2008 IEEE International Conference on
Robotics and Automation
Pasadena, CA, USA, May 19-23, 2008

Three-Dimensional Object Manipulation by Two Robot Fingers with Soft Tips and Minimum D.O.F

Morio Yoshida, Suguru Arimoto, and Zhi-Wei Luo

Abstract— **This paper shows through faithfully deriving Paffian forms of 3-D rolling contact constraints that 3-D pinching can be stabilized by using a pair of robot fingers with a hemispherical soft tip and minimum degrees of freedom under the gravity effect. The proposed control input is based on fingers-thumb opposition without using object information or external sensing. Stability analysis of the closed-loop dynamics is presented by using a Lyapunov method. Finally, for the sake of confirmation of effectiveness of the proposed control signals, numerical simulations are carried out.**

I. Introduction

Since human can walk on only two legs, they can use hands for other effective purposes unlike other living creatures. Hands have evolved to make tools, create gestures, and write pictures and characters for bequeathing ancient wisdom and art. Especially, the primatologist Napier [1] accentuates the importance of fingers-thumb opposition in the progress of humanity. However, there is a dearth of robotics researchers who are interested in control functions of multi-fngered hands based on the opposability.

Robot hands mimicking the human hands attract many robotics researches [2] [3] [4]. However, most researchers are interested in kinematics and planning of motions establishing force/torque closure for secure grasp in a static sense with frictionless contacts. On the other hand, rolling geometry between two objects is investigated in detail [5] [6]. However, all the researches have remained in a kinematic or semi-dynamic meaning. In fact, any explicit 3-D dynamics model of fingers-object interaction, in which effects of rolling constraints should be incorporated as wrench vectors, has been missing.

Very recently in 2000s, however, it was shown by Arimoto *et al*. that pinching of a 2-D rigid object was stabilized by using a pair of robot fingers with hemispherical ends in a dynamic sense [7] [8]. In the research, it is shown that tangential forces are induced by rolling constraints embedded in the overall dynamics and the redundancy of the overall fingers-object system for a desired task is overcome naturally without use of the pseudo inverse of a Jacobian or any artificial cost function. In the year of 2006, a problem of modeling of 3-D object grasping was tackled by Arimoto *et al*. [9] and a mathematical model was derived as a set of motion of the fingers-object system under Pfaffian constraints of rolling contacts and another nonholonomic constraint due to the assumption of the cease of spinning motion around the opposing axis. In that case, the object dynamics is expressed by a five-variables model with six wrench vectors [9]. In this paper, we derive overall fingers-object dynamics as a full-variables model under the assumption that spinning around the opposition axis is possible but it accompanies viscous friction exerting on the rotational motion of the object around the x-axis in the frame coordinates. A pair of robot fingers has hemispherical tips made by soft material and the minimum degrees of freedom (DOF) for desired tasks. Then, we propose a simple control signal based on the opposability without using object information or external sensing as seen when human grasp an object. Finally, we show that any closed-loop solution converges to an equilibrium pose establishing force/torque balance as time tends to infinity.

M. Yoshida, S. Arimoto, and Z.W.Luo are with the Bio-Mimetic Control Research Center, RIKEN, Nagoya, Aichi, 463-0003, Japan yoshida@bmc.riken.jp
S. Arimoto is with the Department of Robotics, Ritsumeikan University, Kusatsu, Shiga, 525-8577, Japan arimoto@se.ritsumei.ac.jp
Z.W. Luo is with the Department of Computer and Systems Engineering, Kobe University, Kobe, Hyogo, 657-8501, Japan luo@gold.kobe-u.ac.jp

II. Dynamics

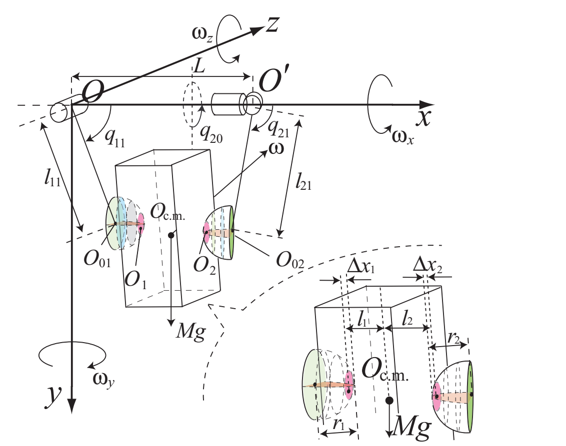


Fig. 1. Two robot fingers pinching an object with parallel flat surfaces under the gravity effect

Consider motion of a rigid object with parallel flat surfaces, which is grasped by a pair of robot fingers with 1 DOF and 2 DOFs as shown in Fig.1. The left finger (finger $i = 1$) is planar with one joint whose rotational axis is in z-direction denoted by angle q_{11}. The root joint with center O' of the right finger is a saddle joint having two rotational axes in x-direction denoted by angle q_{20} and z-direction denoted by angle q_{21}. When the distance from the straight line $\overrightarrow{O_1O_2}$(opposition axis) connecting two contact points denoted by $\boldsymbol{x}_i = (x_i, y_i, z_i)^{\mathrm{T}}$ between finger-ends

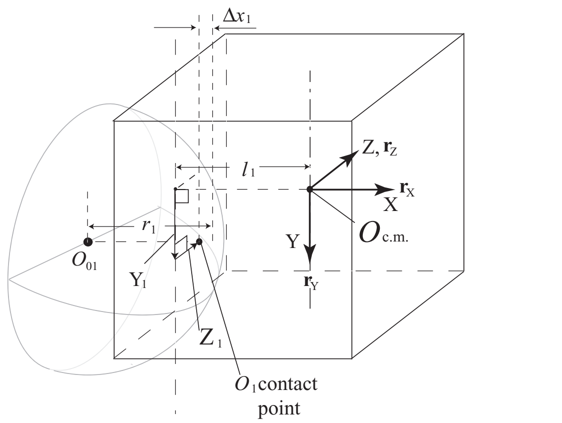


Fig. 2. $\boldsymbol{r}_X, \boldsymbol{r}_Y$, and $\boldsymbol{r}_Z$ are mutually orthogonal vectors express rotational motion of 3-D object

