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Kinematic Modeling of a Bio-Inspired Robotic Fish

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Abstract—**This paper proposes a kinematic modeling method for a bio-inspired robotic fish based on single joint. Lagrangian function of freely swimming robotic fish is built based on a simplified geometric model. In order to build the kinematic model, the fluid force acting on the robotic fish is divided into three parts: the pressure on links, the approach stream pressure and the frictional force. By solving Lagrange's equation of the second kind and the fluid force, the movement of robotic fish is obtained. The robotic fish's motion, such as propelling and turning are simulated, and experiments are taken to verify the model.**

I. INTRODUCTION

THERE are more and more productive underwater activities and research work, which make the existing underwater equipments can not satisfy changing requests, and Autonomous Underwater Vehicle (AUV) is received much attentions. In nature, fish have advantage over conventional marine vehicles powered by rotary propellers with the same power consumption [1]. Attracted by fish with high efficiency, high maneuverability and low noise, roboticists explore fish-like underwater robots or robotic fish to develop small size, efficient, maneuverable and low noise AUVs.

Many theories are proposed to explore the secrets of fish swimming mechanisms and summarize driving modes of fish motions [2]-[7]. Based on these theories, many prototypes of biomimetic robotic fish have been developed [8]-[24]. There are also some work on the motion [23] [24] and control [25] of robotic fish. McIsaac and Ostrowski [23] give a Lagrangian model, reduced by Lie group symmetries, for a symmetrical structure robot eel. F. Boyer, M. Porezand and W. Khalil [25] present the dynamic modeling of a continuous three-dimensional swimming eel-like robot. K. A. Morgansen, et al. [26] study the nonlinear control methods for robot fish locomotion.

In the former project, a miniature biomimetic robotic fish is developed based on single joint undulating [27]. The prototype is only 15 centimeters long and integrates the control system, wireless communication module, battery power, servo, infrared and light intensity sensors. The robotic fish may execute tasks autonomously by the control law.

In this paper, kinematic model is built for this robotic fish. Considering the coupling of the kinematic law in continuous motions and the force acted on robotic fish, which can not be calculated respectively, the Lagrangian function of the freely swimming robotic fish is built. The force acting on robotic fish is divided to the pressure on links, the approach stream pressure and the frictional forces. The movement is calculated by solving the Lagrange's equation of the second kind. The kinematic parameters are calculated.

The rest of the paper is organized as follows. The Lagrange's equation and the fluid model are introduced in Section II. Section III gives simulation and experimental results, and Section IV concludes the paper.

II. THE KINEMATIC MODELING

Two coordinate are given to describe the motion. XO_1Y is the world rectangular coordinate system (WRCS). Another is a polar coordinate system with the pole O at the joint and the polar axis pointing to the tail's initial direction.

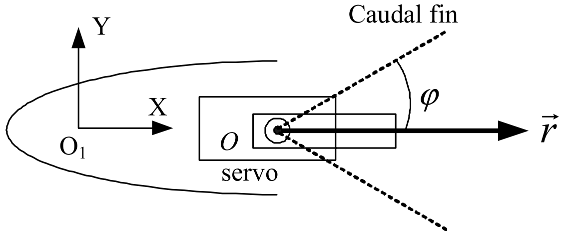

Fig. 1. Coordinates of the robotic fish

A. The Lagrange's equation

The propelling of robotic fish is essentially different from the traditional AUV, and there is no existing model for the movement. For the purpose of analyzing the motion effect, controlling and optimizing, a motion model of robotic fish is needed. This robotic fish is a typical serial robot in structure, and it can be described by the function of Lagrange, analogously. One important difference is that the movement of the fish have neither any fixed points (just like the base frame of the mechanical arm), nor fixed reference system (just like the ground reference frame of mobile robot). The robotic fish swims in the water freely, and the movement of links interacts with the water: the movement decides the amount and direction of the fluid forces and the forces also decide the fish's movement. The kinematic and dynamic problems are coupled, and they can not be calculated respectively.

The basic idea for modeling the miniature biomimetic robotic fish is to build the Lagrange's function unrelated with

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the internal force based on the structure of single joint. The generalized forces got from the Lagrange's equation of the second kind are equal to the fluid forces calculated according to the hydrodynamics. Then a system of partial differential equations can be built to solve the movement of the robotic fish.

Some reasonable assumptions are given to simplify the question:

1) The body of robotic fish can be treated as two plates jointed together [23] [24].
2) The robotic fish swims on still water, and it is not affected by the influence of reflection wave from the environment.
3) The deformation of robotic fish can be ignored except the motion of the joint.

Fig. 2 shows the links at a moment under the coordinate systems described in Fig. 1.

