

Central Axis Approach for Computing n -Finger Force-closure Grasps

B. Bounab, D. Sidobre and A. Zaatri

Abstract—In this paper, we propose a new approach for computing force-closure grasps of two-dimensional and three-dimensional objects. Assuming n hard-finger contact with Coulomb friction model and based on central axes of the grasp wrench (i.e., force and torque), we develop a new necessary and sufficient condition for n -finger grasps to achieve force-closure. We demonstrate that a grasp is force-closure if and only if, its wrench can generate any arbitrary central axis. According to this condition, we reformulate the force-closure test as a linear programming problem without computing the convex hull of the primitive contact wrenches. Therefore, we present an efficient algorithm for computing n -finger force-closure grasps. Finally, we have implemented the proposed algorithm and verified its efficiency through some examples.

Index Terms—multifingered robotic hand, force-closure grasp, grasp optimization.

I. INTRODUCTION

SERVICE robots will be equipped with multifingered hands in order to carry out everyday tasks with common and often irregular shaped objects in human environment. The grasp planning problem is to determine the position of the contact points on the object surface, while satisfying basic grasp properties. Force-closure is an important property in multifingered robotic grasps [1], [2], [3]. Under a force-closure grasp, any external wrench applied on the object can be balanced by applying appropriate grasp forces with the robotic hand at contact points.

In this paper, we focus on the problem of computing n -finger force-closure grasps of arbitrary 2D and 3D objects. Based on the grasp wrench central axes distribution, we propose a general formulation of the force-closure test and we present new force-closure algorithms.

II. RELATED WORKS

A grasp on an object is said to be force-closure if and only if arbitrary forces and torques can be exerted on the object through a set of contact points. Salisbury and Roth [4] have proved that a necessary and sufficient condition for force-closure is that the primitive contact wrenches of contact forces positively span the entire wrench space.

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This condition is equivalent to the origin of wrench space lying strictly inside the convex hull of the primitive contact wrenches. Nguyen [5] demonstrated that non-marginal equilibrium grasps achieve force-closure and presented a geometrical algorithm for computing 2-finger force-closure grasps. Ferrari and Canny [6] have developed an algorithm for computing an optimal grasp. Ponce and Faverjon [7] proposed several sufficient conditions for 3-finger equilibrium grasps of planar object, and implemented an algorithm with Gaussian elimination and linear programming methods. Jia-Wei Li *et al.* [8] have proposed a geometric algorithm for computing 3-finger force-closure 2D grasps. Their method begins by processing friction cones using an operation called disposition H to remove unnecessary regions of these cones. Liu [9] has proposed an algorithm for computing all n -finger force-closure grasps on polygonal object by transforming the problem from \mathbb{R}^3 to \mathbb{R}^1 . Recently, Sudsang and Phoka [10] have developed another algorithm for 3-finger force-closure grasp based on a technique for representing a friction cone as a line segment in a dual plane. Due to complicated geometry, there are only a few examples in the literature for computing 3D force-closure grasps. Ponce *et al.* [11] have illustrated that 4-finger force-closure grasps fall into three classes: concurrent, pencil, and regulus grasps, and developed techniques for computing them. Jia-Wei Li *et al.* [12] have extended their work in [8] and proposed a geometric algorithm for computing 3-finger force-closure grasps. Liu [13] has developed a qualitative test algorithm of n -finger force-closure grasp. He has reformulated the force-closure condition given in [4] as a ray-shooting problem, and he solved it by a linear programming method. In [14], Liu *et al.* have proposed an algorithm for searching force-closure grasps of a 3D object represented by discrete points.

In this work, a general approach for computing n -finger force-closure grasps is proposed. We demonstrate that a grasp achieves force-closure if and only if, its wrench can generate any arbitrary central axis. So, we reformulate the problem as a linear programming one without computing the convex hull of the primitive contact wrenches, which reduces the amount of computation. This approach is applicable to 2D and 3D grasps for any number of contacts points.

The rest of the paper is organized as follows, in Section III, we present the background of grasp wrench and central axis. Using some grasp examples, we illustrate the relationship between central axes and force-closure concept. In Section IV, we demonstrate the proposed equilibrium and force-closure conditions. In Section V, we present the algorithm. Finally, in Section VI we have implemented a comparative study on polygonal and polyhedral objects.

III. GRASP WRENCH AND CENTRAL AXIS

This section includes basic grasping terminologies and then introduces the grasp wrench central axis. Further, we analyze the relationship between these axes and the notion of force-closure via 2D and 3D grasp examples.