and object surfaces to the vertical axis through the object mass center in the direction of gravity becomes large, there arises a spinning rotation of the object around that opposition axis. In the previous paper [10], a problem of modeling of pinching is considered in the situation that this spinning motion has ceased after the center of mass of the object came sufficiently close to a point just beneath the opposing axis and there will no more arise such spinning rotation due to dry friction and micro-deformations near the contact points between the finger-ends and object surfaces. Instead of this assumption, we assume that spinning around the opposition axis is possible to arise but viscosity is exerted on rotational motion of the object around x-axis, that is, about ω_x, where $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)^{\mathrm{T}}$ denotes the vector of rigid body rotation in terms of frame coordinates O-xyz (see Figure 1). At the same time, we introduce, the cartesian coordinates $O_{c.m.}$-XYZ fixed at the object frame and denote three orthogonal unit vectors at the object frame in each corresponding direction X, Y, and Z by $\boldsymbol{r}_X = (r_{Xx}, r_{Xy}, r_{Xz})^{\mathrm{T}}$, $\boldsymbol{r}_Y = (r_{Yx}, r_{Yy}, r_{Yz})^{\mathrm{T}}$, and $\boldsymbol{r}_Z = (r_{Zx}, r_{Zy}, r_{Zz})^{\mathrm{T}}$ as shown in Fig. 2. Note that $\boldsymbol{r}_X$, $\boldsymbol{r}_Y$, and $\boldsymbol{r}_Z$ are fixed at the object but rotate in terms of the frame coordinates, and therefore components r_{Xx}, r_{Xy}, and $r_{Xz}$of vector $\boldsymbol{r}_X$ are denoted in the frame coordinates $O - xyz$. The same applies to $\boldsymbol{r}_Y$ and $\boldsymbol{r}_Z$. Next, denote the cartesian coordinates of the object mass center $O_{c.m.}$ by $\boldsymbol{x} = (x, y, z)^{\mathrm{T}}$ based on the frame coordinates O-xyz and note that three mutually orthogonal unit vectors fixed at the object may rotate dependently on the angular velocity vector $\boldsymbol{\omega}$ of body rotation. Then, it is well known that the 3×3 rotation matrix

$$R(t) = (\boldsymbol{r}_X, \boldsymbol{r}_Y, \boldsymbol{r}_Z) \tag{1}$$

belongs to SO(3) and is subject to the first-order differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t} R(t) = R(t)\Omega(t) \tag{2}$$

where

$$\Omega(t) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \tag{3}$$

Next, denote the position of the center of each hemispherical finger-end by $\boldsymbol{x}_{0i} = (x_{0i}, y_{0i}, z_{0i})^{\mathrm{T}}$. Then, it is possible to notice that (see Figure 2)

$$\boldsymbol{x}_i = \boldsymbol{x}_{0i} - (-1)^i (r_i - \Delta x_i) \boldsymbol{r}_X \tag{4}$$

$$\boldsymbol{x} = \boldsymbol{x}_{0i} - (-1)^i (r_i - \Delta x_i + l_i) \boldsymbol{r}_X - Y_i \boldsymbol{r}_Y - Z_i \boldsymbol{r}_Z \tag{5}$$

Since each contact point O_i can be expressed by the coordinates $((-1)^i l_i, Y_i, Z_i)$ based on the object frame $O_{c.m.}$-XYZ, taking an inner product between Equation (5) and $\boldsymbol{r}_Y$ gives rise to

$$Y_i = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{r}_Y, \ i = 1, 2 \tag{6}$$

Similarly, it follows that

$$Z_i = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{r}_Z, \ i = 1, 2 \tag{7}$$

A rolling constraint between one finger-end and its contacted object surface can be expressed by equality of two contact point velocities expressed on either of finger-end spheres and on its corresponding tangent plane (that is coincident with one of object flat surfaces) as follows [9]:

$$\begin{cases} (r_1 - \Delta x_1) \{\omega_z - r_{Zz} \dot{q}_{10}\} = \dot{Y}_1 \\ (r_1 - \Delta x_1) \{-\omega_y + r_{Yz} \dot{q}_{10}\} = \dot{Z}_1 \end{cases} \tag{8}$$

$$\begin{cases} (r_2 - \Delta x_2) \{-\omega_z + (r_{Zz} \cos q_{20} - r_{Zy} \sin q_{20}) \dot{q}_{21} \\ \qquad + r_{Zx} \dot{q}_{20}\} = \dot{Y}_2 \\ (r_2 - \Delta x_2) \{\omega_y - (r_{Yz} \cos q_{20} - r_{Yy} \sin q_{20}) \dot{q}_{21} \\ \qquad - r_{Yx} \dot{q}_{20}\} = \dot{Z}_2 \end{cases} \tag{9}$$

where $q_{10} = 0$. The rolling constraint conditions expressed through Equations (8) and (9) are non-holonomic but linear and homogeneous with respect to velocity variables. Hence, Equations (8) and (9) can be treated as Pfaffian constraints [2] [11] that can be expressed with accompaning Lagrange's multipliers $\{\lambda_{Y1}, \lambda_{Z1}\}$ for Equation (8) and $\{\lambda_{Y2}, \lambda_{Z2}\}$ for Equation (9) in such forms as

$$\begin{cases} \lambda_{Yi} \left\{ \boldsymbol{Y}_{\boldsymbol{q}i}^{\mathrm{T}} \dot{\boldsymbol{q}}_i + \boldsymbol{Y}_{\boldsymbol{x}i}^{\mathrm{T}} \dot{\boldsymbol{x}} + Y_{\varphi i} \dot{\varphi} + Y_{\psi i} \dot{\psi} + Y_{\theta i} \dot{\theta} \right\} = 0 \\ \lambda_{Zi} \left\{ \boldsymbol{Z}_{\boldsymbol{q}i}^{\mathrm{T}} \dot{\boldsymbol{q}}_i + \boldsymbol{Z}_{\boldsymbol{x}i}^{\mathrm{T}} \dot{\boldsymbol{x}} + Z_{\varphi i} \dot{\varphi} + Z_{\psi i} \dot{\psi} + Z_{\theta i} \dot{\theta} \right\} = 0 \\ \qquad i = 1, 2 \end{cases} \tag{10}$$

where

$$\begin{cases} \boldsymbol{Y}_{\boldsymbol{q}i} = \frac{\partial Y_i}{\partial \boldsymbol{q}_i} - (r_i - \Delta x_i) \left\{ (-1)^i (r_{Zz} \cos q_{i0} \right. \\ \qquad \left. - r_{Zy} \sin q_{i0}) \boldsymbol{e}_i + r_{Zx} \boldsymbol{e}_{0i} \right\} \\ \boldsymbol{Y}_{\boldsymbol{x}i} = \frac{\partial Y_i}{\partial \boldsymbol{x}}, \ Y_{\varphi i} = \frac{\partial Y_i}{\partial \varphi}, \ Y_{\psi i} = \frac{\partial Y_i}{\partial \psi} \\ Y_{\theta i} = \frac{\partial Y_i}{\partial \theta} + (-1)^i (r_i - \Delta x_i), \ i = 1, 2 \end{cases} \tag{11}$$

and

$$\begin{cases} \boldsymbol{Z}_{\boldsymbol{q}i} = \frac{\partial Z_i}{\partial \boldsymbol{q}_i} - (r_i - \Delta x_i) \left\{ (-1)^i (r_{Yz} \cos q_{i0} \right. \\ \qquad \left. - r_{Yy} \sin q_{i0}) \boldsymbol{e}_i + r_{Yx} \boldsymbol{e}_{0i} \right\} \\ \boldsymbol{Z}_{\boldsymbol{x}i} = \frac{\partial Z_i}{\partial \boldsymbol{x}}, \ Z_{\varphi i} = \frac{\partial Z_i}{\partial \varphi}, \\ Z_{\psi i} = \frac{\partial Z_i}{\partial \psi} - (-1)^i (r_i - \Delta x_i), \ Z_{\theta i} = \frac{\partial Z_i}{\partial \theta}, i = 1, 2 \end{cases} \tag{12}$$

