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Stability of a trotting quadruped robot with passive, underactuated legs

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Abstract—**A 2-D dynamic simulation of a quadruped robot with passive, directionally compliant legs has shown that with the proper leg configuration, increased stability of the robot can be achieved passively. Increased stability is defined as decreased pitching motion of the robot. A limited search of the design space of the legs resulted in the finding the leg parameters that minimized the pitching. Steady state trotting, changing speeds, and disturbances such as steps and holes were tested.**

I. INTRODUCTION

LEGGED robots have greater potential to move on rough and discontinuous terrain than wheeled robots. However, legged robots are inherently more complex in structure, actuation, and control [1, 5, 7, 9, 10, 14, 15, 20, 21, 22]. Quadruped robots demonstrate great mobility on different surfaces and at different speeds, a fact which has been supported by biological research [13, 22]. However, quadruped robots present stability and attitude-control problems since they are not statically stable as hexapod or octapod robot can be.

Current research on the quadruped robot has focused on improvement of its stability through manipulation of sensing and control algorithms because improvement of the robot would provide it with greater mobility in various environments [1, 5, 7, 10, 14]. However, the complexity of robotic hardware and software increases proportional to the degrees of freedom (DOF), which significantly increases potential problems. Besides the research focus on sensing and control algorithms in attempts to improve robotic stability, many researchers have looked to the compliance of individual joints or to use of compliant prismatic legs [1, 4, 6, 8, 17, 18].

Several researchers have noted the importance of passive stability in legged animals and robots [8, 11, 23]. None have suggested means of enhancing the passive stability.

The goal of this work has been to investigate passive, directionally-compliant legs to improve the stability of a quadruped trotting robot. The basic question is: How much stability can be provided through properly designed passive legs?

Robotic rationale

Legged robots have many more degrees of freedom than do wheeled or tracked robots. This requires more actuators and sensors to control the extra degrees of freedom. Many researchers are investigating underactuated, compliant legs to reduce the complexity. We propose that by properly designing the compliance, the stability can be improved passively which, in turn, increases the rubustness of the robot. An analogy is that of an airplane. A controller can stabilize an aerodynamically unstable airplane, but an aerodynamically stable airplane is more robust in its flying stability.

Biological inspiration

Biologists have shown that at least three independent evolutionary lines resulted in the same leg configuration for quadrupeds, that of knee pointing forward and the elbow pointing backward. This suggests that there is a mechanical reason for this [6, 13].

$$\text{Froude Number} = \frac{(\text{Speed})^2}{(g)(\text{hip or shoulder height})} \qquad (1)$$

Biological quadrupeds also change their gaits according to the speed and size of the animal [23 11, 19]. A nondimensional parameter that indicates the desired gait for an animal is the Froude number (1). This normalizes the speed across different body sizes. Geometrically different animals have similar gaits at the same Froude number. Animals walk at Froude numbers below 0.5. They transition to trotting at Froude numbers of about 0.5 and transition to galloping at Froude numbers of about 2.5. However, the use of the Froude number as an indicator of gait transitions has been rarely used in the field of robotics [19].

II. PROCEDURES

This problem has been approached in several steps. Its first step consisted of the design of a simulation model that could be used to analyze the pitching performance of a multi-legged robot with a trotting gait. Since this is an exploratory work, we chose Working Model 2D. This simplified the dynamic modeling. It is, however, not efficient to search the design space for optimization. The second step was to use of the two-dimensional models to optimize the design space of directional-compliant legs. This process consisted of the setting the leg parameters, running the model and recording the resulting pitching

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motion. Simulation were made of steady state trotting with equal aerial and ground phases at different speeds, transient pitching and settling times for step changes in velocity, and disturbances due to steps and holes in the ground model.

