\%,\end{aligned} \tag{12}$$

where $PR_{s,c}$ and $PR_{s,p}$ are the production rates obtained by simulation for circulate and priority split policies, respectively.

Remark 3: Assumption 4) defines a block before service (BBS) convention, i.e., a machine will not load a part if it is blocked. In Simul8, a block after service (BAS) is typically used, where a part is still loaded and processed even if no downstream buffer is available. The capacity of buffers under BBS and BAS schemes are related as

$$N_i^{BBS} = N_i^{BAS} + 1, \quad i = 1, 2, 3.$$

The results of this investigation are illustrated in Figures 4 and 5 for Procedures 1 and 2, respectively. It is shown that in all cases we studied, the error is less than 2%. Therefore, Procedures 1 and 2 provide an accurate approximation for system production rates.

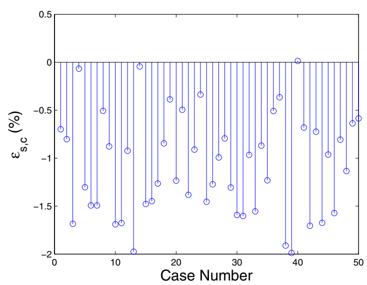

Fig. 4. Accuracy of Procedure 1

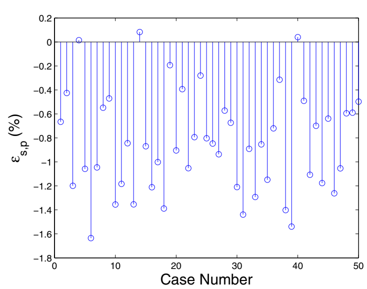

Fig. 5. Accuracy of Procedure 2

E. Structural Properties

1) Conservation of flow:

Corollary 2: Under assumptions 1)-6), the production rates of Lines 1-3 in split system satisfy the following property:

$$\widehat{PR}_1 = \widehat{PR}_2 + \widehat{PR}_3.$$

2) Monotonicity: It has been shown in [7] that monotonicity holds in serial lines and assembly systems, i.e., improving machine reliability and/or increasing buffer capacity lead to improvement of system production rate. Similar properties are observed in split systems as well.

Corollary 3: Under assumptions 1)-6), the system production rates in split systems are monotonically increasing with respect to p_i, $i = 1, \ldots, 4$, and N_i, $i = 1, 2, 3$.

IV. MODELING AND ANALYSIS OF MERGE SYSTEM

A. Idea of the Approach

An idea similar to that of the split system can be applied to the merge system (Figure 6) as well. Line 1 consists of pseudo machine m_3' taking into the account of blockage of buffer b_3 and capacity allocation to buffer b_1, m_3'' in Line 2 considers the blockage of b_3 and capacity allocated to b_2, and finally, m_3''' includes starvation probabilities from b_1 and b_2. A recursive procedure to update these blockage and starvation probabilities is then introduced:

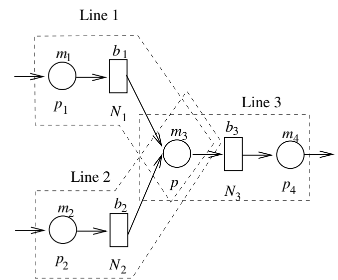

Fig. 6. Overlapping decomposition of merge system

1) Circulate policy: Consider the merge system in Figure 2. Similar to the rationale in circulate split policy, by replacing the blockage with starvation, and vice versa, we obtain

Procedure 3:

$$\begin{aligned}
&\text{Line 1}\\
p_3'(s+1) &= p_3(0.5(1 - X_{20}(s)) + X_{20}(s))\\
&\quad (1 - X_{3N_3}(s)),\\
pr_1(s+1) &= PR(p_1, p_3'(s+1), N_1),\\
X_{10}(s+1) &= 1 - \frac{pr_1(s+1)}{p_3'(s+1)},
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\text{Line 2}\\
p_3''(s+1) &= p_3(0.5(1 - X_{10}(s+1)) + X_{10}(s))\\
&\quad (1 - X_{3N_3}(s)),\\
pr_2(s+1) &= PR(p_2, p_3''(s+1), N_2),\\
X_{20}(s+1) &= 1 - \frac{pr_2(s+1)}{p_3''(s+1)},
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\text{Line 3}\\
p_3'''(s+1) &= p_3(1 - X_{10}(s+1)X_{20}(s+1)),\\
pr_3(s+1) &= PR(p_3'''(s+1), p_4, N_3),\\
X_{3N_3}(s+1) &= 1 - \frac{pr_3(s+1)}{p_3'''(s+1)},\\
s &= 0, 1, 2, \ldots,\\
X_{10}(0) &= X_{20}(0) = 0,
\end{aligned} \tag{15}$$

where X_{10}, X_{20}, X_{3N_3} denote the probabilities that b_1 and b_2 are empty, and b_3 is full, respectively.