TABLE I

PARTIAL DERIVATIVES OF CONSTRAINTS IN (φ, ψ, θ).

$$
\begin{aligned}
&\tfrac{\partial P_{\Delta x1}}{\partial \varphi} = \tfrac{\partial P_{\Delta x2}}{\partial \varphi} = 0 \\
&\tfrac{\partial Y_i}{\partial \varphi} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \tfrac{\partial \boldsymbol{r}_Y}{\partial \varphi} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{r}_Z = Z_i \\
&\tfrac{\partial Y_i}{\partial \psi} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \tfrac{\partial \boldsymbol{r}_Y}{\partial \psi} = 0 \\
&\tfrac{\partial Y_i}{\partial \theta} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \tfrac{\partial \boldsymbol{r}_Y}{\partial \theta} = -(\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{r}_X = -(-1)^i (r_i + l_i) \\
&\tfrac{\partial Z_i}{\partial \varphi} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \tfrac{\partial \boldsymbol{r}_Z}{\partial \varphi} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{r}_Y = -Y_i \\
&\tfrac{\partial Z_i}{\partial \psi} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \tfrac{\partial \boldsymbol{r}_Z}{\partial \psi} = -(\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{r}_X = (-1)^i (r_i + l_i) \\
&\tfrac{\partial Z_i}{\partial \theta} = (\boldsymbol{x}_{0i} - \boldsymbol{x})^{\mathrm{T}} \tfrac{\partial \boldsymbol{r}_Z}{\partial \theta} = 0 \\
&Y_{\varphi i} = \tfrac{\partial Y_i}{\partial \varphi} = Z_i,\ Z_{\varphi i} = \tfrac{\partial Z_i}{\partial \varphi} = -Y_i \\
&Y_{\psi i} = \tfrac{\partial Y_i}{\partial \psi} = 0,\ Z_{\psi i} = \tfrac{\partial Z_i}{\partial \psi} - (-1)^i r_i = (-1)^i l_i \\
&Y_{\theta i} = \tfrac{\partial Y_i}{\partial \theta} + (-1)^i r_i = -(-1)^i l_i,\ Z_{\theta i} = 0
\end{aligned}
$$

and $\boldsymbol{q}_1 = q_{11}$, $\boldsymbol{q}_2 = (q_{20}, q_{21})^{\mathrm{T}}$, $\dot{\varphi} = \omega_x$, $\dot{\psi} = \omega_y$, $\dot{\theta} = \omega_z$, $\boldsymbol{e}_1 = 1$, $\boldsymbol{e}_2 = (0,1)^{\mathrm{T}}$, $\boldsymbol{e}_{01} = 0$, and $\boldsymbol{e}_{02} = (1,0)^{\mathrm{T}}$. To simplify notations, we rewrite Equation (10) into

$$
\left\{ \begin{array}{l} \lambda_{Yi} \boldsymbol{Y}_i^{\mathrm{T}} (\mathrm{d}\boldsymbol{X}/\mathrm{d}t) = 0 \\ \lambda_{Zi} \boldsymbol{Z}_i^{\mathrm{T}} (\mathrm{d}\boldsymbol{X}/\mathrm{d}t) = 0 \end{array} \right., \ i = 1, 2 \tag{13}
$$

where $\boldsymbol{X} = (\boldsymbol{q}_1^{\mathrm{T}}, \boldsymbol{q}_2^{\mathrm{T}}, \boldsymbol{x}^{\mathrm{T}}, \varphi, \phi, \theta)^{\mathrm{T}}$,

$$
\left\{ \begin{array}{l} \boldsymbol{Y}_1 = \left(\boldsymbol{Y}_{\boldsymbol{q}1}^{\mathrm{T}}, 0_2, \boldsymbol{Y}_{\boldsymbol{x}1}^{\mathrm{T}}, Y_{\varphi 1}, Y_{\phi 1}, Y_{\theta 1} \right)^{\mathrm{T}} \\ \boldsymbol{Y}_2 = \left(0, \boldsymbol{Y}_{\boldsymbol{q}2}^{\mathrm{T}}, \boldsymbol{Y}_{\boldsymbol{x}2}^{\mathrm{T}}, Y_{\varphi 2}, Y_{\phi 2}, Y_{\theta 2} \right)^{\mathrm{T}} \end{array} \right. \tag{14}
$$

and

$$
\left\{ \begin{array}{l} \boldsymbol{Z}_1 = \left(\boldsymbol{Z}_{\boldsymbol{q}1}^{\mathrm{T}}, 0_2, \boldsymbol{Z}_{\boldsymbol{x}1}^{\mathrm{T}}, Z_{\varphi 1}, Z_{\phi 1}, Z_{\theta 1} \right)^{\mathrm{T}} \\ \boldsymbol{Z}_2 = \left(0, \boldsymbol{Z}_{\boldsymbol{q}2}^{\mathrm{T}}, \boldsymbol{Z}_{\boldsymbol{x}2}^{\mathrm{T}}, Z_{\varphi 2}, Z_{\phi 2}, Z_{\theta 2} \right)^{\mathrm{T}} \end{array} \right. \tag{15}
$$

The reproducing force due to finger-tip deformation can be described as

$$
f_i(\Delta x_i, \Delta \dot{x}_i) = \bar{f}_i(\Delta x_i) + \xi_i(\Delta x_i) \Delta \dot{x}_i \tag{16}
$$

where

$$
\bar{f}_i(\Delta x_i) = k_i \Delta x_i^2, \ i = 1, 2 \tag{17}
$$

with stiffness parameter $k_i > 0[\mathrm{N/m^2}]$ and $\xi_i(\Delta x_i)[\mathrm{Ns/m}]$ is a positive scalar function of Δx_i.

