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A Local Sensor Based Leader-follower Flocking System

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Abstract— **This paper presents a leader-follower flocking system based on local sensor information. In this system, the leader robot knows global trajectory, tries to track the global trajectory, and has the ability to avoid collisions. The follower robots do not know global trajectory. They only use local sensors to acquire relative information between neighbors. In this paper they use a laser scanner to obtain relative distances and relative angles between neighbors. By using local information, the follower robots maintain the distances between their neighbors and avoid collisions with their neighbors. We prove that such a leader-follower flocking system is stable based on LaSalles invariance principle. To evaluate the performance of the flocking system, we simulate the flocking system tracking a desired trajectory. The flocking algorithm is tested with three Pioneer robots and SICK laser scanners. Both of simulation and real robot experiments successfully show the proposed local sensor based algorithm works in the leader-follower flocking system.**

I. INTRODUCTION

The flocking behavior of living beings, such as flocks of birds, schools of fish, herds of wildebeest, and colonies of bacteria has certain advantages, including avoiding predators, increasing the chance of finding food, saving energy, etc. Inspired by the collective and cooperation behaviors of biology, robot flocking systems have became an active research area during the past few years.

A biological behavior introduced in [1] gives a hint that minority of informed leaders is capable of leading the entire flocking to move to the expected destination. And followers do not need to know which individuals are leaders in flocking. In [3] and [2], a dynamic model of distance and angle between leaders and followers is built. That means the mobile robots must know who are their leaders. The paper [4] investigated leader-follower system in terms of controllability and optimal control in which followers need to know who are the leaders as well. In [7], different leader roles were discussed and a convergent condition was constructed by using the contraction theory. The convergent condition needs the global information and the followers need to know who are leaders. In [8], a leader based containment control strategy was designed. The group members clearly know which members are the leaders and the leaders have a desired formation pattern. The paper [9] applies local information to implement the Vicsek system [10]. Each robot uses a vision sensor to measure the relative distances and relative angles to its neighbors. Depending on the local information, robots gradually update their headings to the same direction. Another experiment achievement introduced in [11] demonstrates a distributed multi-agent cyclic pursuit algorithm which uses the information of relative angles between neighbors. Both of [10] and [11] concern the robot headings given that robot forward velocities are defined as constants. The multi-agent system introduced in [12] is capable of controlling both the distances and angles between neighbors by using local information. But the algorithm is designed for robot formation and the neighbor relationships must be fixed.

It can be seen that, in most of the flocking systems, followers need to know the global information. Only several flocking systems utilize local information to control the robot headings with constant forward velocity. Although local information can be used to control relative distances and relative angles in robot formation, the followers must know which members are their leaders and leader-follower relationships must be fixed in robot formation.

In this paper, a distributed algorithm will be designed to control the leader-follower flocking. Different from other multi-agent systems, the leader-follower relationship in the flocking is no longer fixed, instead the leaders diffuse the navigation information into the flocking by using the local interactions between adjacent flocking members. In other words, there is no constant leader-follower relationship between any two robots, whereas a collective leader-follower behavior emerges from the interaction between neighbors. Furthermore, followers do not know which one is leader and they do not know any global information (global coordinate or navigation information). By using the local information, the relative distance between any neighbors asymptotically reaches a specific value.

To control the distance between adjacent robots, we design a fuzzy force function. Different from the “logarithm” force function introduced in [5] [6], fuzzy logic controller can generate finite control force. It is easier to adjust the attraction and repulsion between robots by selecting proper fuzzy set. Moreover, we can build a continuous system function by using fuzzy logic which is necessary for stability analysis.

The outline of this paper is arranged as: Section II briefly introduces the mobile robot model and the laser scanner. Section III introduces the flocking algorithms for leader and followers. Section IV proves that the flocking algorithms can stabilize the flocking system. In section V, ten simulated robots are used to simulate the flocking system. Three real Pioneer robots are used to test the flocking algorithm. A brief conclusion is given in section VI.

