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Homogeneous Matric Approach for the Operational Space Control of Bipedal Robots with Flexible Feet

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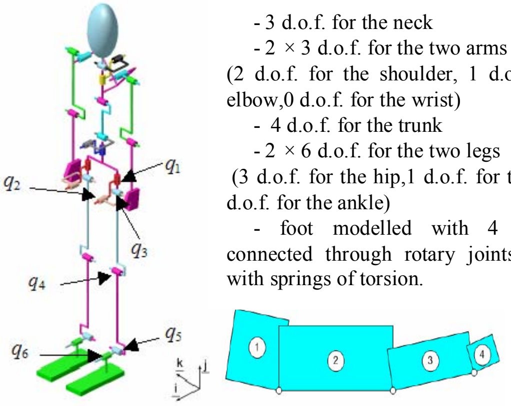
Abstract - **This paper presents a new approach to control gaits of humanoid bipedal robots in the operational space. This strategy based on the homogeneous transformation matrices integrates, in a unified representation, the structural aspects of the robot, the reference trajectories based on motion capture, the measures given by the robot sensors, the locomotion constraints and the high-level control using desired trajectories modified at each sample time. This approach is applied on a biped with flexible feet walking with an average speed equal to 1.2 m/s.**

I. INTRODUCTION

The more advanced projects of humanoid robots develop walking anthropomorphic robots able to move in environments well-adapted to human beings and able to cooperate with them : Asimo [1], the HRP series developed by AIST [2] , the small robot QRIO proposed by Sony [3], the KHR series developed by KAIST [4], the last robot of Waseda University having seven degrees of freedom per leg [5], Johnnie [6], H7 [7], and the Fujitsu robots Hoap [8]. To obtain better stability and larger velocity for a real dynamic walk, three points have to be improved : control methods, gait generation and mechanical design. To exploit the dynamic effects for gait generation, five kinds of approaches are used. The first one uses pragmatic rules based on qualitative studies of human walking gaits [8], [9], [10]. The second one focuses on the mechanical design of the robot in order to obtain natural passive dynamic gaits [11] or to exploit the feet flexibilities [2],[12] and others compliance properties[13], [14], [15]. The third one deals with studies of limit cycles [16]. The fourth one creates various gaits with reference trajectories [12]. The fifth one uses analytical criterions for dynamic gaits generation [17]. However, the framework of theses approaches has not really an unified representation integrating the structural aspects of the robot, the reference trajectories based on motion capture, the measures given by the robot sensors, the locomotion constraints and the high-level control using desired trajectories modified at each sample time. Our objective is to carry out the complementary framework of these approaches and to illustrate it for dynamic walking on a biped robot having flexible feet. This paper is organized as follows: in section II, the modeling of the anthropomorphic biped is briefly presented. In section III, the basic control strategy in the operational space called "reference strategy" is described. In section IV a new control strategy in the operational space with locomotion constraints and high-level control is proposed. The presentation of the simulation results are given in section V. Conclusions and further developments of this approach are finally presented.

II. MODEL OF THE ANTHROPOMORPHIC BIPED

The model is made up of 25 active d.o.f. (degrees of freedom). Some passive joints are also included in the foot. The degrees of freedom are distributed as shown in figure 1.



- 3 d.o.f. for the neck
- 2 × 3 d.o.f. for the two arms (2 d.o.f. for the shoulder, 1 d.o.f. for the elbow,0 d.o.f. for the wrist)
- 4 d.o.f. for the trunk
- 2 × 6 d.o.f. for the two legs (3 d.o.f. for the hip,1 d.o.f. for the knee, 2 d.o.f. for the ankle)
- foot modelled with 4 primitives connected through rotary joints provided with springs of torsion.

