2008 IEEE International Conference on
Robotics and Automation
Pasadena, CA, USA, May 19-23, 2008

An Asymmetric Robotic Catapult based on the Closed Elastica for Jumping Robot

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***Abstract*— In this paper, we propose a new asymmetric robotic catapult based on the closed elastica. The conventional robotic catapults based on the closed elastica which the authors developed are the robotic elements for generating impulsive motions by utilizing a snap-through buckling. In a typical closed elastica, the two ends of an elastic strip are fixed to a passive rotational joint and an active rotational joint, respectively. Here we found that by adding only a range limitation to the passive rotational joint, compared to the conventional type, the deforming shape of the elastic strip becomes more complicated and 40% more elastic energy can be stored. Using this modification, we can develop a compact jumping robot which is able to leap over 700[mm] away and 200[mm] high.**

I. Introduction

Current robot technologies have not succeeded in developing rapid and lithe autonomous compact robots capable of creature-like brisk motions. Rather than hopping like kangaroo or Raibert's one-legged robot [16], for compact-size autonomous robots, as well as tiny creatures like grasshopper, cricket, and flea, large distance jumps by impulsively releasing stored potential energy are required for achieving a useful locomotion. The purpose of this research is robotic realization of such creature-like brisk jumping motions.

The existence of special biological catapults in chameleons, fleas and others [4] [5] [7] [14] indicates that the key to achieve creature-like motions may be the generation of impulsive motions. There are some researches on robotic catapults, such as [8], but here we focus on the robotic catapults for compact mobile robots moving impulsively, which are mainly in the field of jumping robots. Recently, there appeared some robots with impulsive jumping motions. Jumping robot with a piston-driving combustion chamber mounted inside a sphere was created by Sandia National Laboratories [23]. In this robot, a small charge of liquid propane or other fuel ignites within the chamber. It can jump thousands of times on a 20[g] fuel tank, enabling it to wander for at least several kilometers before running out of gas. Hirose developed a quadruped walking robot which can jump with legs driven by strong pneumatic actuators [9] [19]. The recent modified version of this robot with wheels attached to the legs can jump up to the height of about 0.7[m] for its body length of about 0.3[m]. Tsukagoshi et al. developed a jumping rescue robot which can jump over 1.0[m] for its body lengths of about 0.25[m] by using accumulated air as jumping energy source [22]. These robots adopt pneumatic systems which tend to be heavy and large. Therefore, it is difficult to make compact impulsive robots by these mechanisms. Nassiraei et al. developed an acrobat robot which can move with rapid velocity change using an eccentric cam [2] [3]. Recently, Scarfogliero et al. proposed a robotic click mechanism for a jumping robot which aims at a driving frequency of 2[Hz] [17]. In this robot, an eccentric cam loads a torsional spring that actuates the robot legs. However, high frequency impulsive motions by using this mechanism have not been achieved for compact autonomous robot yet. There are deformable robots by Hirai et al. actively using the large deformation of the elastica as well as our robot [18]. Their robot whose total mass is only 4[g] is actuated by SMA and can jump up to 300[mm]. But, it takes over 30 seconds to prepare one jumping motion. Armour et al. also developed a flexible jumping robot [1]. However, the compact size one has not been achieved yet. Further they surveyed many jumping robots in detail.

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In this paper, we propose a new asymmetric robotic catapult based on the closed elastica. Our analysis of the robotic catapult shows the following: from the theoretical analysis, we see that by using only one active rotational joint and a passive rotational joint with a range limitation, the elastic strip can be deformed into the same shape as that of two active rotational joint type robotic catapult [20]. From the numerical simulation results, we see that the stored elastic energy of the proposed robotic catapult is higher than the energy stored by a conventional catapult [20] with the same actuator. Using the proposed catapult with the improved passive joint, we developed a prototype compact jumping robot which is able to leap over 700[mm] away and 200[mm] high.