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II. PIONEER ROBOT AND LASER SCANNER

Pioneer robots are used in our flocking system. Hand position is a point located at the heading axis with distance L to the center of a robot. By taking the hand position as a reference point $q = [x, y]^T$, its velocities along x and y axes are described as $\dot{q} = [\dot{x}, \dot{y}]^T$. Let v denotes the forward velocity and ω the rotation velocity of robot. The robot model can be written as:

$$\begin{aligned} v =& \dot{x}\cos\theta + \dot{y}\sin\theta \\ \omega =& \frac{1}{L}\Big[\dot{x}\cos(\theta + \frac{\pi}{2}) + \dot{y}\cos(\theta + \frac{\pi}{2})\Big] \end{aligned} \quad (1)$$

where θ is the heading of robot.

The laser scanner equipped to the pioneer robot can provide local information including relative distance and relative angle between neighbors. Fig. 1 illustrates a robot i can obtain local information about one of its neighbors j. The local information includes d_{ij}, the length of the link between two robots and β_{ij}, the angle between the link and the heading of robot i.

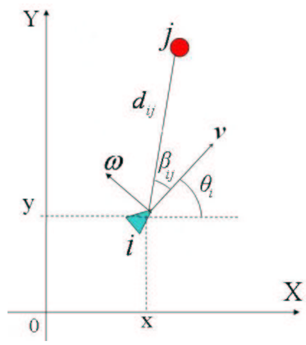
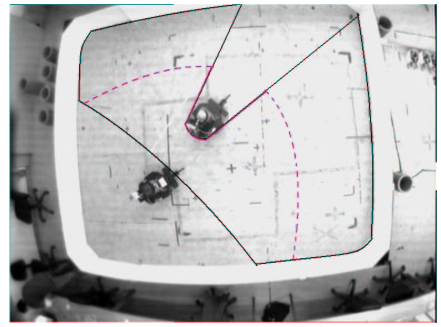


Fig. 1. Local information acquired by robot i

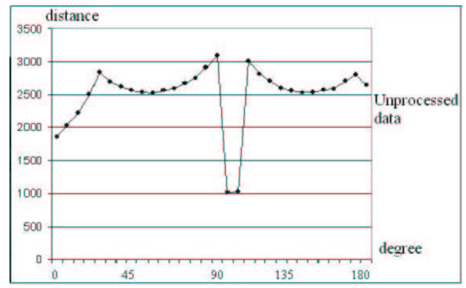
The laser scanner provides a 180° scan of the environment at approximately forty centimeters from the floor and with a half degree resolution. It is known that laser readings are very accurate and the error is in the order of millimeters.

Fig. 2(a) shows a robot uses the laser scanner to measure the relative position to a neighbor. The solid line describes the original pattern of a laser scan. The laser range data can be represented as a function on a X-Y graph in which the x-axis is the angle and the y-axis is the measured distance. In the simplest case, the pattern of one neighbor is illustrated in fig. 2(b).

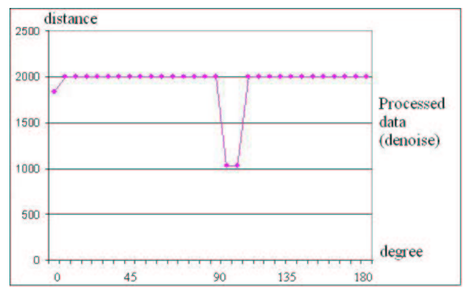
To identify the position of neighbors, the noise must be removed from the laser data. For the purpose of de-noise, a threshold is made to limit the maximal range. In the experiment, the threshold is set as $2m$. The dashed line in fig. 2(a) shows the pattern of the limited scan. The processed data is illustrated in fig. 2(c). It can be seen that the data changes smoothly but decreases suddenly at the position of neighbor.



