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Region Following Formation Control for Multi-Robot Systems

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***Abstract*— In this paper, a region following formation control method for multi-robot systems is proposed. In this control method, the robots move as a group inside a desired region while maintaining a minimum distance among themselves. Various shapes of desired region can be formed by choosing the appropriate objective functions. The robots do not need to have specific identities since the proposed controller does not need specific orders of robots within the group. Therefore, the system is scalable since any robot can come in or go out of the group without affecting the system. Lyapunov-like function is presented for convergence analysis of the multi-robot systems. Simulation results are presented to illustrate the performance of the proposed controller.**

I. INTRODUCTION

Nature provides many examples of biological systems that work cooperatively to accomplish a common goal. For example, a flock of birds flying together in a formation to save energy on long migration, a herd of animals move in a group to stay safe from predators. Multiple robots can be used to accomplish tasks not possible with individual robot acting alone. One important research problem is control of multi-robot systems in maintaining a desired formation during movements. In behavior-based formation control [1]-[6], a desired set of behaviors is implemented onto individual robots. By defining the relative importance of all the behaviors, the overall behavior of the robot is formed. In leader-following control strategy [7]-[11], the leaders are identified and the follower are defined to follow their respective leaders. In virtual structure method [12]-[15], the entire formation is considered as a single entity and desired motion is assigned to the structure.

In general, both leader-following and virtual structure methods belong to centralized control strategy whereas behavior-based control is decentralized. As such, behavior-based can be implemented with significantly less communication as compare to the other two methods. However, it is difficult to analyze the overall system mathematically to gain insights into the formation control problems. It is also not possible to show that the system converges to the desired formation. The leader-following approach is easier to analyze and implement. However, an obvious problem is that the failure of one robot (i.e leader) leads to the failures of the entire system. The formation of the group in virtual structure approach is very rigid as the geometric relationship among the robots in the system must be rigidly maintain during the movement. Therefore, it is generally not possible for the formation to change with time and obstacle avoidance could also be a problem. The leader-following and virtual structure approaches are not suitable for controlling a large group of robots because the constraint relationships among robots become more complicated as the number of robots in the group increases. To alleviate the problem, Belta and Kumar [16] proposed a control method for a large group of robots to move along a specified path. However, this proposed control strategy has no control over the desired formation since the shape of the whole group is dependent on the number of the robots in the group. For large number of robots, the formation is fixed as an elliptical shape whereas for a small number of robots the formation is fixed as a rectangular shape. Moreover, this method does not consider the effects of dynamics on formation control.

In this paper, we propose a region following formation control for a large group of robots. In our proposed formation control method, each robot in the group stays within a moving region as a group (global objective) and at the same time maintains a minimum distance from each other (local objective). The desired region can be specified as various shapes, hence different formations can be formed. The robots in the group only need to communicate with their neighbors and not the entire community. The robots do not have specific identities or roles within the group. Therefore, the proposed method does not require specific orders or positions of the robots inside the region and hence different formations can be formed even for a swarm of robots. The dynamics of the robots are also considered in the stability analysis of the formation control system. The system is scalable in the sense that any robot can move into the formation or leave the formation without affecting the other robots. Lyapunov theory is used to show the stability of the multi-robot systems. Simulation results are presented to illustrate the performance of the proposed formation controller.

II. ROBOT DYNAMICS

We consider a group of N fully actuated mobile robots whose dynamics of the i^{th} robot with n degrees of freedom can be described as [17], [18]:

$$M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i, \dot{x}_i)\dot{x}_i + g_i(x_i) = u_i \quad (1)$$

where $x_i \in R^n$ is a generalized coordinate, $M_i(x_i) \in R^{n\times n}$ is an inertia matrix, $C_i(x_i, \dot{x}_i) \in R^{n\times n}$ is a matrix of Coriolis and centripetal terms, $D_i(x_i, \dot{x}_i) \in R^{n\times n}$ represents the damping force, $g_i(x_i) \in R^n$ denotes a gravitational force vector, and $u_i \in R^n$ denotes the control inputs.

Several important properties of the dynamic equation described by equation (1) are given as follows [17], [18]:

Property 1: The inertia matrix $M_i(x_i)$ is symmetric and positive definite for all $x_i \in R^n$.

Property 2: The Coriolis and centripetal matrix $C(x, \dot{x})$ is characterized by the following property $s^T[\dot{M}_i(x_i) - 2C_i(x_i, \dot{x}_i)]s = 0$ for all $s \in R^n$, $x_i \in R^n$.

Property 3: The damping matrix $D_i(x_i, \dot{x}_i)$ is positive definite for all $x_i \in R^n$.

Property 4: The dynamic model described by equation (1) is linear in a set of unknown parameters $\theta_i \in R^p$ as

$$\begin{aligned} & M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i))\dot{x}_i + D_i(x_i, \dot{x}_i)\dot{x}_i + g_i(x_i) \\ = \; & Y_i(x_i, \dot{x}_i, \dot{x}_i, \ddot{x}_i)\theta_i \end{aligned} \tag{2}$$

where $Y_i(x_i, \dot{x}_i, \dot{x}_i, \ddot{x}_i) \in R^{n\times p}$ is a known regressor matrix.

