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Trajectory Tracking Control of Omnidirecitonal Wheeled Mobile Manipulators: Robust Neural Network based Sliding Mode Approach

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***Abstract*—This paper focuses on developing a robust neural network (NN) based sliding mode controller (NNSMC) to solve the trajectory tracking problem of a redundantly-actuated omnidirectional mobile manipulator. The SMC is designed to be robust to disturbances assuring the stability of the system. The NN is used to identify the unstructured uncertainty of system dynamics. The stability of the closed-loop system, the convergence of the NN weight-updating process, and the boundedness of the NN weight estimation errors are all strictly guaranteed. Through theories analysis, we know the controller is also capable of disturbance-rejection in the presence of time varying disturbances. Finally, simulation results demonstrate the proposed NNSMC approach can guarantee the whole system's convergence to the desired manifold with prescribed performance.**

I. INTRODUCTION

Dynamic modeling and trajectory tracking of omnidirectional mobile manipulator was investigated in [1]. There were also other contributions in this field [2,3]. But because omnidirectional mobile manipulator is characterized as redundantly-actuated mobile platform, and complex coupling mechanism between the platform and its mounted manipulator, it is still a challenging problem to develop effective control strategies for such system.

It is well known that the main advantages of using sliding mode control (SMC) include fast response, good transition, and robustness with respect to system uncertainties and external disturbances. Therefore, it is attractive for many highly nonlinear uncertain systems [4,5], such as the holonomic and nonholonomic constrained mechanical systems. Neural network (NN), one of the most popular intelligent computation approaches, has an inherent learning ability and can approximate a nonlinear continuous function to arbitrary accuracy. This feature is crucial in the controller design for complex model identifying and unstructured uncertainties compensating. Thus, in the past two decades, the development of intelligent control, especially neural network control (NNC) in robotic fields has attracted considerable interest. Polycarpou [6] presented a systematic methodology to identify a nonlinear system using an NN. Lewis [7] proposed an NN control scheme that guaranteed the closed-loop performance in terms of small tracking errors and bounded controls. An NN based control methodology was proposed for the joint space position control of a mobile manipulator in [8], which comprised a linear control term (classical PID) and an NN compensation term. Hu [9] proposed a control scheme which consisted of an SMC and an NNC, and the contribution proportion of these two parts was determined by a fuzzy supervisory controller. Adaptive control scheme of robot manipulators using Chebyshev neural network under actuator constraints was developed in [10].

In this paper, we focus on a redundantly-actuated omnidirectional mobile manipulator with holonomic kinematic constraints. A robust control scheme using an NN combined with an SMC (NNSMC) is proposed to solve trajectory tracking problem. This control scheme not only overcomes the unstructured uncertainties, but also has the capability of disturbance rejection in the presence of unknown bounded disturbances.

II. DYNAMIC MODEL

The mobile manipulator considered here moves on the horizontal plane which is in the global frame OXY. It is cylinder-shaped; three identical castor wheels are axed symmetrically on the bottom of the platform and a two links manipulator locates on the gravity center C (which coordinate is defined as $\begin{bmatrix} x & y & \theta \end{bmatrix}^T$) of the platform, as shown in Fig.1. The angle displacements for wheel rolling and steering are φ_i and η_i , and the details are listed in [1]. l_1 and l_2 are the lengths of the two links, and θ_1 and θ_2 are the angle displacements of this two links, respectively.

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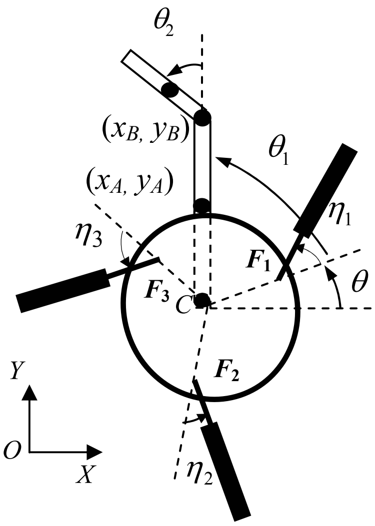

Fig.1 Mobile manipulator assembly

According to the above description, we first define the following state variables for easy references:

$q_1 = \begin{bmatrix} x & y & \theta & \theta_1 & \theta_2 \end{bmatrix}^T$ --- The pose of the mobile manipulator

$q_2 = \begin{bmatrix} \varphi_1 & \eta_1 & \varphi_2 & \eta_2 & \varphi_3 & \eta_3 \end{bmatrix}^T$ --- The drive variables of the platform

$$q = \begin{bmatrix} q_1^T & q_2^T \end{bmatrix}^T = \begin{bmatrix} x & y & \theta & \theta_1 & \theta_2 & \varphi_1 & \eta_1 & \varphi_2 & \eta_2 & \varphi_3 & \eta_3 \end{bmatrix}^T$$