(a) Scan pattern of laser scanner



(b) Original data



(c) Processed data

Fig. 2. SICK laser scanner

III. LEADER-FOLLOWER FLOCKING ALGORITHM

A leader-follower flocking system is composed of minority leaders and majority followers. In this system, leaders use a tracking strategy to lead the flocking to move to the destination. Leaders are the flocking members who can get the navigation information (desired trajectory) and are able to track the desired trajectory; the followers are the flocking members without navigation information and can only interact with neighbors.

A. Follower Algorithm

As a basic requirement of the flocking system, adjacent robots should keep a specific distance. If the distance between adjacent robots is too small, they attempt to separate; If the distance between adjacent robots is too large, the cohesive force will take effect.

Assume the distance between robot i and one of its neighbor j is $d_{ij} = ||q_i - q_j||$. We use $H_s(d_{ij})$ to denote the potential function between them. We denote the gradient of $H_s(d_{ij})$ with respect to d_{ij} as $f_s(d_{ij})$ and denote the gradient of $H_s(d_{ij})$ with respect to q_i as $\nabla H_s(d_{ij})$:

$$\nabla H_s(d_{ij}) = f_s(d_{ij})\frac{q_i - q_j}{d_{ij}} = f_s(d_{ij})\begin{bmatrix} \cos(\beta_{ij} + \theta_i) \\ \sin(\beta_{ij} + \theta_i) \end{bmatrix}$$

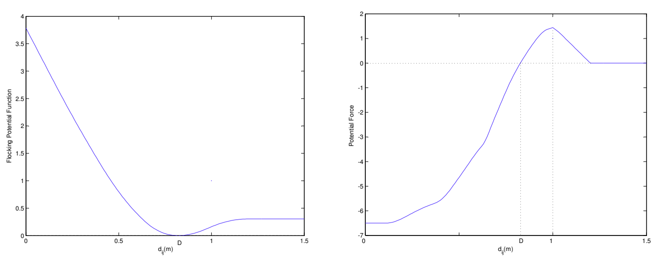
Because of the mutual relationship between neighbors, $\nabla H_s(d_{ij})$ function satisfies:

$$\nabla H_s(d_{ij}) = -\nabla H_s(d_{ji}) \tag{2}$$

Fig. 3 illustrates an example of $H_s(d_{ij})$ and its corresponding $f_s(d_{ij})$ designed by using fuzzy logic. It can be seen that the potential function of robot i is nonnegative. Furthermore, it owns following properties:

- When the distance d_{ij} between robots i and j is smaller than a specific distance, $f_s(d_{ij})$ is negative. Robot i moves away from robot j.
- When the distance d_{ij} between robots i and j is larger than a specific distance, $f_s(d_{ij})$ is positive.
- When the distance d_{ij} between robots i and j is larger than the communication range, $f_s(d_{ij})$ is zero.
- $H_s(d_{ij})$ is a differentiable function.

The first two points in the above can guarantee there is an equilibrium in the flocking system. The third point can guarantee the algorithm matches to sensor limitation. The last point is necessary for stability proof.



(a) $H_s(d_{ij})$ (b) $f_s(d_{ij})$

Fig. 3. Separation potential function and force function

The flocking algorithm of follower i is designed as follows:

$$\begin{aligned}\dot{q}_i =& -\sum_{j\in N_i} \nabla H_s(d_{ij}) + \max(||\dot{q}_r||) \\ & \sum_{j\in N_i} \nabla H_s(d_{ij}) \Big/ \Big\| \sum_{j\in N_i} \nabla H_s(d_{ij}) \Big\| \\ =& -\sum_{j\in N_i} [f_s(d_{ij}) - Q\mathrm{sgn}(f_s(d_{ij}))] \\ & \begin{bmatrix} \cos(\beta_{ij}+\theta_i) \\ \sin(\beta_{ij}+\theta_i) \end{bmatrix}\end{aligned} \tag{3}$$

where $Q = \max(||\dot{q}_r||)$ and q_r is the position of a desired trajectory. Further it can also be written as:

$$\begin{aligned}\dot{x}_i =& -\sum_{j\in N_i} [f_s(d_{ij}) - Q\mathrm{sgn}(f_s(d_{ij}))] \\ & \cos(\beta_{ij}+\theta_i) \\ \dot{y}_i =& -\sum_{j\in N_i} [f_s(d_{ij}) - Q\mathrm{sgn}(f_s(d_{ij}))] \\ & \sin(\beta_{ij}+\theta_i)\end{aligned} \tag{4}$$