Fig. 1. Numerical model of the biped and of the flexible feet

The position and orientation of the foot according to the pelvis are expressed in the form of (1) where each translation or rotation component of each 4*4 homogeneous matrix is expressed as a temporal spline function. The angles q_i described in Fig.1 are calculated according to the coefficients given in (1) with leg inverse kinematics

$$\underline{\underline{M^{u-v}(t)}}=\begin{bmatrix} a(t) & b(t) & c(t) & x(t) \\ e(t) & f(t) & g(t) & y(t) \\ i(t) & j(t) & k(t) & z(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

III. SIMPLE CONTROL IN THE OPERATIONAL SPACE

The basic control strategy in the operational space called

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“reference strategy” is described by Fig. 2. Based on motion capture, the gait generator produces feasible dynamic motions for the numerical biped structure. Relative angular joint motion and direct kinematics of the different parts of the biped (legs, arms, trunk and head) are used to obtain the vector $\underline{V_{HM\mathrm{Ref}}}$ of homogeneous matrix (in the form of (1)) given by (2). This vector is composed by six homogeneous matrix giving the reference position of some key points of the structure and the reference orientation of the frames associated with these points.

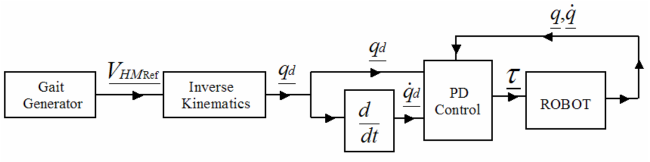

Fig. 2. Simple control in the operational space

$$\underline{V_{HM\mathrm{Ref}}} = \begin{pmatrix} \underline{M_{\mathrm{Ref}}^{hip^{right}-ank^{right}}} \\ \underline{M_{\mathrm{Ref}}^{hip^{left}-ank^{left}}} \\ \underline{M_{\mathrm{Ref}}^{pel^{CM}-torso}} \\ \underline{M_{\mathrm{Ref}}^{torso-hand^{right}}} \\ \underline{M_{\mathrm{Ref}}^{torso-hand^{left}}} \\ \underline{M_{\mathrm{Ref}}^{torso-head}} \end{pmatrix} \qquad (2)$$

The vector of desired joint motions $\underline{q_d}$ in Fig. 2 is obtained with inverse kinematics of the different parts of the biped (legs, arms, trunk and head) . The vector of desired joint velocities $\underline{\dot{q}_d}$ is computed by numerical derivation of $\underline{q_d}$.

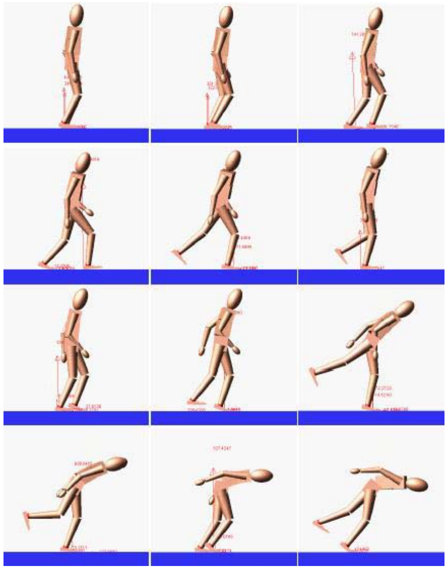

Fig. 3. Falling after a non-desired impact of the swing leg with ground

All the joints are then controlled by a simple PD control. With this basic approach, the numerical model of the humanoid robot is not able to produce walking without falling. Some snapshots of the simulation are given in Fig. 3 for planar motions. The main problems raised by this approach are the following ones:

- the biped robot falls down towards backward or forward because of a too small or too large horizontal acceleration of the pelvis and because of a non desired orientation of the pelvis with regard to the ground implying falling due to gravitational effects.
- the swing foot frequently touches the ground before the right instant and the biped robot naturally falls down (see Fig. 3).

In order to prevent this kind of problems, a new control strategy is proposed which is based on a closed loop in the high-level control with corrections of the desired trajectories in the operational space at each sample time.