III. Formation Control of Multi-Robot System

In this section, we present the region following formation controller for the group of mobile robots. First, a region of specific shape is defined for all the robots to stay inside. This can be viewed as a global objective of all robots. Second, a minimum distance is specified between each robot and its neighboring robots. This can be viewed as a local objective of each robot. Thus, the group of robots will be able to move in a desired formation while maintaining a minimum distance among each other.

Let us define a global objective function by the following inequality:

$$f_G(x_i) = [f_{G1}(\Delta x_{io1}),\ f_{G2}(\Delta x_{io2}),\ ...,\ f_{GM}(\Delta x_{ioM})]^T \leq 0 \tag{3}$$

where $\Delta x_{iol} = x_i - x_{ol}$, $x_{ol}(t)$ is a reference point within the l^{th} desired region, $l = 1, 2, ..., M$, M is the total number of objective functions, $f_{Gl}(\Delta x_{iol})$ are continuous scalar functions with continuous partial derivatives that satisfy $f_{Gl}(\Delta x_{iol}) \to \infty$ as $||\Delta x_{iol}|| \to \infty$. $f_{Gl}(\Delta x_{iol})$ is chosen in such a way that the boundedness of $f_{Gl}(\Delta x_{iol})$ ensures the boundedness of $\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial \Delta x_{iol}}$, $\frac{\partial^2 f_{Gl}(\Delta x_{iol})}{\partial \Delta x_{iol}^2}$. Each reference point of the individual region is chosen to be a constant offset of one another so that $\dot{x}_{ol} = \dot{x}_o$, where $\dot{x}_o$ is the speed of the desired region. Various formations such as circle, ellipse, crescent, ring, triangle, square etc. can be formed by choosing the appropriate functions. For example, a ring formation can be formed by choosing the objective functions as follows:

$$\begin{aligned} f_1(\Delta x_{io1}) = r_1^2 - (x_{i1} - x_{o11})^2 - (x_{i2} - x_{o12})^2 \leq 0 \\ f_2(\Delta x_{io2}) = (x_{i1} - x_{o11})^2 + (x_{i2} - x_{o12})^2 - r_2^2 \leq 0 \end{aligned} \tag{4}$$

where r_1 and r_2 are the constant radii of the two circles such that $r_1 < r_2$, $(x_{o11}(t), x_{o12}(t))$ represents the common center of the two circles. Some examples of the desired regions are shown in figure 1.

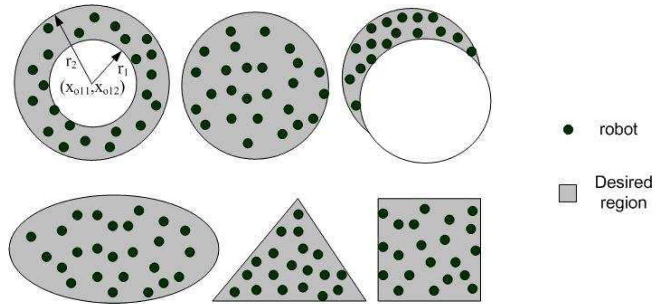

Fig. 1. Examples of Desired Regions

The potential energy function of the global objective functions is defined as follows:

$$P_{Gi}(\Delta x_{iol}) = \sum_{l=1}^{M} P_{Gl}(\Delta x_{iol}), \tag{5}$$

where

$$P_{Gl}(\Delta x_{iol}) = \begin{cases} 0, & f_{Gl}(\Delta x_{iol}) \leq 0 \\ \frac{k_l}{2} f_{Gl}^2(\Delta x_{iol}), & f_{Gl}(\Delta x_{iol}) > 0 \end{cases} \tag{6}$$

and k_l are positive constants. Note that $P_{Gi}(\Delta x_{iol}) = 0$ only if all the objective functions in (3) are satisfied.

Partial differentiating the potential energy function described by equation (5) and equation (6) with respect to Δx_{iol}, we have:

$$\frac{\partial P_{Gi}(\Delta x_{iol})}{\partial \Delta x_{iol}} = \sum_{l=1}^{M} \frac{\partial P_{Gl}(\Delta x_{iol})}{\partial \Delta x_{iol}} \tag{7}$$

where

$$\frac{\partial P_{Gl}(\Delta x_{iol})}{\partial \Delta x_{iol}} = \begin{cases} 0, & f_{Gl}(\Delta x_{iol}) \leq 0 \\ k_l f_{Gl}(\Delta x_{iol})(\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial \Delta x_{iol}})^T, & f_{Gl}(\Delta x_{iol}) > 0 \end{cases}$$

The above equations can be written as:

$$\begin{aligned} \frac{\partial P_{Gi}(\Delta x_{iol})}{\partial \Delta x_{iol}} &= \sum_{l=1}^{M} k_l max(0, f_{Gl}(\Delta x_{iol}))(\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial \Delta x_{iol}})^T \\ &\triangleq \Delta \xi_i \end{aligned} \tag{8}$$

Note that when the robot i is outside the desired region, the control force $\Delta \xi_i$ described by equation (8) is activated to attract the robot toward the desired region. When the robot is inside the desired region, then $\Delta \xi_i = 0$.