2) Priority policy: Assuming buffer b_1 has higher priority than b_2. Analogously to Procedure 2, we have

Procedure 4:

$$
\begin{array}{rcl}
\multicolumn{3}{l}{\text{Line 1}} \\
p_3'(s+1) &=& p_3(1-X_{3N_3}(s)), \\
pr_1(s+1) &=& PR(p_1, p_3'(s+1), N_1), \\
X_{10}(s+1) &=& 1-\dfrac{pr_1(s+1)}{p_3'(s+1)},
\end{array}
\qquad (16)
$$

$$
\begin{array}{rcl}
\multicolumn{3}{l}{\text{Line 2}} \\
p_3''(s+1) &=& p_2 X_{10}(s+1)(1-X_{3N_3}(s+1)), \\
pr_2(s+1) &=& PR(p_2, p_3''(s+1), N_2), \\
X_{20}(s+1) &=& 1-\dfrac{pr_2(s+1)}{p_3''(s+1)},
\end{array}
\qquad (17)
$$

$$
\begin{array}{rcl}
\multicolumn{3}{l}{\text{Line 3}} \\
p_3'''(s+1) &=& p_3(1-X_{10}(s+1))X_{20}(s+1)), \\
pr_3(s+1) &=& PR(p_3'''(s+1), p_4, N_3), \\
X_{3N_3}(s+1) &=& 1-\dfrac{pr_3(s+1)}{p_3'''(s+1)}, \\
s &=& 0,1,2,\ldots, \\
X_{10}(0) &=& X_{20}(0)=0.
\end{array}
\qquad (18)
$$

B. Convergence

Theorem 2: Under assumptions 1)-6), Procedures 3 and 4 are convergent, therefore, the following limits exist:

$$\lim_{s\to\infty} pr_i(s) := \widehat{PR}_i, \qquad i=1,2,3. \qquad (19)$$

Corollary 4: Under assumptions 1)-6), the steady state equations of Procedures 3 and 4 have unique solutions.

Therefore, we obtain the estimates of the production rates, $\widehat{PR}_{m,c}$ for circulate policy, $\widehat{PR}_{m,p}$ for priority policy, of the merge systems in steady state, which are equal to $\widehat{PR}_3$ in their corresponding procedures.

C. Accuracy

The accuracy of the estimation is again investigated numerically. By reversing the split systems, and applying the corresponding parameters, we obtain 50 merge lines. We apply both circulate and priority merge policies to these lines and same simulation setups are carried out. The differences between analytical and simulation results are evaluated as

$$
\begin{array}{rcl}
\epsilon_{m,c} &=& \dfrac{\widehat{PR}_{m,c}-PR_{m,c}}{PR_{m,c}}\cdot 100\%, \\
\epsilon_{m,p} &=& \dfrac{\widehat{PR}_{m,p}-PR_{m,p}}{PR_{m,p}}\cdot 100\%,
\end{array}
\qquad (20)
$$

where $PR_{m,c}$, $PR_{m,p}$ are the production rates obtained by simulation for circulate and priority merge policies, respectively. Again it is shown in Figures 7 and 8 that the error for production rate approximation are typically less than 2%.

D. Structural Properties

1) Conservation of flow:

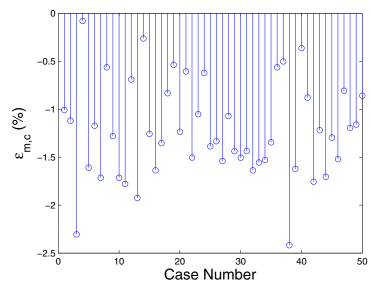

Fig. 7. Accuracy of Procedure 3

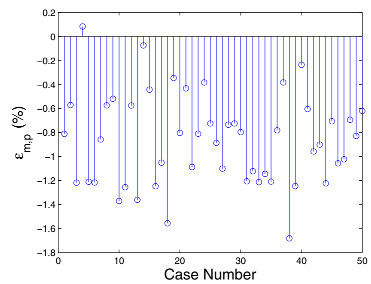

Fig. 8. Accuracy of Procedure 4

Corollary 5: Under assumptions 1)-6), the production rates of Lines 1-3 in merge system satisfy the following property:

$$\widehat{PR}_3 = \widehat{PR}_1 + \widehat{PR}_2.$$

2) Monotonicity:

Corollary 6: Under assumptions 1)-6), the system production rates in merge systems are monotonically increasing with respect to p_i, $i=1,\ldots,4$, and N_i, $i=1,2,3$.