A. Grasp Wrench

Suppose that n hard fingers are grasping a rigid object in 3D workspace (Fig.1-a). Assume that the Coulomb friction exists at contact points c_i . The static coefficient of friction $\mu = \tan \alpha$ depends on materials which are in contact. To ensure non-slipping between fingertips and object, the force f_i must lie inside the friction cone. So, grasp forces f_i must satisfy the following constraints

$$\sqrt{f_{ix}^2 + f_{iy}^2} \leq \mu f_{iz} \quad ; \quad f_{iz} \geq 0 \quad (1)$$

Where (f_{ix}, f_{iy}, f_{iz}) denotes x, y and z components of the grasp force f_i w.r.t. the i th coordinate frame (x_i, y_i, z_i) , z_i is the normal to the surface object at contact point c_i .

To simplify the problem by eliminating the nonlinear constraints given by (1), each friction cone can be linearized by a m -sided polyhedral convex cone (Fig.1-a). Under this approximation, grasp force f_i , expressed in the object coordinate frame, is given by

$$f_i = \sum_{j=1}^m a_{ij} v_{ij} \quad ; \quad v_{ij} = T_i s_{ij} \quad , \quad a_{ij} \geq 0 \quad (2)$$

The matrix T_i specifies the location of the i th coordinate frame with respect to the object coordinate frame. s_{ij} denotes the j th edge vector of the polyhedral convex cone expressed in the i th coordinate frame, vectors s_{ij} are given by

$$s_{ij} = \left(\mu \cos(2\pi j/m), \quad \mu \sin(2\pi j/m), \quad 1 \right)^T \quad (3)$$

The wrench induced on the object by the grasp force f_i , denoted w_i , applied at the origin of the object coordinate frame o is given by

$$w_i = \left\{ \begin{array}{c} f_i \\ \tau_{i/o} = c_i \times f_i \end{array} \right\} \quad (4)$$

Substituting (2) into (4) yields

$$w_i = \sum_{j=1}^m a_{ij} w_{ij} \quad (5)$$

Where w_{ij} denotes the primitive contact wrenches of the i th finger. They are given, w.r.t. the object coordinate frame, by

$$w_{ij} = \left\{ \begin{array}{c} v_{ij} \\ c_i \times v_{ij} \end{array} \right\} \quad (6)$$

The net wrench applied by the hand on the grasped object is the sum of all primitive contact wrenches. It is given by

$$W_g = \sum_{i=1}^n \sum_{j=1}^m a_{ij} w_{ij} = \left\{ \begin{array}{c} F_g \\ \tau_{g/o} \end{array} \right\} \quad (7)$$

The whole external wrench applied on the object is the sum of the grasp wrench W_g , which is applied by the robotic

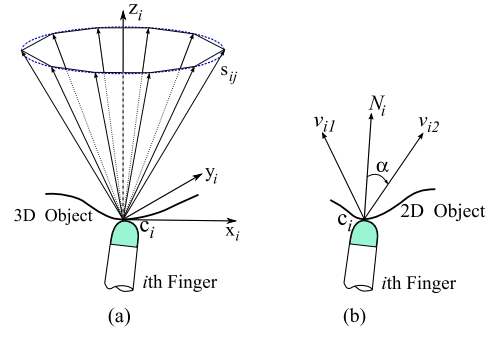


Fig. 1. Geometric interpretation of Coulomb friction model: (a) friction cone approximation in 3D grasps, (b) 2D grasp.

hand, and the task wrench W_t which is required to achieve task (perturbations are included). It is given by

$$W_{ext} = W_g + W_t = \left\{ \begin{array}{c} F_g \\ \tau_{g/o} \end{array} \right\} + \left\{ \begin{array}{c} F_t \\ \tau_{t/o} \end{array} \right\} \quad (8)$$

In 2D grasps (Fig.1-b), the grasp force f_i must lie inside the friction cone defined by two vectors v_{i1} and v_{i2} . N_i is the normal to the surface object at contact point c_i . The force f_i can be represented as follows

$$f_i = a_{i1} v_{i1} + a_{i2} v_{i2} \quad (9)$$

Coefficients a_{i1} and a_{i2} are nonnegative constants.

B. Central Axis

The central axis of a wrench, is the geometric place of the points with respect to which the wrench is reduced to a force and a parallel torque. The central axis is also defined by the following theorem [15].