II. An Asymmetric Robotic Catapult based on the Closed Elastica

Figure 1 is the schematic diagram showing the conventional planar type robotic catapults, which can generate repeated impulsive motions, and the proposed asymmetric robotic catapult. An active-passive type consists of a passive rotational joint and an active rotational joint driven by a motor. When we actuate slowly the active joint in a clockwise direction, the elastic strip changes its shape to a horizontal 'S' shape (the 2^{nd} mode) before snap-through buckling [6]. Continuing to drive the active joint in the same direction generates a snap-through buckling. On the other hand, an active-active type consists of two active rotational joints actuated by motors. The active joints are actuated

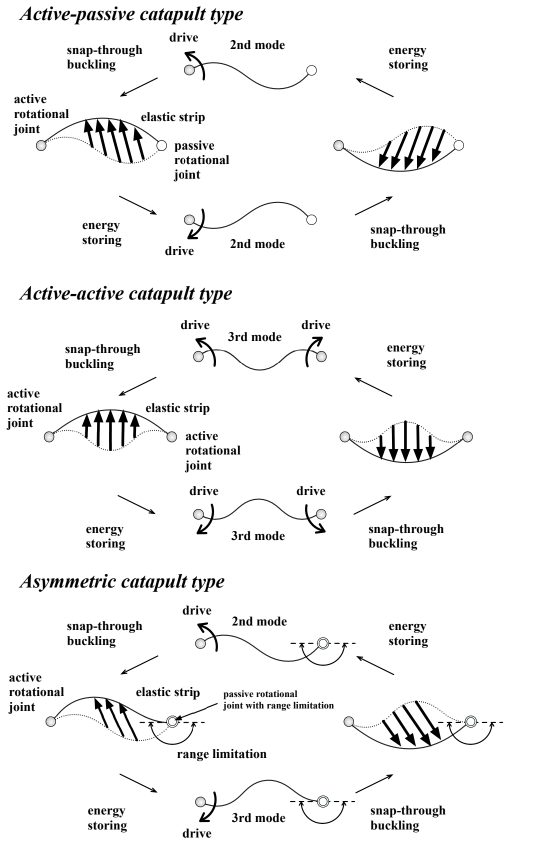

Fig. 1. Shape transitions of conventional planar type robotic catapults based on the closed elastica and its impulsive motions. The shape in the 2^{nd} mode has two extrema and the shape in the 3^{rd} mode has three extrema.

in opposite directions. When the joints on both sides are slowly actuated, the elastic strip changes its shape to a 'W' shape (the 3^{rd} mode) before snap-through buckling. The proposed asymmetric robotic catapult consists of an active rotational joint and a passive rotational joint which has a range limitation. The passive joint behaves like a simple free joint in the rotational range. If the passive joint is at the clockwise limit and we actuate slowly the active joint in a clockwise direction, when the two joints are in symmetric position, the elastic strip shape is the same as that of the active-active catapult type. After generating the snap-through buckling, we can obtain the next impulsive motion by driving the active joint in the counter clockwise direction. However, the next impulsive motion is weaker because of the range limitation of the passive joint.

In this section, we show in theoretical analysis that only introducing a range limitation to the passive rotational joint of an active-passive catapult type leads to more complicated shapes of the elastic strip.

A. Equilibrium Equations

The elastic strip, which is continuum, is approximated by serial chain of rigid bodies connected by elastic joints [20] as

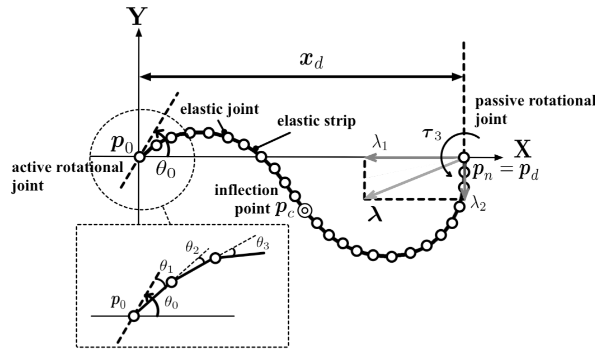

Fig. 2. The relation between shape of the elastic strip and constraint forces of the robotic catapult

shown in Fig.2. Let $\boldsymbol{p}_i \in \Re^3$ be the position vector of the i^{th} link and $\boldsymbol{\Phi}_i \in SO(3)$ [15] be the orientation of the i^{th} joint frame $(i = 1, \cdots, n)$. To express a closed-loop structure of the elastic strip by using the serial chain approximation, the constraint force $\boldsymbol{\lambda} = [\lambda_1\ \lambda_2\ 0]^{\mathrm{T}}$ and the constraint torque $\boldsymbol{\tau} = [0\ 0\ \tau_3]^{\mathrm{T}}$ are applied at the tip of the serial chain. $\boldsymbol{\lambda}$ operates to satisfy $\boldsymbol{p}_n = \boldsymbol{p}_d = [x_d\ 0\ 0]^{\mathrm{T}}$, and $\boldsymbol{\tau}$ operates to satisfy $\boldsymbol{\Phi}_n = \boldsymbol{\Phi}_d$. In static condition, the principle of virtual work leads to the equation as follows.