IV. CONTROL IN THE OPERATIONAL SPACE WITH LOCOMOTION CONSTRAINTS

The new control strategy is presented in Fig. 4. The vector of homogeneous matrix $\underline{V_{HM\mathrm{Ref}}}$ given by (2) is produced by the gait generator with spline functions again. However, three new vectors are introduced to stabilize the pelvis and to avoid premature impacts with the ground, which are due to position and orientation errors of the pelvis according to the ground. The first one is $\underline{V_{HM\mathrm{Des}}}$ which includes a set of locomotion constraints resulting from knowledge and general information on human walk which mainly states for example the necessity:

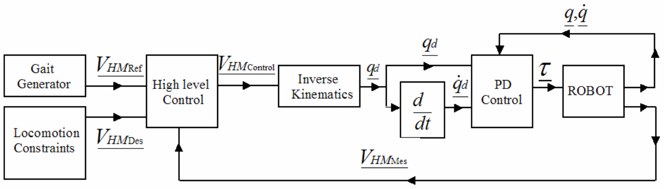

Fig. 4. Control strategy with corrections in the operational space

- to keep the pelvis, parallel on the ground to avoid large pitch of the upper part leading to the biped fall due to gravity.
- to keep the biped mass center projection near the pressure center in order to maximize the whole system equilibrium.
- to avoid the swing foot contact with the ground before that is not traversed a certain distance with regard to the stance foot.

The three kinds of preceding considerations expressed in the operational space are contained in the vector $\underline{V_{HM\mathrm{Des}}}$. The second vector introduced in Fig. 4 is $\underline{V_{HM\mathrm{Mes}}}$ which is composed of a homogeneous matrix set expressed in the operational space resulting from the sensors of the robot. All the data contained in this vector are measurements results. They give for instance the measured position and orientation of the pelvis relatively to the ground, of the swing foot relatively to the pelvis and of the swing foot relatively to the ground. The third vector lately introduced into Fig. 4 is the vector $\underline{V_{HM_{\text{Control}}}}$ which is a composition of the vectors $\underline{V_{HM_{\text{Ref}}}}$, $\underline{V_{HM_{\text{Des}}}}$ and $\underline{V_{HM_{\text{Mes}}}}$. The vector $\underline{V_{HM_{\text{Control}}}}$ includes homogeneous matrices expressing position and orientation of one solid according to another in the operational space, for instance : the swing foot according to the hip of the swing leg or the hip of the stance leg according to the stance foot. The vector $\underline{V_{HM_{\text{Control}}}}$ is the input of the inverse kinematics of all the parts of the biped robot and can be performed with the actuators of these parts. In this paper will be presented one control strategy used for the swing leg (called “AIG”) and two strategies for the stance leg (called ”KPPG” and “SCOP”).

A. Swing leg control strategy AIG

The control strategy AIG (Avoidance of an undesirable Impact with the Ground) uses a critical area that the swing foot has to avoid in order to minimize the risk of fall due to the impact with the ground during the swing phase. This area, shown in Fig. 5, is centered on the middle of the stance foot and is defined with four parameters which are :

1) $H_{\max}$ maximum height of the critical area

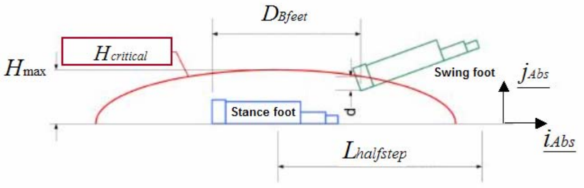

Fig. 5. Critical area for the control strategy AIG of the swing foot

2) D_{Bfeet} distance between the stance foot and the swing foot in the sagittal plane at each sample time.