Poinsot's theorem: "Every collection of wrenches applied to a rigid body is equivalent to a force applied along a fixed axis and a torque around the same axis".

Using this theorem, the central axis Δ_g of the grasp wrench W_g is defined as follows

$$\Delta_g = \left\{ \begin{array}{ll} \frac{F_g \times \tau_{g/o}}{\|F_g\|^2} + \lambda F_g & : \text{if } F_g \neq 0 \\ 0 + \lambda \tau_{g/o} & : \text{if } F_g = 0 \end{array} \right. \quad ; \quad \lambda \in \mathbb{R} \quad (10)$$

The axis Δ_g is a directed line through a point. For $F_g \neq 0$, the axis is a line in the F_g direction going through point $I = \frac{F_g \times \tau_{g/o}}{\|F_g\|^2}$. For $F_g = 0$, the axis is in $\tau_{g/o}$ direction going through the origin [15]. For $F_g \neq 0$, the torque around Δ_g is

$$\tau_{g/I} = \frac{F_g \cdot \tau_{g/o}}{\|F_g\|^2} \cdot F_g \quad (11)$$

In 2D grasps, for any given point on the grasp plane, the torque and the force are orthogonal. When $F_g \neq 0$, the central axes are in the grasp plane and from (11), the torque around these axes is null. When $F_g = 0$, the torque $\tau_{g/o}$ is applied around central axis which is normal to the plane.

C. Central Axes of the Grasp Wrench

In order to illustrate the relationship between grasp central axes and the force-closure concept, we draw central axes of some 2D and 3D grasps [16]. We vary randomly the amplitudes and the orientations of fingertips' forces f_i inside the friction cones using (2) for 3D grasps and (9) for 2D ones. Grasp wrench central axes are computed from (10). We show some examples of force-closure and non-force-closure grasps with their central axes distributions (Fig. 3 and Fig. 4).

Using an appropriate normalization of the central axes equations, we ensure that all this axes are inside a unit circle for 2D grasps (or sphere for the 3D case).

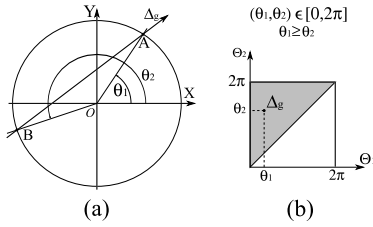


Fig. 2. (a) Central axis parameters in 2D grasp (b) Dual representation.

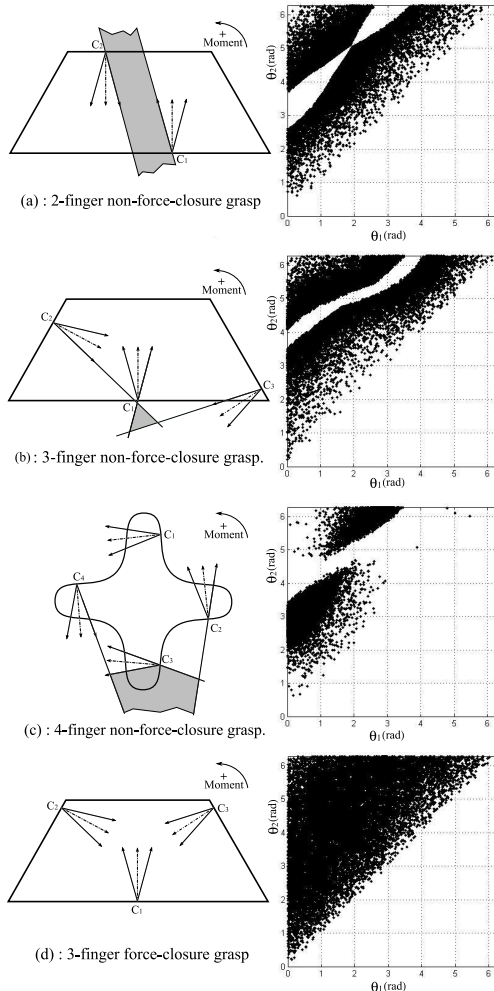


Fig. 3. Examples of 2D grasps and their central axes distributions.