Based on the above result, the velocities of robot in equation (1) can be written as:

$$\begin{aligned}v_i =& \dot{x}_i \cos\theta_i + \dot{y}_i \sin\theta_i \\ =& -\sum_{j\in N_i} \Big[f_s(d_{ij}) - Q\mathrm{sgn}(f_s(d_{ij}))\Big] \cos\beta_{ij} \\ \omega_i =& \frac{1}{L}\Big[\dot{x}_i \cos(\theta_i + \frac{\pi}{2}) + \dot{y}_i \sin(\theta_i + \frac{\pi}{2})\Big] \\ =& -\sum_{j\in N_i} \Big[f_s(d_{ij}) - Q\mathrm{sgn}(f_s(d_{ij}))\Big] \sin\beta_{ij}\end{aligned} \tag{5}$$

It can be seen that the forward velocity and rotation velocity of a follower robot only depend on the local information, namely the relative distance d_{ij} and relative angle β_{ij}. They can obtained from the laser scanner or similar sensors. There are no need for global information. Additional information required is Q, which represents the maximum velocity of the desired trajectory. This requirement is reasonable as the followers can use a large value to replace it when Q is unknown. In other words, the flocking can only track the target with limited maximum speed Q. Otherwise, the flocking system is not stable. In the testing, the maximum speed of virtual leader is $50mm/s$, the Q is set as $60mm/s$.

B. Leader Algorithm

To simplify the analysis, we consider the flocking system with only one leader, which is denoted as robot l. The potential energy of tracking control for the leader l is defined as: $\frac{1}{2}||q_l - q_r||^2$ where q_l, q_r are the positions of the leader and the desired trajectory. Taking the separation potential function into consideration, the leader flocking algorithm is defined as:

$$\dot{q}_l = -\sum_{j\in N_l} \nabla H_s(d_{lj}) - k_r(q_l - q_r) + \dot{q}_r \tag{6}$$

where k_r is the gain of the tracking control.

The leader flocking algorithm can be expressed as:

$$\begin{aligned}\dot{x}_l &= -k_r(x_l - x_r) + \dot{x}_r - \sum_{j\in N_l} f(d_{lj}) \cos(\beta_{lj} + \theta_l) \\ \dot{y}_l &= -k_r(y_l - y_r) + \dot{y}_r - \sum_{j\in N_l} f(d_{lj}) \sin(\beta_{lj} + \theta_l)\end{aligned} \tag{7}$$

Similar to the follower flocking algorithm, we can get v and ω of leader:

$$\begin{aligned}v_l =& -[k_r(x_l - x_r) + \dot{x}_r]\cos\theta_l \\ & - [k_r(y_l - y_r) + \dot{y}_r]\sin\theta_l \\ & - \sum_{j\in N_l} f_s(d_{lj})\cos\beta_{lj} \\ \omega_l =& -\frac{1}{L}[k_r(x_l - x_r) + \dot{x}_r]\cos(\theta_l + \frac{\pi}{2}) \\ & - \frac{1}{L}[k_r(y_l - y_r) + \dot{y}_r]\sin(\theta_l + \frac{\pi}{2}) \\ & - \sum_{j\in N_l} f_s(d_{lj})\sin\beta_{lj}\end{aligned} \tag{8}$$

It can be seen that the forward velocity and rotation velocity of the leader robot depends on not only the local information, the relative distance and relative angle, but also global information, global state (x_l, y_l, θ_l) and the desired trajectory $(x_r, y_r, \dot{x}_r, \dot{y}_r)$.