The design space for the legs is large. We limited the search to distal leg stiffness, elbow/knee torsional stiffness, and unstretched elbow/knee angle and four leg configurations. These leg configurations, which are displayed in Fig. 1, are named Posture (Posture) I, II, III, and IV, respectively. All four configurations used the same properties of the knee/elbow torsional spring constant and the distal spring constant

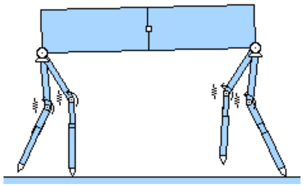

Fig 1a Posture I
‘Natural‘
configuration.
Motion is from left
to right

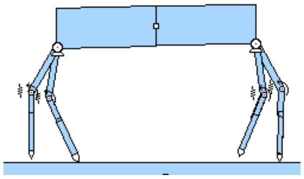

Fig 1.b. Posture II
Reverse of Pos. I.
Motion is from left
to right

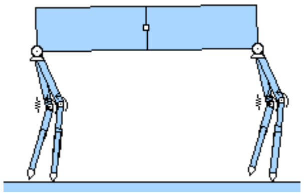

Fig 1 c. Posture III
Knee/elbow
forwards.
Motion is from left
to right

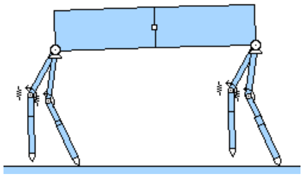

Fig1. D. Posture IV
Knee[elbow
backwards.
Motion is from left
to right

The increase of stability was determined by the decrease in pitching motion. With legged robots, we are ore concerned with attitudinal control. In 2-D, the only attitudinal motion is pitching. It is also interesting to note that Herr and McMahon [11] in their modeling of a horse with pogo-stick compliant legs could not achieve acceptable pitch control with passive compliance alone. In this work we show that you can achieve passive pitch control with passive anisotropic compliant legs.

III. SIMULATION MODEL

Physical Configuration

The physical configuration of the Posture I robot, which is based on the biological model [6, 13], is shown in Fig. 2, and its relevant parameters are indicated in Table 1. In this figure, the knee points forward and the elbow points backward as in the biological systems. The knees and elbows have torsional springs. The distal portions of the legs are prismatic joints with translational springs. The other three configurations share the same characteristics of the Posture I model except for the knee/elbow pointing direction. Posture II is Posture I with negative elbow/knee angles and Posture IV is Posture II with negative angles.

The mass of the robot, which is shown in Table 1, is one of its most important characteristics. The mass reflects an actual robot under design and its properties, the robot’s proximal leg is lighter than its upper and middle distal leg. The total mass of the legs is much less than the weight of the body, so the effect of this mass would be negligible in low speed-locomotion. However, in high-speed locomotion, the overall effect of the leg mass could be significant [12].

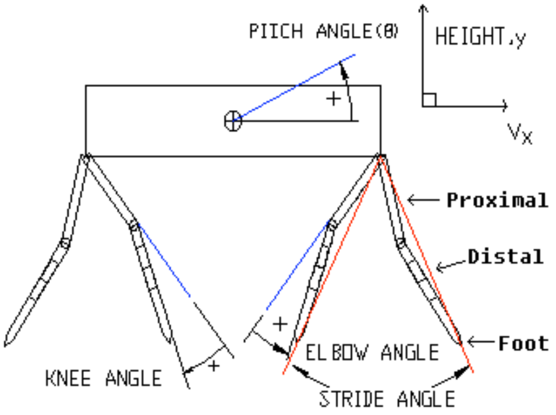

Fig. 2. Physical configuration of simulation model of Posture I

The only actuators in the model were at the shoulder and the hip. In all simulations, the angular velocities of the hip and shoulder were equal and constant except for instant changes in the velocities for the transient analyses. No other control was used in the simulation except for the hip/should velocity control.

The foot contact was modeled as a coefficient of restitution of 0 to prevent bouncing of the feet. The coefficient of friction was .9 in order to avoid issues of foot slip.

Table 1. Variables of a leg in simulation model

		Mass (kg)	Length (m)	Width (m)
Body		8	0.07	0.35
Proximal leg		0.0467	0.09	0.01
Distal leg	upper	0.0876	0.045	0.01
	middle	0.0525	0.045	0.01
	lower	0.13	0.065	0.01
Foot		0.0045	0.015	0.01

Leg Parameters

The robot model used in this study has two sets legs that act out of phase. In a trotting gait, the right front leg is retracted with the left rear leg and the left front leg is retracted with the right rear leg. This means that for straight-ahead trotting only two actuators are required.

The parameters that were varied were the elbow/knee spring constants, the unstretched length of the elbow/knee (nominal bend of the joints), the distal leg spring constant, and the velocities of the robot. The values are shown in Table 2.