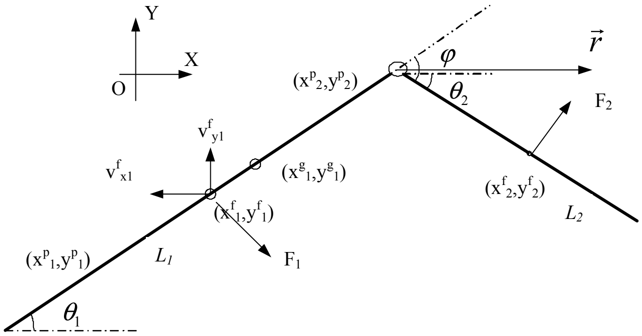

Fig. 2 The simplified robotic fish and some parameters definition

In the figure, L_1 is the first link of robotic fish (the head), and L_2 is second one (the tail and the tail fin). The parameters θ_1, θ_2, φ is anti-clockwise positive. (x_i^f, y_i^f), (x_i^g, y_i^g) and (x_i^p, y_i^p) are the center of figure, the center of gravity and the begin point of i^{th} link, respectively. The joint angle φ is known, and it is given according to the fish swimming mechanisms.

The potential energy is constant E when the robotic fish swims on the water. The kinetic energy of each link is the sum of the kinetic energy of translation under WRCS and the kinetic energy of rotation under the centre-of-mass system, so the Lagrange's function is defined as follows:

$$\begin{aligned} L &= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 \\ &= \frac{1}{2}m_1\left(\left(\dot{x}_1^g\right)^2 + \left(\dot{y}_1^g\right)^2\right) + \frac{1}{2}I_1\left(\dot{\theta}_1\right)^2 \\ &\quad + \frac{1}{2}m_2\left(\left(\dot{x}_2^g\right)^2 + \left(\dot{y}_2^g\right)^2\right) + \frac{1}{2}I_2\left(\dot{\theta}_2\right)^2 + E \end{aligned} \tag{1}$$

where m_i is the mass of the i^{th} link. I_i is the moment of inertia of the i^{th} link under the centre-of-mass system.

x_2^p, y_2^p, θ_2 are selected as the generalized coordinate, let $X = x_2^p, Y = y_2^p, \Theta = \theta_2$. We denote l_i with the length of L_i, and l_i^g is labeled as the length between (x_i^p, y_i^p) and (x_i^g, y_i^g), then we have:

$$\begin{cases} x_1^g = X - (l_1 - l_1^g)\cos\theta_1 \\ y_1^g = Y - (l_1 - l_1^g)\sin\theta_1 \\ \theta_1 = \Theta - \varphi \\ x_2^g = X + l_2^g\cos\theta_2 \\ y_2^g = Y + l_2^g\sin\theta_2 \\ \theta_2 = \Theta \end{cases} \tag{2}$$

Rewrite the Lagrange's function, and we get $L = L(X, Y, \Theta)$, thus the Lagrange's equations of the second kind are:

$$\begin{cases} F_X = \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{X}} - \dfrac{\partial L}{\partial X} \\ F_Y = \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{Y}} - \dfrac{\partial L}{\partial Y} \\ M_\Theta = \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{\Theta}} - \dfrac{\partial L}{\partial \Theta} \end{cases} \tag{3}$$

where F_X , F_Y and M_Θ are the composition of forces at X-axis, Y-axis and composition of moments at the (x_2^p, y_2^p), respectively.

B. The fluid model

The fluid forces acting on the robotic fish are decided by the instantaneous movement. A fluid drag model is employed to analyze forces perpendicular to the surface of swimming robotic fish, which has been used extensively in the case of large Reynolds number in the literature [23] [24][28][29], and it is:

$$F = -\mu\,\mathrm{sgn}(v^\perp)(v^\perp)^2 \tag{4}$$

where $\mu = \frac{1}{2}\rho C S$ is the drag coefficient, ρ is the density of water, C is shape coefficient, and S is effective area. $v^\perp$ is the projection of the velocity along the direction perpendicular to the surface.

The forces acting on the robotic fish are divided to three parts: pressure on links, approach stream pressure and friction drag.

1) The pressure on links: the fluid force on the robotic fish's i^{th} link when it swings:

$$F_i^\perp = -u^\perp\,\mathrm{sgn}(v_i^\perp)\left|v_i^\perp\right|^2 = -u^\perp v_i^\perp\left|v_i^\perp\right| \tag{5}$$

where $F_i^\perp$ is the pressure on i^{th} link, $v_i^\perp$ is the projection of the velocity of i^{th} link along the perpendicular direction, and $u^\perp$ is the drag coefficient with C of flat plate type [28].

$$v_1^{\perp} = \dot{X}\sin\theta_1 - \dot{Y}\cos\theta_1 - (l_1 - l_1^f)\dot{\theta}_1$$
$$v_2^{\perp} = \dot{X}\sin\theta_1 - \dot{Y}\cos\theta_1 - l_2^f\dot{\theta}_2 \tag{6}$$