The Lagrangian for the overall fingers-object system can be expressed by the scalar quantity $L = K - P$, where K denotes the total kinetic energy expressed as

$$
\begin{aligned} K &= \frac{1}{2} \sum_{i=1,2} \dot{\boldsymbol{q}}_i^{\mathrm{T}} H_i(\boldsymbol{q}_i) \dot{\boldsymbol{q}}_i + \frac{1}{2} M \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right) \\ &\quad + \frac{1}{2} \boldsymbol{\omega}^{\mathrm{T}} H_0 \boldsymbol{\omega} \end{aligned} \tag{18}
$$

and P denotes the total potential energy expressed as

$$
P = P_1(\boldsymbol{q}_1) + P_2(\boldsymbol{q}_2) - Mgy + \sum_{i=1,2} P_{\Delta xi} \tag{19}
$$

where $H_i(\boldsymbol{q}_i)$ stands for the inertia matrix for finger i, M the mass of the object, $P_i(\boldsymbol{q}_i)$ the potential energy of finger i, g the gravity constant, and $P_{\Delta xi}(= \int_0^{\Delta x_i} \bar{f}_i(\xi) \mathrm{d}\xi)$ the potential energy of reproducing force for finger i, and H_0 is given in the following:

$$
H_0 = R(t) H R^{\mathrm{T}}(t) \tag{20}
$$

where H stands for the constant inertia matrix of the object that must be evaluated on the basis of fixed body coordinates $O_{\mathrm{c.m.}}$-XYZ, that is,

$$
H = \begin{pmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{YX} & I_{YY} & I_{YZ} \\ I_{ZX} & I_{ZY} & I_{ZZ} \end{pmatrix} \tag{21}
$$

Thus, owing to the variational principle applied to the form

$$
\begin{aligned} -\int_{t_0}^{t_1} \delta L \mathrm{d}t &= \int_{t_0}^{t_1} \sum_{i=1,2} \Big\{ \boldsymbol{u}_i^{\mathrm{T}} \delta \boldsymbol{q}_i \\ &\quad - \left(\lambda_{Yi} \boldsymbol{Y}_i^{\mathrm{T}} + \lambda_{Zi} \boldsymbol{Z}_i^{\mathrm{T}} \right) \delta \boldsymbol{X} \Big\} \mathrm{d}t \\ &\quad - \int_{t_0}^{t_1} \Big\{ c_\varphi \omega_x \delta \varphi + \sum_{i=1,2} \xi_i(\Delta x_i) \Delta \dot{x}_i \frac{\partial \Delta x_i}{\partial \boldsymbol{X}^{\mathrm{T}}} \delta \boldsymbol{X} \Big\} \mathrm{d}t \end{aligned} \tag{22}
$$

we obtain a set of Lagrange's equations of motion of the overall system:

$$
\begin{aligned} & H_i(\boldsymbol{q}_i) \ddot{\boldsymbol{q}}_i + \left\{ \frac{1}{2} \dot{H}_i(\boldsymbol{q}_i) + S_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i) \right\} \dot{\boldsymbol{q}}_i \\ & -(-1)^i f_i J_{0i}^{\mathrm{T}}(\boldsymbol{q}_i) \boldsymbol{r}_X - \lambda_{Yi} \boldsymbol{Y}_{\boldsymbol{q}i} - \lambda_{Zi} \boldsymbol{Z}_{\boldsymbol{q}i} \\ & + \boldsymbol{g}_i(\boldsymbol{q}_i) = \boldsymbol{u}_i \qquad i = 1, 2 \end{aligned} \tag{23}
$$

$$
\begin{aligned} & M\ddot{\boldsymbol{x}} - (f_1 - f_2) \boldsymbol{r}_X + (\lambda_{Y1} + \lambda_{Y2}) \boldsymbol{r}_Y \\ & \qquad + (\lambda_{Z1} + \lambda_{Z2}) \boldsymbol{r}_Z - Mg \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \end{aligned} \tag{24}
$$

$$
\begin{aligned} & H_0 \dot{\boldsymbol{\omega}} + \left(\frac{1}{2} \dot{H}_0 + S \right) \boldsymbol{\omega} + c_\varphi \begin{pmatrix} \omega_x \\ 0 \\ 0 \end{pmatrix} + f_1 \begin{pmatrix} 0 \\ -Z_1 \\ Y_1 \end{pmatrix} \\ & + f_2 \begin{pmatrix} 0 \\ Z_2 \\ -Y_2 \end{pmatrix} - \lambda_{Y1} \begin{pmatrix} Z_1 \\ 0 \\ l_1 \end{pmatrix} - \lambda_{Y2} \begin{pmatrix} Z_2 \\ 0 \\ -l_2 \end{pmatrix} \\ & - \lambda_{Z1} \begin{pmatrix} -Y_1 \\ -l_1 \\ 0 \end{pmatrix} - \lambda_{Z2} \begin{pmatrix} -Y_2 \\ l_2 \\ 0 \end{pmatrix} = 0 \end{aligned} \tag{25}
$$

where calculation of partial differentiations of Y_i, Z_i in φ, ψ, θ is presented in Table 1. Further, it is possible to see from $P_{\Delta xi} (i = 1, 2)$ and Equations (6) and (7) that

$$
\left\{ \begin{array}{l} \frac{\partial (P_{\Delta x1} + P_{\Delta x2})}{\partial \boldsymbol{q}_i} = -(-1)^i f_i J_i^{\mathrm{T}}(\boldsymbol{q}_i) \boldsymbol{r}_X, \ i = 1, 2 \\ \frac{\partial (P_{\Delta x1} + P_{\Delta x2})}{\partial \boldsymbol{x}} = -(f_1 - f_2) \boldsymbol{r}_X \\ \frac{\partial (P_{\Delta x1} + P_{\Delta x2})}{\partial \varphi} = 0, \ \frac{\partial (P_{\Delta x1} + P_{\Delta x2})}{\partial \psi} = -f_1 Z_1 + f_2 Z_2 \\ \frac{\partial (P_{\Delta x1} + P_{\Delta x2})}{\partial \theta} = f_1 Y_1 - f_2 Y_2 \end{array} \right. \tag{26}
$$

$$
\left\{ \begin{array}{l} \boldsymbol{Y}_{\boldsymbol{x}i} = \partial Y_i / \partial \boldsymbol{x} = -\boldsymbol{r}_Y \\ \boldsymbol{Z}_{\boldsymbol{x}i} = \partial Z_i / \partial \boldsymbol{x} = -\boldsymbol{r}_Z \end{array} \right. \tag{27}
$$