$$\frac{\partial K}{\partial \boldsymbol{\theta}}^{\mathrm{T}} - \boldsymbol{J}_b^{\mathrm{T}} \begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\tau} \end{bmatrix} = \boldsymbol{0}, \qquad (1)$$

$$\boldsymbol{p}_n = \boldsymbol{p}_d, \qquad (2)$$

$$\boldsymbol{\Phi}_n = \boldsymbol{\Phi}_d, \qquad (3)$$

where $\boldsymbol{J}_b \in \Re^{6\times n}$ represents the Jacobian matrix that transforms the constraint force and torque to the equivalent joint torques. K is an elastic energy given as follows.

$$K(\boldsymbol{\theta}) = \frac{k}{2}\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\theta}, \qquad (4)$$

where $\boldsymbol{\theta} = [\theta_1\ \cdots\ \theta_n]^{\mathrm{T}} \in \Re^n$ and the angle of the base joint is θ_0 (Fig.2). k is the elastic coefficient of the bend of the elastic strip. From (1), the statics of each joint are given as follows.

$$k\theta_i - \boldsymbol{e}_{\mathrm{z}}^{\mathrm{T}}\{(\boldsymbol{p}_n - \boldsymbol{p}_{i-1}) \times \boldsymbol{\lambda} + \boldsymbol{\tau}\} = 0,\ i = 1, \cdots, n \qquad (5)$$

The equation (5) denotes the relation between the stiffness of the i^{th} joint and rotational forces composed of the constraint torque $\boldsymbol{\tau}$ and the momentum by the constraint force $\boldsymbol{\lambda}$ generated by rotating around the vector $\boldsymbol{e}_{\mathrm{z}}$. The driving torque τ_0 at the base joint of the serial chain is represented by a restoration force of the stiffness at the first joint of the elastic strip as follows.

$$\tau_0 = -k\theta_1. \qquad (6)$$

In the case of an active-passive type, since the tip joint of the serial chain is free joint, $\boldsymbol{\tau} = \boldsymbol{0}$ is satisfied. In an active-active type, since both joints are actuated in opposite directions, the constraint condition at the first joint is as follows.

$$\tau_3 = -\tau_0. \qquad (7)$$

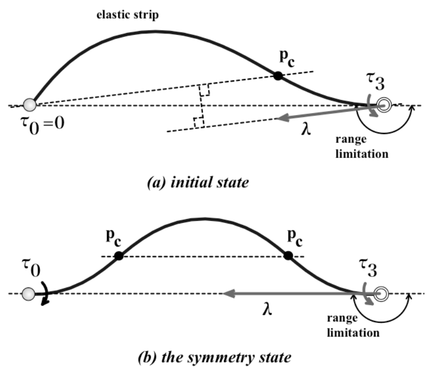

Fig. 3. Deformation of an asymmetric catapult type

B. A Passive Rotational Joint with A Range Limitation

Consider a serial-chain with applied constraint torque τ_3. The constraint torque τ_3 is added such that the tip angle of the serial chain is always a constant value θ_d. We substitute (6) into the statics (5) at the first joint of the serial chain,

$$\tau_0 = -\boldsymbol{e}_{\mathrm{z}}^{\mathrm{T}}(\boldsymbol{p}_n \times \boldsymbol{\lambda} + \boldsymbol{\tau}). \tag{8}$$

Since the tip angle of the elastic strip is fixed about θ_d, there is an inflection point $\boldsymbol{p}_c = [x_c\ y_c\ 0]^{\mathrm{T}}$ on the elastic strip at the initial state ($\tau_0 = 0$) as shown in Fig.3(a). The statics at the inflection point $\boldsymbol{p}_c$ on the elastic strip is as follows.

$$\boldsymbol{e}_{\mathrm{z}}^{\mathrm{T}}\{(\boldsymbol{p}_n - \boldsymbol{p}_c) \times \boldsymbol{\lambda} + \boldsymbol{\tau}\} = 0. \tag{9}$$