2) The approach stream pressure: It is introduced because the water pushes on the cross section of the robotic fish when the robotic fish advances:

$$F_1^{=} = -u^{=} v_1^{=} \left| v_1^{=} \right| \tag{7}$$

where $v_1^{=} = \dot{X}\cos\theta_1 + \dot{Y}\sin\theta_1$ is the projection of the velocity of first link along the parallel direction, and $u^{=}$ is drag coefficient with the type of bullet [28]. Considering the cross-sectional area of the second link is much smaller than the first, the flow's effect is reduced and $F_2^{=}$ is ignored.

3) The friction drag: There is friction drag acting on the surface of the robotic fish, which is parallel to the body. It is often evaluated empirically (30%-50% of the approach stream pressure). In this paper, we use 50% because of the unsmooth surface of the robotic fish.

$$F_f = 50\% F_1^{=} = -\frac{u^{=}}{2} v_1^{=} \left| v_1^{=} \right|$$
(8)

Therefore, the composition of forces on the X-axis and Y-axis are:

$$F_x = -F_1^{\perp}\sin\theta_1 + F_2^{\perp}\sin\theta_2 + \left(F_1^{=} + F_f\right)\cos\theta_1 \tag{9}$$

$$F_y = F_1^{\perp}\cos\theta_1 - F_2^{\perp}\cos\theta_2 + \left(F_1^{=} + F_f\right)\sin\theta_1 \tag{10}$$

The composition of moment act on the joint is:

$$M_{\Theta} = -F_1^{\perp}(l_1 - l_1^f) - F_2^{\perp} l_2^f \tag{11}$$

Based on (4) and (9)-(11), a system of partial differential equations of X, Y, Θ, t can be obtained.

The movement of the robotic fish is described by $X(t), Y(t), \Theta(t)$, which are all determined by the tail's motion law $\varphi(t)$. The equations are complex, and they are solved by numerical method with boundary conditions at initial time.

III. SIMULATIONS AND EXPERIMENTS

The developed robotic fish prototype is shown in Fig. 3.

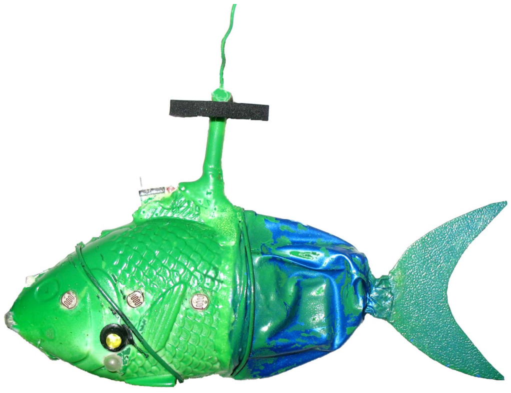

Fig.3. The prototype of the robotic fish

Experiments are conducted in an experiment pool of 1.80m*1m to testify the miniature biomimetic robotic fish. An overhead camera is introduced to capture the motion of the robotic fish and the video information is sent to the upper computer to recognize and record the realtime positions of the robotic fish

A. Forward motion

The head of robotic fish is assumed to point to the positive direction of X axis, and the position is at origin point at initial time. Equation (4) is solved to obtain $X(t)$, $Y(t)$, $\Theta(t)$, which are the functions of *Amp, f* and A_{turn}. The Fig. 4 gives the simulation results of forces act on the robotic fish in forward motion with $Amp = \pi/4,\ \ A_{turn} = 0,\ \ f = 2.14Hz$.

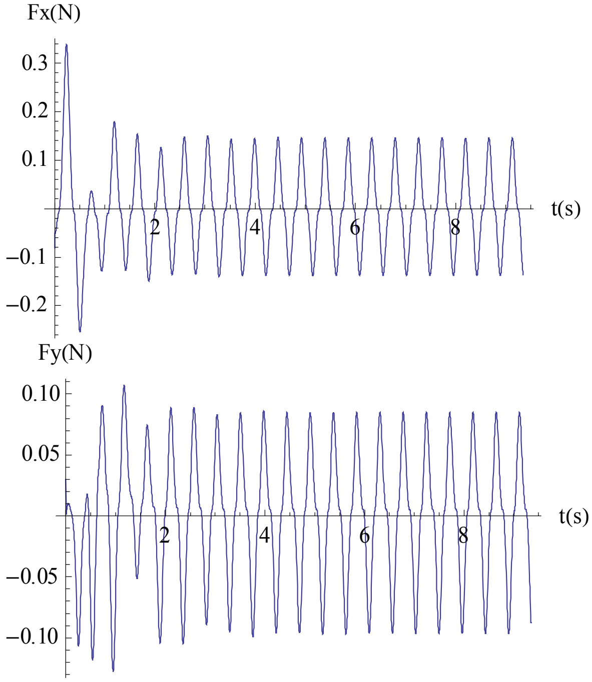

Fig. 4. Simulated results of forces acting on robotic fish.