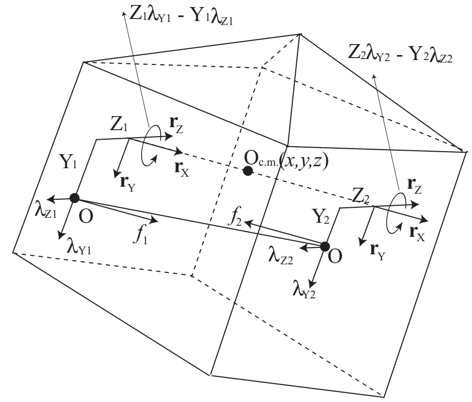


Fig. 3. Normal forces arising from deformation of the finger ends and tangential forces from rolling constraints at contact points O_1 and O_2

From taking inner products between $\dot{\boldsymbol{q}}_i$ and Equation (23), $\dot{\boldsymbol{x}}$ and Equation (24), and $\boldsymbol{\omega}$ and Equation (25), we obtain the relation

$$\sum_{i=1,2} \dot{\boldsymbol{q}}_i^{\mathrm{T}} \boldsymbol{u}_i = \frac{\mathrm{d}}{\mathrm{d}t}(K+P) + c_\varphi \dot{\varphi}^2 + \sum_{i=1,2} \xi_i(\Delta x_i)\Delta \dot{x}_i^2 \quad (28)$$

Finally, it is intersting to note that six vectors associated with f_1, f_2, λ_{Y1}, λ_{Y2}, λ_{Z1}, and λ_{Z2} from the fourth term to the ninth term in the right hand side of Equation (25) constitute, together with corresponding six vectors of Equation (24), a set of wrench vectors exerted on the 3-dimension rigid object (see Fig.3). The last term of the left hand side of Equation (24) is regarded as an external force vector caused by the gravity.

III. Control signals

We consider the stability of control of "blind grasping" under the gravity effect. The control signal is defined as follows:

$$\begin{aligned} \boldsymbol{u}_i &= -C_i\dot{\boldsymbol{q}}_i + (-1)^i \frac{f_d}{r_1+r_2} J_{0i}^{\mathrm{T}}(\boldsymbol{x}_{01}-\boldsymbol{x}_{02}) \\ &\quad -\frac{\hat{M}g}{2}\frac{\partial y_{0i}}{\partial \boldsymbol{q}_i} - r_i\hat{N}_i\boldsymbol{e}_i - r_i\hat{N}_{0i}\boldsymbol{e}_{0i}, \ i=1,2 \quad (29) \end{aligned}$$

where

$$\begin{aligned} \hat{M} &= \hat{M}(0) + \int_0^t \frac{g\gamma_M^{-1}}{2} \sum_{i=1,2} \left(\frac{\partial y_{0i}}{\partial \boldsymbol{q}_i}\right)^{\mathrm{T}} \boldsymbol{q}_i \mathrm{d}\tau \\ &= \hat{M}(0) + \frac{g\gamma_M^{-1}}{2}(y_{01}(t)+y_{02}(t) \\ &\quad -y_{01}(0)-y_{02}(0)) \quad (30) \\ \hat{N}_i &= \gamma_{Ni}^{-1}\int_0^t \left(r_i\boldsymbol{e}_i^{\mathrm{T}}\dot{\boldsymbol{q}}_i\right)\mathrm{d}\tau \\ &= r_{Ni}^{-1} r_i \boldsymbol{e}_i^{\mathrm{T}}(\boldsymbol{q}_i(t)-\boldsymbol{q}_i(0)), \ (i=1,2) \quad (31) \\ \hat{N}_{01} &= 0 \quad (32) \\ \hat{N}_{02} &= \gamma_{N02}^{-1}\int_0^t (r_i\dot{q}_{20})\mathrm{d}\tau \\ &= \gamma_{N02}^{-1} r_2 (q_{20}(t)-q_{20}(0)) \quad (33) \end{aligned}$$

and γ_M, $\gamma_{Ni}(i=1,2)$, and γ_{N02} are positive constants. In this form, nothing differes from that of control signal proposed in the rigid contact case [9]. second term of the right hand side of eq.(29) is a signal based upon the opposable force between O_{01} and O_{02} (not between O_1 and O_2, because positions of O_1 and O_2 can not be measured). third term stands for compensation for the object mass based upon its estimator. The fourth and fifth terms are introduced for saving excess movements of finger joints from the initial pose. Next define

$$f_0 = f_d\left(1+\frac{l_1+l_2-\Delta x_1-\Delta x_2}{r_1+r_2}\right) \quad (34)$$

Differently from the case of rigid finger-ends [8], f_0 is not a constant but dependent on the magnitude of $\Delta x_1+\Delta x_2$. Nevertheless, it is possible to find $\Delta x_{di}(i=1,2)$ for a given $f_d>0$ so that they satisfy

$$\bar{f}_i(\Delta x_{di}) = \left(1+\frac{l_1+l_2-\Delta x_{d1}-\Delta x_{d2}}{r_1+r_2}\right) f_d, \ i=1,2 \quad (35)$$

because $\bar{f}_i(\Delta x_i)$ is of the form of $\bar{f}_i(\Delta x)=k_i\Delta x^2$(eq.(17)). Substituting this control signals (eq.(29)) into Equation (23) yields

$$\begin{aligned} & H_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \left\{\frac{1}{2}\dot{H}_i(\boldsymbol{q}_i) + S_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i) + C_i\right\}\dot{\boldsymbol{q}}_i \\ & -(-1)^i J_{0i}^{\mathrm{T}}(\boldsymbol{q}_i)\Delta f_i \boldsymbol{r}_X - \Delta\lambda_{Yi}\boldsymbol{Y}\boldsymbol{q}_i - \Delta\lambda_{Zi}\boldsymbol{Z}\boldsymbol{q}_i \\ & +\frac{\Delta Mg}{2}\left(\frac{\partial y_{0i}}{\partial \boldsymbol{q}_i}\right) + r_i\Delta N_i\boldsymbol{e}_i + r_i\Delta N_{0i}\boldsymbol{e}_{0i} + \boldsymbol{g}(\boldsymbol{q}_i) = 0 \\ & \qquad i=1,2 \quad (36) \end{aligned}$$

where

$$\begin{aligned} \Delta\bar{f}_i &= \bar{f}_i - f_0 - (-1)^i \frac{Mg}{2} r_{Xy} \quad (37) \\ \Delta\lambda_{Yi} &= \lambda_{Yi} + (-1)^i \frac{f_d}{r_1+r_2}(Y_1-Y_2) - \frac{Mg}{2} r_{Yy} \quad (38) \\ \Delta\lambda_{Zi} &= \lambda_{Zi} + (-1)^i \frac{f_d}{r_1+r_2}(Z_1-Z_2) - \frac{Mg}{2} r_{Zy} \quad (39) \end{aligned}$$