In spite of the different configurations that arise from different pointing directions, the models in all four postures share the same problem: the hind distal spring is more compressed, and the movement from the fore leg is greater than that of the hind leg. This causes the quadruped robot to nose up and also to trot in an unstable way until it reaches the proper running speed. In order to reduce the nose up problem, the hind distal joint requires a higher spring constant than the fore distal joint. Consequently, the ratio of the fore and hind limb distal spring constant, $R_{r/f}$, shown in Table 3, has been used in all simulations. That ratio is 1.2, which means that the hind distal spring is 20 percent stronger than the fore distal spring.

Table 2. Variables in simulation

Variables	Values
Angular velocity (deg/s)	295, 335, 375, 415
Distal spring constant (N/m)	2790, 3170, 3550, 3930, 4310, 4690, 5070, 5450, 5830
Knee/elbow spring constant (N-m/deg)	0.4, 1.0, 2.0, 4.0
Knee/elbow spring angle (deg)	40, 30, 20, 10, 0, -10, -20, -30,-40

Also, it is desired to match the vibrational characteristics of the different models. This was done by matching the damping ratio of the legs for all variations of the leg parameters. The attenuation rates are defined in Table 3.

Table 3. Scale factor and the parameters

$R_{k/d} = b^2/k$	Spring ratio between stiffness and the damper constant
$R_{r/f} = K_r/K_f$	Distal spring ratio between hind and fore leg

The b is the damper constant, k is the spring stiffness constant.

IV. SIMULATION RESULTS

This project is based on the supposition that it is possible to improve the stability of a quadruped robot through the choice of an appropriate set of parameters for the legs, parameters such as the knee/elbow and distal spring constants, and the knee/elbow joint angle itself. Within these parameters, this project has focused on two key factors: 1) study of the relationship between the leg components and the stability of the quadruped robot; and 2) determination of the optimal leg configuration for the quadruped robot.

The measure of stability for the robot is the pitching angle; this angle was defined as the difference between the maximum and minimum of the angle of movement of the robot.

Steady-state responses

The simulations were run until steady state pitching was achieved - typically in two or three steps. Four velocities were simulated. These corresponded to Froude numbers of 0.5, 0.75, 0.9, and 1.1, all within the range for trotting.

Table 4 shows the results of these simulations. The leg parameters that resulted in the lowest pitching over all speeds are given.

There are several interesting results: 1) the best leg configuration is posture I - the 'biological' configuration (knee forward/elbow back), 2) the best torsional spring constant 1.0 N-m/deg for all leg configurations, and 3) the best knee/elbow angle is 30° (except for posture II). Note that negative angles of postures II and IV are the same as reversing the knee/elbow directions of postures I and III.

Table 4. Optimal leg parameters for steady-state response

Posture	Distal Spring (N/m)	knee/elbow spring (N-m/deg)	knee/elbow angle (deg)	pitching angle (deg)
I	3170	1.0	30	1.6
II	3170	1.0	-10	2.2
III	5070	1.0	30	1.9
IV	3170	1.0	-30	2.2

Transient responses

The transient responses to an instant changes in the velocity of the shoulder/hip joints response were studied in the same way as the steady-state responses in the first simulation set, and similarly involved the same groups of simulation models. The magnitude of the transient and the settling time for the pitching to return to steady state after the velocity change were used in a comparison of their response.

The same range of leg configurations and parameters that were used in the steady state simulations were used for the transient simulations. The results were plotted in a similar manner. Table 5 summarizes the optimal parameters for each leg configuration. Interestingly, the settling time was the same for all of the simulations. This is probably due to the damping ratio being held constant for all of the models. The robot achieved steady state within three steps.

Table 5. Optimal parameters for transient response

Posture	Distal Spring (N/m)	knee/elbow spring (N-m/deg)	knee/elbow angle (deg)	pitching angle (deg)
I	2790	1.0	30	2.8
II	3170	1.0	-10	2.6
III	3170	1.0	30	3.1
IV	3170	1.0	-10	2.1

Just as the steady-state response provided several interesting results, the transient response resulted in several interesting observations: The optimal knee/elbow torsional spring constant was 1 N-m/deg for all leg configurations. Posture I model showed low transient pitching throughout the design area that featured middle and high transient speeds whereas the other postures showed more variation in the range of pitching angles over the design space(not shown by the table). This would hint that Posture I is more tolerant of parameter variation.