For 2D grasps, we use variables θ_1 and θ_2 to draw central axes. Thus, a central axis Δ_g of equation $y = ax + b$ in (x, y) plane has a corresponding point (θ_1, θ_2) in the dual plane (Θ_1, Θ_2) . A and B are the intersection points with the unit circle of origin o (Fig.2). The friction angle is set to 10° and taking $(\theta_2 \geq \theta_1)$, all central axes are presented in the upper triangle of the (Θ_1, Θ_2) plane (Fig.2-b). In 3D workspace, we use the coordinates of the two intersection points between central axes and a unit sphere of origin o , to represent the distribution of grasp central axes for 3D grasps (Fig. 4). The friction angle is set to 20° .

In 2D grasps (Fig. 3), the first example is a 2-finger grasp (Fig. 3-a), it cannot balance external positive torques with center lines passing through gray region. The representation in dual plane shows that the grasp wrench cannot generate all central axes configurations. Example shown in (Fig. 3-b) is a 3-finger non-force-closure grasp, this grasp cannot resist to negative torques applied in gray region. Hence, there is no central axis passing through this region, which can be clearly deduced from dual representation. In example shown in (Fig. 3-c), the 4-fingers cannot produce negative torques in gray region. When a grasp is force-closure (Fig. 3-d), grasp wrench generates all possible central axes, we see in the dual representation that the upper triangle is wholly colored.

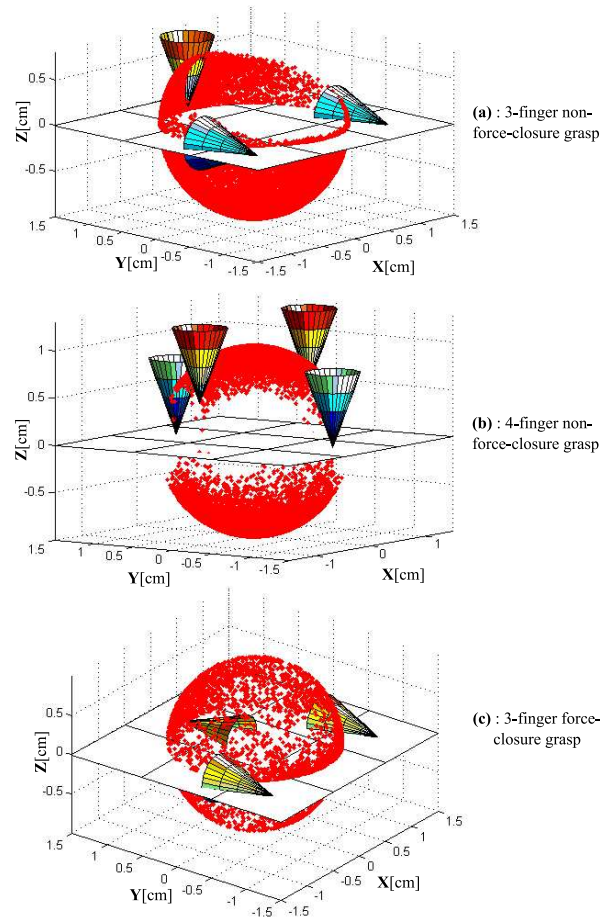


Fig. 4. Examples of 3D grasps and their central axes distributions.

For 3D grasps (Fig. 4), in the first example (Fig. 4-a), the 3-fingered grasp is non-force-closure because the three friction cones cannot all intersect with the plane formed by contact points [12]. We observe that the grasp wrench cannot generate all central axes. Example shown in (Fig. 4-b) is a 4-finger non-force-closure grasp. It cannot produce forces along $-Z$ axis and we see that this grasp cannot produce all possible central axes. When a grasp is force-closure (Fig. 4-c), grasp wrench generates all central axes.

From the examples shown in Fig. 3 and Fig. 4, we conclude that if a grasp is non-force-closure, its wrench cannot generate all central axes. When the grasp achieves force-closure, its wrench can generate any arbitrary central axis. Hence, we can derive a force-closure test based on the grasp wrench central axes independently of fingers' number.

IV. FORCE-CLOSURE AND EQUILIBRIUM CONDITIONS

In the present section, we demonstrate the proposed equilibrium and the force-closure conditions.