$$\left\{ \begin{array}{l} \Delta M = \hat{M} - M, \ \Delta N_i = \hat{N}_i - N_i, \ i=1,2 \\ \Delta N_{0i} = \hat{N}_{0i} - N_{0i} \end{array} \right. \quad (40)$$

$$\begin{aligned} N_i &= \frac{f_d}{r_1+r_2}\frac{(r_i-\Delta x_i)}{r_i}\Big\{(Y_1-Y_2)r_Z(q_{i0}) \\ &\quad -(Z_1-Z_2)r_Y(q_{i0})\Big\} \\ &\quad -(-1)^i\frac{(r_i-\Delta x_i)}{r_i}\frac{Mg}{2}\Big\{r_{Yy}r_Z(q_{i0}) \\ &\quad -r_{Zy}r_Y(q_{i0})\Big\}, \ i=1,2 \quad (41) \\ N_{01} &= 0 \quad (42) \\ N_{02} &= \frac{f_d}{r_1+r_2}\frac{(r_2-\Delta x_2)}{r_2}\Big\{(Y_1-Y_2)r_{Zx} \\ &\quad -(Z_1-Z_2)r_{Yx}\Big\} \end{aligned}$$

$$-\frac{(r_2-\Delta x_2)}{r_2}\frac{Mg}{2}\Big\{r_{Yy}r_{Zx} - r_{Zy}r_{Yx}\Big\} \quad (43)$$

$$\left\{\begin{array}{l} r_Z(q_{i0}) = r_{Zz}\cos q_{i0} - r_{Zy}\sin q_{i0} \\ r_Y(q_{i0}) = r_{Yz}\cos q_{i0} - r_{Yy}\sin q_{i0} \end{array}\right. ,i=1,2 \quad (44)$$

In derivation of this form, we used the relation

$$f_0\boldsymbol{r}_X + \frac{f_d}{r_1+r_2}(\boldsymbol{x}_{01}-\boldsymbol{x}_{02}) = \frac{f_d(Y_1-Y_2)}{r_1+r_2}\boldsymbol{r}_Y + \frac{f_d(Z_1-Z_2)}{r_1+r_2}\boldsymbol{r}_Z \quad (45)$$

which immediately follows from Equations (4), (5), (6), and (7). To the accompaniment of closed-loop expression of fingers' dynamics, we rewrite Equations (24) and (25) into the followings:

$$M\ddot{\boldsymbol{x}} - (\Delta f_1 - \Delta f_2)\boldsymbol{r}_X + (\Delta\lambda_{Y1}+\Delta\lambda_{Y2})\boldsymbol{r}_Y + (\Delta\lambda_{Z1}+\Delta\lambda_{Z2})\boldsymbol{r}_Z = 0 \quad (46)$$

$$H_0\dot{\boldsymbol{\omega}} + \left(\frac{1}{2}\dot{H}_0 + S\right)\boldsymbol{\omega} + c_\varphi\begin{pmatrix}\omega_x\\0\\0\end{pmatrix} + \Delta f_1\begin{pmatrix}0\\-Z_1\\Y_1\end{pmatrix} + \Delta f_2\begin{pmatrix}0\\Z_2\\-Y_2\end{pmatrix} - \Delta\lambda_{Y1}\begin{pmatrix}Z_1\\0\\l_1\end{pmatrix} - \Delta\lambda_{Y2}\begin{pmatrix}Z_2\\0\\-l_2\end{pmatrix} - \Delta\lambda_{Z1}\begin{pmatrix}-Y_1\\-l_1\\0\end{pmatrix} - \Delta\lambda_{Z2}\begin{pmatrix}-Y_2\\l_2\\0\end{pmatrix} - \begin{pmatrix}S_X\\S_Y\\S_Z\end{pmatrix} = 0 \quad (47)$$

where

$$S_X = \frac{Mg}{2}\{(Z_1+Z_2)r_{Yy} - (Y_1+Y_2)r_{Zy}\} \quad (48)$$

$$S_Y = \frac{f_d}{r_1+r_2}(r_1-\Delta x_1+r_2-\Delta x_2)(Z_1-Z_2) - \frac{Mg}{2}\{r_{Xy}(Z_1+Z_2)+r_{Zy}(l_1-l_2)\} \quad (49)$$

$$S_Z = -\frac{f_d}{r_1+r_2}(r_1-\Delta x_1+r_2-\Delta x_2)(Y_1-Y_2) + \frac{Mg}{2}\{r_{Xy}(Y_1+Y_2)+r_{Yy}(l_1-l_2)\} \quad (50)$$

Since Equation (43) implies

$$\|\boldsymbol{x}_{01}-\boldsymbol{x}_{02}\| = l_w^2 + (Y_1-Y_2)^2 + (Z_1-Z_2)^2 \quad (51)$$

where $l_w = l_1+l_2+r_1+r_2-(\Delta x_1+\Delta x_2)$, it follows from Equations (28) and (29) that

$$\frac{\mathrm{d}}{\mathrm{d}t}E = -c_\varphi\dot{\varphi}^2 - \sum_{i=1,2}\dot{\boldsymbol{q}}_i^{\mathrm{T}}C_i\dot{\boldsymbol{q}}_i - \sum_{i=1,2}\xi_i(\Delta x_i)\Delta\dot{x}_i^2 \quad (52)$$

where

$$E = K + \Delta P + W \quad (53)$$

and K is defined as in Equation (18), and ΔP and W are defined as follows:

$$\Delta P = \sum_{i=1,2}\int_0^{\delta x_i}\left\{\bar{f}_i(\Delta x_{di}+\xi) - \bar{f}_i(\Delta x_{di})\right\}\mathrm{d}\xi \quad (54)$$

$$W = \frac{f_d}{2(r_1+r_2)}\left\{(Y_1-Y_2)^2+(Z_1-Z_2)^2\right\} + Mg\tilde{y} + \frac{\gamma_M}{2}\Delta M^2 + \sum_{i=1,2}\frac{\gamma_{Ni}}{2}\hat{N}_i^2 + \frac{\gamma_{N02}}{2}\hat{N}_{02}^2 \quad (55)$$

where $\delta x_i = \Delta x_i - \Delta x_{di}$ and $\tilde{y} = (y_{01}+y_{02})/2 - y$. From eq.(5), it follows that

$$\frac{y_{01}+y_{02}}{2} - y = -\frac{1}{2}\{(r_1-r_2)+(l_1-l_2) - (\Delta x_1-\Delta x_2)\}r_{Xy} + \frac{Y_1+Y_2}{2}r_{Yy} + \frac{Z_1+Z_2}{2}r_{Zy} \quad (56)$$