IV. SYSTEM STABILITY ANALYSIS

In our flocking system, the system energy is defined as $H(q)$. We use $H(q_i)$ to denote the energy of robot i, the relationship between $H(q)$ and $H(q_i)$ is:

$$H(q) = \sum_{i=1}^{N} H(q_i) \tag{9}$$

where N is the total number of robots in the flocking system. As shown in the previous section, the leader and followers use different flocking algorithms. So the energy function $H(q_i)$ is different from leader to followers:

$$\begin{aligned} H(q_l) =& \frac{1}{2}\sum_{j\in N_l} H_s(d_{lj}) + \frac{1}{2}d_{lr}^2 \\ H(q_i) =& \frac{1}{2}\sum_{j\in N_i} H_s(d_{ij}) \end{aligned} \tag{10}$$

Their derivatives are:

$$\begin{aligned} \dot{H}(q_l) =& \frac{1}{2}\sum_{j\in N_l} \nabla H_s(d_{lj})^T(\dot{q}_l - \dot{q}_j) + (q_l - q_r)^T(\dot{q}_l - \dot{q}_r) \\ \dot{H}(q_i) =& \frac{1}{2}\sum_{j\in N_i} \nabla H_s(d_{ij})^T(\dot{q}_i - \dot{q}_j) \end{aligned} \tag{11}$$

Then, according to the mutual relationship (2), we have

$$\begin{aligned} \dot{H}(q) =& \dot{H}(q_l) + \sum_{i\neq l}\dot{H}(q_i) \\ =& \sum_{j\in N_l} \nabla H_s(d_{lj})^T\dot{q}_l + (q_l - q_r)^T(\dot{q}_l - \dot{q}_r) \\ &+ \sum_{i\neq l}\sum_{j\in N_i} \nabla H_s(d_{ij})^T\dot{q}_i \end{aligned} \tag{12}$$

By using the flocking algorithm of leader (6), we have

$$\begin{aligned} \dot{H}(q) =& -\Big|\Big|\sum_{j\in N_l} \nabla H_s(d_{lj}) - k_r(q_l - q_r)\Big|\Big|^2 \\ &+ \sum_{j\in N_l} \nabla H_s(d_{lj})^T\dot{q}_r \\ &+ \sum_{i\neq l}\sum_{j\in N_i} \nabla H_s(d_{ij})^T\dot{q}_i \end{aligned} \tag{13}$$

By using the flocking algorithm of follower (3), we have

$$\begin{aligned} \dot{H}(q) =& -\Big|\Big|\sum_{j\in N_l} \nabla H_s(d_{lj}) - k_r(q_l - q_r)\Big|\Big|^2 \\ &+ \sum_{j\in N_l} \nabla H_s(d_{lj})^T\dot{q}_r \\ &- \sum_{i\neq l}\Big|\Big|\sum_{j\in N_i} \nabla H_s(d_{ij})\Big|\Big|^2 \\ &+ \sum_{i\neq l}\Big|\Big|\sum_{j\in N_i} \nabla H_s(d_{ij})\Big|\Big|Q \end{aligned} \tag{14}$$

To simplify the presentation, we define:

$$\begin{aligned} E =& \Big|\Big|\sum_{j\in N_l} \nabla H_s(d_{lj}) - k_r(q_l - q_r)\Big|\Big|^2 \\ &+ \sum_{i\neq l}\Big|\Big|\sum_{j\in N_i} \nabla H_s(d_{ij})\Big|\Big|^2 \end{aligned} \tag{15}$$