$\zeta = \begin{bmatrix} \theta_1 & \theta_2 & \varphi_1 & \eta_1 & \varphi_2 & \eta_2 & \varphi_3 & \eta_3 \end{bmatrix}^T$ --- The drive variables of the mobile manipulator

$0_{m\times n}$ --- The $m\times n$ zero matrix

$I_{n\times n}$ --- The $n\times n$ identity matrix

$\lambda_{\min}(*)$ and $\lambda_{\max}(*)$ --- The minimum and maximum eigenvalues of $(*)$.

Based on the kinematic model proposed in [1], the dynamic equations of this mechanical system with external disturbances can be derived as:

$$M(q)\ddot{q} + C(q,q)\dot{q} + G(q) + d(t) = B(q)u + A^T(q)\lambda \quad (1)$$

The kinematic constraints are considered independent of time and can be expressed as:

$$A(q)\dot{q} = 0 \quad (2)$$

Here $u \in R^n$ is the vector of the generalized input torques; $M(q) \in R^{n\times n}$ is the symmetric bounded positive definite inertia matrix; $C(q,\dot{q})\dot{q} \in R^n$ represents the vector of centripetal and Coriolis torques; $G(q) \in R^n$ is the gravitational torque vector; $d(t) \in R^n$ denotes the external disturbances; $B(q) \in R^{n\times n}$ is a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of the system; $A(q) \in R^{m\times n}$ is the kinematic constraints matrix, $\lambda \in R^m$ is a constraint force vector.

Here, the effect of the kinematic constraints can be viewed as restricting the dynamics on a constraint manifold as:

$$\Omega = \{(q,\dot{q}) \in R^n \times R^n : A(q)\dot{q} = 0 \} \quad (3)$$

The presence of m kinematic constraints causes the system to lose m degrees of freedom; hence, the system only has $n-m$ degrees of freedom left. Based on the kinematic model, we know $q_1(t) \in R^{n-m}$ is the vector of the independent generalized coordinates of the mobile manipulator system.

Property 1: Define $H = \begin{bmatrix} I_3 & 0_{3\times 2} \end{bmatrix}$ and $R(q) = \begin{bmatrix} H^T & J_{q1}^T \end{bmatrix}^T$. From [1], the matrix $R^T(q)B(q)$ is of full rank, so the Moore-Penrose inverse matrix of $R(q)$ always exists.

In order to reduce complexity of the whole system, applying the projection of the dynamic model into the null space of the constraints, we can obtain

$$\begin{aligned} &\dot{q} = R(q)\dot{q}_1 \\ &\bar{M}(q)\ddot{q}_1 + \bar{C}(q,\dot{q})\dot{q}_1 + \bar{G}(q) + \bar{d}(t) = \bar{E}u \end{aligned} \quad (4)$$

where $\bar{M}(q) = R^T(q)M(q)R(q)$, $\bar{C}(q,\dot{q}) = R^T(q)C_1(q,\dot{q})$, $C_1(q,\dot{q}) = M(q)\dot{R}(q) + C(q,\dot{q})R(q)$, $\bar{G}(q) = R^T(q)G(q)$, $\bar{d}(t) = R^T(q)d(t)$, $\bar{E}(q) = R^T(q)B(q)$. (2) and (3) lead to $A(q)R(q)\dot{q}_1 = 0$. In other words, $A(q)R(q) = 0$, since $A(q)R(q)$ is of full columns and the configuration of the whole system has $n-m$ degrees of freedom. For (4), some fundamental properties are addressed below.

Property 2: The matrix $\bar{M}(q) = R^T(q)M(q)R(q)$ is symmetric and positive definite.

Property 3: $\dot{\bar{M}}(q) - 2\bar{C}(q,\dot{q})$ is skew-symmetric, i.e., $\xi^T(\dot{\bar{M}}(q) - 2\bar{C}(q,\dot{q}))\xi = 0$, $\forall \xi \in R^{n-m}$.

Property 4: There exist positive scalars $\beta_i (i = 1,\cdots,5)$ such that $\forall q \in R^n, \forall \dot{q} \in R^n$: $\|M(q)\| \le \beta_1 < \infty$, $\|C(q,\dot{q})\| \le \beta_2 + \beta_3\|\dot{q}\|$, $\|G(q)\| \le \beta_4$ and $\sup_{t\ge 0}\|d(t)\| \le \beta_5$.