We substitute (8) and (6) into (9),

$$\tau_0 \quad = \quad -\boldsymbol{e}_{\mathrm{z}}^{\mathrm{T}}(\boldsymbol{p}_c \times \boldsymbol{\lambda}). \tag{10}$$

Consider the case that there is the inflection point $\boldsymbol{p}_c$ in the range of $x_c > 0$ and $y_c > 0$ at the initial state. Then, (10) represents that the position vector $\boldsymbol{p}_c$ and the constraint force vector $\boldsymbol{\lambda}$ at the tip of the serial chain are parallel at the initial state (Fig.3(a)). Since the distance between the two joints is shorter than the length of the elastic strip, λ_1 is always negative. Then, the direction of the constraint force vector $\boldsymbol{\lambda}$ at the initial state is shown in Fig.3(a), that is, λ_2 is negative. Equation (8) can be written as follows.

$$\tau_0 + \tau_3 = -x_d \lambda_2. \tag{11}$$

In the initial state ($\tau_0 = 0$), the constraint torque τ_3 is positive from (11). From the initial shape, since the active joint is driven at a clockwise direction, the elastic strip deforms into a symmetric shape about the perpendicular line crossing the center of the elastic strip.

Note that τ_0 is always negative while the elastic strip deforms into the symmetric shape. In the symmetric shape, since (7) is satisfied as in the case of the active-active type, τ_3 is positive and $\lambda_2 = 0$ from (11). The constraint force $\boldsymbol{\lambda}$ changes according to the deformation of the elastic strip. The condition of $\lambda_2 = 0$ is satisfied only when the shape of the elastic strip is symmetric, and for λ_2 to change from negative to positive, the elastic strip has to go through the symmetric shape. Therefore, the λ_2 is always negative in the interval between the initial shape and the symmetric shape. Consequently, from (11), τ_3 is always positive between the initial shape and the symmetric shape. However, if θ_d is chosen such that $y_c < 0$ is satisfied during the deformation, λ_2 is positive and there is possibility that the elastic strip never forms a symmetric shape from (11).

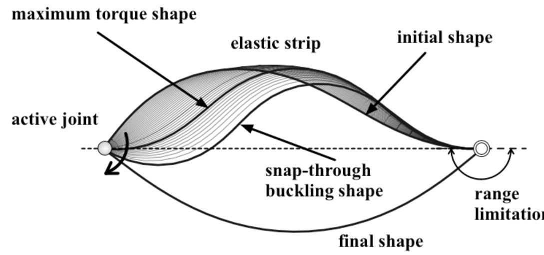

Fig. 4. Stick diagram of an asymmetric catapult type

Noticing that because the sign of the constraint torque τ_3 is constant between the initial shape and the symmetric shape, we realized that we do not need the second actuator and τ_3 can be generated by a passive joint with a range limitation. Therefore, we propose a passive rotational joint which has a range limitation until θ_d. The deforming shape of the elastic strip is shown in Fig.4 which can be obtained by using (2), (3), and (5) for Newton's method [20]. In this simulation, the degree of freedom is $n = 30$. Using the proposed passive rotational joint with a range limitation, the elastic strip deformed into a symmetric shape in spite of having only one active joint. Since the 'W' shape is more complicated than 'S' shape, the stored elastic energy in the proposed catapult is higher than that of a conventional active-passive one. By driving the active rotational joint continuously from the symmetric shape, the snap-through buckling is generated. If we choose a sufficient rotational range to generate snap-through buckling, the proposed passive rotational joint behaves like a conventional free joint between the shape of starting to generate snap-through buckling and the arch shape after snap-through buckling. Consequently, the proposed robotic catapult can release more stored elastic energy than the conventional active-passive type robotic catapult.