Next, we define a minimum distance between robots by the following inequality:

$$g_{Lij}(\Delta x_{ij}) = r^2 - ||\Delta x_{ij}||^2 \leq 0 \tag{9}$$

where $\Delta x_{ij} = x_i - x_j$ is the distance between robot i and robot j and r is a minimum distance between the two robots as illustrated in figure 2. For simplicity, the minimum distance between robots is chosen to be the same for all the robots. Note from the above inequality that the function $g_{Lij}(\Delta x_{ij})$ is twice partially differentiable. From equation (9), it is clear that

$$g_{Lij}(\Delta x_{ij}) = g_{Lji}(\Delta x_{ji}) \tag{10}$$

and

$$\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} = -\frac{\partial g_{Lji}(\Delta x_{ji})}{\partial \Delta x_{ji}} \tag{11}$$

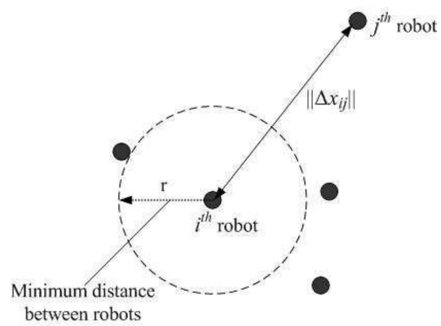

Fig. 2. Minimum Distance between Robots

A potential energy for the local objective function (9) is defined as:

$$Q_{Lij}(\Delta x_{ij}) = \sum_{j\in N_i} \frac{k_{ij}}{2}[max(0, g_{Lij}(\Delta x_{ij}))]^2 \tag{12}$$

where k_{ij} are positive constants and N_i is a set of neighbors around robots i. Any robot that is at a distance smaller than r_N from robot i is called neighbor of robot i. r_N is a positive number satisfy the condition $r_N > r$.
Partial differentiating equation (12) with respect to Δx_{ij}, we get

$$\frac{\partial Q_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} = \sum_{j\in N_i} k_{ij} max(0, g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}})^T \triangleq \Delta\rho_{ij} \tag{13}$$

Note that $\Delta\rho_{ij}$ is a resultant control force acting on robot i by its neighboring robots. Similarly, when robot i maintains minimum distance r from its neighboring robots, then $\Delta\rho_{ij} = 0$. The control force $\Delta\rho_{ij}$ is activated only when the distance between robot i and any of its neighboring robots is smaller than the minimum distance r. We consider a bidirectional interactive force between each pair of neighbors. That is, if robot i keeps a distance from robot j then robot j also keeps a distance from robot i. Next, we define a vector $\dot{x}_{ri}$ as

$$\dot{x}_{ri} = \dot{x}_o - \alpha_i\Delta\xi_i - \gamma\Delta\rho_{ij} \tag{14}$$

where $\Delta\xi_i$ is defined in equation (8), $\Delta\rho_{ij}$ is defined in (13), α_i and γ are positive constants. Differentiating equation (14) with respect to time we get

$$\ddot{x}_{ri} = \ddot{x}_o - \alpha_i\Delta\dot{\xi}_i - \gamma\Delta\dot{\rho}_{ij} \tag{15}$$

A sliding vector for robot i is then defined as:

$$s_i = \dot{x}_i - \dot{x}_{ri} = \Delta\dot{x}_i + \alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij} \tag{16}$$

where $\Delta\dot{x}_i = \dot{x}_i - \dot{x}_o$. Differentiating equation (16) with respect to time yields

$$\dot{s}_i = \ddot{x}_i - \ddot{x}_{ri} = \Delta\ddot{x}_i + \alpha_i\Delta\dot{\xi}_i + \gamma\Delta\dot{\rho}_{ij} \tag{17}$$

where $\Delta\ddot{x}_i = \ddot{x}_i - \ddot{x}_o$. Substituting equations (16) and (17) into equation (1), and using property 4 we have

$$M_i(x_i)\dot{s}_i + C_i(x_i,\dot{x}_i)s_i + D_i(x_i,\dot{x}_i)s_i + Y_i(x_i,\dot{x}_i,\dot{x}_{ri},\ddot{x}_{ri})\theta_i = u_i \tag{18}$$