III. Neural Network Based Sliding Mode Control Design (NNSMC)

This section considers the trajectory tracking problem of the above redundantly-actuated mobile manipulator system discussed above.

Assumption 1: The vectors q_1, $\dot{q}_1$ are bounded and uniformly continuous derivatives up to the second order. Moreover, the matrices $R(q)$ and $\dot{R}(q)$ are also bounded as $\|R(q)\| \le \beta_6$, $\|\dot{R}(q)\| \le \beta_7\|\dot{q}\|$, where $\beta_i (i = 6,7)$ are positive constants.

Remark 1: According to Wang's claim [12], if $q_1(t)$ is bounded, then $R(q)$ is bounded. If $\dot{q}_1(t)$ is bounded, then $\dot{R}(q)$ is bounded.

The trajectory tracking control objective is: on the basis of the vector q_1 and $\dot{q}_1$, given a desired q_{1d} and $\dot{q}_{1d}$, develop a controller such that for any $(q(0),\dot{q}(0)) \in \Omega$ in (3), $q_1(t)$ and $\dot{q}_1(t)$ can asymptotically converge to a manifold Ω_d specified as:

$$\Omega_d = \{\{q,\dot{q}\} \mid q_1 = q_{1d}, \dot{q}_1 = \dot{q}_{1d}, q = R(q)q_1\} \quad (5)$$

Here, the vector $q_1(t)$ can be considered as $n-m$ "output equations" of the system.

Assumption 2: The desired reference trajectory q_{1d} is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. So there exists the constant q_B, that $\|q_{1d}^T \quad \dot{q}_{1d}^T \quad \ddot{q}_{1d}^T\| \le q_B$ always holds.

In the following, some variables are defined as:

$$e = q_1 - q_{1d} \quad (6)$$

$$\dot{q}_{1r} = \dot{q}_{1d} - \Lambda e \quad (7)$$

where e and q_{1r} denote the tracking error and a set of auxiliary signals, respectively. Λ is a positive definite matrix which eigenvalues are strictly in the right-hand of complex plane. Then a sliding variable is defined as:

$$s = \dot{q}_1 - \dot{q}_{1r} = \dot{e} + \Lambda e \quad (8)$$

Remark 2: From Assumptions 1 and 2, there exist positive scalars $\beta_i (i = 8,9)$ such that $\forall \dot{q}_{1r} \in R^{n-m}$, $\forall \ddot{q}_{1r} \in R^{n-m}$: $\|\dot{q}_{1r}\| \le \beta_8 + \Lambda\|e\|$, $\|\ddot{q}_{1r}\| \le \beta_9 + \Lambda\|\dot{e}\|$.

When the sliding surface $s = 0$, according to the theory of SMC, the sliding mode is governed by the following differential equation:

$$\dot{e} = -\Lambda e \quad (9)$$

Obviously, the behavior of the system on the sliding surface is determined by the structure of the matrix Λ. In other words, when $s = 0$, the tracking error transient response is completely governed by the above equation.

Based on (1) and the first equation of (4), the system in terms of the sliding variable s can be obtained by:

$$\begin{aligned} M(q)R(q)\dot{s} &= B(q)u - (M(q)R(q)\ddot{q}_{1r} + C_1(q,\dot{q})\dot{q}_{1r} \\ &+ G(q) + d(t)) - C_1(q,\dot{q})s + A^T(q)\lambda \end{aligned} \quad (10)$$

Multiplying left $R^T(q)$ and using Property 2 lead to:

$$\bar{M}(q)\dot{s} = \bar{E}(q)u - \bar{H}(\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q,t) - \bar{C}(q,\dot{q})s \quad (11)$$

$$\begin{aligned} &\bar{H}(\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q,t) = \underline{H}(\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q) + R^T(q)d(t) \\ &\underline{H}(\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q) = R^T(q)(M(q)R(q)\ddot{q}_{1r} + C_1(q,\dot{q})\dot{q}_{1r} + G(q)) \\ &= R^T(q)M(q)R(q)(\ddot{q}_{1d} - \Lambda\dot{e}) + R^T(q)C_1(q,\dot{q})(\dot{q}_{1d} - \Lambda e) \\ &+ R^T(q)G(q) \\ &= \underline{H}(x) \end{aligned} \quad (12)$$

where $x = [\ddot{q}_{1d}^T \quad \dot{q}_{1d}^T \quad q_{1d}^T \quad \dot{q}^T \quad q^T]^T \in R^{3\times(n-m)+2\times n}$.