3) $L_{halfstep}$ average half-step length in the sagittal plane

4) λ coefficient without unit included in [0.85, 0.95] which makes it possible to change on-line the length of the half-step and the limit of critical area whose area limit expression is:

$$H_{critical}=H_{\max}\sqrt{1-\frac{D_{Bfeet}^{\;2}}{(\lambda L_{halfstep})^2}} \qquad (3)$$

At each sample time, the maximum penetration **d** of the swing foot in the critical area along the vertical axis $\underline{j_{Abs}}$ is measured (Fig. 5). To avoid the critical area, the swing foot reference trajectory has to be displaced of distance **d** along $\underline{j_{Abs}}$ with:

$$Trans(\underline{j_{Abs}},d)=\begin{bmatrix}1&0&0&0\\0&1&0&d\\0&0&1&0\\0&0&0&1\end{bmatrix} \qquad (4)$$

Each homogeneous matrix in the form (1) can be written as the product of two pure homogeneous matrices of translation **T** and rotation **R** respectively :

$$\underline{\underline{M^{u-v}}}(t)=T\left(\underline{\underline{M^{u-v}(t)}}\right).R\left(\underline{\underline{M^{u-v}(t)}}\right)=\begin{bmatrix}1&0&0&x(t)\\0&1&0&y(t)\\0&0&1&z(t)\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}a(t)&b(t)&c(t)&0\\e(t)&f(t)&g(t)&0\\i(t)&j(t)&k(t)&0\\0&0&0&1\end{bmatrix} \qquad (5)$$

Finally, the homogeneous matrix $\underline{\underline{M_{\text{control}}^{pel^{hip\_sw}-ank_{\text{d}}^{sw}}}}$ used in the AIG control strategy for the swing leg (part of $\underline{V_{HM_{\text{Control}}}}$), is written:

$$\underline{\underline{M_{\text{control}}^{pel^{hip\_sw}-ank_{\text{d}}^{sw}}}}=\underline{\underline{M_{\text{Ref}}^{pel^{hip\_sw}-ank^{sw}}}}.T\left(\underline{\underline{M^{ank^{sw}-ank_d^{sw}}}}\right) \qquad (6)$$

with $\underline{\underline{M^{ank^{sw}-ank_d^{sw}}}}=R\left(\underline{\underline{M^{ank^{sw}-Abs}}}\right).Trans(j_{Abs},d)$

(7)

$\underline{\underline{M_{\text{control}}^{pel^{hip\_sw}-ank_{\text{d}}^{sw}}}}$ represents the desired position of the ankle and the orientation of the swing foot according to the hip center of the swing leg. This matrix is equal to $\underline{\underline{M_{\text{Ref}}^{pel^{hip\_sw}-ank^{sw}}}}$ multiplied by the translation part of $\underline{\underline{M^{ank^{sw}-ank_d^{sw}}}}$, which is given by (7). This last matrix gives the **required** position and orientation of the swing foot according to the **current** position and orientation of the swing foot. $\underline{\underline{M^{ank^{sw}-ank_d^{sw}}}}$ is obtained by using the rotation part of the matrix $\underline{\underline{M^{ank^{sw}-Abs}}}$ (current position and orientation of the swing foot according to an absolute frame) multiplied by the pure translation matrix $Trans(j_{Abs},d)$ given by (4). However, the coefficients of the matrix $\underline{\underline{M^{ank^{sw}-Abs}}}$ cannot be directly measured by the sensors of the robot. In order to evaluate $\underline{\underline{M^{ank^{sw}-Abs}}}$ the following decomposition is performed:

$$\underline{\underline{M^{ank^{sw}-Abs}}}=\underline{\underline{M_{mes}^{ank^{sw}-pel^{hip\_sw}}}}.\underline{\underline{M_{struct}^{pel^{hip\_sw}-pel^{CM}}}}.\left(\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}\right)^{-1} \qquad (8)$$

The matrix $\underline{\underline{M_{mes}^{ank^{sw}-pel^{hip\_sw}}}}$, which is a composition of the 6 elementary homogeneous matrices from the ankle to the hip, is easily obtained by the measures of all joints angular positions of the swing leg. The matrix $\underline{\underline{M_{struct}^{pel^{hip\_sw}-pel^{CM}}}}$ is known and all its coefficients are constant because it is related to the biped structure (two frames attached to the same body): it gives the position and the orientation of the frame attached to the pelvis and placed in mass center with regard to the frame attached to the pelvis and placed in the swing leg hip center. $\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}$ is obtained with position and orientation sensors placed on the pelvis: one 3-axis accelerometer sensor, one laser sensor to obtain the distance between the pelvis and the ground, and one 3-axis gyroscope sensor. We have to notice that this matrix is known **independently of the flexion of the toes of the foot**. Finally all terms of (8) are known and $\underline{\underline{M^{ank^{sw}-Abs}}}$ can be obtained. $\underline{\underline{M_{\mathrm{control}}^{pel^{hip\_sw}-ank_{d}^{sw}}}}$, fully defined with (6), (7) and (8), is used as swing leg inverse kinematics input.