A. Necessary and Sufficient Equilibrium Condition

In the study of the equilibrium, we consider only the grasp forces applied by the n fingers of the robotized hand. We divide these forces into two parts: force F_i which is applied by the i th finger at the contact point c_i , and forces F_r applied by the remainder of the $n - 1$ fingers. Therefore, the object is subjected to two external wrenches $w_i = (F_i, \tau_{i/c_i})^T$ and $w_r = (F_r, \tau_{r/c_i})^T$ w.r.t the point c_i . Torques τ_{i/c_i} produced by F_i w.r.t. c_i are null. So, the equilibrium condition is

$$W_g = w_i + w_r = 0 \implies \begin{cases} F_r = -F_i \\ \tau_{r/c_i} = 0 \end{cases} \quad (12)$$

According to Poinso's theorem, we divide the central axes Δ_g given by (10) into two classes. Δ_i are the central axes of the wrench w_i , and Δ_r central axes of w_r . The second condition ($\tau_{r/c_i} = 0$) in (12), defines a subclass of central axis Δ_r^* with null torques and passing through c_i . We denote Δ_g^* the union of Δ_i and Δ_r^* ($\Delta_g^* = \Delta_i \cup \Delta_r^*$). Now, we put forward the following proposition for n -finger equilibrium grasps

Proposition 1: A 3D (res. 2D) n -finger grasp is said to achieve equilibrium if and only if, all grasp wrench central axes of class Δ_g^* can positively span \mathbb{R}^3 (res. \mathbb{R}^2) at c_i .

Proof: *i) Sufficient Condition:* A set of vectors positively span \mathbb{R}^n if any vector in \mathbb{R}^n can be written as a positive combination of the given vectors [9]. Hence, the central axes Δ_g^* can positively span \mathbb{R}^3 (\mathbb{R}^2 for 2D grasps) at c_i if and only if, it exist at least one central axis of class Δ_r^* that pass through point c_i and pointing inside the negative i th friction cone (cone pointing outside the object). Hence, there exist force F_i inside the i th friction cone that is opposite to the force F_r (Fig. 5-a and 5-b) and the grasp is in equilibrium.

ii) Necessary Condition: Now, we consider that the wrench w_r cannot generate central axis Δ_r^* , that pass through point c_i and inside the negative i th friction cone. Hence, the axes Δ_g^* do not positively span \mathbb{R}^3 (\mathbb{R}^2 for 2D grasp) and, the force F_i cannot be balanced. Therefore, the grasp is not in equilibrium and the proposition 1 is necessary. \square

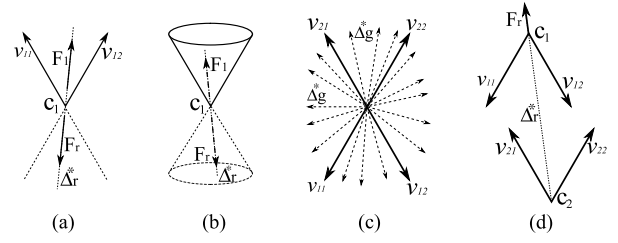


Fig. 5. Axis Δ_g^* positively span (a) \mathbb{R}^2 (b) \mathbb{R}^3 , (c) 2-finger 2D grasp achieves equilibrium but non-force-closure, (d) 2-finger force-closure grasp.

B. Necessary and Sufficient Force-Closure Condition

Force-closure is the ability to constrain completely the motion of the grasped object via contact forces. The grasp is said to be force-closure, if the grasp wrench W_g can balance any task wrench W_t . The following definition gives the relation between equilibrium and force-closure grasps [5]

Definition 2: A grasp is said to achieve force-closure if and only if, it is in equilibrium for any arbitrary task wrench.

As noted in [7], force-closure grasps always achieve equilibrium but equilibrium grasps are not always force-closure. Example shown in (Fig. 5-c), is an equilibrium grasp but it is a non-force-closure grasp because fingertips forces cannot generate torques with center lines passing through contact points. Figure 5-d shows a 2-finger force-closure grasp. Forces in the second friction cone can generate one central axis Δ_r^* passing through point c_1 , and inside the negative side of the first friction cone. The difference between grasp shown in (Fig. 5-c) and (Fig. 5-d) is that, in the second example the grasp torque w.r.t. c_1 can positively span \mathbb{R}^2 . Hence, we complete the proposition 1 by the following condition.

Proposition 2: A 3D (res. 2D) n -finger grasp is force-closure if and only if, w.r.t. one arbitrary point (eg. c_1):

- 1) the proposition 1 is satisfied at c_1 , and
- 2) the torque applied by the n fingers positively span \mathbb{R}^3 (res. \mathbb{R}^2) at c_1 .