where $Y_i(x_i,\dot{x}_i,\dot{x}_{ri},\ddot{x}_{ri})\theta_i = M_i(x_i)\ddot{x}_{ri} + C_i(x_i,\dot{x}_i)\dot{x}_{ri} + D_i(x_i,\dot{x}_i)\dot{x}_{ri} + g_i(x_i)$. The region following controller for multi-robot systems is proposed as

$$u_i = -K_{si}s_i - K_p(\alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij}) + Y_i(x_i,\dot{x}_i,\dot{x}_{ri},\ddot{x}_{ri})\hat{\theta}_i \tag{19}$$

where K_{si} are positive definite matrices, $K_p = k_pI$, k_p is a positive constant and I is an identity matrix. The estimated parameters $\hat{\theta}_i$ are updated by

$$\dot{\hat{\theta}}_i = -L_iY_i^T(x_i,\dot{x}_i,\dot{x}_{ri},\ddot{x}_{ri})s_i \tag{20}$$

where L_i are positive definite matrices.
The closed-loop dynamic equation is obtained by substituting equation (19) into equation (18):

$$M_i(x_i)\dot{s}_i + C_i(x_i,\dot{x}_i)s_i + D_i(x_i,\dot{x}_i)s_i + K_{si}s_i + Y_i(x,\dot{x},\dot{x}_{ri},\ddot{x}_{ri})\Delta\theta_i + K_p(\alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij}) = 0 \tag{21}$$

where $\Delta\theta_i = \theta_i - \hat{\theta}_i$. Let us define a Lyapunov-like function for multi-robot systems as

$$\begin{aligned} V &= \sum_{i=1}^{N}\frac{1}{2}s_i^TM_i(x_i)s_i + \sum_{i=1}^{N}\frac{1}{2}\Delta\theta_i^TL_i^{-1}\Delta\theta_i \\ &+ \sum_{i=1}^{N}\frac{1}{2}\alpha_ik_p\sum_{l=1}^{M}k_l[max(0,f_{Gl}(\Delta x_{iol})]^2 \\ &+ \frac{1}{2}\sum_{i=1}^{N}\frac{1}{2}\gamma k_p\sum_{j\in N_i}k_{ij}[max(0,g_{Lij}(\Delta x_{ij}))]^2 \end{aligned} \tag{22}$$

Differentiating equation (22) with respect to time, we get

$$\begin{aligned} \dot{V} =& \sum_{i=1}^{N}s_i^TM_i(x_i)\dot{s}_i + \sum_{i=1}^{N}\frac{1}{2}s_i^T\dot{M}_i(x_i)s_i - \sum_{i=1}^{N}\dot{\hat{\theta}}_i^TL_i^{-1}\Delta\theta_i \\ &+ \sum_{i=1}^{N}\alpha_ik_p\sum_{l=1}^{M}k_l\Delta\dot{x}_{iol}^Tmax(0,f_{Gl}(\Delta x_{iol}))(\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}})^T \\ &+ \frac{1}{2}\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_{ij}^Tmax(0,g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial\Delta x_{ij}})^T \end{aligned} \tag{23}$$

Substituting $\dot{\hat{\theta}}_i$ from equation (20) and $M_i(x_i)\dot{s}_i$ from equation (21) into equation (23) and using property 2 we get

$$\begin{aligned} \dot{V} =& -\sum_{i=1}^{N}s_i^TK_{si}s_i - \sum_{i=1}^{N}s_i^TD_i(x_i,\dot{x}_i)s_i \\ &- \sum_{i=1}^{N}s_i^Tk_p(\alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij}) \\ &+ \sum_{i=1}^{N}\alpha_ik_p\sum_{l=1}^{M}k_l\Delta\dot{x}_{iol}^Tmax(0,f_{Gl}(\Delta x_{iol}))(\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}})^T \end{aligned}$$

$$+\frac{1}{2}\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_{ij}^T max(0,g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial\Delta x_{ij}})^T \quad (24)$$

Using equation (8), equation (24) can be written as

$$\begin{aligned}\dot{V}=&-\sum_{i=1}^{N}s_i^TK_{si}s_i-\sum_{i=1}^{N}s_i^TD_i(x_i,\dot{x}_i)s_i\\&-\sum_{i=1}^{N}s_i^Tk_p(\alpha_i\Delta\xi_i+\gamma\Delta\rho_i)+\sum_{i=1}^{N}\alpha_ik_p\Delta\dot{x}_i^T\Delta\xi_i\\&+\frac{1}{2}\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_{ij}^T max(0,g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial\Delta x_{ij}})^T\end{aligned} \quad (25)$$

where $\Delta\dot{x}_{iol}=\dot{x}_i-\dot{x}_{ol}=\Delta\dot{x}_i$ since $\dot{x}_o=\dot{x}_{ol}$.
Next, since $\Delta\dot{x}_{ij}=\dot{x}_i-\dot{x}_j=(\dot{x}_i-\dot{x}_o)-(\dot{x}_j-\dot{x}_o)=\Delta\dot{x}_i-\Delta\dot{x}_j$, using equation (13), the last term of equation (25) can be written as

$$\begin{aligned}&\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_{ij}^T max(0,g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial\Delta x_{ij}})^T\\=&\sum_{i=1}^{N}\gamma k_p\Delta\dot{x}_i^T\Delta\rho_{ij}\\&-\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_j^T max(0,g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial\Delta x_{ij}})^T\end{aligned} \quad (26)$$