B. Stance leg control strategy

Two control strategies can be used at the same time for the stance leg: the KPPG control in order to Keep the Pelvis Parallel with the Ground (small variations according to the reference trajectories) and the SCOP control in order to Stabilize the robot with the relation between the center of mass and the Center Of Pressure (possible large variations according to the reference trajectories).

1) KPPG control: Keep Pelvis Parallel with the Ground

The aim of this control strategy is to obtain the matrix $\underline{\underline{M_{control\_KPPG}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ to control the stance leg by introducing some measures given by the sensors and the two following matrices: $\underline{\underline{M_{\mathrm{Ref}}^{pel^{hip\_sta}-ank^{sta}}}}$ obtained with the gait generator to produce the reference trajectory of the stance leg (Fig. 4) and $\underline{\underline{M_{d\_KPPG}^{Abs-pel_{d}^{CM}}}}$ which namely gives the desired orientation of the pelvis with regard to the absolute frame. The matrix $\underline{\underline{M_{control\_KPPG}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ is also written:

$$\begin{aligned} M_{control\_KPPG}^{pel_{d}^{hip\_sta}-ank^{sta}} &= \underline{\underline{M_{struct}^{pel_{d}^{hip\_sta}-pel_{d}^{CM}}}}.\left(\underline{\underline{M_{d\_KPPG}^{Abs-pel_{d}^{CM}}}}\right)^{-1} \\ &.\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}\,\underline{\underline{M_{struct}^{pel^{CM}-pel^{hip\_sta}}}}.\underline{\underline{M_{\mathrm{Ref}}^{pel^{hip\_sta}-ank^{sta}}}} \end{aligned} \qquad (9)$$

The measures given by the sensors are included in the matrix $\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}$ (pelvis position and rotation with regard to the ground). Finally, $\underline{\underline{M_{struct}^{pel_{d}^{hip\_sta}-pel_{d}^{CM}}}}$ and $\underline{\underline{M_{struct}^{pel^{CM}-pel^{hip\_sta}}}}$ are known and all their coefficients are constant because they are related to the biped structure. Furthermore, $\underline{\underline{M_{d\_KPPG}^{Abs-pel_{d}^{CM}}}}$, since the aim is to keep the pelvis parallel with the ground, has the form:

$$\underline{\underline{M_{d\_KPPG}^{Abs-pel_{d}^{CM}}}} = T\left(\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}\right)\underline{\underline{I_4}} \qquad (10)$$

the 4*4 identity matrix $\underline{\underline{I_4}}$ allows to maintain a pelvis desired orientation parallel with the ground. The translation part of $\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}$ represents the current pelvis mass center position.

2) SCOP control : Stabilize the robot with the relation between the center of mass and the Center Of Pressure

The objective of SCOP control strategy is to produce the matrix $\underline{\underline{M_{control\_Stab}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ to control the pelvis center mass position with regard to the center of pressure with the stance leg. This control strategy is able to produce large variations of the reference trajectories according to the imbalance of the system represented by the distance between the projection of the mass center on the ground and the center of pressure:

$$\begin{aligned} \underline{\underline{M_{control\_Stab}^{pel_{d}^{hip\_sta}-ank^{sta}}}} &= \mathrm{H}_{1}\left(T\left(\underline{\underline{M_{\mathrm{Ref}}^{pel^{hip\_sta}-ank^{sta}}}}\right)\right) \\ &.\mathrm{H}_{2}\left(T\left(\underline{\underline{M_{\mathrm{closed\_des\_mes}}^{pel^{hip\_sta}-ank^{sta}}}}\right)\right).R\left(\underline{\underline{M_{\mathrm{Ref}}^{pel^{hip\_sta}-ank^{sta}}}}\right) \end{aligned} \qquad (11)$$