V. DISCUSSIONS AND EXTENSIONS

A. Reversibility

It has been shown that the reversibility exists in Bernoulli serial production lines ([7]). For the split and merge systems with circulate and priority policies considered in this paper, such property still holds. To illustrate this, denote the machine and buffer parameters in Figure 1 as p_i^s, $i=1,\ldots,4$, N_i^s, $i=1,2,3$, and in Figure 2 as p_i^m, and N_i^m. In addition,

$$
\begin{array}{rcl}
p_1^s = p_4^m, & & p_2^s = p_3^m, \\
p_3^s = p_1^m, & & p_4^s = p_2^m, \\
N_1^s = N_3^m, & & N_2^s = N_1^m, \\
N_3^s = N_2^m. & &
\end{array}
\qquad (21)
$$

Corollary 7: Under assumptions 1)-6) and condition (21), the system production rates in split and merge systems with circulate and priority policies have identical production rates. In other words

$$\widehat{PR}_{m,c} = \widehat{PR}_{s,c}, \qquad \widehat{PR}_{m,p} = \widehat{PR}_{s,p}.$$

B. Comparisons

A comparison between the circulate and priority policies has been carried out. The results show that the difference in system production rates between systems with circulate and priority policies is typically small. In other words,

$$|\widehat{PR}_{s,c} - \widehat{PR}_{s,p}| \ll 1, \quad |\widehat{PR}_{m,c} - \widehat{PR}_{m,p}| \ll 1.$$

In addition, numerical results suggest that it is always beneficial to assign more reliable machine with higher priority. In other words, if $p_1 > p_2$, then a merge system with machine m_1 having higher priority will achieve better production rate than a system where m_2 has higher priority, i.e.,

$$\begin{aligned} &\widehat{PR}(p_1, p_2, p_3, p_4, N_1, N_2, N_3) \\ &\quad > \widehat{PR}(p_2, p_1, p_3, p_4, N_1, N_2, N_3). \end{aligned}$$

Based on reversibility, similar argument applies to split system as well.

C. Extensions to Larger Systems

The methods introduced here can be easily extended to split and merge systems with longer lines and multiple branches (see Figures 9 and 10 for illustrations of split systems, similar figures for merge systems can be found in [19]). The preliminary studies have been carried out to analyze such systems. Overlapping decomposition procedures can be implemented to evaluate the system performance. For example, for long split lines, overlapped Lines 1-3 become $(m_{11}, \ldots, m'_{1M_1}, b_{11}, \ldots, b_{1M_1-1})$, $(m''_{1M_1}, m_{21}, \ldots, m_{2M_2}, b_{21}, \ldots, b_{2M_2})$ and $(m'''_{1M_1}, m_{31}, \ldots, m_{3M_3}, b_{31}, \ldots, b_{3M_3})$. Long serial line analysis procedure ([7]) will be applied here. For multiple split lines, M lines are introduced, (m_1, m'_2, b_1), (m''_2, m_3, b_2), $\ldots$, $(m_2^{'\cdots'}, m_{M+1}, b_M)$. Machine m_2 is the overlapping machine.

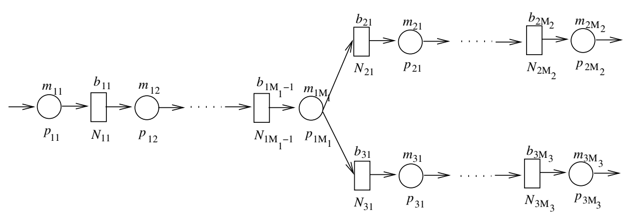

Fig. 9. Long split lines

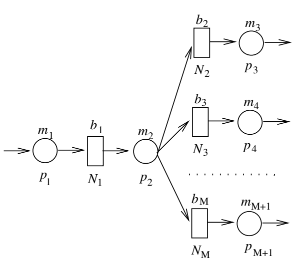

Fig. 10. Multiple split lines

The preliminary results show that the proposed methods still achieve acceptable accuracy in production rate estimation. All structural properties hold for such systems as well.

VI. CONCLUSIONS

Split and merge are widely used in many manufacturing systems. In this paper, we present analytical methods to approximate the system production rates of split and merge systems with Bernoulli reliability machines. Two split and merge policies are addressed: circulate and priority. It is shown that these methods can provide an accurate precision for system production rate estimation. In future work, these methods will be extended to other machine reliability models (e.g., exponential). The successful development of such methods will provide production engineers a quantitative tool for design and continuous improvement of complex production systems.

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