Proof: *i) Sufficient Condition:* When all central axes of class Δ_g^* positively span \mathbb{R}^3 (res. \mathbb{R}^2) at c_1 and the torque produced by the n fingers w.r.t. c_1 can positively span \mathbb{R}^3 (res. \mathbb{R}^2), the wrench at c_1 is

$$W_{g/c_1}^* = \begin{Bmatrix} F_g^* \\ \tau_{g/c_1} \end{Bmatrix} \quad (13)$$

Where F_g^* denote grasp forces applied along central axes Δ_g^* , and τ_{g/c_1} is the torque applied by all contact forces w.r.t. c_1 .

The forces F_g^* and the torques τ_{g/c_1} are independent entities because; torques produced by F_g^* w.r.t. c_1 are nulls. So, if τ_{g/c_1} and F_g^* positively span \mathbb{R}^3 (res. \mathbb{R}^2), the wrench W_{g/c_1}^* can balance any task wrench W_{t/c_1} applied at c_1 . With respect to any other point p , the wrench W_{g/c_1}^* is given by

$$W_{g/p}^* = \begin{Bmatrix} F_g^* \\ \tau_{g/c_1} + \tau_{g/p}^* \end{Bmatrix} \quad (14)$$

The forces F_g^* are passing through c_1 . Their torque w.r.t. any point p is given by $\tau_{g/c_1}^* = (c_1 - p) \times F_g^*$. The torque

τ_{g/c_1}^* can balance any external torque; because the forces F_g^* positively span \mathbb{R}^3 (res. \mathbb{R}^2). We conclude that for any point p , $W_{g/p}^*$ can balance any arbitrary task wrench. So, the grasp is force-closure and the proposition 2 is sufficient.

ii) *Necessary Condition*: Obviously, when the torques produced by the n fingers w.r.t. an arbitrary point cannot have both signes (around any direction), the grasp is non-force-closure. Hence, proposition 2-(1) is necessary. Also, if proposition 2-(2) is not satisfied, there exist task forces that cannot be balanced by F_g^* . So, proposition 2 is necessary. \square

V. ALGORITHMS

The problem of computing 2D and 3D n -finger force-closure grasps is simplified by using proposition 2.

The proposed algorithm has two steps. We start by computing the torque applied by the n friction cones w.r.t. the first contact point c_1 . If this torque cannot positively span \mathbb{R}^3 (\mathbb{R}^2 for 2D grasp) the grasp is non-force-closure. Furthermore, the second step consist of verifying that all central axes Δ_g^* produced by the grasp wrench can positively span \mathbb{R}^3 (\mathbb{R}^2 for 2D grasps). Else, the grasp is non-force-closure.

A. Force-Closure Algorithm

For 3D grasps, the torque applied by the n friction cones w.r.t. the point c_1 can positively span \mathbb{R}^3 if the following inequalities are satisfied

$$\left\{ \begin{array}{l} \left| \sum_{i=2}^n \sum_{j=1}^m \tau_{ijx} \right| - \sum_{i=2}^n \sum_{j=1}^m \left| \tau_{ijx} \right| < 0 \\ \left| \sum_{i=2}^n \sum_{j=1}^m \tau_{ijy} \right| - \sum_{i=2}^n \sum_{j=1}^m \left| \tau_{ijy} \right| < 0 \\ \left| \sum_{i=2}^n \sum_{j=1}^m \tau_{ijz} \right| - \sum_{i=2}^n \sum_{j=1}^m \left| \tau_{ijz} \right| < 0 \end{array} \right. \quad (15)$$

Where, $||$ denotes the absolute value and, $(\tau_{ijx}, \tau_{ijy}, \tau_{ijz})$ denotes x, y and z components of the torque τ_{ij} w.r.t. the object coordinate frame (x, y, z) . They are given as follows

$$\left\{ \begin{array}{l} \tau_{ijx} = x \cdot ((c_i - c_1) \times v_{ij}) \\ \tau_{ijy} = y \cdot ((c_i - c_1) \times v_{ij}) \\ \tau_{ijz} = z \cdot ((c_i - c_1) \times v_{ij}) \end{array} \right. \quad (16)$$

The second step verifies that all central axes Δ_g^* can positively span \mathbb{R}^3 . Using the friction cones linearization procedure, we can formulate the equilibrium condition given by (12) as the relaxed 6 equations system

$$\left\{ \begin{array}{l} \sum_{i=1}^n \sum_{j=1}^m a_{ij} v_{ij} = -\delta Z_1 \\ \sum_{i=2}^n \sum_{j=1}^m ((c_i - c_1) \times a_{ij} v_{ij}) = 0 \end{array} \right. ; a_{ij} \geq 0, \delta > 0 \quad (17)$$

Central axes Δ_g^* can positively span \mathbb{R}^3 if and only if, their positive combination can produce the unit vector $-Z_1$, inverse of the normal at the point c_1 . Therefore, if the system given by (17) has solutions, the 3D grasp is force-closure.