C. A Change of the Stored Elastic Energy

Figure 5 shows the change of the driving torque and Fig.6 shows the energy change against the angle of the driving rotational joint of the proposed asymmetric type and two conventional type robotic catapults. Figure 5 shows that robot catapults had maximum torques at some θ_0 and driving torques decrease before the snap-through buckling. However, elastic energy can be stored with almost same increasing rates as shown Fig.6 [20]. It can be seen that the stored elastic energy of an active-active type is higher than that of an active-passive type when the maximum driving torques of both types active rotational joints are made uniform. The stored elastic energy of the proposed asymmetric robotic catapult is higher than that of the conventional active-passive

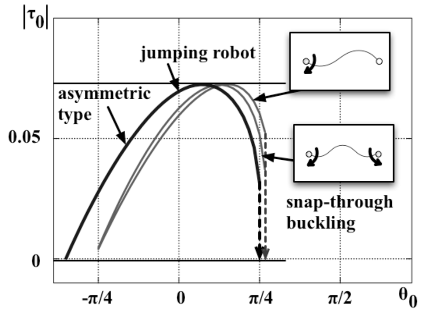

Fig. 5. Driving angle θ_0 vs driving torque τ_0 of an asymmetric catapult type

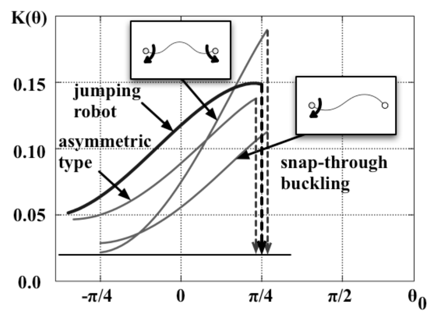

Fig. 6. Driving angle θ_0 vs stored elastic energy $K(\boldsymbol{\theta})$ of an asymmetric catapult type

type. Consequently, we can generate more impulsive motion than conventional active-passive type with the same actuator driving the active rotational joint.

As the robotic catapult has a very simple mechanical structure and the proposed improvement of the passive joint is not complex, the features of compactness and lightness are not lost. In addition, the feature of generating repeated impulsive motions also remains even though the proposed robotic catapult is an asymmetric type.

III. A Jumping Robot based on The Closed Elastica

We can utilize the robotic catapult as a device for jumping if we can reduce its weight. Fortunately, the simple mechanical structure of the robotic catapult allows us to make it very light. We have already shown, as phenomena, aspects of various types of jumping robots powered by the robotic catapults based on the closed elastica in [21]. Here we analyze the functions of the proposed asymmetric robotic catapult for the jumping task with some simulation results.

A. A Prototype Jumping Robot

Figure 7 shows the prototype jumping robot. The elastic strip of the robotic catapult is a hardening, cold rolled, special steel strip (JIS G3311) with 0.012[m] width, 0.002[m] thickness and 0.20[m] length. The distance between the two joints is of 0.150[m]. The active joint is driven by a relatively small RC servo motor (RB90, MiniStudio Inc., China). A

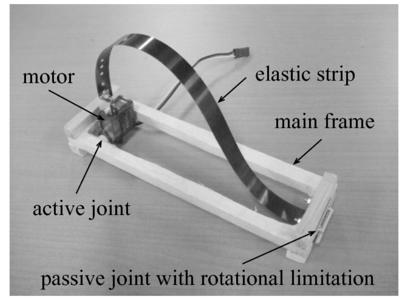

Fig. 7. A prototype jumping robot

TABLE I
Parameters of a Prototype Jumping Robot

Robot parameter	Value
Mass	30 [g]
Body length	170 [mm]
Body height	90 [mm]
Body thickness	50 [mm]
Elastic strip length	200 [mm]
Elastic strip width	12 [mm]
Elastic strip thickness	0.20 [mm]
Arm link	23 [mm]

hinge is used as a passive joint which has a range limitation as shown in Fig.8. Wood is used for the main frame to make the robot as light as possible. A PIC micro computer (12f675) and 9[V] dry cell battery are used as a controller and an energy source, respectively. We attached a rubber sheet to the elastic strip contact area with the ground to prevent slipping. The specification of this robot is summarized in TABLE I. Note that, in this experiment, we did not embed the battery and controller into the robot. The total weight of the robot without a battery and controller is only of 30 [g] (96 [g] including battery and controller board). A dry cell battery is heavier than the robot itself. However, we expect to use lithium-polymer battery which is lighter than the robot. The one end of the elastic strip is fixed at the plastic body of the motor. This structure helps to prevent a rolling motion of the robot because the center of gravity is almost on the long axis of the elastic strip. However, as shown in Fig.9, the structure has a considerable offset between the active rotational joint axis and the end of the elastic strip which differs from the conventional active-passive type, also from the proposed asymmetric catapult type.