Because the omnidirectional mobile manipulator moving in the horizontal plane, $R^T(q)G(q) \equiv 0$. For the omnidirectional mobile manipulator system, there is a unique function $\vartheta : R^{n-m} \to R^m$, such that the holonomic constraint can always be expressed explicitly as [13]: $q_2 = \vartheta(q_1)$. Then we can easily obtain that the uncertain $\underline{H}(x)$ defined in (12) can be rewritten as this form

$$\underline{H}(\bar{x}) = R^T(q_1)M(q_1)R(q_1)\ddot{q}_{1r} + R^T(q_1)C_1(q_1,\dot{q}_1)R(q_1)\dot{q}_{1r} \quad (13)$$

where $\bar{x} = [\ddot{q}_{1r}^T \quad \dot{q}_{1r}^T \quad \dot{q}_1^T \quad q_1^T]^T \in R^{4\times(n-m)}$.

During the development of the controller, we assume $M(q_1)$ and $C_1(q_1,\dot{q}_1)$ are completely unknown. In respect that NN has ability to approximate a nonlinear continuous function to arbitrary accuracy and can be used to identify the unstructured uncertainty of system dynamics, a NN approximator is derived to approximate the integrated uncertain term $\underline{H}(x)$ in (12). Then an effective control method NNSMC is addressed.

A. Robust NNSMC Scheme Design

For any given real continuous functions on a compact set U, define a smooth function $f(\cdot): U \to R^n$. If we choose $f(x) \in C^r(U)$, where $C^r(U)$ is the space of the continuous functions, there exists a radial basis function neural network (RBFNN) system in the following form

$$f(x) = W^T\phi(x) + \varepsilon(x) \quad (14)$$

$$\|\varepsilon\| < \varepsilon_N \quad (15)$$

where W is the weight matrix of the RBFNN, $\phi(x)$ is the excitation function vector, ε_N is a constant.

The universal approximation ability of the RBFNN makes it possible to solve almost any nonlinear modeling problems. Thus, $\underline{H}(x)$ defined in (12) can be identified using the RBFNN with enough number of hidden layer neurons such that

$$\underline{H}(x) = \underline{H}(\bar{x}) = W^T\phi(\bar{x}) + \varepsilon(\bar{x}) \quad (16)$$

where $W \in R^{h\times k}$ is assumed to be constant and bounded by

$$\|W\| \le W_B \quad (17)$$

where W_B is one known positive constant. The basis function vector $\phi(\bar{x})$ is usually chosen as the *Gaussian* functions.

The estimate of the uncertain term $\underline{H}(x)$ is expressed as

$$\hat{\underline{H}}(x) = \hat{\underline{H}}(\bar{x}) = \hat{W}^T\phi(\bar{x}) \quad (18)$$

where $\hat{W}$ is the tuning parameter matrix of the network and is adjusted in the learning process.

Assumption 3: In the RBFNN estimator, the input vector $\bar{x}$ (defined in (13)) is used to identify the uncertain term $\underline{H}(\bar{x})$, i.e. $\underline{H}(x)$. There exist positive constants c_0 and c_1, such that the following expression holds:

$$\|\bar{x}(t)\| \le q_B + c_0 \|s(0)\| + c_1 \|s(t)\| \tag{19}$$

where s is defined in (8). According to the definitions in (6) and (7), assuming that (16) holds, we know that all $\bar{x}$ are in the compact set $\Omega_{\bar{x}} \equiv \{\bar{x} \mid \|\bar{x}\| < b_x\}$, where $b_{\bar{x}} > q_B$. Define the compact set as $\Omega_s \equiv \{s \mid \|s\| < (b_{\bar{x}} - q_B)/(c_0 + c_1)\}$ with $s(0) \in \Omega_s$. Thus the NN approximation property always holds for all s in the compact set Ω_s.

Using the NN on-line estimator to identify the uncertain term $\underline{H}(x)$. For any $(q(0), \dot{q}(0)) \in \Omega$ and desired trajectory $q_{1d}(t)$, we take the control scheme as

$$\bar{E}(q)u = -Ks - K_s \operatorname{sgn}(s) + \hat{W}^T \phi(\bar{x}) \tag{20}$$

$$\dot{\hat{W}} = -\alpha \phi s^T - \mu\alpha \|s\| \hat{W} \tag{21}$$

Where K and K_s are $(n-m)\times(n-m)$ positive definite gain matrixes determined by the designer, α is a positive constant representing the learning rate of the network, μ is a small positive design constant.

Theorem: For the uncertain dynamic system (4), there exists appropriate parameters in the NNSMC, and the following performance can be achieved.