The velocity is decided by the amplitude and the frequency of the tail oscillation, and $v = v(Amp, f)$. The *Amp* and *f* should satisfy the constraint of the servo [27]. Simulations are carried out at the entire feasible region and the result is shown in Fig. 5.

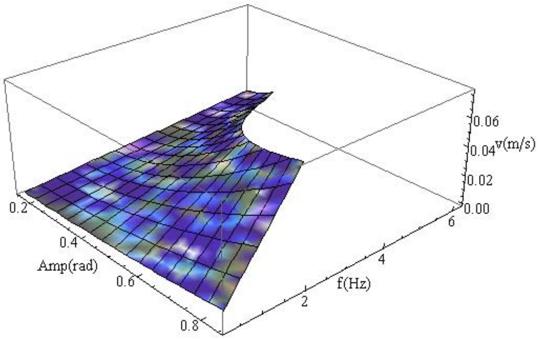

Fig. 5 The stable average velocity at the feasible region of *Amp* and *f*

The velocity increases with the increase of *Amp* and *f*. The contribution to the velocity of *Amp* is more than that of *f*, and the max velocity is got at $Amp = A\max = 0.849rad$, $f = 1.98Hz$, $v_{\max} = 0.0786m/s$.

Multiple velocity experiments, at different *Amp* and *f*, were carried out and the velocity was measured several times to calculate the average value. The comparison between simulations and experiments is given in Fig. 6 and the largest error is 0.0047m/s.

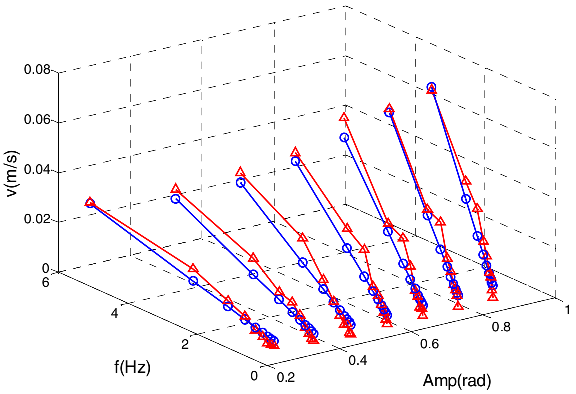

Fig. 6 The simulated and experimental velocity at different Amp and f. Simulated data is shown 'o', and experimental data 'Δ'

B. *Turning motion*

The turning motion is simulated with $Amp = \pi / 5$ and $f = 2.50Hz$. Fig. 7 gives the simulated and experimental results of turning, with $A_{turn} = \pi / 18$. The average turning radius R was 0.196m, the average velocity 0.0604m/s and the average angular velocity was 0.309rad/s.

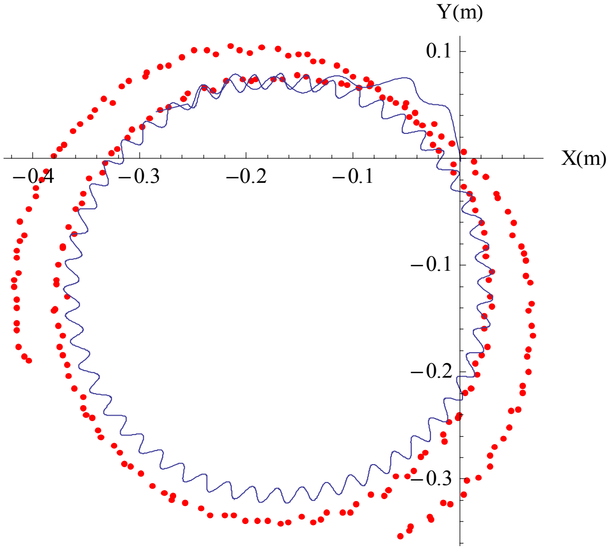

Fig. 7 Simulated and experimental data of turning motion. Simulated data is shown solid, and experimental data dot.

IV. CONCLUSION

This paper describes the modeling method of a Bio-Inspired Robotic Fish. The Lagrangian function of the freely swimming robotic fish is built to describe the movement. The fluid forces are simplified, and the movement of robotic fish is obtained by solving the Lagrange's equation of the second kind. Simulations and Experiments have been carried out to verify the model of the robotic fish. The model may be used to select the optimal parameters of motion, the design of motion controller and other prototype design. The future work will focus on the more accurate fluid force and optimization of the robotic fish.

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