The following matrices are used in (11) :
$T\left(\underline{\underline{M_{\mathrm{Ref}}^{pel^{hip\_sta}-ank^{sta}}}}\right)$ and $R\left(\underline{\underline{M_{\mathrm{Ref}}^{pel^{hip\_sta}-ank^{sta}}}}\right)$ are translational part and rotational part of matrix $\underline{\underline{M_{\mathrm{Ref}}^{pel^{hip\_sta}-ank^{sta}}}}$ which represents the stance foot reference trajectories with regard to the pelvis. H_{i} is an operator which transforms a matrix $\underline{\underline{M(t)}}$ of the form given in (1) into a matrix $\underline{\underline{M'(t)}}$ in the following way:

$$\underline{\underline{M'(t)}} = \mathrm{H}_{i}\left(\underline{\underline{M(t)}}\right) = \begin{bmatrix} a(t) & b(t) & c(t) & H_{ix}(t)x(t) \\ e(t) & f(t) & g(t) & H_{iy}(t)y(t) \\ i(t) & j(t) & k(t) & H_{iz}(t)z(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (12)$$

where the coefficients H_{1} and H_{2} in (11) and (12) are:

$$H_{1x}(t) = H_{1y}(t) = H_{1z}(t) = 1-\kappa \qquad (13)$$

$$H_{2x}(t) = H_{2y}(t) = H_{2z}(t) = \kappa \qquad (14)$$

where $\kappa \in]0;1[$ is a coefficient used to perform the transition between two control strategies according to the biped robot state. More precisely: $\kappa = 0$ corresponds to a perfectly stable gait performed with reference trajectories. This implies that it is not useful to use the SCOP control. $\kappa = 1$ corresponds to a critical situation where the robot enters an unstable phase with risk of fall. In this case, the reference trajectories strategy is not useful and the SCOP control strategy is fully exploited. $\kappa \in]0;1[$ corresponds to a transition phase between the two preceding cases where the two strategies are used together. In (11), $T\left(\underline{\underline{M_{\mathrm{closed\_des\_mes}}^{pel^{hip\_sta}-ank^{sta}}}}\right)$ is the $\underline{\underline{M_{\mathrm{closed\_des\_mes}}^{pel^{hip\_sta}-ank^{sta}}}}$ translation part which represents the closed chain between the pelvis and the ankle and allowing to generate the required translation in order to modify the pelvis mass center position according to the ground. In this case, the aim is to include, in $\underline{\underline{M_{\mathrm{closed\_des\_mes}}^{pel^{hip\_sta}-ank^{sta}}}}$, the matrix $\underline{\underline{M_{d\_cop}^{Abs-pel_{d}^{CM}}}}$ which represents the desired pelvis mass center position according to the center of pressure. The matrix $\underline{\underline{M_{\text{closed\_des\_mes}}^{pel^{hip\_sta}-ank^{sta}}}}$ is composed as follows:

$$\underline{\underline{M_{\text{closed\_des\_mes}}^{pel^{hip\_sta}-ank^{sta}}}}=\underline{\underline{M_{\text{struct}}^{pel_{\text{d}}^{hip\_sta}-pel_{\text{d}}^{CM}}}}.\left(\underline{\underline{M_{d\_cop}^{Abs-pel_{d}^{CM}}}}\right)^{-1}.\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}.\underline{\underline{M_{\text{struct}}^{pel^{CM}-pel^{hip\_sta}}}}.\underline{\underline{M_{\text{mes}}^{pel^{hip\_sta}-ank^{sta}}}} \qquad (15)$$

We have to notice that the matrix $\underline{\underline{M_{d\_cop}^{Abs-pel_{d}^{CM}}}}$ in (15) is written in the following way :

$$\underline{\underline{M_{d\_cop}^{Abs-pel_{d}^{CM}}}}=\left[\begin{array}{c|c} I_3 & \underline{V} \\ \hline 000 & 1 \end{array}\right].R\left(\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}\right) \qquad (16)$$

In (16) the rotational part of $\underline{\underline{M_{d\_cop}^{Abs-pel_{d}^{CM}}}}$ is obtained with sensors allowing to build the measured matrix $\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}$.