The overall fingers-object system depicted in Fig. 1 has superficially the 9 DOFs since the pair of fingers has 3 joints and the object does 3 independent translational variables (x,y,z) and 3 independent angular velocity variables $(\omega_x,\omega_y,\omega_z)$. On the other hand, it is subject to four rolling contact constraints as shown in Equations (8) and (9). These four constraints are nonholonomic, but they are Pfaffian, that is, they are linear and homogeneous in velocity variables (components of $\dot{\boldsymbol{X}} = (\dot{\boldsymbol{q}}_1^{\mathrm{T}}, \dot{\boldsymbol{q}}_2^{\mathrm{T}}, \dot{\boldsymbol{x}}^{\mathrm{T}}, \omega_x, \omega_y, \omega_z)^{\mathrm{T}}$). Hence, in the sense of infinitesimal displacements $\delta\boldsymbol{X}$, these Pfaffian constraints can be written as

$$\boldsymbol{Y}_i^{\mathrm{T}}\delta\boldsymbol{X} = 0,\ \boldsymbol{Z}_i^{\mathrm{T}}\delta\boldsymbol{X} = 0,\ i=1,2 \quad (57)$$

where $\boldsymbol{Y}_1 = (\boldsymbol{Y}_{\boldsymbol{q}1}^{\mathrm{T}}, 0_2, \boldsymbol{Y}_{\boldsymbol{x}1}^{\mathrm{T}}, Y_{\varphi1}, Y_{\psi1}, Y_{\theta1})^{\mathrm{T}}$, $\boldsymbol{Y}_2 = (0, \boldsymbol{Y}_{\boldsymbol{q}2}^{\mathrm{T}}, \boldsymbol{Y}_{\boldsymbol{x}2}^{\mathrm{T}}, Y_{\varphi2}, Y_{\psi2}, Y_{\theta2})^{\mathrm{T}}$, and $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ signify similar meanings as treated in derivation of the variational form described by Eq.(22). Hence, the total DOF of the fingers-object system is five. However, one of the five DOFs as the spinning motion of the object is uncontrollable, but this rotational motion is dampened by viscouse force $C_\varphi(\omega_x,0,0)^{\mathrm{T}}$. That is, it is possible to expect that $\omega_x(=\dot{\varphi})$ converges asymptotically to zero as $t\to\infty$. Thus, ΔP plus W becomes positive definite with respect to independent four position variables Y_1-Y_2, Z_1-Z_2, δx_1, and δx_2 corresponding to the remaining four DOFs. This implies that the scalar function defined by Equation (53) can be regarded as a Lyapunov function and it satisfies a Lyapunov relation of Equation (52).

In this case, it is possible to prove that the closed-loop dynamics of Equations (36), (46), and (47) converge asymptotically to the equilibrium state satisfying

$$\ddot{\boldsymbol{X}}(t)\to 0, \dot{\boldsymbol{X}}(t)\to 0, \boldsymbol{X}(t)\to\boldsymbol{X}_d \quad (58)$$

as $t\to\infty$, where $\boldsymbol{X}_d$ minimizes the artificial potential defined in eq.(53).

IV. Numerical simulation results and Initial values

We carry out computer simulation in order to confirm the theoretical prediction that the control signal of eq.(29) leads to asymptotic convergence of the trajectory $\boldsymbol{X}(t)$ to

TABLE II

PARAMETERS OF CONTROL SIGNALS.

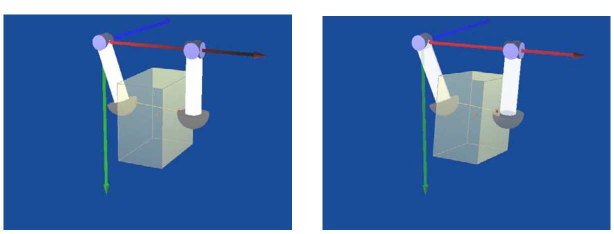
f_d	internal force	1.000 [N]
$c_1 = c_2$	damping coefficient	0.001 [Nms]
c_{20}	damping coefficient	0.006 [Nms]
γ_M	regressor gain	0.050 [m^2/kgs^2]
$\gamma_{Ni}(i = 1, 2)$	regressor gain	5.000×10^{-4} [s^2/kg]
γ_{N02}	regressor gain	5.000×10^{-4} [s^2/kg]
$\hat{M}(0)$	initial estimate value	0.020 [kg]
$\hat{N}_i(0)(i = 1, 2)$	initial estimate value	0.000 [N]
$\hat{N}_{02}(0)$	initial estimate value	0.000 [N]
$Y_1(0) - Y_2(0)$	initial value	-2.200×10^{-3} [m]
$Z_1(0) - Z_2(0)$	initial value	-3.491×10^{-4} [m]

TABLE III

PHYSICAL PARAMETERS OF THE FINGERS AND OBJECT.