Using equation (10) and (11), equation (26) can be written as

$$\begin{aligned}&\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_{ij}^T max(0,g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial\Delta x_{ij}})^T\\=&\sum_{i=1}^{N}\Delta\dot{x}_i^T\gamma k_p\Delta\rho_{ij}\\&+\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_j^T max(0,g_{Lji}(\Delta x_{ji}))(\frac{\partial g_{Lji}(\Delta x_{ji})}{\partial\Delta x_{ji}})^T\end{aligned} \quad (27)$$

Let $k_{ij}=k_{ji}$ then

$$\begin{aligned}&\sum_{i=1}^{N}\gamma k_p\sum_{j\in N_i}k_{ij}\Delta\dot{x}_j^T max(0,g_{Lji}(\Delta x_{ji}))(\frac{\partial g_{Lji}(\Delta x_{ji})}{\partial\Delta x_{ji}})^T\\=&\sum_{j=1}^{N}\gamma k_p\sum_{i\in N_j}k_{ji}\Delta\dot{x}_j^T max(0,g_{Lji}(\Delta x_{ji}))(\frac{\partial g_{Lji}(\Delta x_{ji})}{\partial\Delta x_{ji}})^T\\=&\sum_{j=1}^{N}\gamma k_p\Delta\dot{x}_j^T\Delta\rho_{ji}\\=&\sum_{i=1}^{N}\gamma k_p\Delta\dot{x}_i^T\Delta\rho_{ij}\end{aligned} \quad (28)$$

where N_j is the set of neighbors around robot j. Therefore,

$$\begin{aligned}\dot{V}=&-\sum_{i=1}^{N}s_i^TK_{si}s_i-\sum_{i=1}^{N}s_i^TD_i(x_i,\dot{x}_i)s_i\\&-\sum_{i=1}^{N}s_i^Tk_p(\alpha_i\Delta\xi_i+\gamma\Delta\rho_{ij})\\&+\sum_{i=1}^{N}\Delta\dot{x}_i^T\alpha_ik_p\Delta\xi_i+\sum_{i=1}^{N}\Delta\dot{x}_i^T\gamma k_p\Delta\rho_{ij}\end{aligned} \quad (29)$$

Substituting s_i from equation (16) into equation (29) we get

$$\begin{aligned}\dot{V}=&-\sum_{i=1}^{N}s_i^TK_{si}s_i-\sum_{i=1}^{N}s_i^TD_i(x_i,\dot{x}_i)s_i\\&-\sum_{i=1}^{N}k_p(\alpha_i\Delta\xi_i+\gamma\Delta\rho_{ij})^T(\alpha_i\Delta\xi_i+\gamma\Delta\rho_{ij})\end{aligned} \quad (30)$$

We are ready to state the following theorem:
Theorem: *Consider a group of N robots with dynamic described by equation (1), the adaptive control law (19) and the parameter update laws (20) give rise to the convergence of $\Delta\xi_i\rightarrow 0$ and $\Delta\rho_{ij}\rightarrow 0$ for all $i=1,2,...,N$, as $t\rightarrow\infty$.*
Proof: Since $M_i(x_i)$ are uniformly positive definite, V in equation (22) is positive definite in s_i, $\Delta\theta_i$, $\sum_{i=1}^{N}[max(0,f_{Gl}(\Delta x_{iol}))]^2$ and $\sum_{i=1}^{N}\sum_{j\in N_i}[max(0,g_{Lij}(\Delta x_{ij}))]^2$. Hence, s_i, $\Delta\theta_i$, $f_{Gl}(\Delta x_{iol})$ and $g_{Lij}(\Delta x_{ij})$ are bounded. The boundedness of $f_{Gl}(\Delta x_{iol})$ ensures the boundedness of $\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}}$, $\frac{\partial^2 f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}^2}$. Therefore, $\Delta\xi_i$ is bounded. From equation (9), $max(0,g_{Lij}(\Delta x_{ij}))(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial\Delta x_{ij}})^T$ is always bounded. Hence, $\Delta\rho_{ij}$ is bounded. Next, $\dot{x}_{ri}$ is bounded if $\dot{x}_o$ is bounded as can be seen from equation (14). From equation (16) $\Delta\dot{x}_i$ is bounded since s_i, $\Delta\xi_i$ and $\Delta\rho_{ij}$ are bounded. Hence $\Delta\dot{x}_{iol}$ is bounded. The boundedness of $\Delta\dot{x}_i$ implies the boundedness of $\dot{x}_i$ for all $i=1,2,...,N$ if $\dot{x}_o$ is bounded. Differentiating equation (8) with respect to time yields