1. With suitable positive gain constants K and K_s, the tracking error e will be uniformly ultimately bounded.
2. With sufficient large K_s, the tracking errors e and $\dot{e}$ will asymptotically converge to 0.

Proof:

1. Consider a Lyapunov candidate function as

$$V = \frac{1}{2} s^T \bar{M} s + \frac{1}{2\alpha} tr\{\tilde{W}^T \tilde{W}\} \tag{22}$$

where $\tilde{W} = W - \hat{W}$. According to the matrix trace, inner product, and Frobenius norm theory, the following property holds

$$tr\{\tilde{W}^T \hat{W}\} = tr\{\tilde{W}^T (W - \tilde{W})\} \le \|\tilde{W}\|_F \|W\|_F - \|\tilde{W}\|_F^2 \tag{23}$$

By substituting (20) and (21) into (11), the time derivative of V leads to:

$$\begin{aligned}
\dot{V} &= \frac{1}{2} s^T \dot{\bar{M}} s + s^T \bar{M} \dot{s} + \frac{1}{\alpha} tr\{\tilde{W}^T \dot{\tilde{W}}\} \\
&= s^T(-Ks - K_s \operatorname{sgn}(s) - \varepsilon(\bar{x}) - R^T(q)d(t)) \\
&\quad + \mu \|s\| tr\{\tilde{W}^T \hat{W})\} \\
&\le s^T(-Ks - K_s \operatorname{sgn}(s) - \varepsilon(\bar{x}) - R^T(q)d(t)) \\
&\quad + \mu \|s\| (\|\tilde{W}\|_F \|W\|_F - \|\tilde{W}\|_F^2) \\
&\le -\|s\| (\lambda_{\min}(K)\|s\| + K_s - \varepsilon_N - \beta_6\beta_5 \\
&\quad + \mu(\|\tilde{W}\|_F^2 - \|\tilde{W}\|_F W_B)) \\
&= -\|s\| (\lambda_{\min}(K)\|s\| + K_s - \varepsilon_N - \beta_6\beta_5 \\
&\quad + \mu(\|\tilde{W}\|_F - \frac{W_B}{2})^2 - \mu \frac{W_B^2}{4})
\end{aligned} \tag{24}$$

From the above proof process, in order to ensure the approximation effect of the NN estimator, we know that if the following expression (25) holds, the sliding variable should be always constrained in the compact set Ω_s.

$$\|s\| > \frac{\varepsilon_N + \beta_6\beta_5 + \mu \frac{W_B^2}{4} - K_s}{\lambda_{\min}(K)} \equiv b_s \tag{25}$$

This can be achieved by choosing suitable positive constants to construct the gain matrix K to satisfy

$$\lambda_{\min}(K) > \frac{(\varepsilon_N + \beta_6\beta_5 + \mu \frac{W_B^2}{4} - K_s)(c_0 + c_1)}{b_{\bar{x}} - q_B} \tag{26}$$

Therefore, the compact set defined by $\|s\| > b_s$ is constrained by Ω_s. As a result, the approximation property of the NN estimator always holds.

2. Furthermore, if we choose a sufficient large K_s such that

$$\frac{\varepsilon_N + \beta_6\beta_5 + \mu \frac{W_B^2}{4} - K_s}{\lambda_{\min}(K)} \le 0 \tag{27}$$

Since $\|s\| \ge 0$ and $\lambda_{\min}(K) > 0$, we know

$$\dot{V} \le -\lambda_{\min}(K)\|s\|^2 \le 0 \tag{28}$$

Define

$$V_R = V(t) - \int_0^t (\dot{V}(\nu) + \lambda_{\min}(K)\|s(\nu)\|^2) d\nu \tag{29}$$

Its time derivatives up to the second order are

$$\begin{aligned}
\dot{V}_R &= \dot{V}(t) - \dot{V}(t) - \lambda_{\min}(K)\|s\|^2 \\
&= -\lambda_{\min}(K)\|s\|^2 \\
\ddot{V}_R &= -2\lambda_{\min}(K) s^T \dot{s}
\end{aligned} \tag{30}$$

We can conclude that $|\ddot{V}_R|$ is bounded and $\dot{V}_R$ is continuous. We know that $V(t)$ is bounded from (22) and the second term of V_R is a finite integral. Thus, V_R is bounded. In addition, $\int_0^t \dot{V}_R(\nu) d\nu = V_R(t) - V_R(0)$ is also bounded. Then, according to Barbalat's lemma, $|\dot{V}_R| = \lambda_{\min}(K)\|s\|^2 \to 0$, when $t \to \infty$.