In (16) the desired position of the pelvis center of mass is given by the translational part of $\underline{\underline{M_{d\_cop}^{Abs-pel_{d}^{CM}}}}$ that is to say the vector $\underline{V}=\begin{pmatrix} X_{cop} \\ Y_{\text{Ref}} \\ Z_{cop} \end{pmatrix}$ where X_{cop} and Z_{cop} are the current coordinates of the center of pressure and Y_{Ref} is the vertical position of the pelvis center of mass given by the reference trajectories. In (16), the three matrices $\underline{\underline{M_{struct}^{pel_{d}^{hip\_sta}-pel_{d}^{CM}}}}$, $\underline{\underline{M_{struct}^{pel^{CM}-pel^{hip\_sta}}}}$ and $\underline{\underline{M_{mes}^{Abs-pel^{CM}}}}$ are the same known matrices as in the equation(10). $\underline{\underline{M_{\text{mes}}^{pel^{hip\_sta}-ank^{sta}}}}$ represents the measured trajectories of the stance foot with regard to the pelvis. In order to fix the value of κ in (14) and (15), two circular zones are defined whose center is current center of pressure. If we call r the distance between current **cop** (center of pressure) and current projection of the **cm** (center of mass) on the ground, three cases are possible:

- $r<r_1$ the current **cm** is in the safety zone and $\kappa=0$: the reference strategy with splines and capture motions is used only (the SCOP strategy is not used).
- $r>r_2$ the current **cm** is in the dangerous zone and $\kappa=1$: only the SCOP strategy is used in order to balance the large disequilibrium of the biped robot. In this case $\kappa=1$ is maintained until the full stabilization of the robot.
- $r_1<r<r_2$ the current **cm** is in the intermediate zone and $\kappa\in]0;1[$: the two strategies (SCOP and reference) are used together according to the value of κ which has to be a function of r, r_1 and r_2 and which must be equal to 0 when $r=r_1$ and equal to 1 when $r=r_2$. The chosen form is :

$$\kappa=\sqrt{\frac{r-r_1}{r_2-r_1}} \quad \text{for } r_1<r<r_2 \qquad (17)$$

Finally, (11) used to control the stabilization of the robot is now fully defined. The two strategies KKPG and SCOP based on (9) and (11) respectively are used to control the stance leg. However they have to be used at the same time to produce a dynamic stable walk of the biped robot. An hybrid control strategy is also proposed.

3) Hybrid control strategy: KPPG and SCOP synergy

The aim of the hybrid control strategy is to exploit the properties of the two control strategies at the same time for the stance leg. The final control matrix $\underline{\underline{M_{control}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ has to include the matrices $\underline{\underline{M_{control\_KPPG}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ (9) and $\underline{\underline{M_{control\_Stab}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ (11). Since the KPPG strategy aims at keeping the pelvis parallel with the ground, the main functionality of this strategy is related with the rotation of the pelvis, not with its position. Since the objective of the SCOP strategy is to stabilize the robot with the relation between the center of mass and the center of pressure, the functionality is to preserve or to find a desired distance between the **cm** and the **cop**. So this strategy is related with the translation of the body, not with its orientation. In this case, the final control matrix $\underline{\underline{M_{control}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ is written in the following way for the stance leg:

$$\underline{\underline{M_{control}^{pel_{d}^{hip\_sta}-ank^{sta}}}}=T\left(\underline{\underline{M_{control\_Stab}^{pel_{d}^{hip\_sta}-ank^{sta}}}}\right)R\left(\underline{\underline{M_{control\_KPPG}^{pel_{d}^{hip\_sta}-ank^{sta}}}}\right) \qquad (18)$$

The composition of the $\underline{\underline{M_{control\_Stab}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ translation part and of the $\underline{\underline{M_{control\_KPPG}^{pel_{d}^{hip\_sta}-ank^{sta}}}}$ rotational part allows to exploit the best properties of each control strategy.