$$\begin{aligned}\Delta\dot{\xi}_i=&\sum_{l=1}^{M}k_l\dot{f}_{Gl}(\Delta x_{iol})(\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}})^T\\&+\sum_{l=1}^{M}k_l max(0,f_{Gl}(\Delta x_{iol}))\frac{\partial^2 f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}^2}\Delta\dot{x}_{iol}\end{aligned} \quad (31)$$

where

$$\dot{f}_{Gl}(\Delta x_{iol})=\begin{cases}0, & f_{Gl}(\Delta x_{iol})\le 0\\(\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}})\Delta\dot{x}_{iol}, & f_{Gl}(\Delta x_{iol})>0\end{cases} \quad (32)$$

Since $\frac{\partial f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}}$, $\Delta\dot{x}_{iol}$ and $\frac{\partial^2 f_{Gl}(\Delta x_{iol})}{\partial\Delta x_{iol}^2}$ are bounded, $\Delta\dot{\xi}_i$ is therefore bounded. Similarly, differentiating equation (13) with respect to time yields

$$\Delta\dot{\rho}_{ij}=\sum_{j\in N_i} k_{ij}\dot{g}_{Lij}(\Delta x_{ij})(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}})^T + \sum_{j\in N_i} k_{ij} max(0, g_{Lij}(\Delta x_{ij}))\frac{\partial^2 g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}^2}\Delta\dot{x}_{ij} \quad (33)$$

where

$$\dot{g}_{Lij}(\Delta x_{ij}) = \begin{cases} 0, & g_{Lij}(\Delta x_{ij}) \leq 0 \\ (\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}})\Delta\dot{x}_{ij}, & g_{Lij}(\Delta x_{ij}) > 0 \end{cases} \quad (34)$$

From equation (9), Δx_{ij} is bounded if $g_{Lij}(\Delta x_{ij}) > 0$. $\Delta\dot{x}_{ij}$ is bounded since $\dot{x}_i$ is bounded for all i. Hence, $\dot{g}_{Lij}(\Delta x_{ij})(\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}})^T$ is always bounded. Therefore, $\Delta\dot{\rho}_{ij}$ is bounded since $\frac{\partial^2 g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}^2}$ is bounded (from equation (9)). From equation (15), $\ddot{x}_{ri}$ is bounded if $\ddot{x}_o$ is bounded. From the closed-loop equation (21), we can conclude that $\dot{s}_i$ is bounded. Differentiating equation (30) with respect to time we get

$$\begin{aligned} \ddot{V} = & -2\sum_{i=1}^{N} \dot{s}_i^T K_{si} s_i \\ & -2\sum_{i=1}^{N} k_p(\alpha_i\Delta\dot{\xi}_i + \gamma\Delta\dot{\rho}_{ij})^T(\alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij})] \\ & -2\sum_{i=1}^{N} \dot{s}_i^T D_i(x_i,\dot{x}_i)s_i - \sum_{i=1}^{N} s_i^T \dot{D}_i(x_i,\dot{x}_i)s_i \quad (35) \end{aligned}$$

Hence, $\ddot{V}$ is bounded since $\Delta\xi_i$, $\Delta\dot{\xi}_i$, $\Delta\rho_{ij}$, $\Delta\dot{\rho}_{ij}$, s_i and $\dot{s}_i$ are bounded. Therefore, $\dot{V}$ is uniformly continuos. Applying Barbalat's lemma [18], we have $\alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij} \rightarrow 0$ as $t \rightarrow \infty$. Next, we proceed to show that $\Delta\xi_i \rightarrow 0$ and $\Delta\rho_{ij} \rightarrow 0$ seperately as $t \rightarrow \infty$. Since

$$\alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij} = 0 \quad (36)$$

as $t \rightarrow \infty$. Therefore,

$$\sum_{i=1}^{N} \alpha_i\Delta\xi_i + \sum_{i=1}^{N} \gamma\Delta\rho_{ij} = 0 \quad (37)$$

Note that the interactive forces between robots are bi-directional. These forces cancel out each other and the summation of all the interactive forces in the multi-robot systems is zero. That is $\sum_{i=1}^{N} \Delta\rho_{ij} = 0$. From equation (37), we have

$$\sum_{i=1}^{N} \alpha_i\Delta\xi_i = 0 \quad (38)$$

A trivial solution of equation (38) is $\Delta\xi_i = 0$ for all $i = 1, 2, 3...N$. Now we proceed to prove by contradiction that $\Delta\xi_i = 0$ is the only solution of the above equation. Assume to the contrary that $\Delta\xi_i \neq 0$ is another solution of equation (37). If $\Delta\xi_i \neq 0$, then the summation of all the forces are zero if and only if all the forces cancel out each other. This means that some robots must be at the opposite sides of the desired region as illustrated in figure 3. However, when robots are at the opposite sides of the region, the interactive forces $\Delta\rho_{ij}$ between robots are not activated because the desired region must be large as compare to the minimum distance between the robots in order for all robots to stay inside. It means that $\Delta\rho_{ij} = 0$ and from equation (36), we get $\Delta\xi_i = 0$. This contradicts the assumption that $\Delta\xi_i \neq 0$. Therefore, $\Delta\xi_i = 0$ and hence $\Delta\rho_{ij} = 0$.