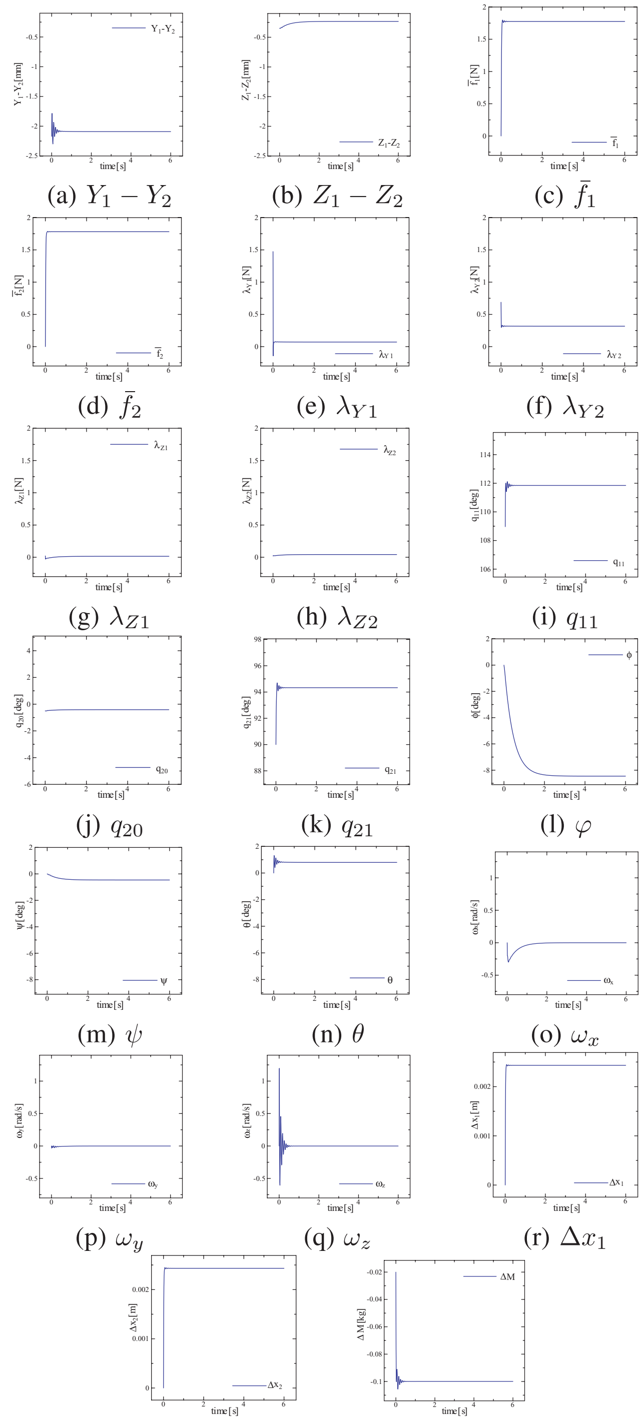
$l_{11} = l_{21}$	length	0.040 [m]
m_{11}	weight	0.043 [kg]
l_{20}	length	0.000 [m]
m_{20}	weight	0.000 [kg]
m_{21}	weight	0.060 [kg]
I_{XX11}	inertia moment	5.375×10^{-7}[kgm^2]
$I_{YY11} = I_{ZZ11}$	inertia moment	6.002×10^{-6}[kgm^2]
I_{XX21}	inertia moment	7.500×10^{-7}[kgm^2]
$I_{YY21} = I_{ZZ21}$	inertia moment	8.375×10^{-6}[kgm^2]
$I_{XX} = I_{ZZ}$	inertia moment(object)	1.133×10^{-5}[kgm^2]
I_{YY}	inertia moment(object)	6.000×10^{-6}[kgm^2]
r_0	link radius	0.005 [m]
$r_i(i = 1, 2)$	radius	0.010 [m]
L	base length	0.063 [m]
M	object weight	0.040 [kg]
$l_i(i = 1, 2)$	object width	0.015 [m]
h	object height	0.050 [m]
$k_i(i = 1, 2)$	stiffness	3.000×10^5[N/m^2]
$c_{\Delta i}(i = 1, 2)$	viscosity	1000.0[Ns/m^2]
c_φ	viscosity	0.001 [Nms]

the equilibrium state $\boldsymbol{X}_d$ that minimizes $\Delta P + W$ as defined in eqs.(54) and (55). Physical parameters of the fingers-object system model are given in Table III. Parameters of control gains are given in Table II. It is confirmed that key physical variables of the fingers-object system converge to some constant values as shown in Figs.4 and 5. We can see through Figs.4 and 5 that spinning motion occurs but eventually it stops after 2 seconds, that is, $\boldsymbol{\omega} \to 0$ as $t \to \infty$. On the other hand, it is confirmed that it continues to occur if we set the damping parameter c_φ zero. In the present simulaiton, the numerical order of the object width $l_1 + l_2$ is similar to that of the radii r_1, r_2 of the robot fingers. Then,



(a) Initial pose (b) After 6 seconds

Fig. 4. Motions of pinching a 3-D object under the gravity effect



(a) $Y_1 - Y_2$ (b) $Z_1 - Z_2$ (c) $\bar{f}_1$ (d) $\bar{f}_2$ (e) λ_{Y1} (f) λ_{Y2} (g) λ_{Z1} (h) λ_{Z2} (i) q_{11} (j) q_{20} (k) q_{21} (l) φ (m) ψ (n) θ (o) ω_x (p) ω_y (q) ω_z (r) Δx_1 (s) Δx_2 (t) ΔM

Fig. 5. Transient responses of physical variables

it can be shown theoretically that the overall fingers-object system is stabilized.On the other hand, viscosity coefficients c_ψ, c_θ around the y-axis, the z-axis in the frame coordinates respectively must have adequate positive constant values respectively under the circumstances that a thin object like a business card is grasped. From these results, the visco-elastic characteristics of the fingertip material play an important role in stabilization of the overall fingers-object system.

Furthermore, we suggest that human can easily grasp an object by adjusting the visco-elastic force between a fingertip and an object surface.

V. Conclusion

It is shown that a 3-D pinching is realized by using a pair of robot fingers with hemispherical soft tips and minimum DOF under the gravity effect. The overall fingers-object dynamics is derived as a full-variables model under the assumption that spinning around the opposition axis accompanies viscous friction exerted on the rotational motion of the object around the x-axis in the frame coordinates. It is shown that any solution of the closed-loop dynamics converges to an equilibrium point establishing force/torque balance. Finally, it is shown that the visco-elastic characteristics of the fingertip material play an important role in stabilization of the overall system.

References

[1] John Napier, *Hands*, Princeton Univ. Press, Princeton, NJ, USA,1993.

[2] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, CRC Press, Boca Raton,FL, USA and Tokyo, Japan, 1994.

[3] M. R.Cutkovsky, *Robotic Grasping and Fine Manipulation*, Kluwer Academic, Dordrecht, Netherlands, 1985.

[4] A. Bicchi, "Hands for dexterous manipulation and robust grasping: A difficult road toward simplicity," *IEEE Trans. on Robotics and Automation*, Vol.16, No. 6, pp.652–662, 2000.

[5] D.J. Montana, The kinematics of contact and grasp, *Int. J.of Robotics Research*, Vol. 7, No. 3, pp.17-32, 1988.

[6] D.J. Montana, "Contact stability for two-fingered grasps", *IEEE Trans. on Robotics and Automation*, vol.8, no.4, pp.421-430,1992.

[7] S. Arimoto, K. Tahara, M. Yamaguchi, P.T.A. Nguyen, and H.-Y. Han, Principle of superposition for controlling pinch motions by means of robot fingers with soft tips, *Robotica*, Vol. 19, No1, pp. 21-28, 2001.

[8] S.Arimoto, K.Tahara, Bae, J.-H, M.Yoshida, A stability theory of a manifold: concurrent realization of grasp and orientation control of an object by a pair of robot fingers, *Robotica*, Vol.21, No.2, pp.163-178, 2003.

[9] S.Arimoto, M.Yoshida, Ji-Hun Bae, Modeling of 3-D object manipulation by multi-joint robot fingers under non-holonomic constraints and stable blind grasping, *JSME Journal of System Design and Dynamics*, Vol.1, No.3, pp.434-446, 2007.

[10] S. Arimoto, A differential-geometric approach for 2-D and 3-D object grasping and manipulation, *Annual Review in Control*, Vol.31, No.2, pp.189-209, 2007.

[11] L. Meirovitch, *Methods of Analytical Dynamics*, McGraw-Hill, New York, USA, 1970.