Therefore, according to the above analysis, the sliding variable $s \to 0$, when $t \to \infty$. Thus, the tracking error e and $\dot{e}$ asymptotically converge to 0 as $t \to \infty$.

Remark 4: For the controller (20), once the control parameters such as K, K_s are determined, the input torque $u(t)$ is bounded for all $t > 0$. If we choose a sufficient large K_s, we can ensure the asymptotically stability, but $u(t)$ may be larger than the maximum output torque of driving motors. In addition, large K_s also causes severe chattering. Therefore, when we design a suitable controller, there is a critical trade-off between the actual input torque and the tracking performance.

B. Partitioned Neural Networks

Given a holonomic constrained mechanical system, it is desired to select a set of basis functions and determine the NN reconstruction error bound so that (16) holds; that is, determine a basis function vector $\phi(\bar{x})$ so that the uncertain term defined in (12) can be expressed by (16). But when we face a complex mechanical system like the omnidirectional mobile manipulator studied in this paper, the inputs and outputs dimensions of the NN are large, and the computation burden of the learning process is big. These above two problems severely prohibit the application of single NN in practical system.

From [14] we know that the NN controller allows one to design partitioned NNs to achieve the same performance as single NN. For (13), there are two separated parts ($R^T(q_1)M(q_1)R(q_1)\ddot{q}_{1r}$, $R^T(q_1)C_1(q_1,\dot{q}_1)\dot{q}_{1r}$) in the $\underline{H}(\bar{x})$, i.e., $\underline{H}(x)$. Therefore, we can apply two separated NNs to identify them respectively. Using the similar analysis method as [14,15], we can derive that

$$\hat{\underline{H}}(\bar{x}) = \begin{bmatrix} \hat{W}_M^T & \hat{W}_C^T \end{bmatrix} \begin{bmatrix} \phi_M \\ \phi_C \end{bmatrix} = \hat{W}_M^T \phi_M + \hat{W}_C^T \phi_C \qquad (31)$$

It is easy to know that the partitioned decoupled NNs can be tuned respectively. Then the learning algorithm (21) can be rewritten as

$$\dot{\hat{W}}_M = -\alpha_M \phi_M s^T - \mu_M \alpha_M \|s\| \hat{W}_M \qquad (32)$$

$$\dot{\hat{W}}_C = -\alpha_C \phi_C s^T - \mu_C \alpha_C \|s\| \hat{W}_C \qquad (33)$$

Remark 5: In essence, the uncertain term $\underline{H}(x)$ that we pay attention to is a vector, not a matrix. Using the partitioned structure is to simplify the NN design process and accelerate the tuning speed. The partitioned NNs in [14] and partitioned fuzzy logic systems in [15] require Kronecker product to construct the new basis function for identifying the uncertain matrix ($R^T(q_1)M(q_1)R(q_1)$). Compared to directly using NN to identify the uncertain integrated vector ($R^T(q_1)M(q_1)R(q_1)\ddot{q}_{1r}$), employing Kronecker product to estimate the whole uncertain matrix increases the computation burden of tuning weight matrixes obviously, especially to the complex mechanical systems. Thus, their method only partitions the structure of the NN approximator, but does not simplify the tuning procedure. In this paper, we employ two separated NNs to directly approximate two decoupled parts ($R^T(q_1)M(q_1)R(q_1)\ddot{q}_{1r}$, $R^T(q_1)C_1(q_1,\dot{q}_1)\dot{q}_{1r}$) of the uncertain term $\underline{H}(x)$. This method does not need Kronecker product so the weight matrixes structures are more compact and tuning speed is faster.

IV. Simulation Results

As an example to verify the proposed approach, this section discusses the simulation of the dynamic model and trajectory tracking controller. The physical parameters are the same with that in [1].

Let the desired output trajectory γ_d be: $x_d = 2\cos(t/2)$, $y_d = 2\sin(t/2)$, $\theta_d = 2\sin(t/2)$ $\theta_{1d} = \sin(t)$, $\theta_{2d} = \cos(t)$ and assume the disturbances is $d = \begin{bmatrix} 20\sin(t) & 20\cos(t) & 20\cos(t) & 3\sin(t) & 2\cos(t) & 0_{1\times 6} \end{bmatrix}^T$.

A NNSMC controller is designed to be comprised of two separative NNs approximating the uncertain terms $R^T(q_1)M(q_1)R(q_1)\ddot{q}_{1r}$ and $R^T(q_1)C_1(q_1,\dot{q}_1)\dot{q}_{1r}$.