V. SIMULATION RESULTS

The simulation of the biped dynamic behavior is carried out using the Adams software. The classical motion equations of the biped robot are written as follows:

$$\underline{\underline{M(q)}}\,\underline{\ddot{q}}+\underline{C(q,\dot{q})}+\underline{G(q)}=\underline{\underline{D_1^T}}\,\underline{F_{C1}}+\underline{\underline{D_2^T}}\,\underline{F_{C2}}+\underline{\tau} \qquad (19)$$

The left term of (19) contains matrix of inertia, vector of acceleration, vector of centrifugal and De Coriolis forces, and vector of gravity. The right term is composed of the contacting forces applied to the feet $\underline{F_{C1}}$ and $\underline{F_{C2}}$ (multiplied by the transpose of the jacobian matrix of each leg $\underline{\underline{D_1}}$ and $\underline{\underline{D_2}}$ respectively)and of $\underline{\tau}=\underline{\underline{K_p}}\left(\underline{q_d}-\underline{q}\right)+\underline{\underline{K_v}}\left(\underline{\dot{q}_d}-\underline{\dot{q}}\right)$ the joint torques vector which is obtained with diagram of

Fig. 4 and based on (6) for the swing leg and on (18) for the stance leg. The control strategy AIG (Avoidance of an undesirable Impact with the Ground) for the swing leg and the combined strategy based on the KPPG and SCOP strategies for the stance leg allow to produce a highly dynamic two-dimensional stable walk (average speed equal to 1.2 m/s) without falling for an infinite number of steps on a flat ground. One of the key points of the approach is that it is elaborated **independently of the flexion of the feet toes what allow us to exploit this flexibility without its disadvantages.** The Fig. 6 shows some snapshots of the simulation of the dynamic behavior during one step.

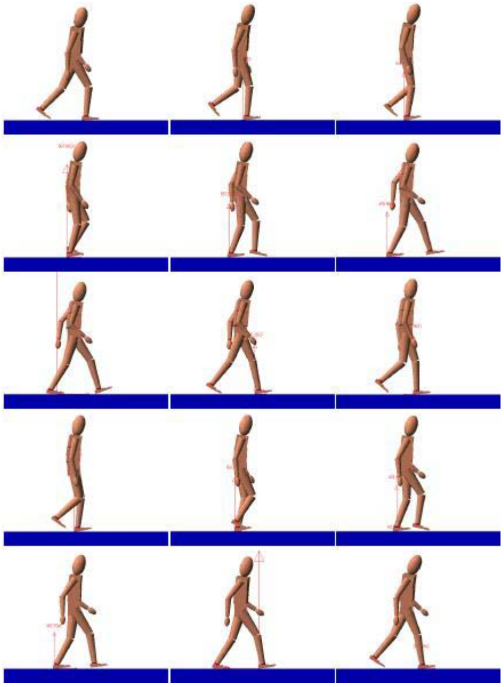

Fig. 6. Snapshots of the simulation during one step

VI. CONCLUSIONS AND FUTURE WORK

This paper presented a new approach to control gaits of humanoid bipedal robots in the operational space based on an unified representation. It was applied on a bipedal robot with flexible feet walking dynamically with an average speed equal to 1.2 m/s. One major interest of the method is that all errors in the operational space and expressed in the joint space are measured by the sensors and taken into account at each sample time to modify desired trajectories satisfying locomotion constraints before low-level control. Although the approach was presented in this paper with the use of inverse kinematics and criterions based on the center of pressure, it gives the framework to include desired joint or operational trajectories resulting from dynamic considerations and others criterions of stability. Work currently under development consists in coordinating the two legs during the double support phase to validate this control strategy on the real robot called ROBIAN developed at the LISV and having flexible feet in order to produce highly dynamic gaits.

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