So

$$\begin{aligned} \dot{H}(q) =& -E + \sum_{j\in N_l} \nabla H_s(d_{lj})^T\dot{q}_r \\ &+ \sum_{i\neq l}\Big|\Big|\sum_{j\in N_i} \nabla H_s(d_{ij})\Big|\Big|Q \\ \leq& -E + \mathbf{1}\Big|\sum_{j\in N_l} \nabla H_s(d_{lj})\Big|Q \\ &+ \sum_{i\neq l}\mathbf{1}\Big|\sum_{j\in N_i} \nabla H_s(d_{ij})\Big|Q \\ \leq& -E + \sum_{i=1}^{N}\mathbf{1}\Big|\sum_{j\in N_i} \nabla H_s(d_{ij})\Big|Q \\ \leq& -E \leq 0 \end{aligned} \tag{16}$$

where $\mathbf{1} = [1, 1]$. We have that $\dot{H}(q)$ is non-positive, which means all the robots attempt to approach to the state where $E = 0$. For the followers,

$$\sum_{i\neq l}\sum_{j\in N_i} \nabla H_s(d_{ij}) = 0 \tag{17}$$

Based on the properties of H_s function mentioned in section III, it means $||q_i - q_j|| = d_{ij}$ for all the followers.

For the leader, the stable state should satisfy the following condition:

$$k_r(q_l - q_r) - \sum_{j\in N_l} \nabla H_s(d_{lj}) = 0 \tag{18}$$

Different from the followers, leader can only stable when sum of potential force from neighbours equals tracking force from the virtual leader.

V. SIMULATION AND EXPERIMENT

As a local sensor based flocking system, the communication network must keep connected at initial state. In the simulation, ten robots are placed randomly in a limited area at the initial place in which each agent can interact with all the others via flocking network. Two desired trajectories are simulated:

- ten robots track a circular trajectory.
- ten robots track a sine shape trajectory.

In both of the simulations, the flocking system is led by only one leader which is represented by the solid circle.

Fig. 4 shows the simulation result of a circular trajectory tracking. It can be seen all ten robots can work together as a group to flock. The trajectories slightly fluctuate at the beginning of the flocking, but finally a stable pattern is formed and all the robots can track the circle. The relative