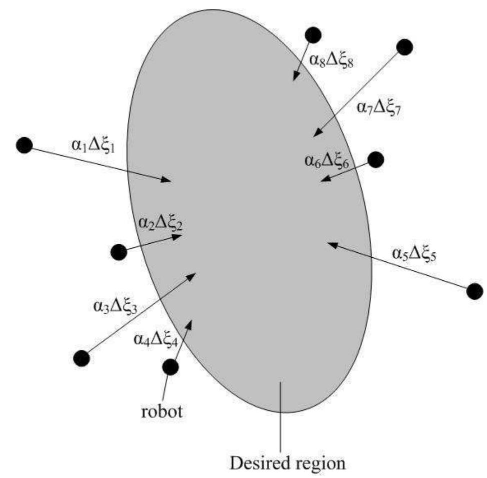

Fig. 3. Illustration of the case when $\sum_{i=1}^{N} \alpha_i\Delta\xi_i = 0$ and $\Delta\xi_i \neq 0$

IV. SIMULATION

This section presents some simulation results to illustrate the performance of the proposed formation controller. In the simulation, 100 robots are used to form different formations while moving along a specified path. The mass of each robot is set as 1 kg. The desired region is moving along a path specified by $x_{o11} = \frac{t}{2}$ and $x_{o12} = 2sin(\frac{t}{2})$.

A. Desired Region as a Ring

The desired region is set as a ring with $r_1 = 0.8\ m$ and $r_2 = 1.7\ m$ given by the following inequalities:

$$f_1(\Delta x_{io1}) = r_1^2 - (x_{i1} - x_{o11})^2 - (x_{i2} - x_{o12})^2 \leq 0$$
$$f_2(\Delta x_{io2}) = (x_{i1} - x_{o11})^2 + (x_{i2} - x_{o12})^2 - r_2^2 \leq 0$$

The minimum distance between robots is chosen to be 0.3 m and $r_N = 0.5m$. The proposed controller is used with $K_{si} = diag\{10, 10\}$, $k_p = 10$, $k_{ij} = 25$, $k_1 = k_2 = 1$, $\gamma = 1$ and $\alpha_i = 1$. The system converges after 7 seconds.

B. Desired Region as a Crescent

The desired region in this section is set as a crescent characterized by the following inequalities:

$$f_1(\Delta x_{io1}) = (x_{i1} - x_{o11})^2 + (x_{i2} - x_{o12})^2 - r_1^2 \leq 0$$
$$f_2(\Delta x_{io2}) = r_2^2 - (x_{i1} - x_{o21})^2 - (x_{i2} - x_{o22})^2 \leq 0$$

where $r_1 = 1.8\ m$, $r_2 = 1.1\ m$, $x_{o21} = x_{o11} - 0.8$ and $x_{o22} = x_{o12} - 0.8$. The minimum distance between robots is chosen to be 0.3 m and $r_N = 0.5m$. The proposed controller is used with $K_{si} = diag\{5, 5\}$, $k_p = 5$, $k_{ij} = 50$, $k_1 = k_2 = 1$, $\gamma = 1$ and $\alpha_i = 1$. The system converges after 10 seconds.