Disturbance responses

In the steady state and transient models the ground was flat. Real robots do not trot on flat ground. To get an idea of the tolerance to roughness and disturbances on the ground, a set of simulations were run to see the effects of sudden changes in the ground height. Two simulations were run. One had the robot step on a block on the ground. The second had the robot step into a hole. The magnitude of the step/hole was set as a ratio between the robot's height and the obstacle. In this simulation, the ratio was 0.06 (≈0.015/0.25).

The optimal leg parameters found using were found for the disturbance simulations. The measure of stability during the simulations was the maximum the pitching motion. When the robot leg passed over the obstacle, the pitching angles showed sharp up-and-down peaks. However, the robot's motion stabilized after a short while. Table 6 summarizes the results of stepping on a black. Table 7 summarizes the results for stepping in a hole.

Table 6. Optimal leg parameters for stepping on a block

Posture	Distal Spring (N/m)	knee/elbow spring (N-m/deg)	knee/elbow angle (deg)	pitching angle (deg)
I	2790	2.0	20	8.5
II	2790	2.0	-10	11.6
III	2790	2.0	30	9.6
IV	2790	2.0	-30	9.3

Table 7. Optimal leg parameters for stepping in a hole

Posture	Distal Spring (N/m)	knee/elbow spring (N-m/deg)	knee/elbow angle (deg)	pitching angle (deg)
I	3170	2.0	30	6.7
II	2790	2.0	-10	8.2
III	2790	2.0	10	7.8
IV	2790	2.0	-20	7.7

The results indicate that the Posture I model handles the disturbances the best. The optimal knee/elbow torsional spring stiffness is higher for all the leg configurations than that found in the steady state results. This makes sense in that a stiffer joint helps stop the robot from pitching forward in the disturbance. This would suggest that stiffening the knee/elbow should be controlled to help stabilize the robot to disturbances. From an observational standpoint, one does stiffen up when a disturbance occurs when you accidentally step in a hole or stumble on an object. It seems that biology uses this strategy.

V. CONCLUSIONS

The ultimate goal of this project was to determine the optimal passive leg configuration for a trotting quadruped robot. To achieve this configuration, three responses — steady-state, transient, and disturbance — were utilized. First, the steady-state pitching performance of the robot models was evaluated. The simulations demonstrated that the Posture I model with the knee/elbow spring constant of 1.0 and 2.0 N-m/deg had the lowest pitching motion. Additionally, the Posture III and the Posture IV models had a similar steady-state pitching response when the knee/elbow distal-spring constants of 1.0 and 2.0 N-m/deg were chosen.

When the velocity was suddenly changed, the results showed that the best spring constants were about the same as found with the steady state simulations. The elbow/knee angle of the Posture IV configuration changed from 30° to 10°. The pitching motion of Posture I was very close to that of Posture II. This means that a leg designed with fixed springs in Posture I could be used for trotting over a range of speeds.

Finally, the disturbance responses were observed in two situations in which an obstacle consisting of a block or a hole was utilized. The pitching results indicated that the Posture I model had the lowest pitching motion and the distal spring and knee/elbow angles were close to that of the steady state and transient results. In order to best respond to a disturbance in the ground height, the knee/elbow torsional spring should stiffen. This would be difficult for a passive leg, but further work may find the best compromise in the leg parameters.

Overall, the optimal leg parameters at different running speeds was achieved with the Posture I model, the 'biological' model with the knee pointing forward and the elbow pointing backward. For the robot being simulated, the best knee/elbow angle unstretched angle was 30 degrees, the knee/elbow torsional spring constant was 1.0 N-m/deg, and the distal-spring constant was 3170 N/m. These would be the fixed parameters for a passive leg to minimize pitching motion (maximize stability) of the robot.

The next phase of this work is to expand the models to three dimensions and to use a direct search optimization such as a Nelder-Mead Simplex Method or possibly a genetic algorithm as the full leg design space is very large.

As the design space is large, simulation rather than analytical solutions will be pursued. We will also use Poincaré mapping to determine more rigorously the stability [24].

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