In the grasp planning processus, we must quantify the force-closure in order to optimize the generated grasps. So, we reformulate the equilibrium condition given by the system (17) as the following linear programming problem

$$\min_a f^T a \quad \text{such that} \quad \left\{ \begin{array}{l} A a = 0 \\ l a \leq a \leq u a \end{array} \right. \quad (18)$$

$$\left\{ \begin{array}{l} a = (a_{11}, a_{12}, \dots, a_{1m}, a_{21}, \dots, a_{2m}, \dots, a_{nm}, \delta)^T \\ l a = (0, \dots, 0, \xi > 0)^T ; u a = (1, \dots, 1)^T \\ f = (1, \dots, 1, -1)^T \end{array} \right. \quad (19)$$

The matrix A of dimension $(6 \times (mn + 1))$ is given by

$$A = \begin{pmatrix} v_{11} & \dots & v_{1m} & v_{21} & \dots & \dots & v_{nm} & Z_1 \\ \tau_{11} & \dots & \tau_{1m} & \tau_{21} & \dots & \dots & \tau_{nm} & 0 \end{pmatrix} \quad (20)$$

In the case of 2D grasps, The torque applied by the n friction cones with respect to the point c_1 has both signs if the following inequality is satisfied

$$\left| \sum_{i=2}^n (\tau_{i1} + \tau_{i2}) \right| - \sum_{i=2}^n (|\tau_{i1}| + |\tau_{i2}|) < 0 \quad (21)$$

Where, $\tau_{i1} = z \cdot ((c_i - c_1) \times v_{i1})$ and $\tau_{i2} = z \cdot ((c_i - c_1) \times v_{i2})$, are torques of frictions cones limits w.r.t. the point c_1 and around axis z which denotes the normal on the grasp plane.

When the left side of (21) is strictly negative, the force-closure test is concluded if all the central axes Δ_g^* can positively span \mathbb{R}^2 . The equilibrium condition can be algebraically reformulated by the following 3 equations

$$\left\{ \begin{array}{l} \sum_{i=1}^n (a_{i1} v_{i1} + a_{i2} v_{i2}) = -\delta N_1 \\ \sum_{i=2}^n ((c_i - c_1) \times (a_{i1} v_{i1} + a_{i2} v_{i2})) = 0 \end{array} \right. \quad (22)$$

Where, $(a_{i1}, a_{i2}) \geq 0$ and $\delta > 0$. Like the 3D case, we can solve the system (22) by reformulating it as a linear programming problem given by (18), where $m = 2$ and the dimension of the matrix A is $3 \times (2n + 1)$.

B. Force-Closure Quality

Using the two phases simplex method, the first phase, also known as the initialization step [17], can determines in a finite number of iterations the basic feasible solutions. If this phase has no solution, we conclude that the original problem (18) is inconsistent [17], and the grasp is non-force-closure.

When the initialization step has a solution, the given grasp is force-closure. Then, we begin the main step with this feasible solution to obtain the optimal one given by the vector $a^*(i)$, where $(i = 1, \dots, 2n + 1)$ for 2D grasps and $(i = 1, \dots, mn + 1)$ for 3D grasps. The proposed force-closure qualities depend on the optimal vector $a^*(i)$. It mesures the minimal contact forces that contribute to obtain the maximum of δ . We reformulate the qualities as follows

$$Q_{2d} = \frac{a^*(2n+1)=\delta^*}{\sum_{i=1}^{2n} a^*(i)} ; Q_{3d} = \frac{a^*(nm+1)=\delta^*}{\sum_{i=1}^{nm} a^*(i)} \quad (23)$$

VI. IMPLEMENTATION AND RESULTS

We have implemented the proposed algorithm in Matlab. In the first example (Fig. 6), we have computed multi-finger grasps of polygonal objects. The friction cone is set to 20° . We choose the first point c_1 and, we generate randomly the $n - 1$ contacts. The force-closure qualities Q_{2d} for the quadrilateral grasps are: (a) 0.940, (b) 0.766, (c) 0.538. We compare these qualities with the largest ball inscribed inside the convex hull of the primitive wrenches. This quality is one of the most popular criterion [1]-[18]. Qualities given in [12] are: (a) 0.173, (b) 0.088, (c) 0.026. We remark that the order of the three grasps (a, b and c) in terms of quality is respected with the quality given in (23). We show in (Fig. 6-d, 6-e and 6-f), 4-finger grasps of a star.