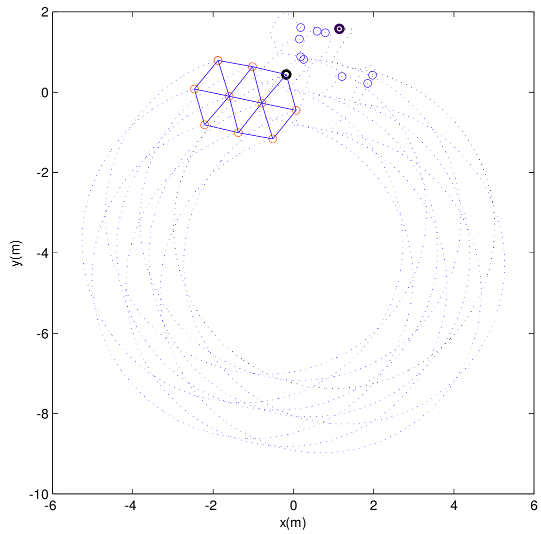


Fig. 4. Ten robots track a circle

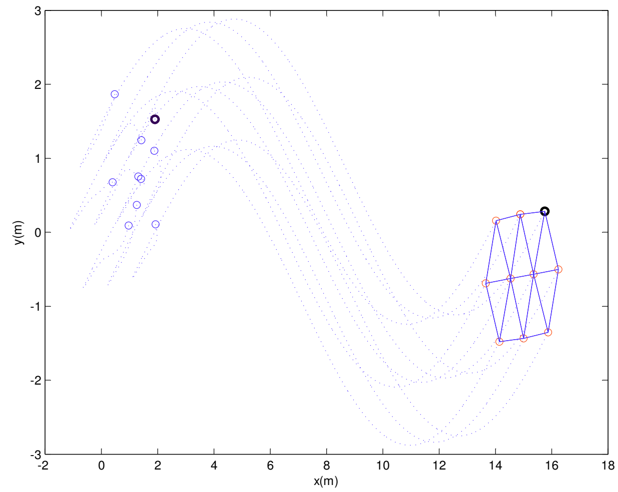


Fig. 5. Ten robots track a sine shape trajectory

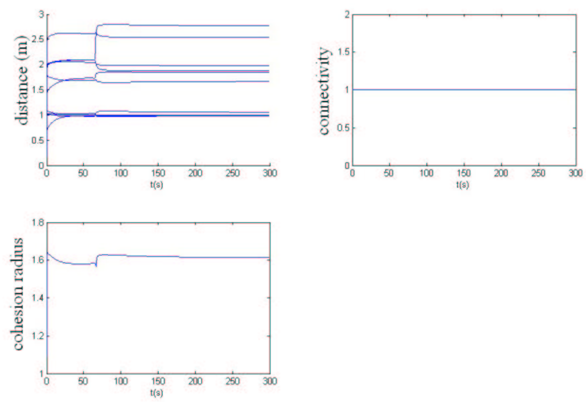


Fig. 6. Distance between robots(top left); Graph connectivity (top right); Cohesive radius (bottom left)

distances between robots keep stable at the specific distance during most of the tracking process.

In the second simulation, the leader robot tracks a sine shape trajectory and the follower robots follow the leader. Fig. 5 shows the process of the sine shape trajectory tracking. It is the same as that of the first simulation that the trajectories slightly fluctuate at the beginning of the tracking, but finally a stable pattern is formed and all the robots can track the trajectory.

For the purpose of connectivity analyzing, the graph connectivity is used. It is the second smallest eigenvalue of graph Laplacian matrix [14]. It can be seen from the top right of fig. 6 that the graph connectivity keeps 1 during the whole process. That means the flocking network keeps connected and the fluctuation does not affect the connectivity of the flocking system. The cohesive radius is the maximum radius of the flocking group. In the bottom left of fig. 6, the cohesive radius of the flocking also reaches a stable level. The performance in the top left of fig. 6 shows that relative distances between robots are influenced at the beginning, but keep stable at the specific distance very quickly.

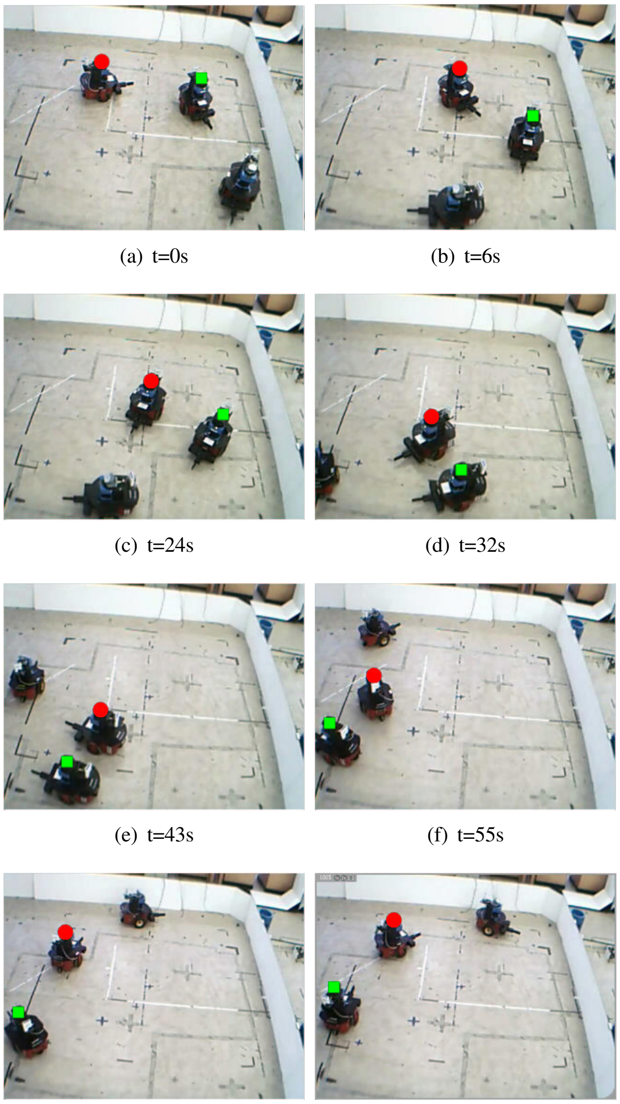
In the real robot test, three Pioneer robots are used, each of which is equipped with a SICK laser scanner. The programs of the robots are implemented in C++ and run in real-time on the robot onboard computers.