B. The Efficiency of the Offset at the Active Joint

For the conventional and the asymmetric catapult structures in which the end of the elastic strip is directly fixed to the active rotational joint, it is possible to regard the offset shown in Fig.9 as a link type active joint such that the active joint is fixed to the end of the link and the tip of the link is fixed to the elastic strip.

The tip position of the link is given using the driving joint angle θ_0 as follows.

$$\boldsymbol{p}_0 = [L\cos\theta_0 \;\; L\sin\theta_0 \;\; 0]^{\mathrm{T}}. \tag{12}$$

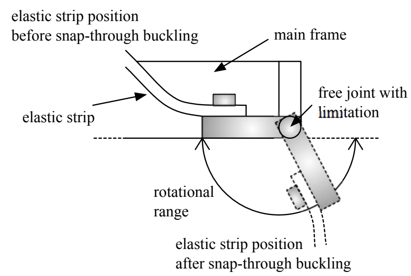

Fig. 8. Passive joint with rotational limitation

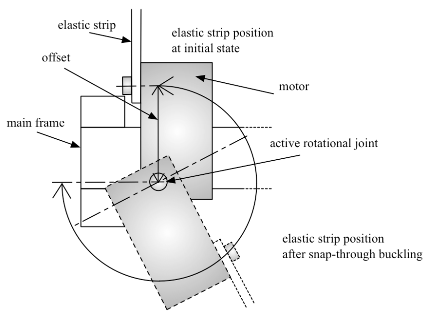

Fig. 9. Active joint with considerable offset

where, L is the length of the link as shown in Fig.10. In this structure, the constraint condition at the first link and the condition at the inflection point $\boldsymbol{p}_c$ are given as follows.

$$\tau_0 = -\boldsymbol{e}_z^{\mathrm{T}}\{(\boldsymbol{p}_n - \boldsymbol{p}_0) \times \boldsymbol{\lambda} + \boldsymbol{\tau}\}, \tag{13}$$

$$\boldsymbol{e}_z^{\mathrm{T}}\{(\boldsymbol{p}_n - \boldsymbol{p}_c) \times \boldsymbol{\lambda} + \boldsymbol{\tau}\} = 0. \tag{14}$$

We substitute (14) into (13),

$$\tau_0 = -\boldsymbol{e}_z^{\mathrm{T}}\{(\boldsymbol{p}_c - \boldsymbol{p}_0) \times \boldsymbol{\lambda}\}. \tag{15}$$

Let us consider in the initial state, that is, $\tau_0 = 0$. Since the distance between the two joints is shorter than the length of the elastic strip, λ_1 is always negative. Then, from (13), the constraint torque τ_3 at the tip of the serial chain is positive if λ_2 is negative. On the other hand, (15) represents that the position vector $\boldsymbol{p}_c - \boldsymbol{p}_0$ and the constraint force vector $\boldsymbol{\lambda}$ at the tip of the serial chain are parallel in the initial state (Fig.10). From the fact that $\lambda_1 < 0$, λ_2 is always negative in the initial state if

$$y_c > L \sin\theta_0 \tag{16}$$

is satisfied. Consequently, we expect to use the passive joint with range limitation if (16) is satisfied.

The deformation of the elastic strip is shown in Fig.11. It can be seen that the link type active joint and the passive joint with range limitation of the jumping robot lead to more complicated shapes of the elastic strip. As shown in Fig.5 and Fig.6, the stored elastic energy of the jump robot is higher than that of the conventional active-passive type and the asymmetric type with a range limitation on the passive joint using the same maximum value of the driving torque.