The basic function of the first NN is designed as

$$\phi_{Mj} = \exp[-\frac{\|x_M - c_j\|^2}{2\sigma_j^2}] \qquad (j = 1, 2, \cdots, 5) \qquad (34)$$

where input state vector $x_M = \begin{bmatrix} q_1^T & \ddot{q}_{1r}^T \end{bmatrix}^T$, the centre vector is $[c_1, c_2, \cdots, c_5]^T = [-3, -1.5, 0, 1.5, 3]^T$, and the width vector is $[\sigma_1, \sigma_2, \cdots, \sigma_5]^T = [3, 3, 3, 3, 3]^T$. $\hat{W}_M$ is a 5×11 matrix and all elements are initialized to zero.

The basic function of the second NN is designed as

$$\phi_{Cj} = \exp[-\frac{\|x_C - c_j\|^2}{2\sigma_j^2}] \qquad (j = 1, 2, \cdots, 7) \qquad (35)$$

where input state vector $x_C = \begin{bmatrix} q_1^T & \dot{q}_1^T & \dot{q}_{1r}^T \end{bmatrix}^T$, the centre vector is $[c_1, c_2, \cdots, c_7]^T = [-3, -2, -1, 0, 1, 2, 3]^T$, and the width vector is $[\sigma_1, \sigma_2, \cdots, \sigma_7]^T = [2, 2, 2, 2, 2, 2, 2]^T$. $\hat{W}_C$ is a 7×11 matrix and all elements are initialized to zero.

Now the estimate of the uncertain term $\underline{H}(x)$ is concretely described by (31). Then the control law is modified as

$$\bar{E}(q)u = -Ks + (\hat{W}_M^T \phi_M + \hat{W}_C^T \phi_C) - K_s \,\mathrm{sgn}(s) \qquad (36)$$

For the convenience of simulations, choose the control parameters as $\Lambda = diag[2,2,2,2,2]$, $\alpha_M = 100$, $\mu_M = 0.1$ $\alpha_C = 200$, $\mu_C = 0.1$, $K = diag[300,300,300,20,20]$, and $K_s = diag[10,10,10,5,5]$, respectively.

The designed approach is applied to the omnidirectional mobile manipulator system and the simulation results are shown in Figs.2-4. The computed trajectory of the platform is shown in Fig.2. Position tracking errors are shown in Fig.3 and Fig.4. From the above explanations, it is clear that the system converges to the desired trajectory quickly and achieves good tracking performance.

V. Conclusion

In this paper, the trajectory tracking problem is addressed for a redundantly-actuated omnidirectional mobile manipulator system with uncertainties and external disturbances. Then an neural network based sliding model control scheme is presented. The stability and convergence of the control system are proved using Lyapunov theory and related lemmas. From the discussion and simulation results, the following conclusions can be reached.

1. The NNSMC can take advantage of the fast transient response of SMC and the self-learning and nonlinear mapping properties of NNs to deal with both structured and unstructured uncertainties.
2. Learning processes of NNs are online. It is effective in dealing with unpredictable disturbance.
3. The NNSMC combines SMC and NNC to take advantage of both of them to get better trajectory tracking performance.
4. The NNSMC requires no information of the mathematical model and/or the parameterization of the mechanical dynamics.
5. Although the overall structure of the NNSMC looks complicated, it is very suitable for real-time application after adopting the partitioned structure, because the partitioned structure developed in this paper simplifies the NN design process and accelerates the tuning speed.

REFERENCES

[1] D. Xu, D. B. Zhao, J. Q. Yi, X. M. Tan, "Robust adaptive control for omnidirectional mobile manipulator," *Proc of the 2007 IEEE International Conference on Intelligent Robots and Systems*, pp.3598-3603, 2007.

[2] J. D. Tan, N. Xi, "Unified model approach for planning and control of mobile manipulators," *Proc of the 2001 IEEE International Conference on Robotics and Automation*, pp.3145-3152, 2001.

[3] R. Holmberg, O. Khatib, "Development and control of a holonomic mobile robot for mobile manipulation tasks," *International Journal of Robotics Research*, vol.19.no.11, pp.1066-1074, 2000.

[4] J. J. Slotine, S. S. Sastry, "Tracking control of nonlinear system using sliding surface with application to robot manipulators," *International Journal of Control*, vol.48, pp.465-492, 1983.

[5] D. K. Chwa, "Sliding-mode tracking control of nonholonomic wheeled mobile robots in polar coordinates," *IEEE Transactions on Control System Technology*, vol.12, pp.637-644, 2004.

[6] M. Polycarpou, "On-line approximators for nonlinear system identification: A unified approach," in *Control and Dynamic Systems: Neural Network Systems. Techniques and Application*, X. Lenodes, Ed., vol. 7, pp.191–230, 1998.