Three robots are moving along a square trajectory with side length $2.5m$. One of the robots is used as leader, the other two robots are followers. Fig. 7 illustrates the flocking process. In fig. 7, the two followers are marked with a rectangle and a circle respectively (on the top of robots). We define the robot with rectangular mark as $followerA$, and the robot marked by circle is called $followerB$.

At the initial place at $t = 0$ (fig. 7(a)), only the follower A can detect the leader, whereas the follower B does not know where the leader is because of the limited sense range. The follower B can find the follower A. So the follower B can track the follower A.

When the leader turns an angle at the corner at $t = 6s$ (fig. 7(b)), the follower B moves close to the leader. At $t = 24s$ (fig. 7(c)), the follower B detects the leader and starts to follow the leader. At the same time, the follower A is pushed away from the leader due to the potential force generated between the two followers (fig. 7(d))(fig. 7(e)). At $t = 55s$ (fig. 7(f)), the follower A loses the contact with the leader and starts to track the follower B. Finally, the new leader-follower relationship is built up and the flocking system is stable again (fig. 7(g))(fig. 7(h)). The positions of robots were tracked by an overhead camera and the trajectories are illustrated in fig. 8. In fig. 8, the start places are marked by stars and the final places are marked by circles. It can be seen that the neighbor relationship at the final place is different from that of the initial place. The experiment result shows that the robots can behave like a flock by using the local flocking algorithm.

The experiment of three robots flocking has been recorded by a overhead camera. The whole process can be viewed in the accompanying video.



(a) t=0s (b) t=6s

(c) t=24s (d) t=32s

(e) t=43s (f) t=55s

(g) t=63s (h) t=71s

Fig. 7. Three robot flocking

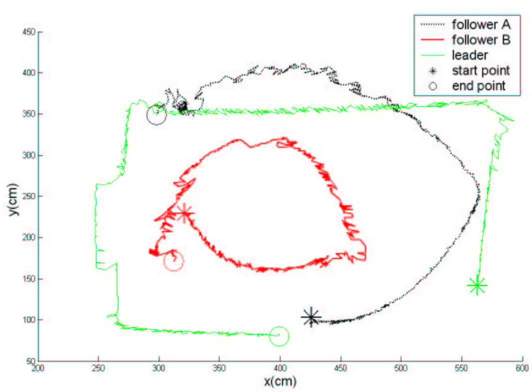


Fig. 8. The trajectories of robots

VI. CONCLUSIONS

This paper demonstrates that a flocking can be led by minority leaders. To do so, a potential function is needed to keep the relative distance between neighbors and the fuzzy logic controller can be used to design the potential function. We convert the follower flocking algorithm into a local form which only uses the local sensor information to achieve the flocking control.

The flocking algorithm is tested with three Pioneer robots. We use the SICK laser scanner to get the local information of neighbors. The experiment result proves that the local flocking algorithm can be used in the real robots. The distributed flocking algorithm provides a more flexible leader-follower strategy. The followers do not need to know which one is leading the flocking and they do not need to follow any specific leader.

In our further work, the flocking algorithm will be tested with more robots. Furthermore, the connectivity problem during flocking will be investigated.

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