We present in Fig. 7 some grasps on 3D objects (glass of 9486 triangular facets, mechanical part of 2824 facets).

The friction cones are linearized with $m = 25$ sides. Friction angles are set to 20° . Generating the $n - 1$ contacts on the facets centroid at random, we confirm that the qualitative force-closure test (initialisation step) is applicable for any number of contacts points. To compare the proposed quality Q_{3d} with the radius ε of the largest ball inscribed inside the convex hull, we use the qhull library [19] to compute the 6-dimensional convex hull of the primitif wrenches. Table 1 summarizes the computed grasps qualities of Fig.7. We also present the volume of the hull V_{ch} which is directly reported by qhull. When the number of contacts is fixed, we remark that qualities ε and Q_{3d} are generally equivalent. But if we compare two grasps with different number's finger, there exist some differences between the measures of Q_{3d} and ε . For example, grasps illustrated in Fig.7 (b), (c) and (d), ε show that (c) and (d) are better than (b), but the measures of Q_{3d} show the inverse because the proposed quality Q_{3d} determines the minimum forces that contribute in force-closure. Hence, it measures the force-closure taking into consideration the number of contacts.

VII. CONCLUSIONS AND FUTURE WORKS

The main contribution of this paper is the development of a general approach for computing 2D and 3D force-closure grasps without computing the convex hull of the

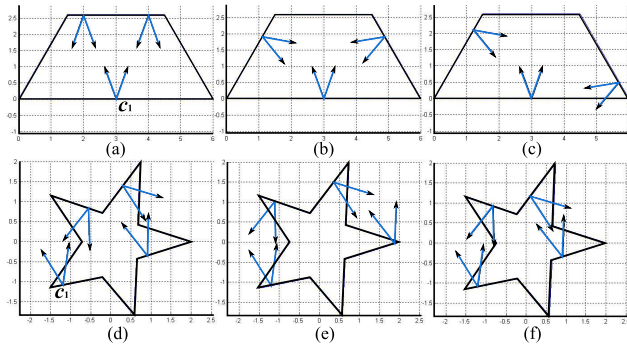


Fig. 6. Examples of 2D grasps. Qualities Q_{2d} are: (a) 0.940, (b) 0.766, (c) 0.538, (d) 0.394, (e) 0.651, (f) 0.939.

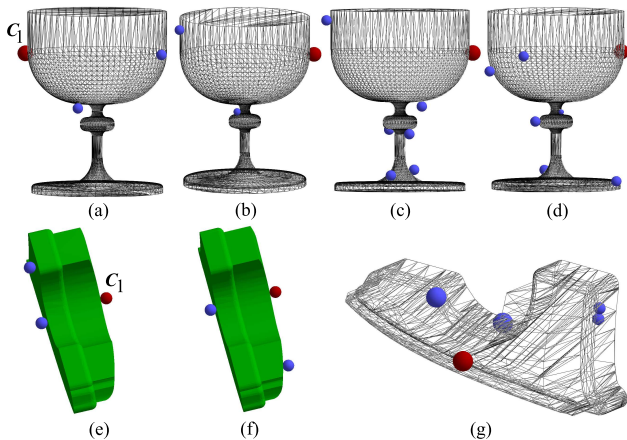


Fig. 7. Generated force-closure grasps on 3D objects: (a,b) 3-finger, (c,d) 7-finger, (e,f) 3-finger, (g) 5-finger.

TABLE I
COMPARISON BETWEEN ε AND Q_{3d} FOR GRASPS SHOWN IN FIG. 7

Grasp	a	b	c	d	e	f	g
Q_{3d}	0.538	0.885	0.627	0.794	0.761	0.831	0.806
ε	0.024	0.062	0.081	0.156	0.022	0.032	0.087
$V_{ch}10^{-1}$	0.088	0.099	0.449	0.604	0.059	0.036	0.225

primitive contact wrenches. Based on grasp wrench central axes, we have proposed necessary and sufficient conditions to achieve equilibrium and force-closure grasps. Further, we have presented an efficient algorithm for computing n -finger force-closure grasps and demonstrated its efficient implementation through examples. Future work will be concentrated on development of oriented task qualities and on grasp planning.

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