[7] F. L. Lewis, S. Jagannathan, A. Yesildirek, Neural Network Control of Robot Manipulators and Nonlinear Systems. London, U.K.: Taylor and Francis, 1999.

[8] S. Lin, A. A. Goldenberg., "Neural-network control of mobile manipulators," *IEEE Transactions on Neural networks*, vol. 12, no.5, pp.1121-1133, 2001.

[9] H. Hui, P. Y. Woo., "Fuzzy supervisory sliding-mode and neural-network control for robotic manipulators," *IEEE Transactions on Industrial Electronics*, vol.53, no.3, pp.929-940, 2006.

[10] S. Purwar, I. N. Kar, A. N. Jha., "Adaptive control of robot manipulators using CNN under actuator constraints," *Proc of the 2005 IEEE International Conference on Robotics and Automation*, pp.1362-1367, 2005.

[11] V. I. Arnold, Mathematical Methods of Classical Mechanics, 2nd ed., New York: Springer-Verlag, 1989.

[12] Z. P. Wang, S. S. Ge, T. H. Lee, "Robust motion/force control of uncertain holonomic/holonomic mechanical system," *IEEE Transactions on Mechatronics*, vol. 9, pp.118-123, 2004.

[13] C. Y. Su, Y. Stepanenko, T. P. Leung, "Combined adaptive and variable structure control for constrained robots," *Automatica*, vol. 31, pp.483–488, 1995.

[14] F. L. Lewis, K. Liu, A. Yesildirek, "Neural net robot controller with guaranteed tracking performance," *IEEE Transactions on Neural Networks*, vol. 6, no.3, pp.703-715, 1995.

[15] Y. C. Chang, B. S. Chen, "Robust tracking designs for both holonomic and nonholonomic constrained mechanical systems: adaptive fuzzy approach," *IEEE Transactions on Fuzzy Systems*, vol.8, no.1, pp.46-66, 2000.

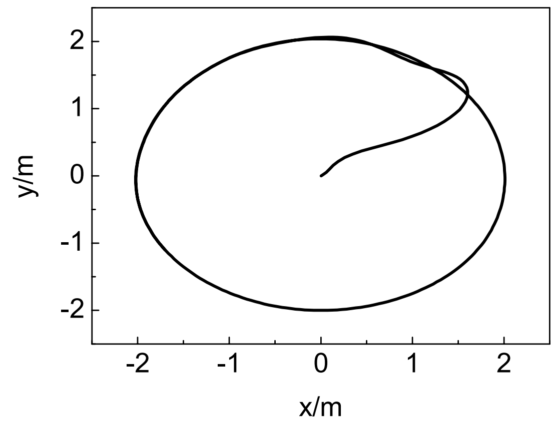

Fig.3 Trajectory of the mobile platform

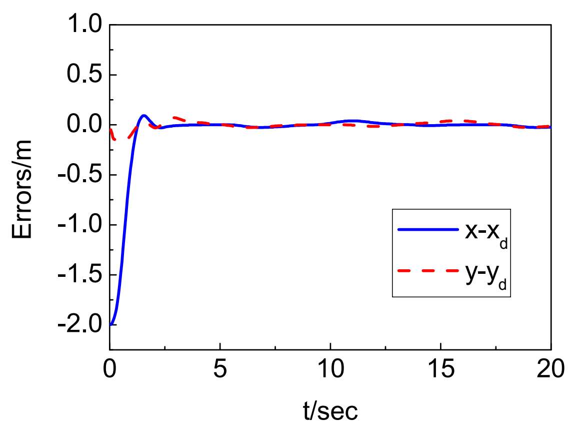

Fig.4 Position errors of X and Y

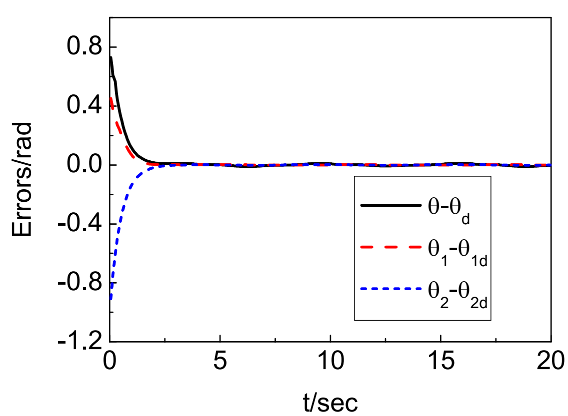

Fig.5 Position errors of θ, θ_1 and θ_2