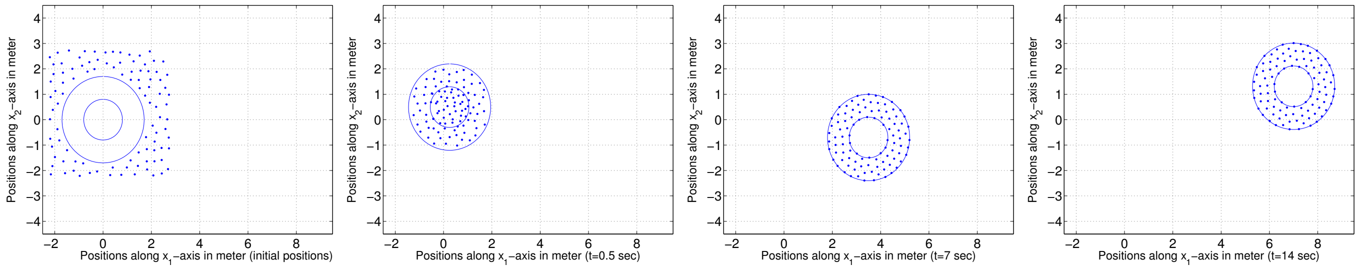

Fig. 4. A group of 100 robots moving together in a ring formation

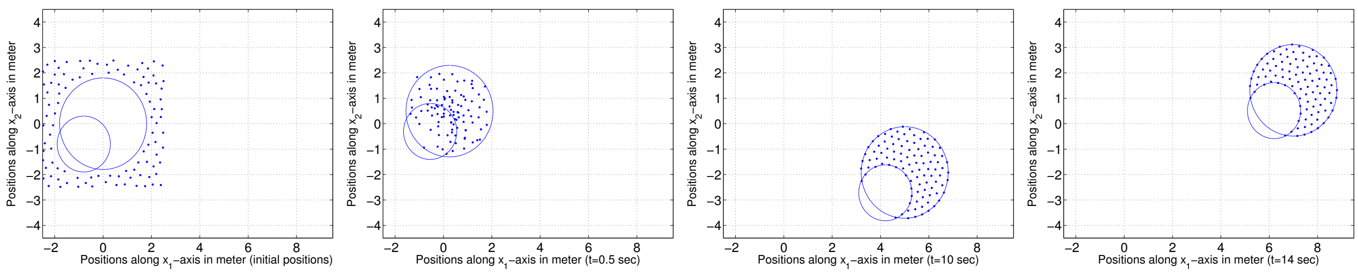

Fig. 5. A group of 100 robots moving together in a crescent formation

V. Conclusion

In this paper, we have proposed a region following formation control method for multi-robot systems. It has been shown that all the robots are able to move as a group inside the desired region while maintaining minimum distance from each other. Lyapunov-like function has been proposed for the stability analysis of the multi-robot systems. Simulation results have been presented to illustrate the performance of the proposed formation controller.

References

[1] C. Reynolds, "Flocks, herds and schools: A distributed behavioral model,". *Computer Graphics*, vol. 21, pp. 25–34,1987.

[2] X. Tu and D. Terzopoulos, "Artificial fishes: physics, locomotion, perception, behavior," *In SIGGRAPH 94 Conference Proceedings*, pp. 43–50, Orlando, FL, USA, July 1994. ACM.

[3] D.C. Brogan and J.K. Hodgins, "Group behaviors for systems with significant dynamics," *Autonomous Robots*, vol. 4, pp. 137–53, March 1997.

[4] T. Balch and R.C. Arkin, "Behavior-based formation control for multi-robot systems," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, Dec. 1998.

[5] J.H. Reif and H. Wang, "Social potential fields: A distributed behavioral control for autonomous robots," *Robotics and Autonomous Systems,* vol. 27, pp. 171-194, 1999.

[6] J.R. Lawton,R.W. Beard, and B.J. Young, "A Decentralized Approach to Formation Maneuvers," *IEEE Transactions on Robotic and Automation*, vol. 19, no. 6, pp. 933–941,Dec. 2003.

[7] P.K.C Wang, "Navigation strategies for multiple autonomous robots moving in formation," *Journal of Robotics Systems*, vol. 8, no. 2, pp. 177–195, 1991.

[8] J.P. Desai, V. Kuma, and P. Ostrowski, "Modeling and control of formations of nonholonomic mobile robots,", *IEEE Transaction on Robotic and Automation*, vol.17, pp. 905–908, Dec. 2001.

[9] A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, "A vision-based formation control framework," *IEEE Transaction on Robotic and Automation*, vol. 18, no. 5, pp. 813–825, 2002.

[10] P. Ogren, M. Egerstedt and X. Hu, "A control Lyapunov function approach to multi-agent coordination," *IEEE Transaction on Robotic and Automation*, vol. 18, no. 5, pp. 847–851, October 2002.

[11] J. Fredslund and M. J. Mataric, "A general algorithm for robot formations using local sensing and minimal communication," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 837–846, October 2002.

[12] M.A. Lewis and K.H. Tan, "High Precision Formation Control of Mobile Robots Using Virtual Structures," *Autonomous Robots*, vol. 4, no. 4, pp. 387–403, 1997.

[13] M. Egerstedt and X. Hu, "Formation constrained multi-agent control," *IEEE Transactions on Robotics and Automation*, vol.17, no. 6, pp. 947–951, 2001.

[14] N. E. Leonard and E. Fiorelli, "Virtual leaders, artificial potentials and coordinated control of groups," in *Proceeding of Decision and Control Conference*, pp. 2968-2973, Orlando, FL, Dec. 2001.

[15] R.W. Beard, J. Lawton, and F.Y. Hadaegh, "A coordination architecture for spacecraft formation control," *IEEE Transactions on Control Systems Technology,* vol. 9, no. 6, pp. 777–790, Nov. 2001.

[16] C. Belta and V. Kumar, "Abstraction and control for groups of robots," *IEEE Transactions on Robotics*, vol. 20, no. 5, pp. 865–875, 2004.

[17] T.I. Fossen, *Guidance and Control of Ocean Vehicles*. Baffins Lane, Chichester: John Wiley & Sons Ltd. 1994

[18] J.J.E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, New Jersy: Prentice Hall, 1991.