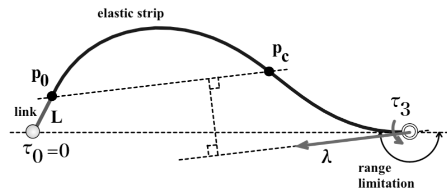

Fig. 10. Initial shape of a jumping robot

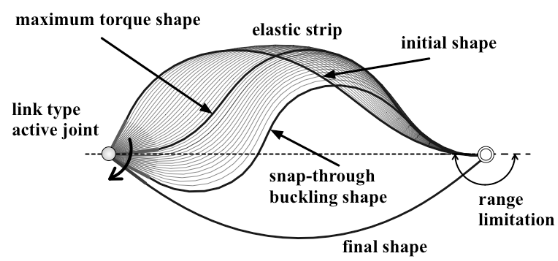

Fig. 11. Stick diagram of a jumping robot

C. Jumping Motion

Figure 12 is a series of photos at $1/15$[s] intervals showing the jumping of the robot. Driving the active rotational joint at a clockwise direction, the elastic strip changes its shape to a 'W' like shape. Then, the main frame of the jumping robot is picked up by the deformed elastic strip contacting the ground. This means that the jumping robot is ready to push the ground efficiently. If we continue to drive the joint, the contact point changes from the left side to the center of the elastic strip. Then, we can obtain the impulsive motion of the elastic strip , i.e., the elastic strip suddenly changes its shape to an arch by snap through buckling. Consequently, the elastic strip strongly pushes the ground, that is, the robot is catapulted by the elastic strip and leaps away. Fig.13 is a series of photos showing the jumping of the robot. The robot has leaped 700[mm] away and 200[mm] high. Since the elastic strip pushes the ground without slip due to the efficiency of the rubber sheet, the robot can leap 4.0 times its body length. Though the jumping robot uses a large deformation of the elastic strip for generating impulsive motion, it takes only about 2[sec] from initial shape to snap-through buckling shape. In addition, because of the proposed mechanism is quite simple, it is suitable for making compact jumping robot.

IV. Conclusion

In this paper, we proposed a new asymmetric robotic catapult based on the closed elastica which the authors developed as a robotic element for generating impulsive motions by utilizing the snap-through buckling. The robotic catapult has a passive rotational joint with a range limitation and a normal active one. The theoretical analysis showed that by using only one active rotational joint and the passive rotational joint with range limitation, the elastic strip could be deformed into the same shape as that of two active rotational joints

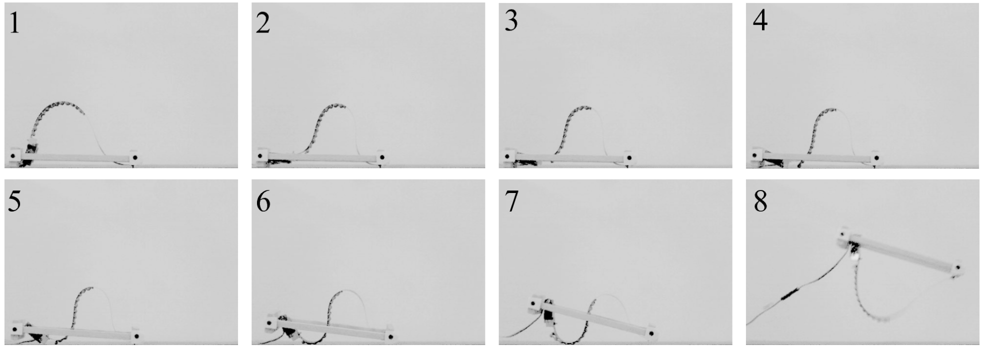

Fig. 12. A series of photos of a robot's jumping motion using high-speed camera

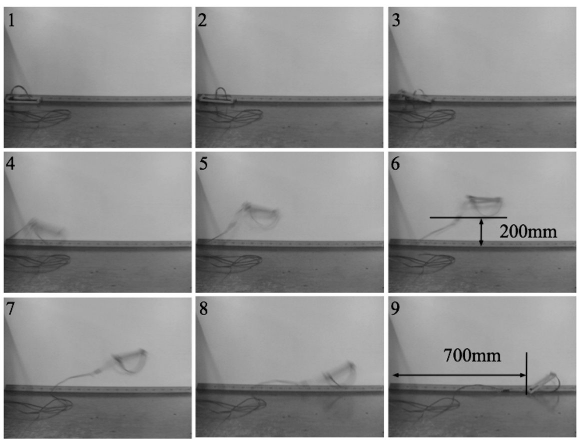

Fig. 13. A series of photos of the jumping robot that leaps over 700 [mm] away

type robotic catapult. The more sophisticated analysis on the energy such as energy efficiency should be addressed in the future. Optimal mechanical design of the proposed robotic catapult including rotational range and offset of the passive joint is also one of the most important future works. We need to built up theory of the robotic closed elastica for these future works which will be based on the theoretical researches on continuum flexible robots [10] [11] [12].

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